FOL II: universal intro

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management
Rensselaer Polytechnic Institute (RPI)
Troy NY 12180 USA

Intro to Logic
2/20/2024

Logic-&-AI In The News

The New York Times

Inside the Funding Frenzy at Anthropic, One of A.I.'s Hottest Start-Ups

The company raised $7.3 billion over the last year, as the lure of artificial intelligence changes Silicon Valley deal-making.

5 MIN READ

Massimo Berruti for The New York Times

Re Test I …

(More-Forgiving) Grading Scheme

C: 2/6

B: 3/6

A: 4/6

A+: 6/6

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C: 2/6

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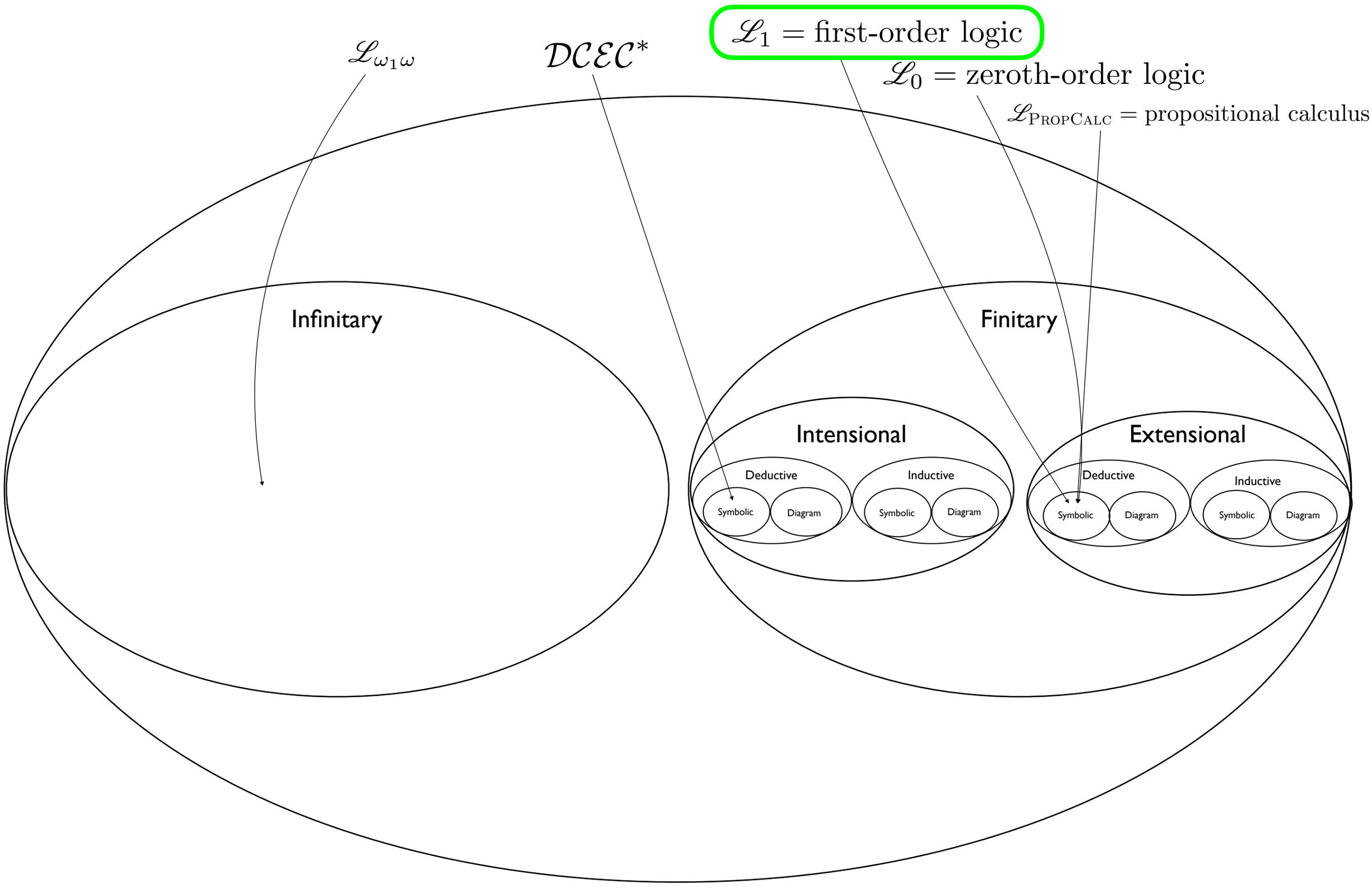
Part 2 Today for Help etc; remarks on DeMorgan's Theorem.

