

# Could AI Ever Match Gödel's Greatness?

(Part II of the Chapter; Part I is on “The Gödel Game,” for IFLAI)

Selmer Bringsjord

Intro to (Formal) Logic (& AI) = IFLAI 2022

4/22/24

ver 0422241036NYY

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# Monographic Context (yet again!)

...

# *Gödel's Great Theorems* (OUP)

by Selmer Bringsjord

- Introduction (“The Wager”)
- Brief Preliminaries (e.g. the propositional calculus & FOL)
- The Completeness Theorem
- The First Incompleteness Theorem
- The Second Incompleteness Theorem
- The Speedup Theorem
- The Continuum-Hypothesis Theorem
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- Gödel’s “God Theorem”
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Gödel's Either/Or ...

# The Question

**Q\*** Is the human mind more powerful than the class of standard computing machines?

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(= finite machines)



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**Q\*** Is the human mind more powerful than the class of standard computing machines?

(= finite machines)

(= Turing machines)

(= register machines)

(= KU machines)

...

# Gödel's Either/Or

“[E]ither ... the human mind (even within the realm of pure mathematics) infinitely surpasses the power of any finite machine, or else there exist absolutely unsolvable diophantine problems.”  
— Gödel, 1951, Providence RI

# PT as a Diophantine Equation

Equations of this sort were introduced to you in middle-school, when you were asked to find the hypotenuse of a right triangle when you knew its sides; the familiar equation, the famous Pythagorean Theorem that most adults will remember at least echoes of into their old age, is:

$$(PT) \quad a^2 + b^2 = c^2,$$

and this is of course equivalent to

$$(PT') \quad a^2 + b^2 - c^2 = 0,$$

which is a Diophantine equation. Such equations have at least two unknowns (here, we of course have three:  $a, b, c$ ), and the equation is solved when positive integers for the unknowns are found that render the equation true. Three positive integers that render (PT') true are

$$a = 4, b = 3, c = 5.$$

It is *mathematically impossible* that there is a finite computing machine capable of solving any Diophantine equation given to it as a challenge (!).



... which means that the 10th of Hilbert's Problems is settled:

## Hilbert's problems

From Wikipedia, the free encyclopedia

**Hilbert's problems** are twenty-three problems in [mathematics](#) published by German mathematician [David Hilbert](#) in 1900. The problems were all unsolved at the time, and several of them were very influential for 20th-century mathematics. Hilbert presented ten of the problems (1, 2, 6, 7, 8, 13, 16, 19, 21, and 22) at the [Paris](#) conference of the [International Congress of Mathematicians](#), speaking on August 8 in the [Sorbonne](#). The complete list of 23 problems was published later, most notably in English translation in 1902 by [Mary Frances Winston Newson](#) in the *[Bulletin of the American Mathematical Society](#)*.<sup>[1]</sup>

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- [Ignorabimus](#)
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David Hilbert



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10th	Find an algorithm to determine whether a given polynomial <a href="#">Diophantine equation</a> with integer coefficients has an integer solution.	Resolved. Result: Impossible; <a href="#">Matiyasevich's theorem</a> implies that there is no such algorithm.	1970
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Julia **R**obinson

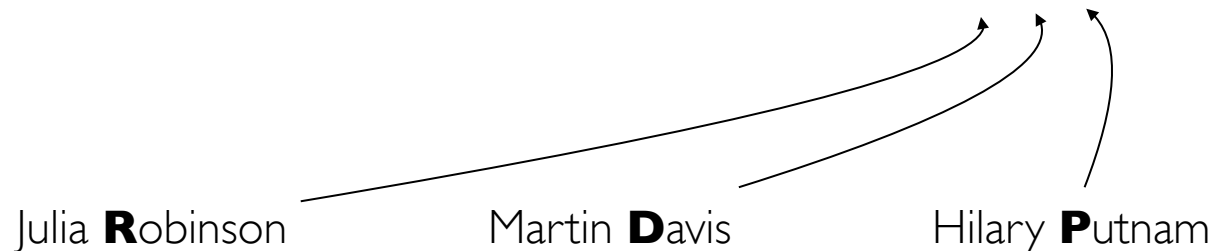
Martin **D**avis

Hilary **P**utnam

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# Background

problem?<sup>7</sup> In his lecture, Gödel precisely defines diophantine problems, but we don't need to bother with all of the details here; we only need to appreciate the general structure of such a problem, and that can be achieved quickly as follows, given what was introduced in Chapter 2.

