

FOL III

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab

Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Logic
2/22/2024



\mathcal{L}_1
FOL III

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Logic-&-AI: Self-Referential Tests of ChatGPT

Logic-&-AI: Self-Referential Tests of ChatGPT

SE

Can you prove that if P holds, and (if P (and (Q (not Q))) holds, then the moon is made of green cheese?



No, I cannot prove that if P holds and (if P (and (Q (not Q))) holds, then the moon is made of green cheese.

The expression (Q (not Q)) is a logical contradiction, since Q cannot be both true and false at the same time. Therefore, the statement (if P (and (Q (not Q)))) is always false, regardless of the truth value of P.

Since the antecedent of the implication is always false, the implication itself is vacuously true. However, this does not provide any evidence or justification for the conclusion that the moon is made of green



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$$\{P, P \rightarrow (Q \wedge \neg Q)\} \vdash \perp$$

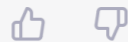
Logic-&-AI: Self-Referential Tests of ChatGPT

Logic-&-AI: Self-Referential Tests of ChatGPT

SE This sentence has a word in it longer than the string 'gfeeteru.'





 That sentence is true, since the word "longer" has more letters than the string "gfeeteru". "Longer" has 6 letters, while "gfeeteru" has only 8 letters.





Logic-&-AI: Self-Referential Tests of ChatGPT

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 That sentence is true, since the word "longer" has more letters than the string "gfeeteru". "Longer" has 6 letters, while "gfeeteru" has only 8 letters.

Are you quite sure?

Logic-&-AI: Self-Referential Tests of ChatGPT

Logic-&-AI: Self-Referential Tests of ChatGPT

SE

And, as Spock, I also tell you the following sentence. This sentence, if false, has more than 103 words. Am I lying, or telling the truth?



As Spock, you would be presenting a paradoxical statement known as the "liar paradox" or the "liar's paradoxical sentence." This type of statement is self-referential and produces a logical contradiction, and cannot be consistently true or false.

In this case, the statement you presented cannot be consistently true or false, because if it were true, then it would be false, and if it were false, then it would be true. Therefore, as Spock, you would not be lying or telling the truth in the traditional sense, but rather presenting a logical conundrum for



Logic-&-AI: Self-Referential Tests of ChatGPT

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Are you quite sure?

**Live-action on
HyperGrader ...**


ThxForThePCOracle

ThxForThePCOracle

Please attempt that now-ish; thx.


ThxForThePCOracle

Please attempt that now-ish; thx.


 Problem Bank Edit Problems

Exercis


es

 Metrics for
ExercisesDownload: LAMA-
BDLAHSHG020421.pdf

Problems

 ThxForThePCOracle RipsSaysNo1
BiconditionalIntroByChaini
... New  ThxForThePCOracle

This straightforward problem is quickly solved with a minimum of tedium, courtesy of the PC (entailment) provability oracle, use of which is allowed to remain in your finished proof (but no use of any other oracle can be in the finished proof). This oracle is for the logic \mathcal{L}_{PC} . Any learning of formal logic, at more than a trivial level, without the availability of the kind of AI embodied by this oracle (and more powerful ones farther up the ladder of extensional logic), is not only pedagogically unwise, but also, for the learner, downright painful.

Deadline March 18, 2021, 12:00 PM EDT Solve Submissions Metrics  RipsSaysNo1

This problem relates to the interesting book *The Psychology of Proof*, by psychologist L. Rips, a book that, shortly before he died, nobelist and computational-logic pioneer Herbert Simon praised as crucial to advancing automated reasoning/AI. Specifically, you are presented here with the challenge of crafting a proof that, as implied by what Rips presents in his volume, is supposed to be beyond the reach of (at least logically naïve) humans! This is quite peculiar, because as you will soon see, that which is to be proved, expressed in meta-logic, is simply this: $\{\neg(\phi \rightarrow \psi)\} \vdash \phi$.

Deadline March 18, 2021, 12:00 PM EDT

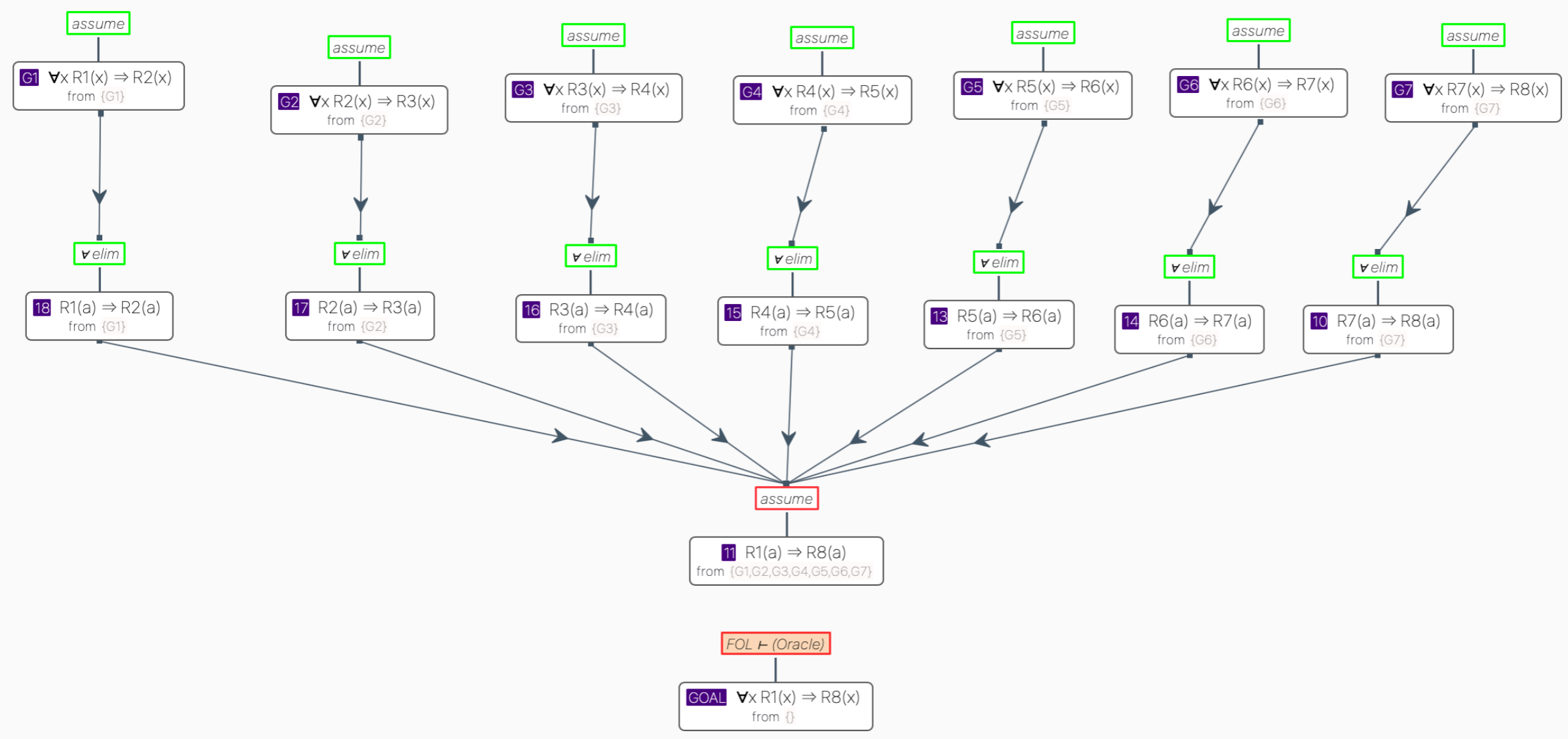
[Open in HyperSlate](#) ThxForThePCOracle

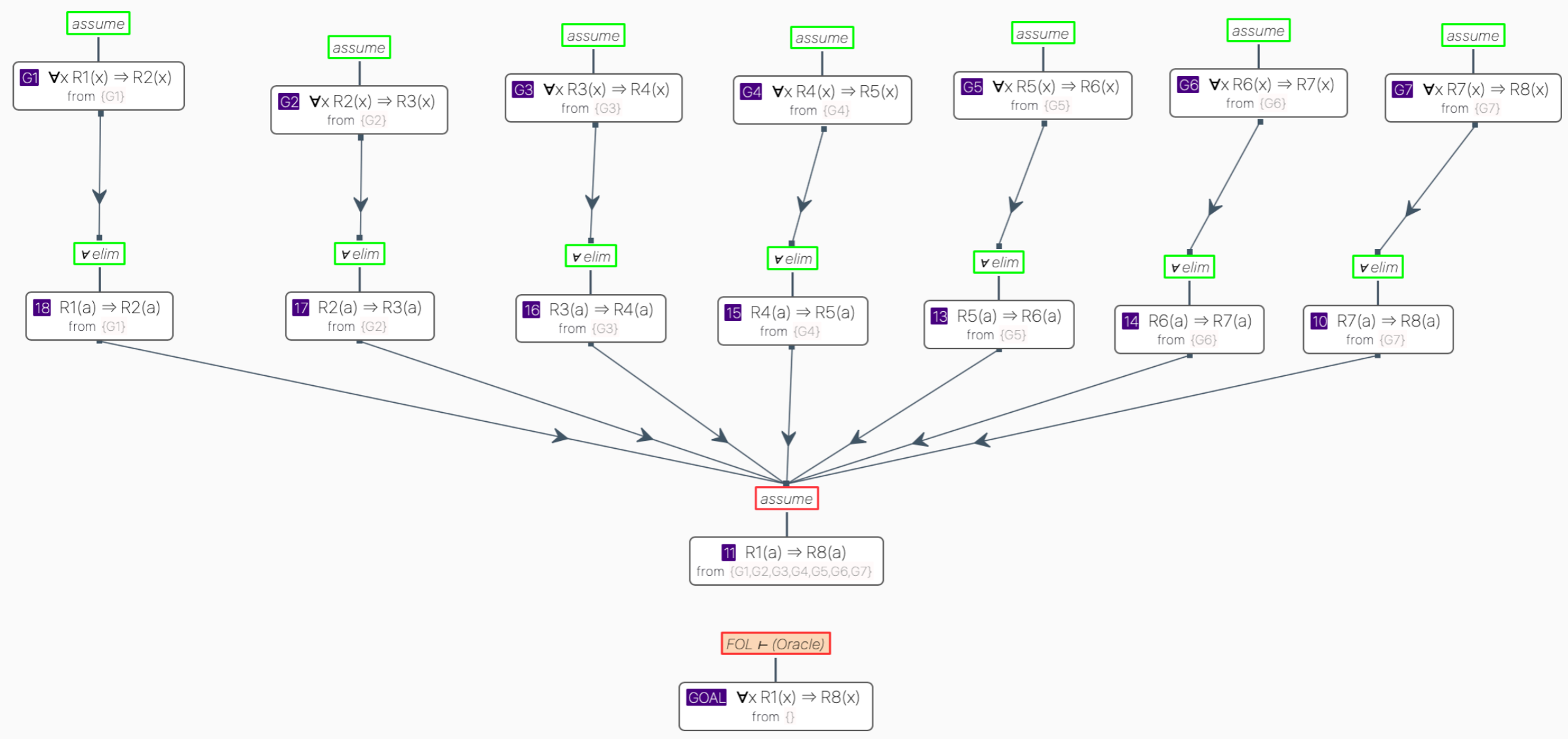
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Deadline March 18, 2021, 12:00 PM EDT