The Universe of Logics



Next New (*Not*-So-Easy!) Inference Rule in FOL

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- `universal introduction`

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  - If something *a* is an *R*, and the constant/name *a* is *genuinely arbitrary*, then we can deduce that everything is an *R*.

The Inference Schema

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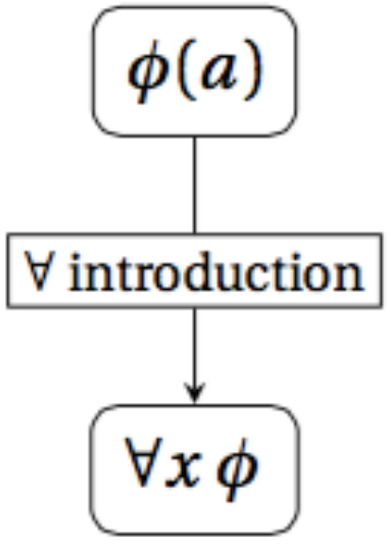
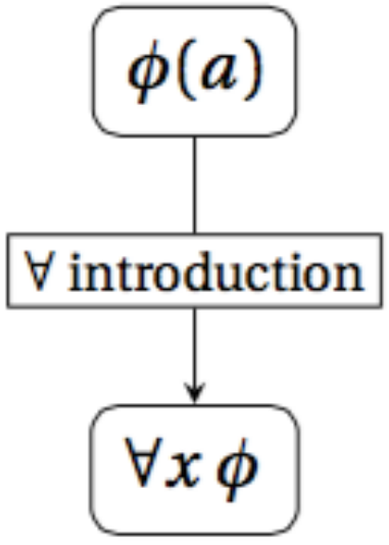


provided that a does not appear free in any in-scope assumption of ϕ, and that no occurrence of a appear in the inferred $\forall x\,\phi$ (3.16)

The Inference Schema



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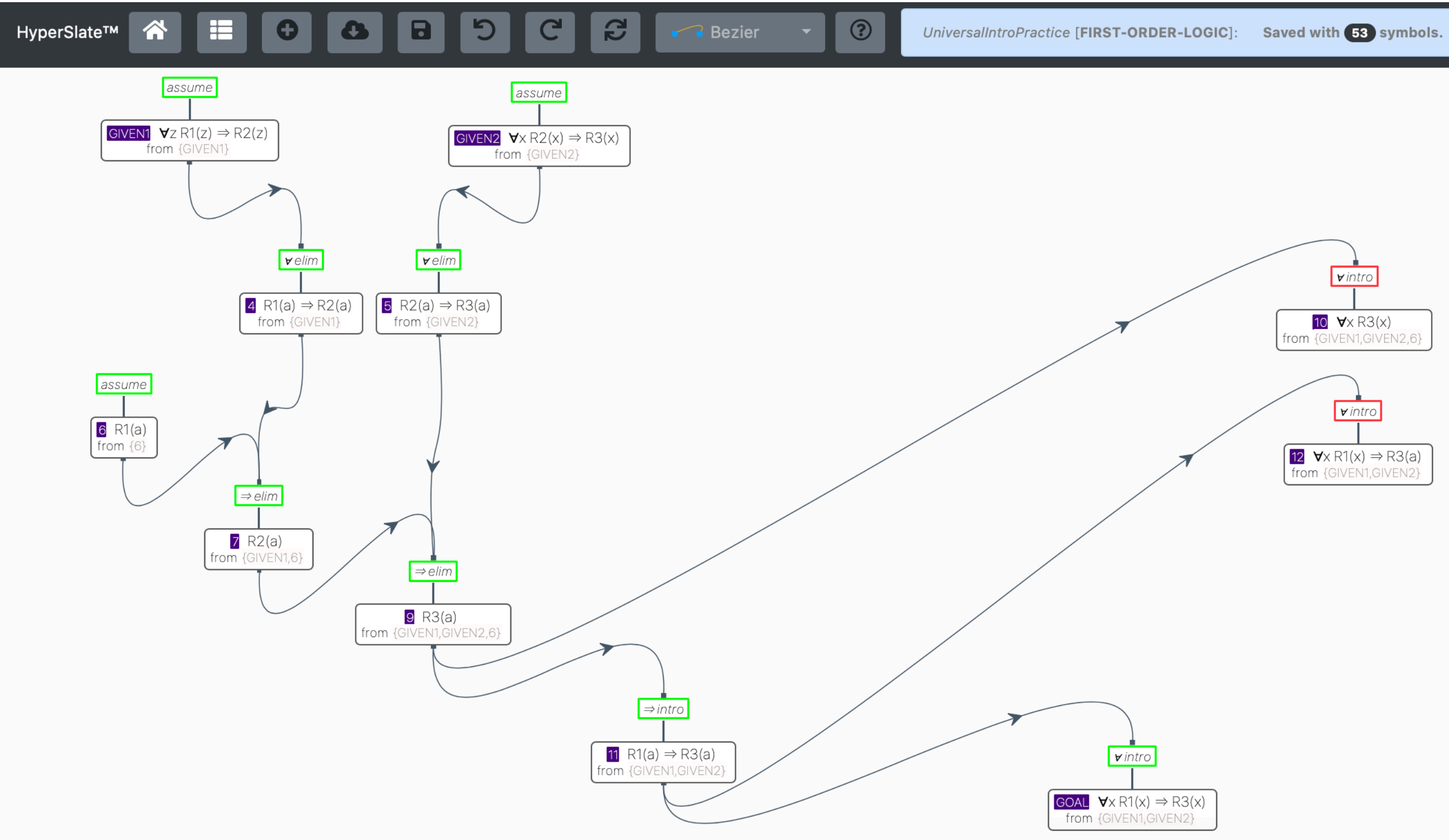
(Why the provisos?)