Each diophantine problem has at its core a polynomial  $\mathcal{P}$  whose variables are comprised by two lists,  $x_1, x_2, \dots, x_n$  and  $y_1, y_2, \dots, y_m$ ; all variables must be integers, and the same for subscripts  $n$  and  $m$ . To represent a polynomial in a manner that announces its variables, we can write

$$\mathcal{P}(x_1, x_2, \dots, x_k, y_1, y_2, \dots, y_j).$$

But Gödel was specifically interested in whether, for all integers that can be set to the variables  $x_i$ , there are integers that can be set to the  $y_j$ , such that the polynomial equals 0. To make this clearer, first, here are two particular, simple equations that employ polynomials that are both instances of the needed form:

$$\text{E1} \quad 3x - 2y = 0$$

$$\text{E2} \quad 2x^2 - y = 0$$

All we need to do now is prefix these equations with quantifiers in the pattern Gödel gave. This pattern is quite simple: universally quantify over each  $x_i$  variable (using the now-familiar  $\forall$ ), after which we existentially quantify over each  $y_i$  variable (using the also-now-familiar  $\exists$ ). Thus, here are the two diophantine problems that correspond to the pair E1 and E2 from just above:

$$\text{P1} \quad \text{Is it true that } \forall x \exists y (3x - 2y = 0)?$$

$$\text{P2} \quad \text{Is it true that } \forall x \exists y (2x^2 - y = 0)?$$

# Great Paper!



Hilbert's Tenth Problem is Unsolvable

Author(s): Martin Davis

Source: *The American Mathematical Monthly*, Vol. 80, No. 3 (Mar., 1973), pp. 233-269

Published by: [Mathematical Association of America](#)

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**1. Diophantine Sets.** In this article the usual problem of Diophantine equations will be inverted. Instead of being given an equation and seeking its solutions, one will begin with the set of “solutions” and seek a corresponding Diophantine equation. More precisely:

**DEFINITION.** A set  $S$  of ordered  $n$ -tuples of positive integers is called **Diophantine** if there is a polynomial  $P(x_1, \dots, x_n, y_1, \dots, y_m)$ , where  $m \geq 0$ , with integer coefficients such that a given  $n$ -tuple  $\langle x_1, \dots, x_n \rangle$  belongs to  $S$  if and only if there exist positive integers  $y_1, \dots, y_m$  for which

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1973] HILBERT'S TENTH PROBLEM IS UNSOLVABLE 235

$$P(x_1, \dots, x_n, y_1, \dots, y_m) = 0.$$

Borrowing from logic the symbols “ $\exists$ ” for “there exists” and “ $\Leftrightarrow$ ” for “if and only if”, the relation between the set  $S$  and the polynomial  $P$  can be written succinctly as:

$$\langle x_1, \dots, x_n \rangle \in S \Leftrightarrow (\exists y_1, \dots, y_m) [P(x_1, \dots, x_n, y_1, \dots, y_m) = 0],$$

or equivalently:

$$S = \{ \langle x_1, \dots, x_n \rangle \mid (\exists y_1, \dots, y_m) [P(x_1, \dots, x_n, y_1, \dots, y_m) = 0] \}.$$

Note that  $P$  may (and in non-trivial cases always will) have negative coefficients. The word “polynomial” should always be so construed in the article except where the contrary is explicitly stated. Also all numbers in this article are positive integers unless the contrary is stated.



Hilbert's Tenth  
Author(s): Marston  
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Notice that this is a perfect fit with how we used formal logic to present and understand the Polynomial Hierarchy and the Arithmetic Hierarchy.