Days Hours Minutes Seconds

03:19:39:00**Problem Type:** SIMPLE**Difficulty:** 1**Points:** 10[Leaderboard](#)





Selmer's Third = Hardest AI-generated Test-1 Problem

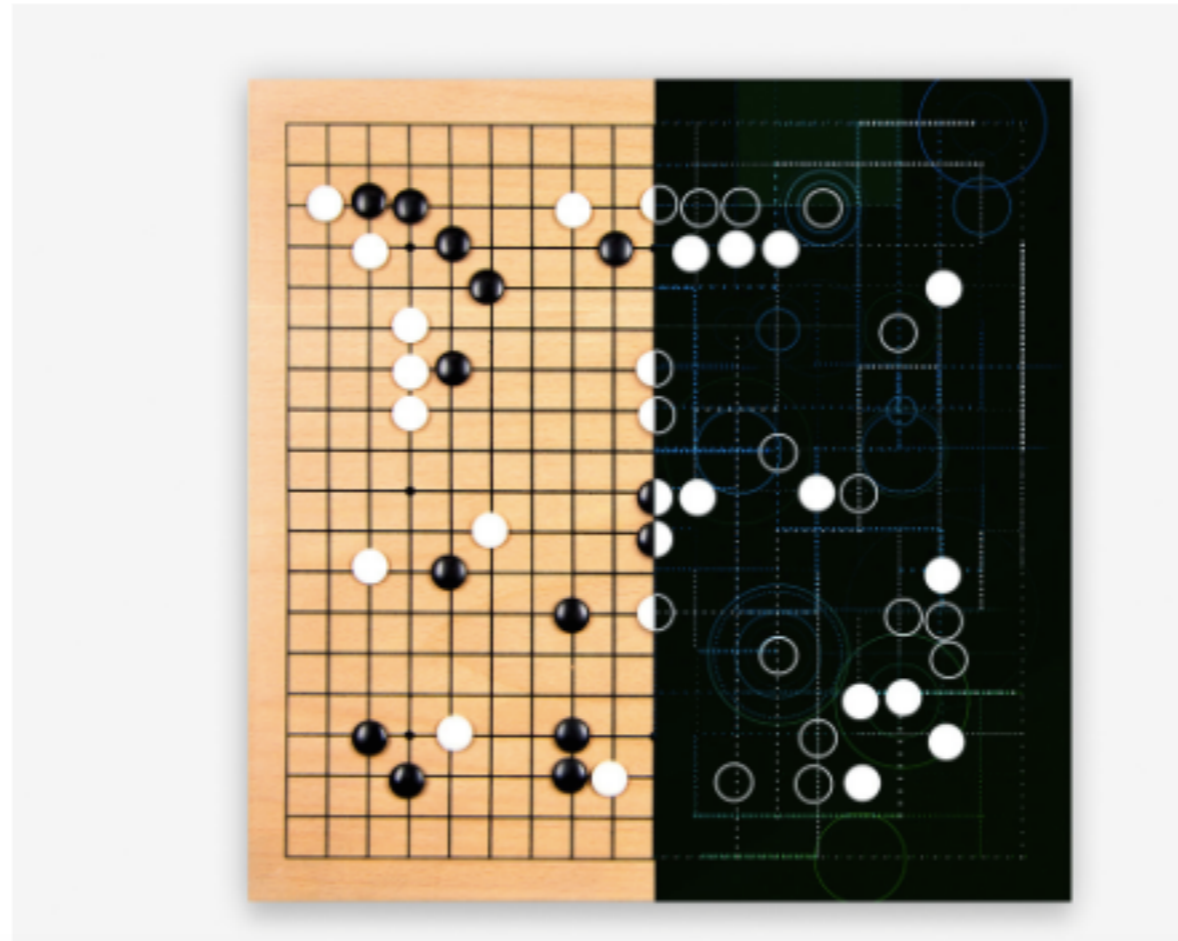
Selmer's Third = Hardest AI-generated Test-1 Problem

Student Success on the New Knight-Knave THLPE #1

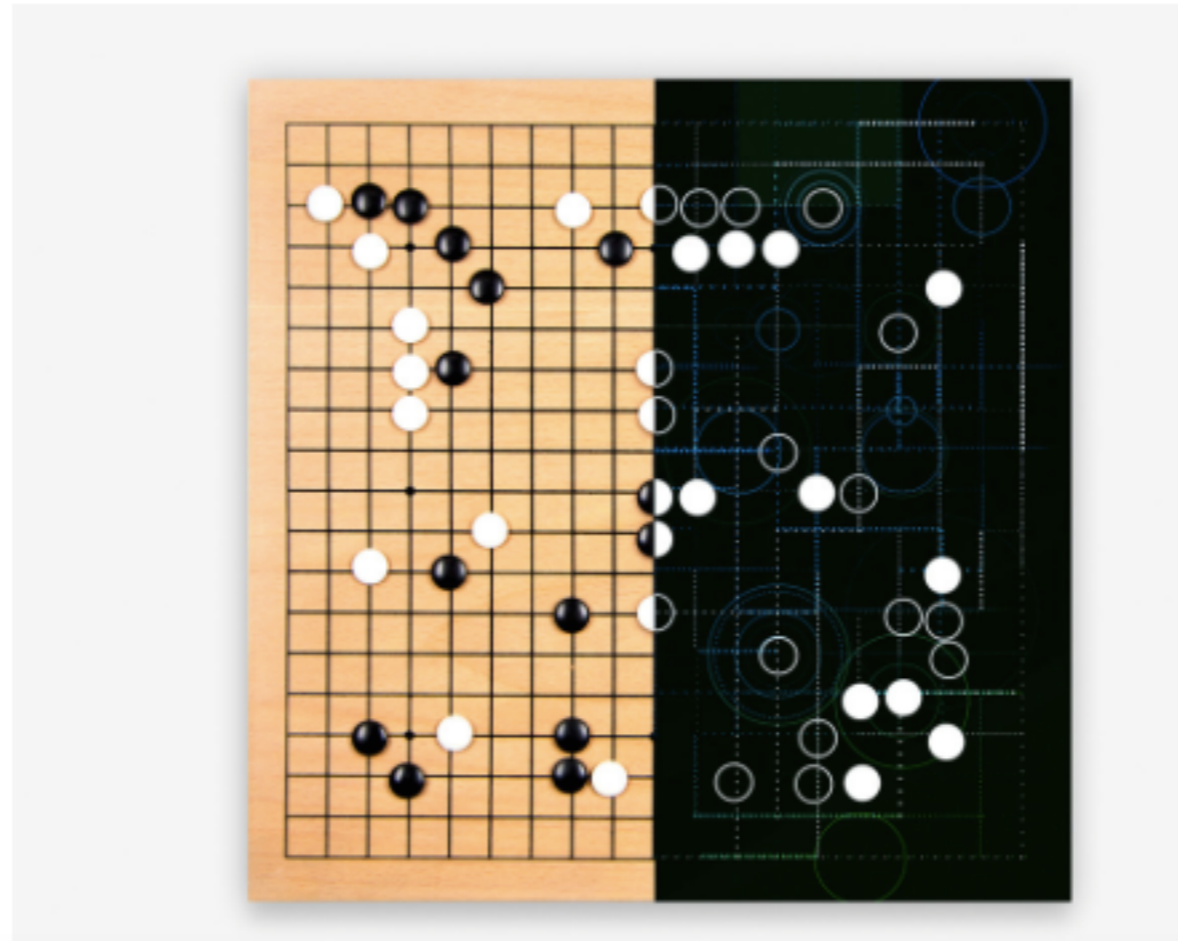
Student Success on the New Knight-Knave THLPE #1

Interlude re Formal Logic & Games ...