universal intro Example/Tutorial



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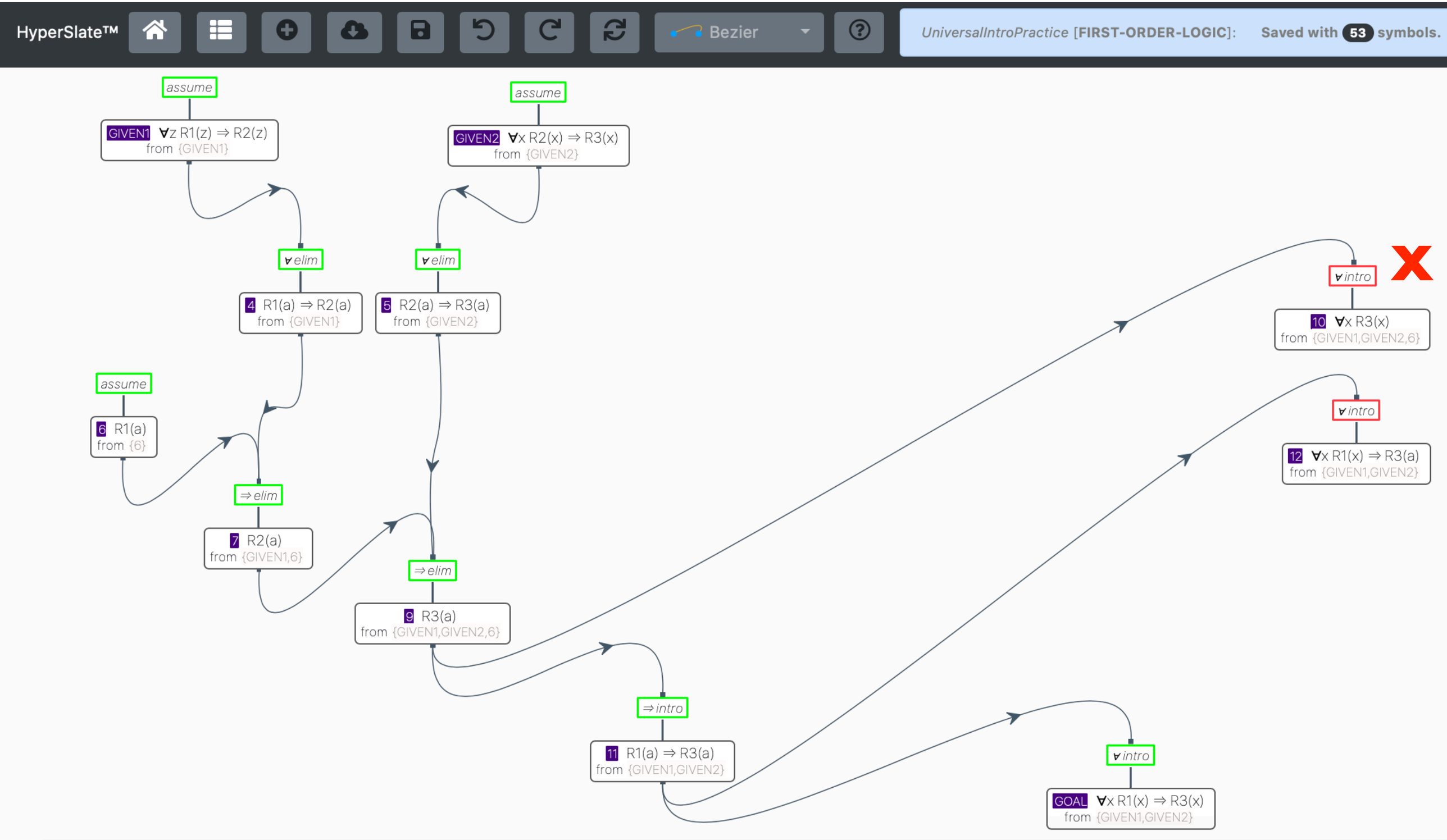