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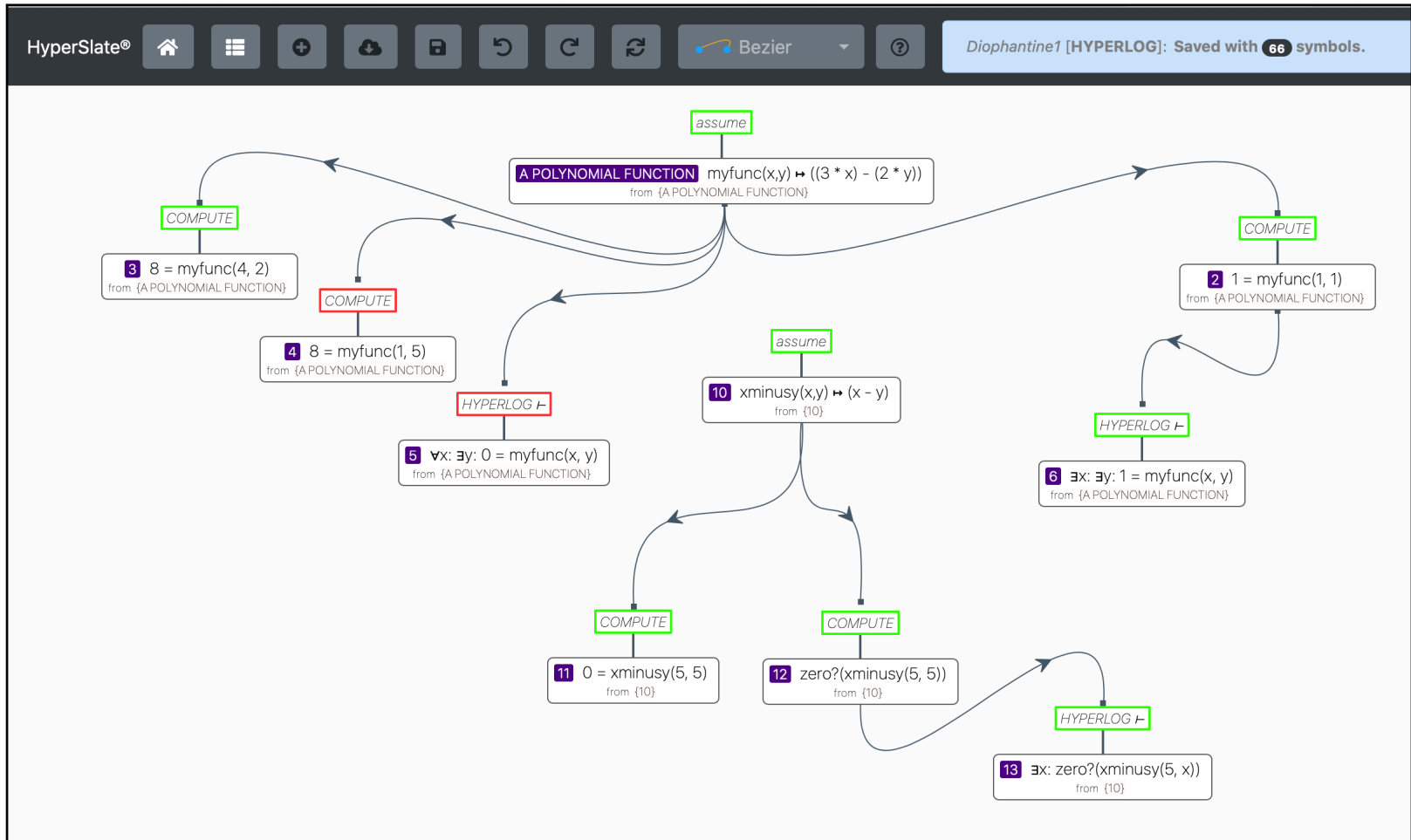
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# Diophantine “Threat” in the Programming Language Hyperlog<sup>®</sup>



# The Cruc

$\exists \mathcal{P}$  s.t. no human mind could ever decide  $\forall x_1 \forall x_2 \cdots \forall x_k \exists y_1 \exists y_2 \cdots \exists y_j (\mathcal{P}(x_1, x_2, \dots, x_k, y_1, y_2, \dots, y_j))$ ?

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**Yes.**

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Yes.



The human mind is *not* infinitely more powerful than any standard computing machine.



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No.



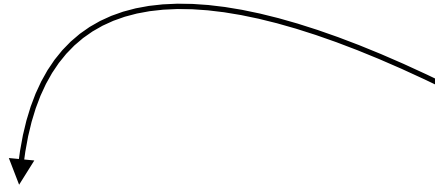
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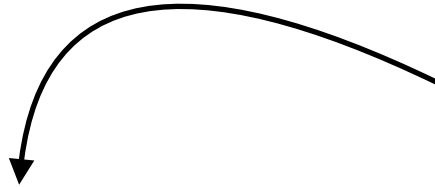
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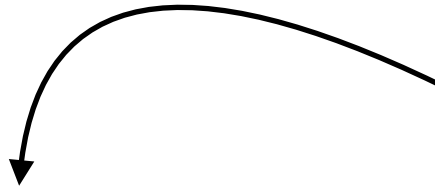
# The Crucial Question