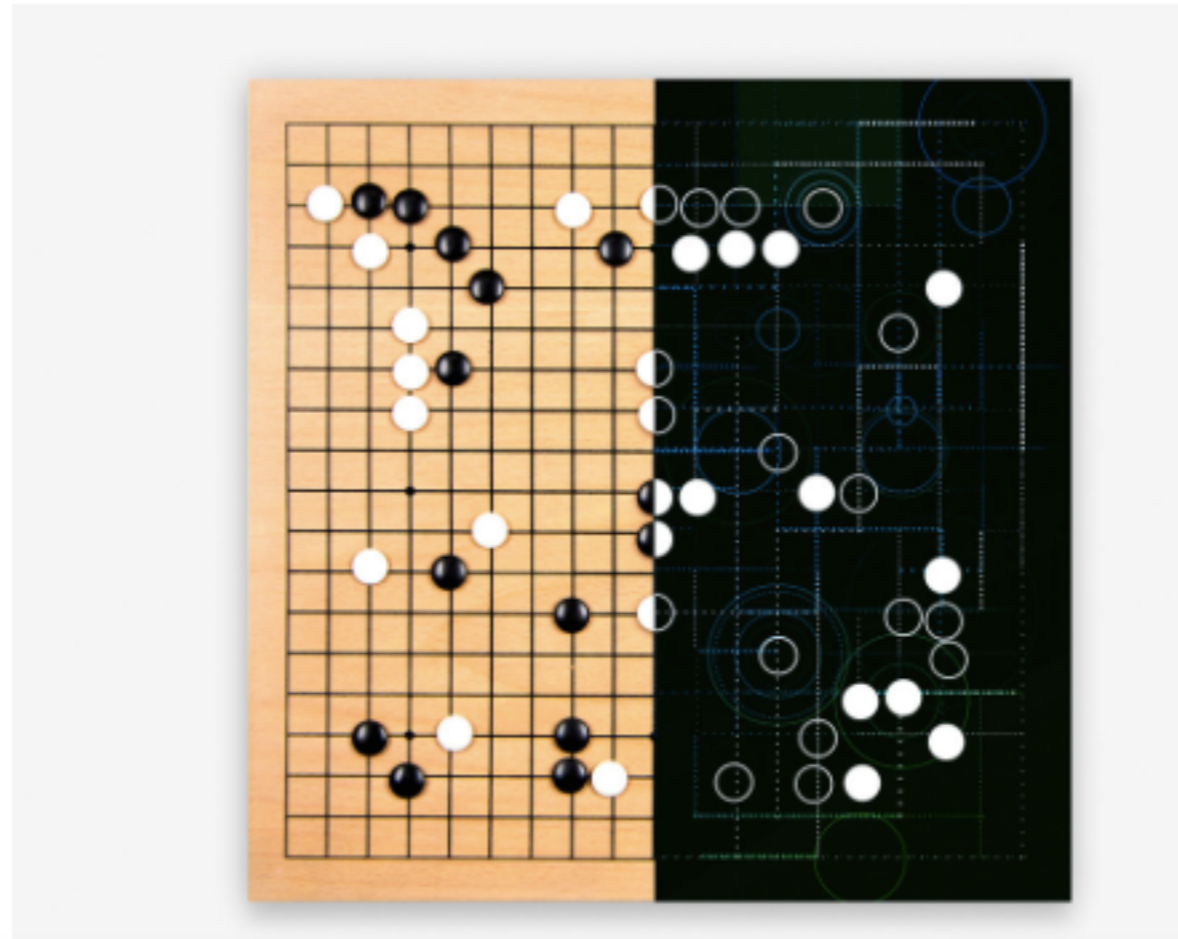
IN A HUGE BREAKTHROUGH, GOOGLE'S AI BEATS A TOP PLAYER AT THE GAME OF GO



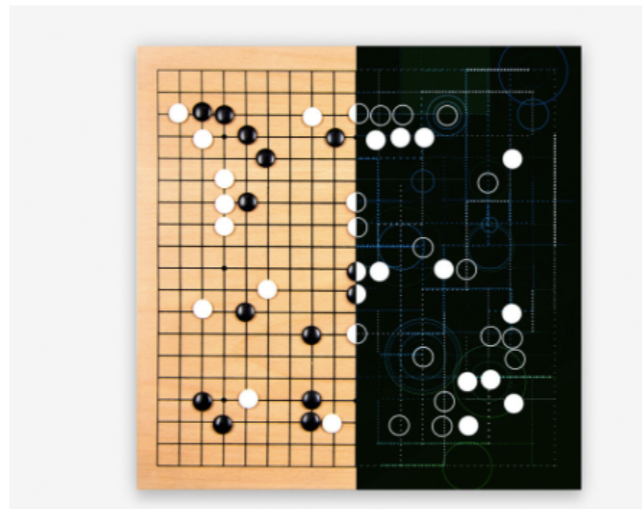
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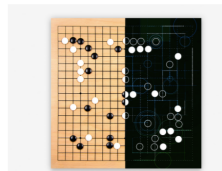
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**IN A HUGE BREAKTHROUGH,
GOOGLE'S AI BEATS A TOP
PLAYER AT THE GAME OF GO**



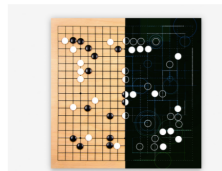
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The *Entscheidungsproblem*

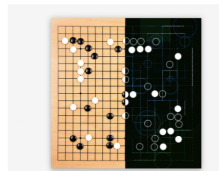
The *Entscheidungsproblem*

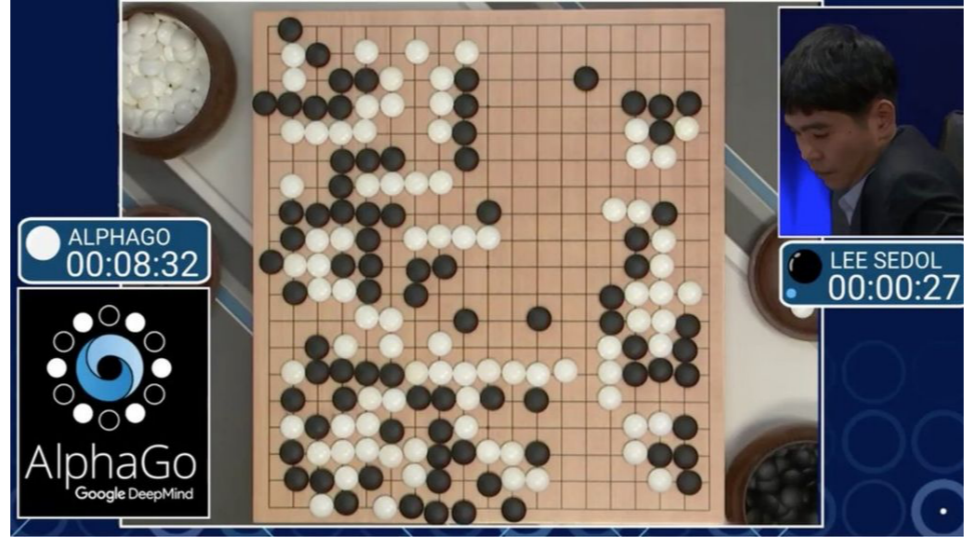
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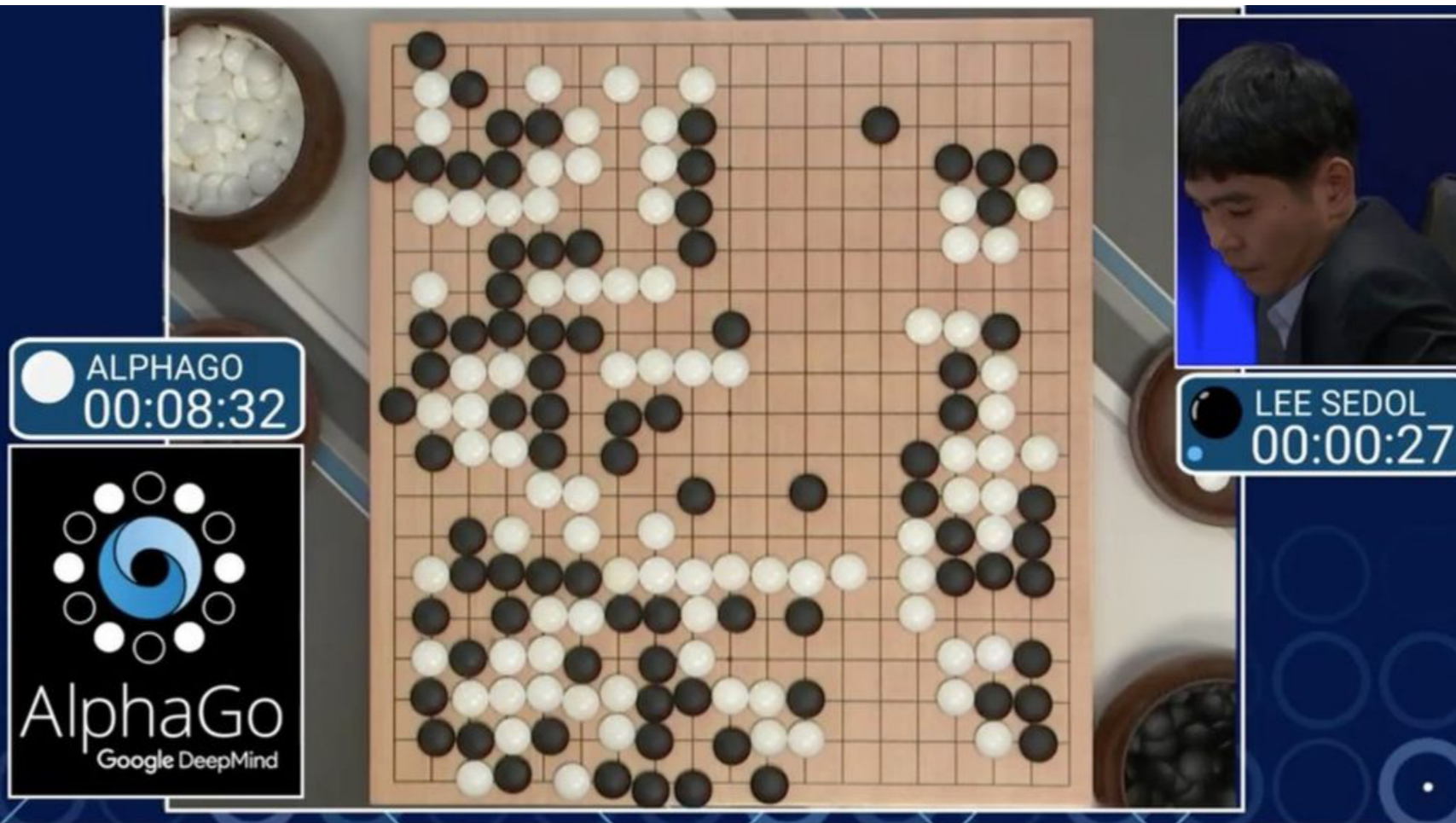