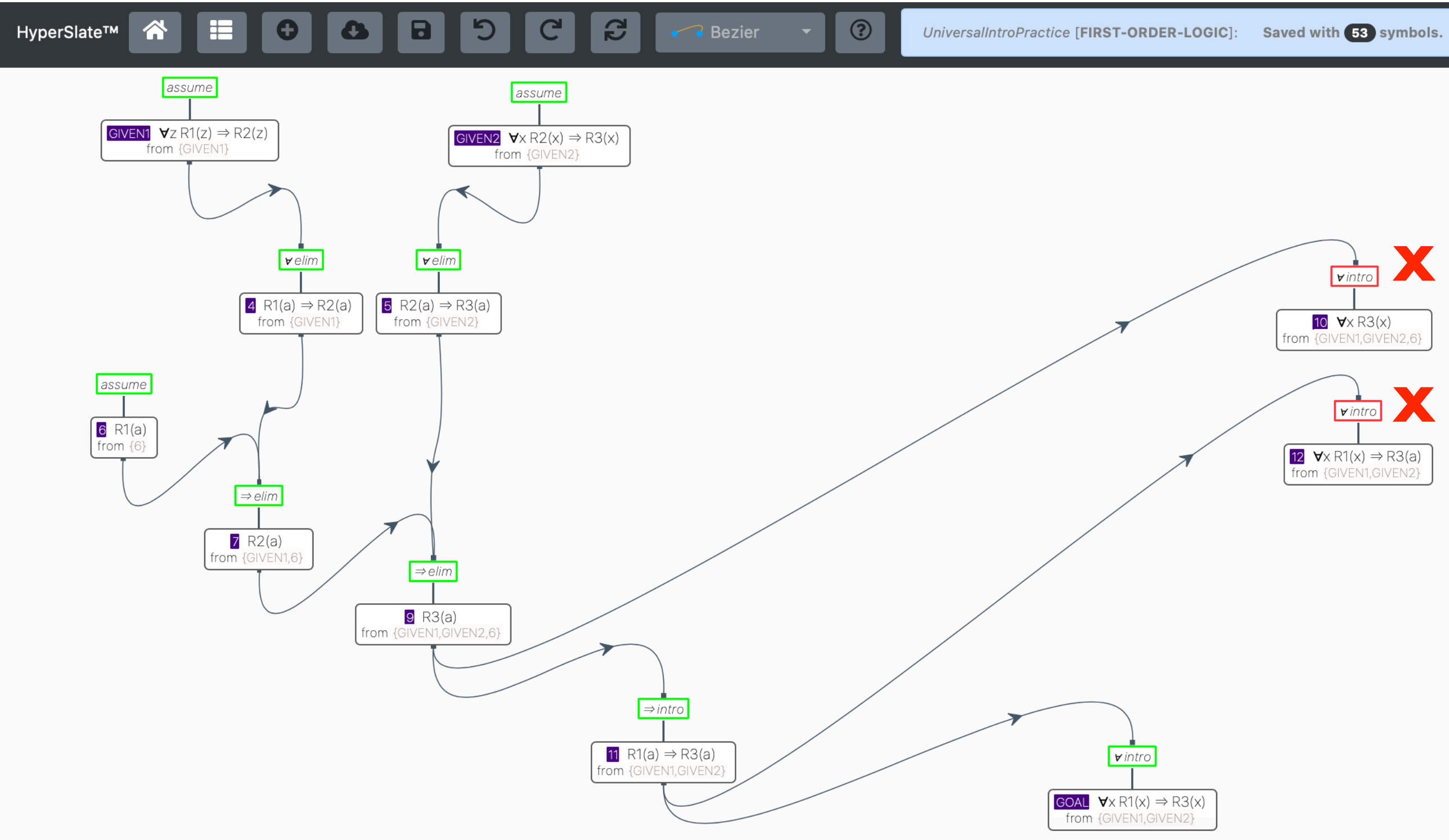
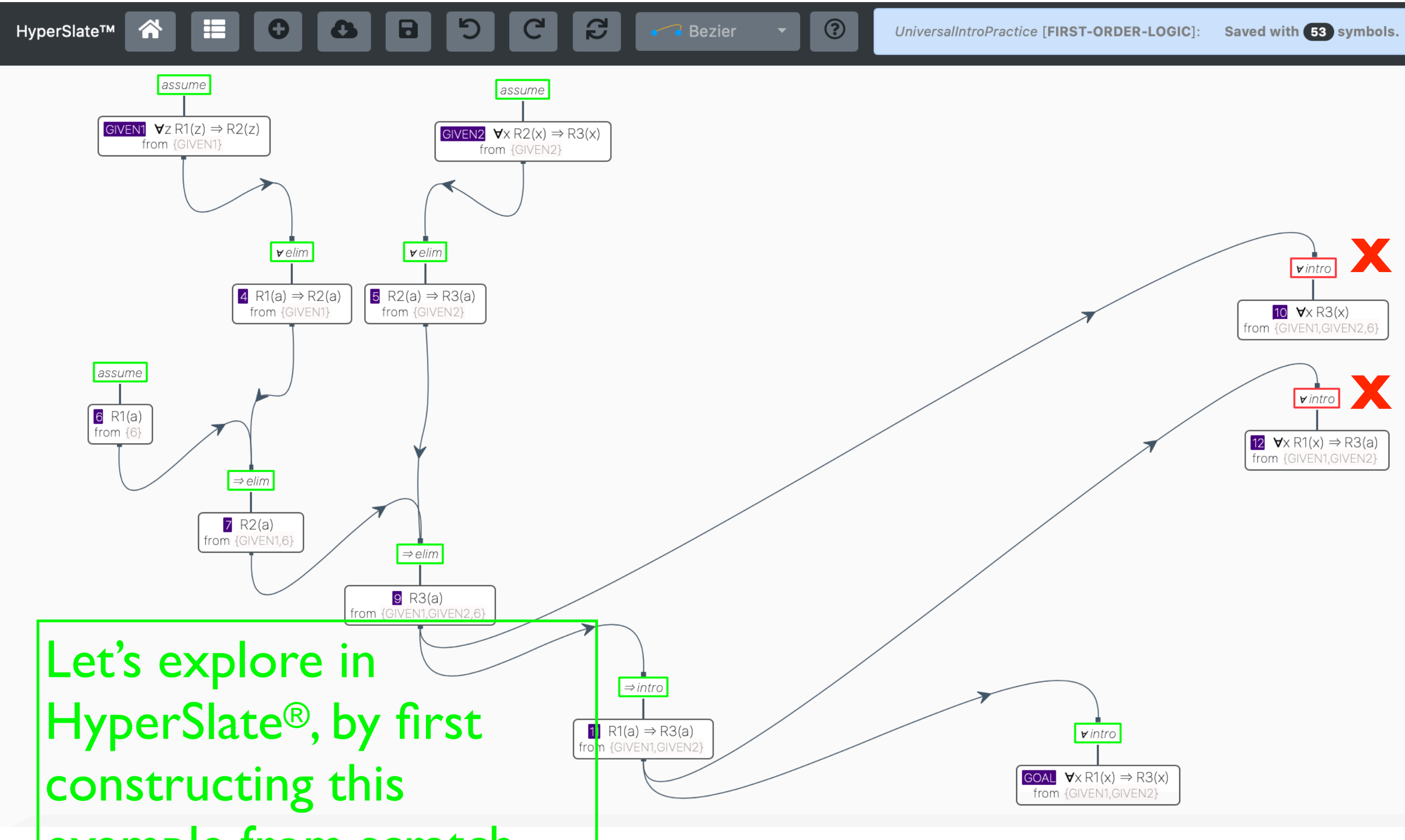