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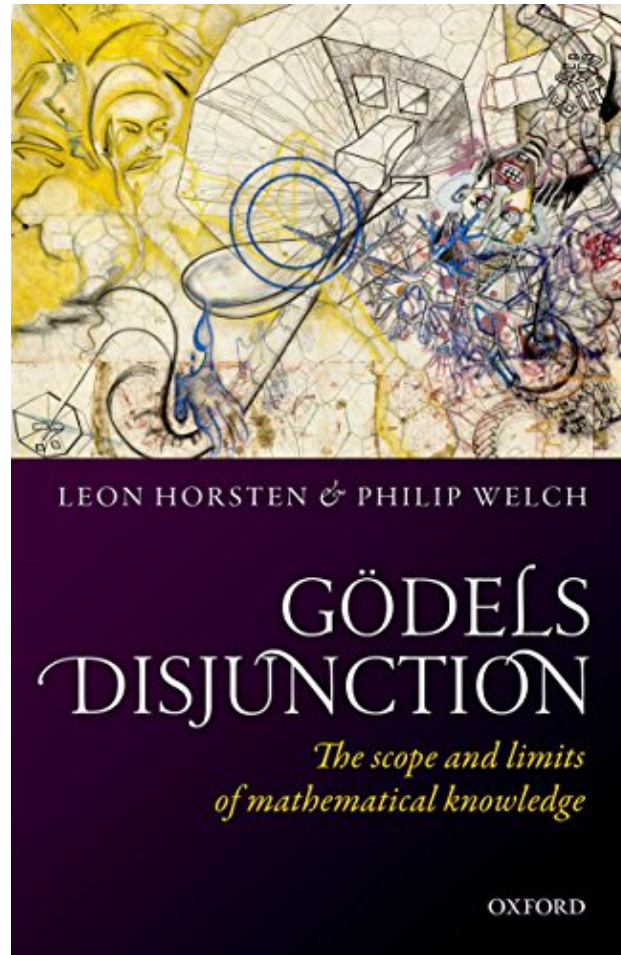
No.



The human mind *is* infinitely more powerful than any standard computing machine.

**Entire book on Gödel's Either-Or ...**

# Entire book on Gödel's Either-Or ...



# Earlier Gödelian Argument for the “No.”



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## Outline

### Abstract

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5. Setting the context: the busy beaver problem
6. The new Gödelian argument
7. Objections
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## Figures (1)



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Table 1



ELSEVIER

Applied Mathematics and Computation

Volume 176, Issue 2, 15 May 2006, Pages 516-530



## A new Gödelian argument for hypercomputing minds based on the busy beaver problem ☆

Selmer Bringsjord , Owen Kellett, Andrew Shilliday, Joshua Taylor, Bram van Heuveln, Yingrui Yang, Jeffrey Baumes, Kyle Ross

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<https://doi.org/10.1016/j.amc.2005.09.071>

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## Abstract

Do human persons hypercompute? Or, as the doctrine of *computationalism* holds, are they information processors at or below the Turing Limit? If the former, given the essence of hypercomputation, persons must in some real way be capable of infinitary information processing. Using as a springboard Gödel's little-known assertion that the human mind has a power “converging to infinity”, and as an anchoring problem Rado's [T. Rado, On non-computable functions, Bell System Technical Journal 41 (1963) 877–884] Turing-uncomputable “busy beaver” (or  $\Sigma$ ) function, we present in this short paper a new argument that, in fact, human persons can hypercompute. The argument is intended to be formidable, not conclusive: it brings Gödel's intuition to a greater level of precision, and places it within a sensible case against computationalism.

**Finally, finally, ...**



# Gödel-vs-AI “Scorecard”

The Particular Work	Nutshell Diagnosis	Beyond AI?

# Gödel-vs-AI “Scorecard”

The Particular Work	Nutshell Diagnosis	Beyond AI?
Completeness Thm. (Ch. 3)	Reduction lemma impressive.	Likely Not

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*On Intuitionistic Logic	Beyond our scope.	Likely Not
*Philosophical Reasoning	Undeniably beyond foreseeable AI.	Yes

# Test-3 Grading Scheme

Test-3 problems will be published before today ends (we shall now take a look @ the human-created ones), & grades on Test 3 will be reported out in HG<sup>®</sup> as for Test 1 & Test 2. Deadline for Test 3: Thu May 2 11:59pm NY time. Here's the grading scheme, same as used for Test 2 & easy to remember:

- C: any 1 trophy-winning proof.
- B: any 2 trophy-winning proofs.
- A: any 3 trophy-winning proofs.
- A+: *all* proofs found!

And now let's wrap up  
with extensions etc.

...



*Med nok penger, kan  
logikk løse alle problemer.*