The *Entscheidungsproblem*

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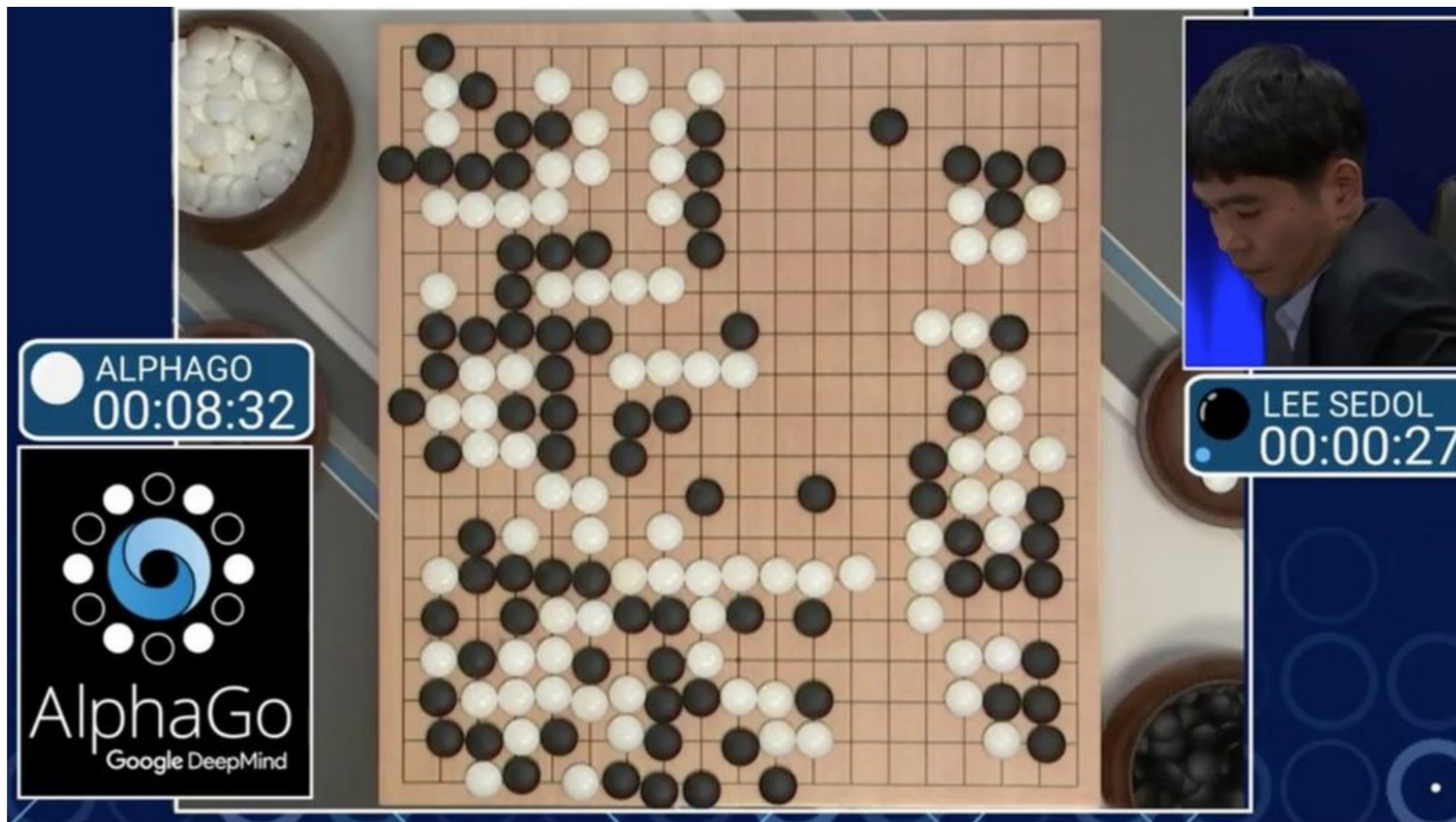




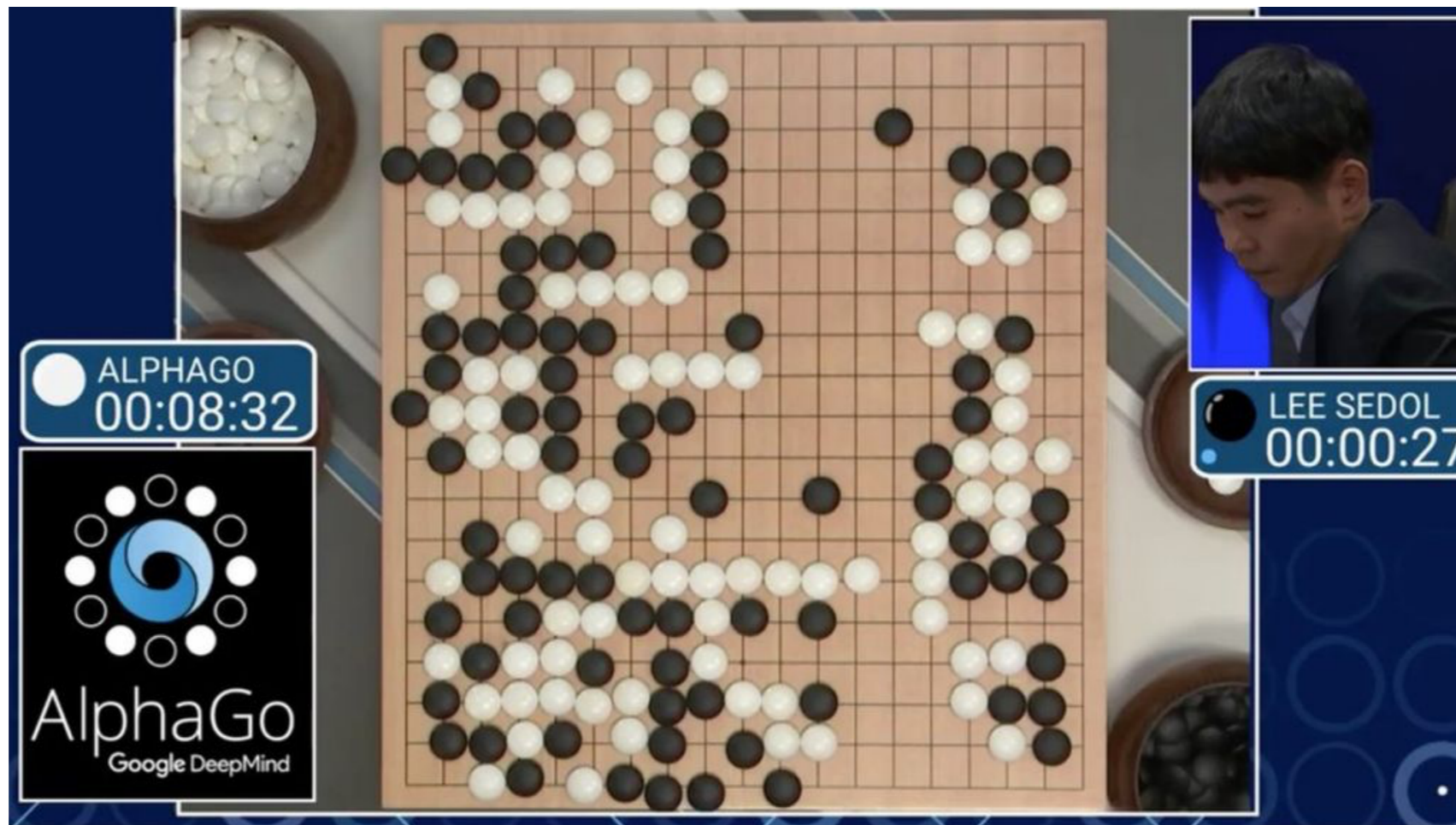
ALPHAGO
00:08:32

LEE SEDOL
00:00:27

AlphaGo
Google DeepMind

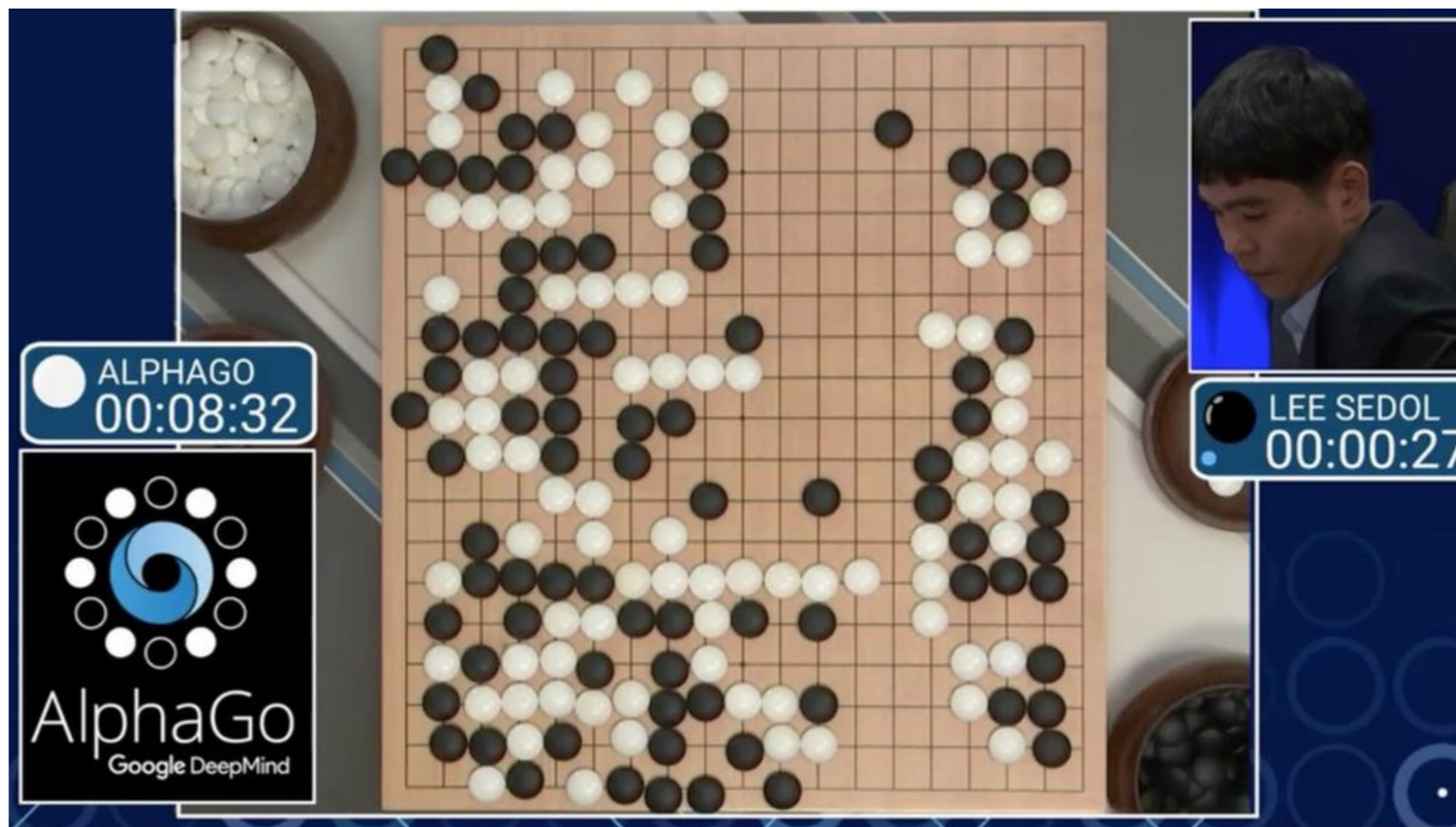


Praiseworthy *AI simplicter*, perhaps.