universal intro Example/Tutorial

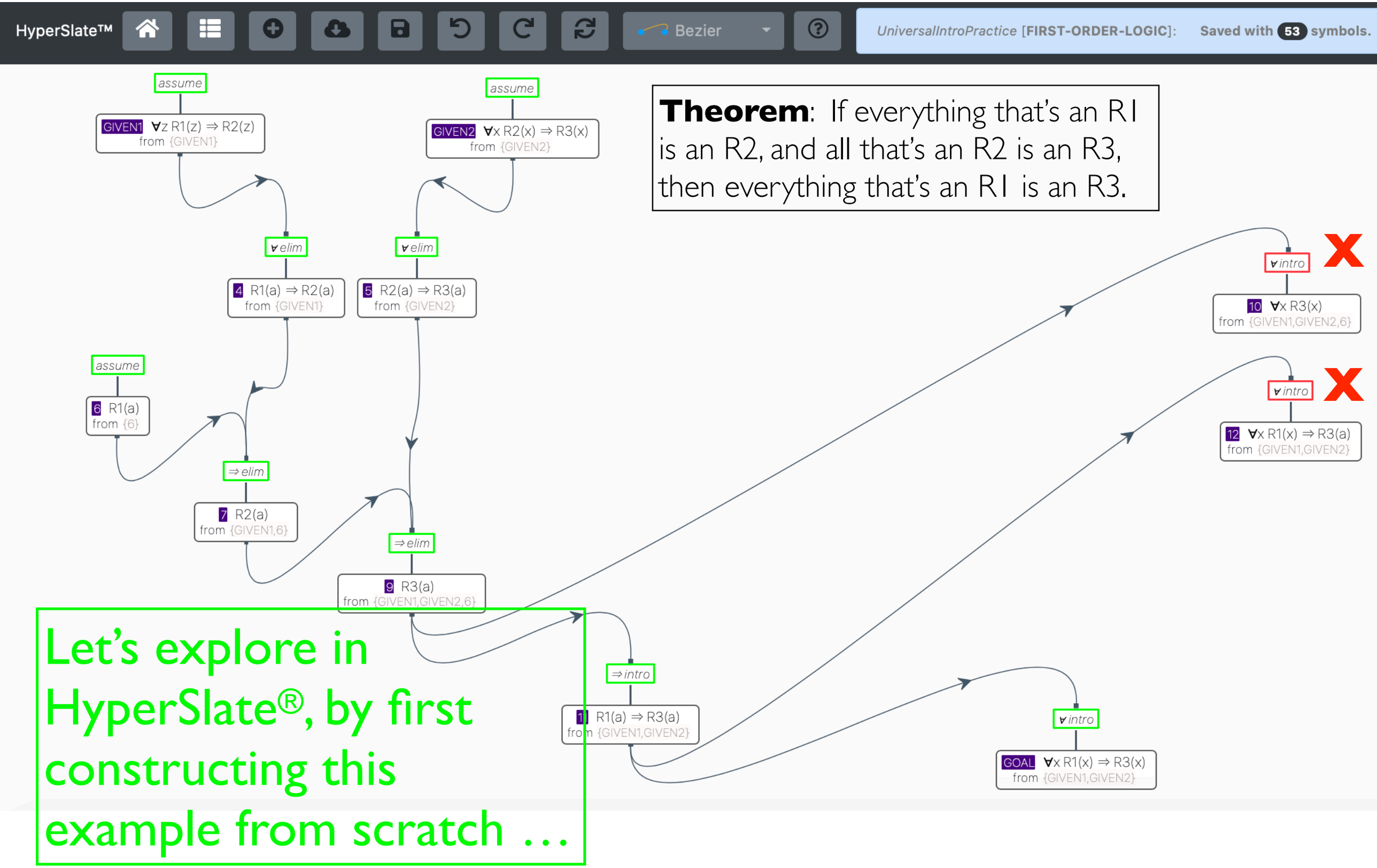


universal intro Example/Tutorial



Let's explore in HyperSlate®, by first constructing this example from scratch …

universal intro Example/Tutorial



Theorem: If everything that's an R1 is an R2, and all that's an R2 is an R3, then everything that's an R1 is an R3.