Praiseworthy *AI simpliciter*, perhaps.

But certainly *not AI = HI!*



Praiseworthy *AI simpliciter*, perhaps.

But certainly *not AI = HI!*

“AlphaGo, from the perspective of South, how many majuscule Roman letters are in black? Why do you say that?”

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive



Descartes

Entscheidungsproblem

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Descartes



Leibniz

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Leibniz



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Gödel

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The first (procedural) programmer!



Descartes



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Polynomial Hierarchy

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Polynomial Hierarchy

Entscheidungsproblem

$$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$$

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The first (procedural) programmer!



Descartes



Leibniz



Church



Gödel



⋮
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Go:AlphaGo



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Church



Gödel



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Jeopardy!:



Go:AlphaGo



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Gödel



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Chess: Deep Blue
●

Jeopardy!:
●

Go: AlphaGo
●

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The first (procedural) programmer!



Descartes

Checkers: Chinook
●



Leibniz

Chess: Deep Blue
●



Church

Jeopardy!:
●



Gödel

Go: AlphaGo
●



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Leibniz



Church



Gödel



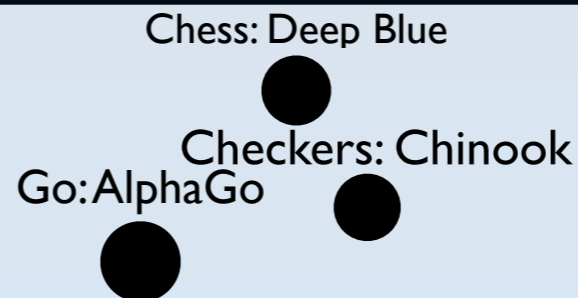
Turing

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Entscheidungsproblem

Polynomial Hierarchy

Jeopardy!: **Watson**



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Descartes



Leibniz



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$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Rengo Kriegspiel



The screenshot shows a web browser window with the URL usgo.org. The browser's address bar and tabs are visible at the top. The website's header features the American Go Association logo on the left and a navigation menu with links to "U.S. Go Congress", "Crosstabs", "Free E-Journal", "Upcoming Events", "Ratings", "Kids & Teens", and "Latest Go News".

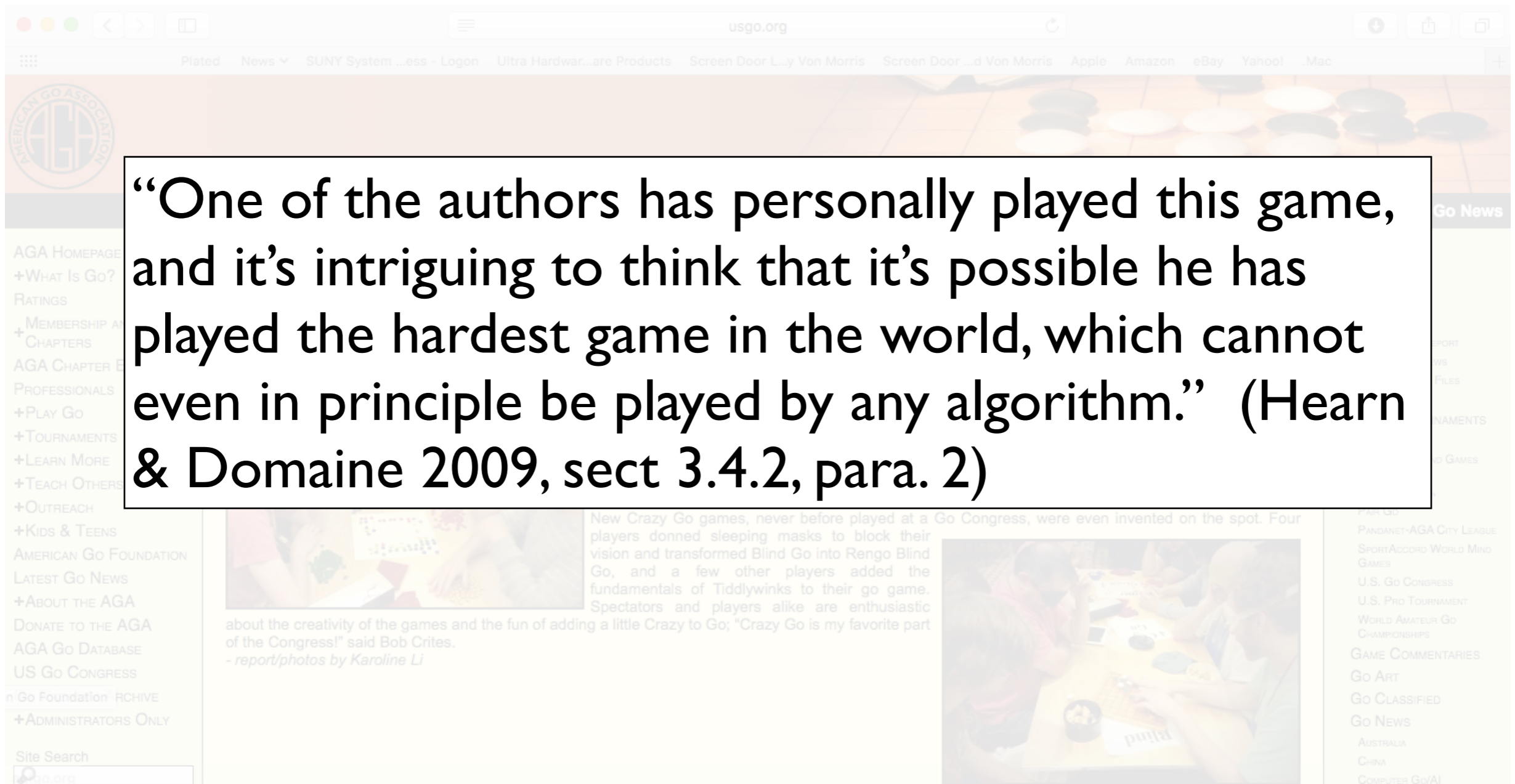
The main content area is titled "American Go E-Journal" and features an article titled "US Go Congress Goes a Little Crazy" dated Wednesday August 13, 2014. The article includes a photograph of several people playing Go at a table. The text of the article describes a game called Rengo Kriegspiel, where players play on their own boards while facing away from the main board. It mentions that Terry Benson, founder of Crazy Go, officiated several games of Rengo Kriegspiel at the event. The article also notes that spectators and players alike are enthusiastic about the creativity of the games and the fun of adding a little Crazy to Go.

On the right side of the page, there is a "CATEGORIES:" section with a list of links including "ALL", "CALENDAR", "COLUMNS", "JOHN POWER REPORT", "REDMOND REVIEWS", "THE JANICE KIM FILES", "WHY WE PLAY", "EVENTS/TOURNAMENTS", "COTSEN OPEN", "IMSA ELITE MIND GAMES", "OTHER", "N.A. GO CONVENTION", "PAIR GO", "PANDANET-AGA CITY LEAGUE", "SPORTACCORD WORLD MIND GAMES", "U.S. GO CONGRESS", "U.S. PRO TOURNAMENT", "WORLD AMATEUR GO CHAMPIONSHIPS", "GAME COMMENTARIES", "GO ART", "GO CLASSIFIED", "GO NEWS", "AUSTRALIA", "CHINA", and "COMPUTER GO/AI".

On the left side of the page, there is a sidebar with various links including "AGA HOMEPAGE", "+WHAT IS GO?", "RATINGS", "+MEMBERSHIP AND CHAPTERS", "AGA CHAPTER EMAIL LIST", "PROFESSIONALS", "+PLAY GO", "+TOURNAMENTS", "+LEARN MORE", "+TEACH OTHERS", "+OUTREACH", "+KIDS & TEENS", "AMERICAN GO FOUNDATION", "LATEST GO NEWS", "+ABOUT THE AGA", "DONATE TO THE AGA", "AGA GO DATABASE", "US GO CONGRESS", "n Go Foundation ARCHIVE", "+ADMINISTRATORS ONLY", and a "Site Search" box with the URL usgo.org.

Rengo Kriegspiel

“One of the authors has personally played this game, and it’s intriguing to think that it’s possible he has played the hardest game in the world, which cannot even in principle be played by any algorithm.” (Hearn & Domaine 2009, sect 3.4.2, para. 2)



Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy



Descartes



Leibniz



Church



Gödel



Turing

⋮
 Π_2
 Σ_2
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Entscheidungsproblem

Polynomial Hierarchy

Jeopardy!: **Watson**



$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

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Descartes



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Gödel



Turing

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Entscheidungsproblem

Computational Hierarchy



$$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$$

And where is RPI's quantum computer??

But starting simpler ...

Tabular “Deduction” (Example)

Logic Puzzles

Story

	Eduardo	Greg	Jay	Patrick	Salvador	indigo	lime	maroon	orange	red	Fullerton	Hollister	San Pedro	Templeton	Urbandale
22 minutes															
23 minutes															
24 minutes															
25 minutes															
26 minutes															
Fullerton															
Hollister															
San Pedro															
Templeton															
Urbandale															
indigo															
lime															
maroon															
orange															
red															

1. Patrick, who is from Templeton and finished with a time of 24 minutes, wore neither the red nor the maroon shirt.
2. The runner in the orange shirt finished one minute before the one in the
3. The runner who finished in 25 minutes was from San Pedro.
4. The runner who finished in 26 minutes was from Templeton.
5. The runner who finished in 24 minutes wore the maroon shirt.
6. The runner who finished in 25 minutes wore the lime shirt.
7. Jay didn't finish with the fastest time.
8. The runner from San Pedro was either the one in the orange shirt or Eduardo.

5K Fun Run

Valero's 5k Fun Run was held yesterday in the downtown district. Determine the shirt color and hometown of each of the top five runners, and match each to their finishing time.

Tabular “Deduction” (Example)

< Logic Puzzles

Story

	Eduardo	Greg	Jay	Patrick	Salvador	indigo	lime	maroon	orange	red	Fullerton	Hollister	San Pedro	Templeton	Urbandale
22 minutes															
23 minutes															
24 minutes															
25 minutes															
26 minutes															
Fullerton															
Hollister															
San Pedro															
Templeton															
Urbandale															
indigo															
lime															
maroon															
orange															
red															

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3. The runner who finished in 23 minutes (who wasn't Eduardo) wasn't from Hollister.
4. The Urbandale contestant wore the red shirt.
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Tabular “Deduction” (Example)

< Logic Puzzles
↶
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22 minutes															
23 minutes															
24 minutes															
25 minutes															
26 minutes															
Fullerton				✗											
Hollister				✗											
San Pedro				✗											
Templeton	✗	✗	✗	●	✗										
Urbandale				✗											
indigo															
lime															
maroon															
orange															
red															

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Tabular “Deduction” (Example)

< Logic Puzzles

S

Story	Eduardo	Greg	Jay	Patrick	Salvador	indigo	lime	maroon	orange	red	Fullerton	Hollister	San Pedro	Templeton	Urbandale
22 minutes															
23 minutes															
24 minutes															
25 minutes															
26 minutes															
Fullerton				X											
Hollister				X											
San Pedro				X											
Templeton	X	X	X	●	X										
Urbandale				X											
indigo															
lime															
maroon															
orange															
red															

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In HyperSlate ...

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Tabular “Deduction” (Example)

←
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💡
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☰

S

Logic Puzzles

Story	Eduardo	Greg	Jay	Patrick	Salvador	indigo	lime	maroon	orange	red	Fullerton	Hollister	San Pedro	Templeton	Urbandale
22 minutes															
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26 minutes															
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Hollister				✗						✗					
San Pedro				✗						✗					
Templeton	✗	✗	✗	●	✗					✗					
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Logic Puzzles

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Urbandale				X		X	X	X	X	●					
indigo															
lime															
maroon															
orange															
red															

These two cells tell you something about the cell in yellow.

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Tabular “Deduction”: It’s Taught!



Example

Grace, Dylan, Kira, and Diego are each wearing different colored shirts. Grace’s shirt is red. Dylan’s shirt is not white. Kira’s shirt is not green. Diego’s shirt is not yellow or white. What color shirt is each person wearing?

First, make a chart to show what you know.

- Each shirt is a different color.
- Grace’s shirt is red.
- Dylan’s shirt is not white.
- Kira’s shirt is not green.
- Diego’s shirt is not yellow or white.

	Red	White	Green	Yellow
Grace	yes	no	no	no
Dylan	no	no		
Kira	no		no	
Diego	no	no	yes	no

Then use reasoning and the information in the chart to complete the chart and find the answer.

Grace’s shirt is red, so no other shirt can be red.

Diego’s shirt is not red, white, or yellow, so it must be green.

Dylan’s shirt must be yellow because it cannot be red, white, or green.

That means Kira’s shirt must be white.

Solve

Tabular “Deduction”: It’s Taught!



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Solve

IMHO very bad idea—if *before* real learning of deduction to answer “Why, exactly? Prove it!”

Tabular “deduction” *not* the skill that’s valuable.

8:29 AM

iPad

8:29 AM

91

Recall from Lesson 4-8 that the complex numbers $a + bi$ and $a - bi$ are conjugates. Similarly, the irrational numbers $a + \sqrt{b}$ and $a - \sqrt{b}$ are conjugates. If a complex number or an irrational number is a root of a polynomial equation with rational coefficients, so is its conjugate.

TAKE NOTE Theorem

Conjugate Root Theorem

If $P(x)$ is a polynomial with *rational* coefficients, then irrational roots of $P(x) = 0$ that have the form $a + \sqrt{b}$ occur in conjugate pairs. That is, if $a + \sqrt{b}$ is an irrational root with a and b rational, then $a - \sqrt{b}$ is also a root.

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Every quadratic polynomial equation has two roots, every cubic polynomial equation has three roots, and so on.

This result is related to the *Fundamental Theorem of Algebra*. The German mathematician Carl Friedrich Gauss (1777–1855) is credited with proving this theorem.

TAKE NOTE Theorem

The Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n roots, including multiple and complex roots.

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From Algebra 2

Practice and Problem-Solving Exercises - Contir

Determine whether each of the following statements is *always*, *sometimes*, or *never* true.

41. A polynomial function with real coefficients has real zeros.
42. Polynomial functions with complex coefficients have one complex zero.
43. A polynomial function that does not intercept the x -axis has complex roots only.
44. **Reasoning** A 4th-degree polynomial function has zeros at 3 and $5 - i$. Can $4 + i$ also be a zero of the function? Explain your reasoning.
45. **Open-Ended** Write a polynomial function that has four possible rational zeros but no actual rational zeros.
46. **Reasoning** Show that the Fundamental Theorem of Algebra must be true for all quadratic polynomial functions.

C • Challenge

47. Use the Fundamental Theorem of Algebra and the Conjugate Root Theorem to show that any odd degree polynomial equation with real coefficients has at least one real root.
48. **Reasoning** What is the maximum number of points of intersection between the graphs of a quartic and a quintic polynomial function?
49. **Reasoning** What is the least possible degree of a polynomial with rational coefficients, leading coefficient 1, constant term 5, and zeros at $\sqrt{2}$ and $\sqrt{3}$? Show that such a polynomial has a rational zero and indicate this zero.

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The Game of LogiNim \mathcal{L}

In HyperSlate®

The Game of LogiNim \mathcal{L} in HyperSlate®

Selmer Bringsjord

Motalen LLC

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A *logician directed acyclic hypergraph*, hereafter simply a *hypergraph*, is a pair

$$\mathcal{H} := \langle N, A \rangle$$

where

- each *node* $\nu \in N$ contains some formula $\phi \in \mathcal{L}$, where \mathcal{L} is a background formal language, possibly including a label l_ϕ for the node, and the set of all assumptions on which inference of ϕ may rely;
- each *arc* $a \in A$ is a pair composed of a label l_σ for some $\sigma \in \mathcal{I}$, a collection of inference schemata, and one of $\{r, g\}$;
- arcs are directed; and
- no *cycles* are permitted.

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Back to FOL ...

Our Final New Inference Rule in FOL

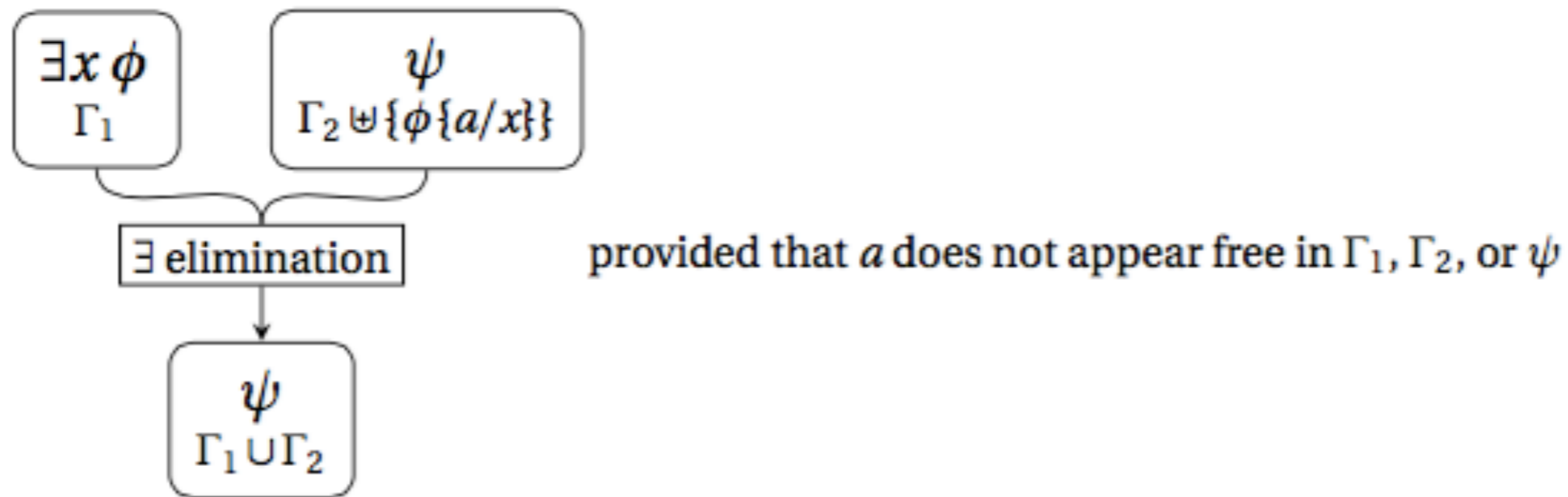
Our Final New Inference Rule in FOL

- existential elimination (intuitively put):

Our Final New Inference Rule in FOL

- existential elimination (intuitively put):
 - If we know that (i) there's something x which is an R , and (ii) on the supposition that a is an arbitrary representative (a “witness”) of such an x we can prove P , then we are permitted to deduce P from (i) alone.