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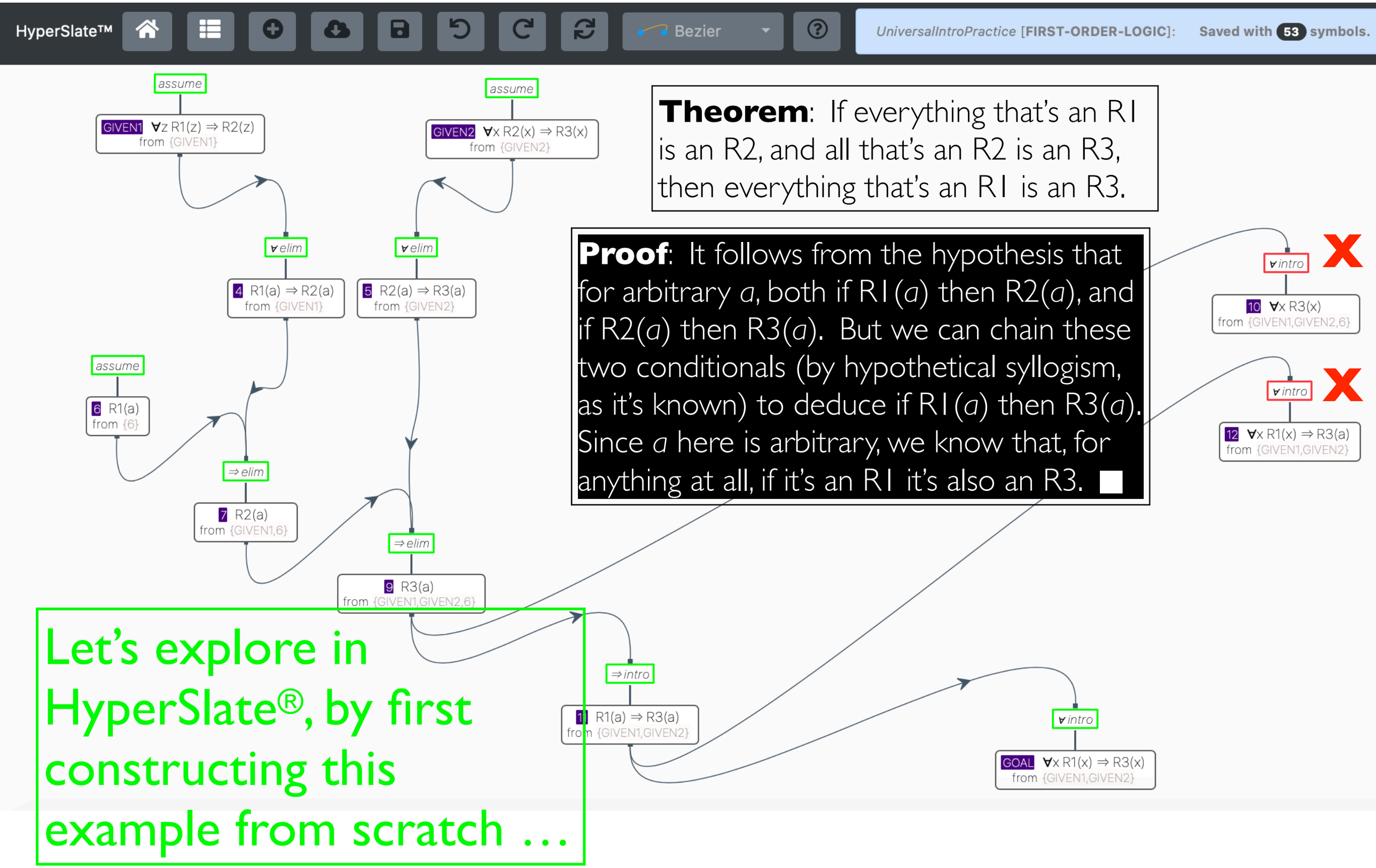
universal intro Example/Tutorial



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Proof: It follows from the hypothesis that for arbitrary *a*, both if R1(*a*) then R2(*a*), and if R2(*a*) then R3(*a*). But we can chain these two conditionals (by hypothetical syllogism, as it's known) to deduce if R1(*a*) then R3(*a*). Since *a* here is arbitrary, we know that, for anything at all, if it's an R1 it's also an R3. ■