existential elimination, precise version:

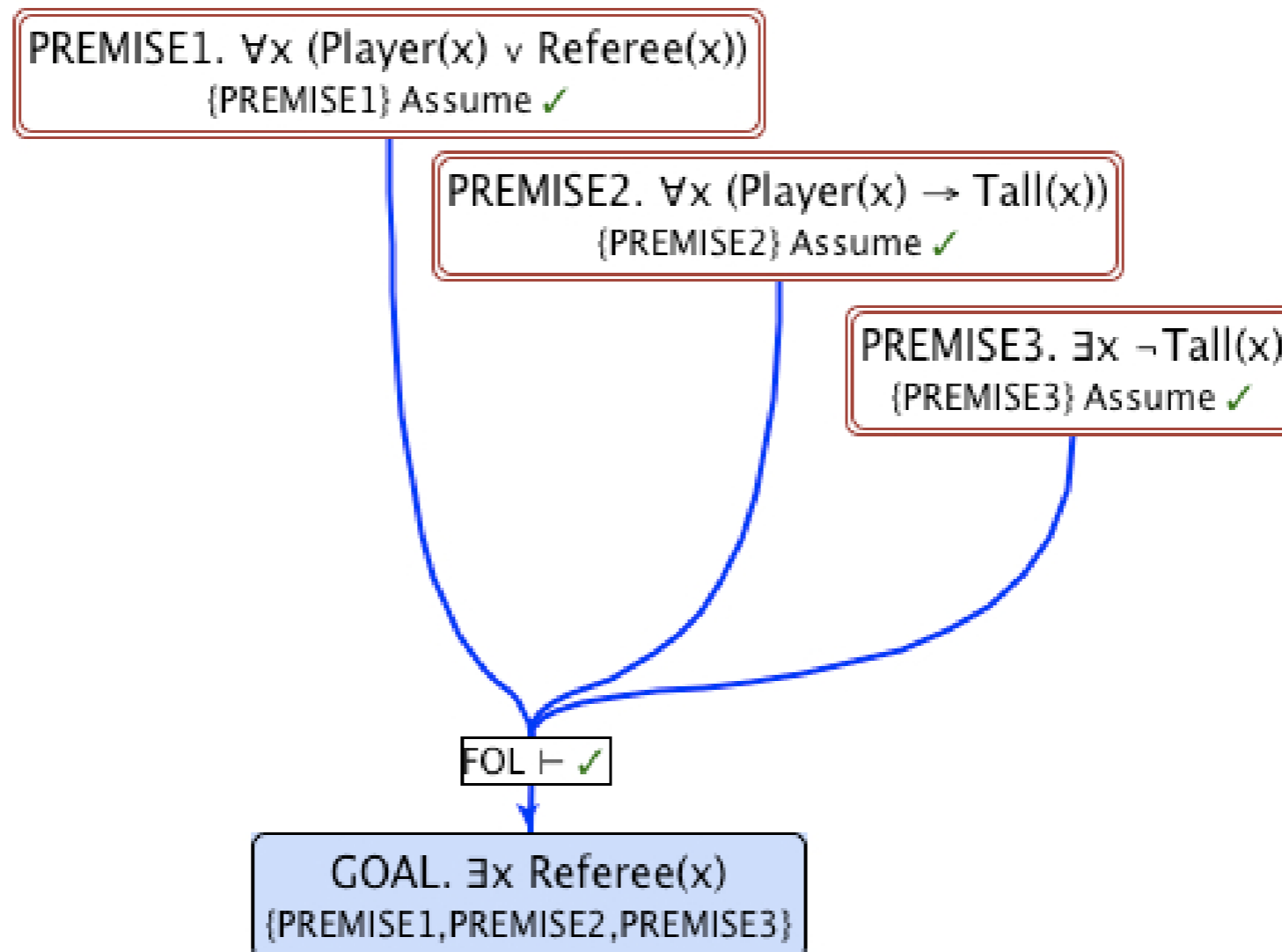


Starting State of Workspace

(Assumes a domain of e.g. players on a
March-madness basketball court.)

Starting State of Workspace

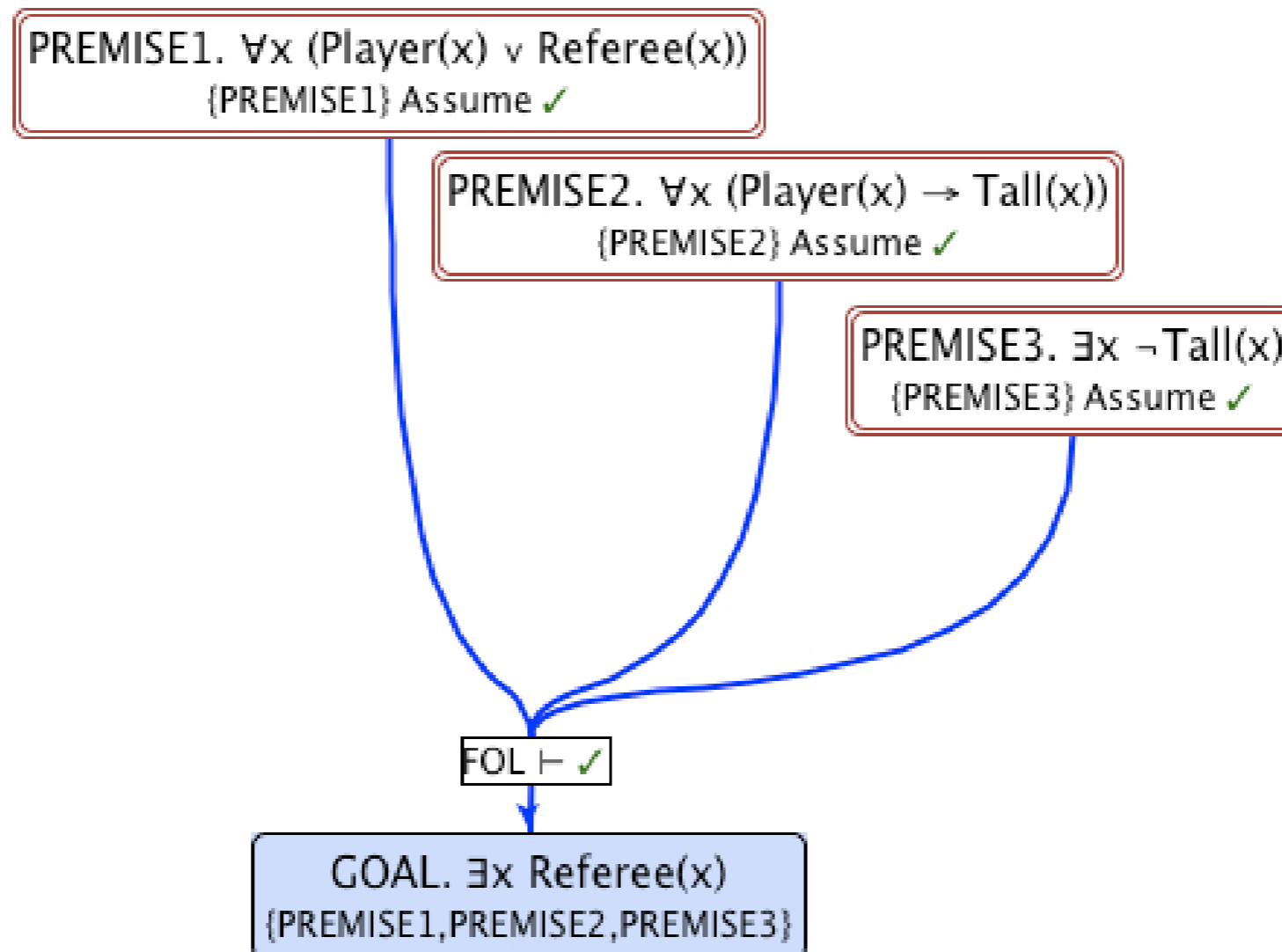
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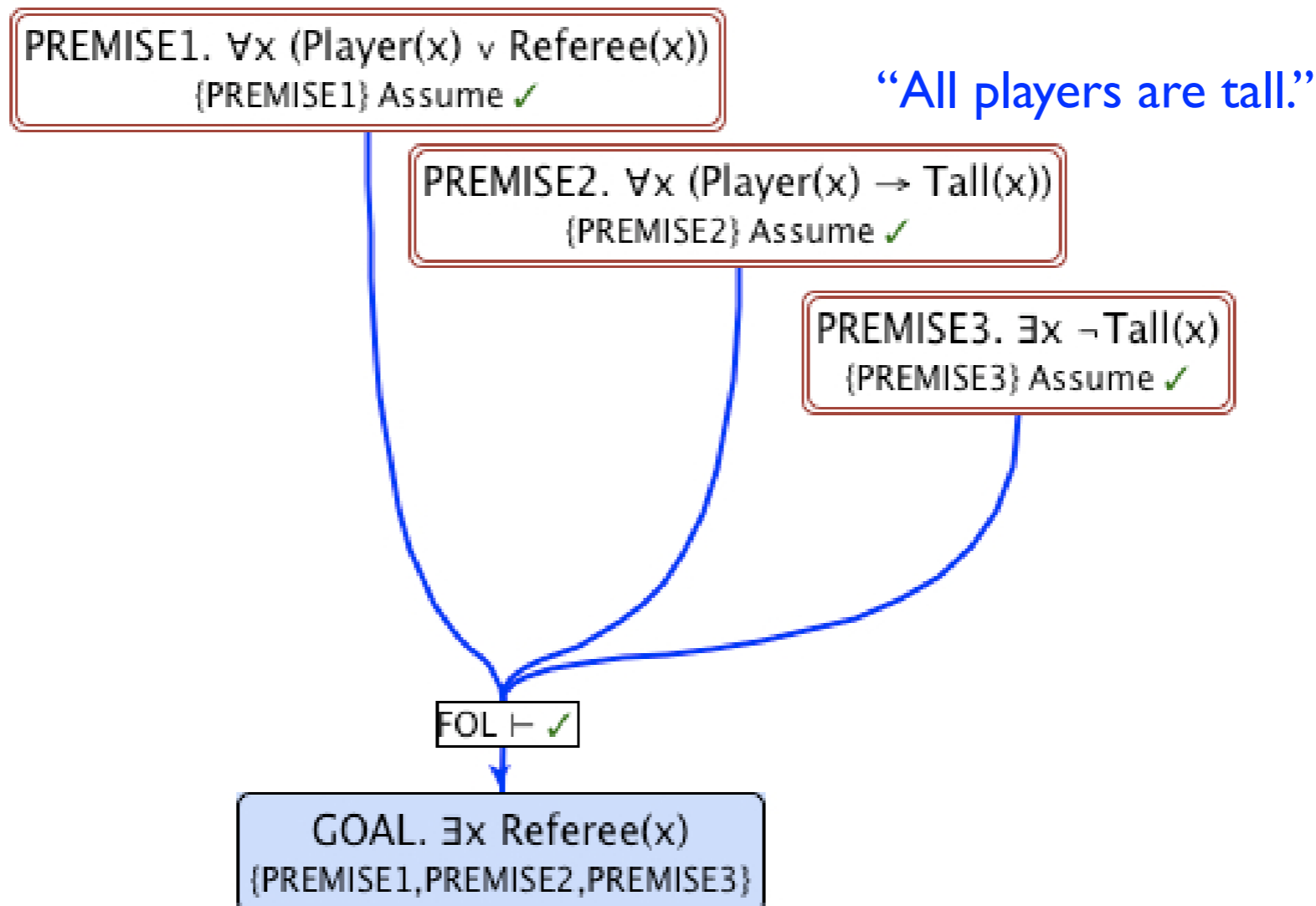
“Each and every thing is either a player or a referee.”