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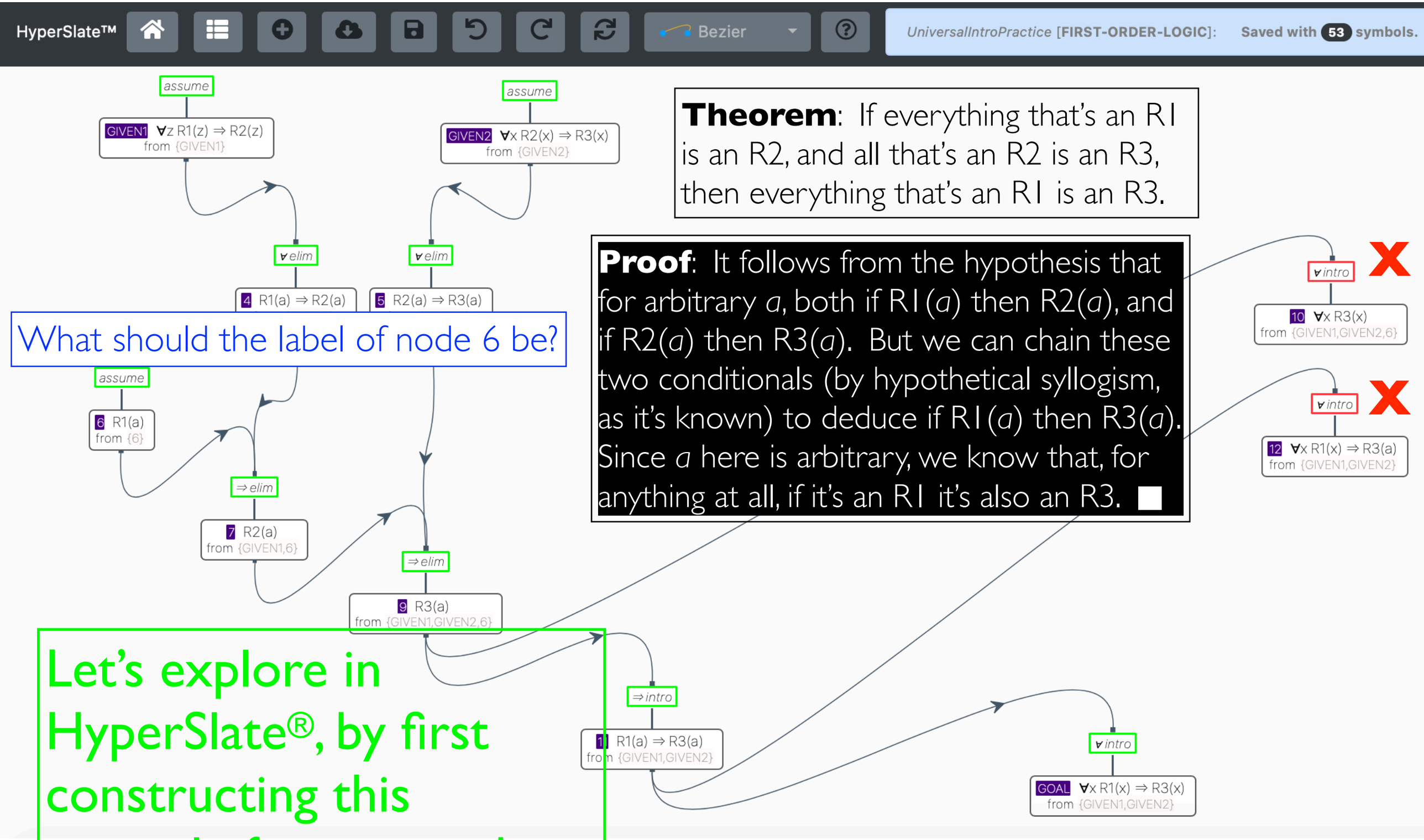
universal intro Example/Tutorial



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Suggested Practice Problems in HyperSlate®!

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$$\{\forall \mathtt{x}(\mathtt{R}(\mathtt{x}) \leftrightarrow \mathtt{S}(\mathtt{x})), \forall \mathtt{x}\mathtt{R}(\mathtt{x})\} \vdash \forall \mathtt{x}\mathtt{S}(\mathtt{x})\} \textbf{ ?}$$

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Hvis du forstår det, kan du bevise det.

Part I: *Slutten — for i dag.*

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Part II: Hands-on Q&A & Review …