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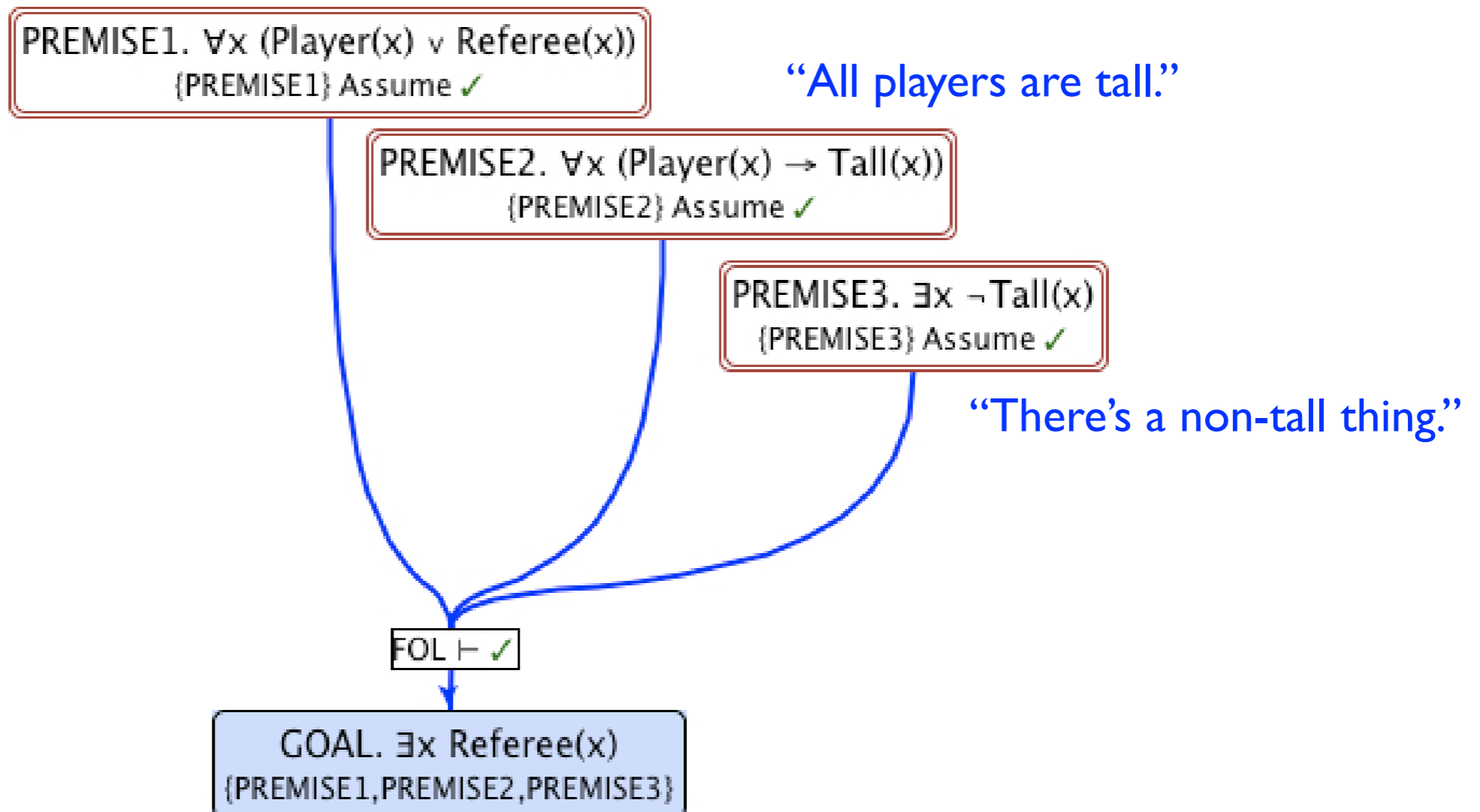
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“Each and every thing is either a player or a referee.”



Step 1

PREMISE1. $\forall x (\text{Player}(x) \vee \text{Referee}(x))$
{PREMISE1} Assume ✓

PREMISE2. $\forall x (\text{Player}(x) \rightarrow \text{Tall}(x))$
{PREMISE2} Assume ✓

PREMISE3. $\exists x \neg \text{Tall}(x)$
{PREMISE3} Assume ✓

5. $\neg \text{Tall}(a)$
{5} Assume ✓

GOAL. $\exists x \text{Referee}(x)$
FOL \vdash ✗

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PREMISE3. $\exists x \neg \text{Tall}(x)$
{PREMISE3} Assume ✓

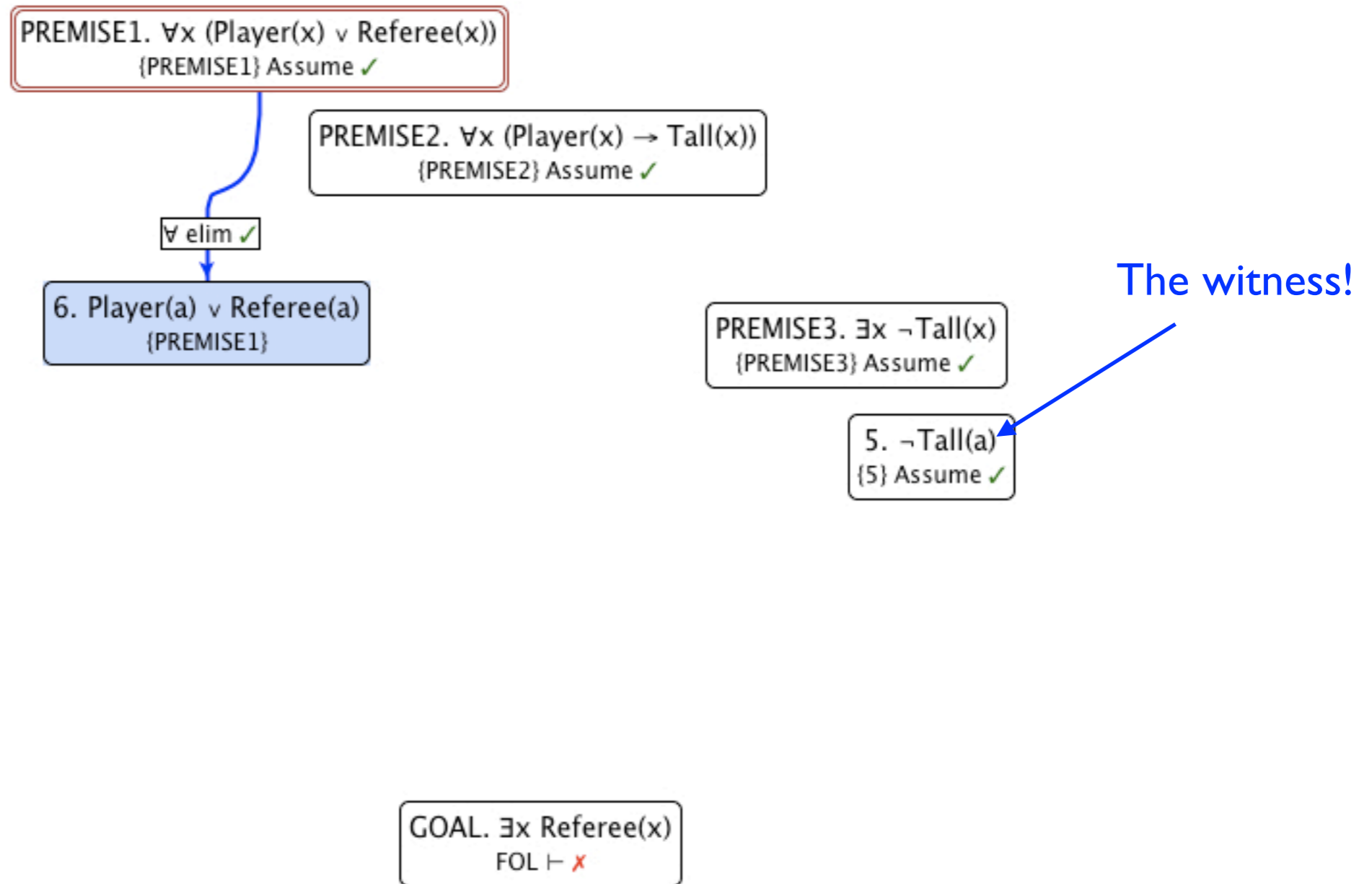
5. $\neg \text{Tall}(a)$
{5} Assume ✓

The witness!

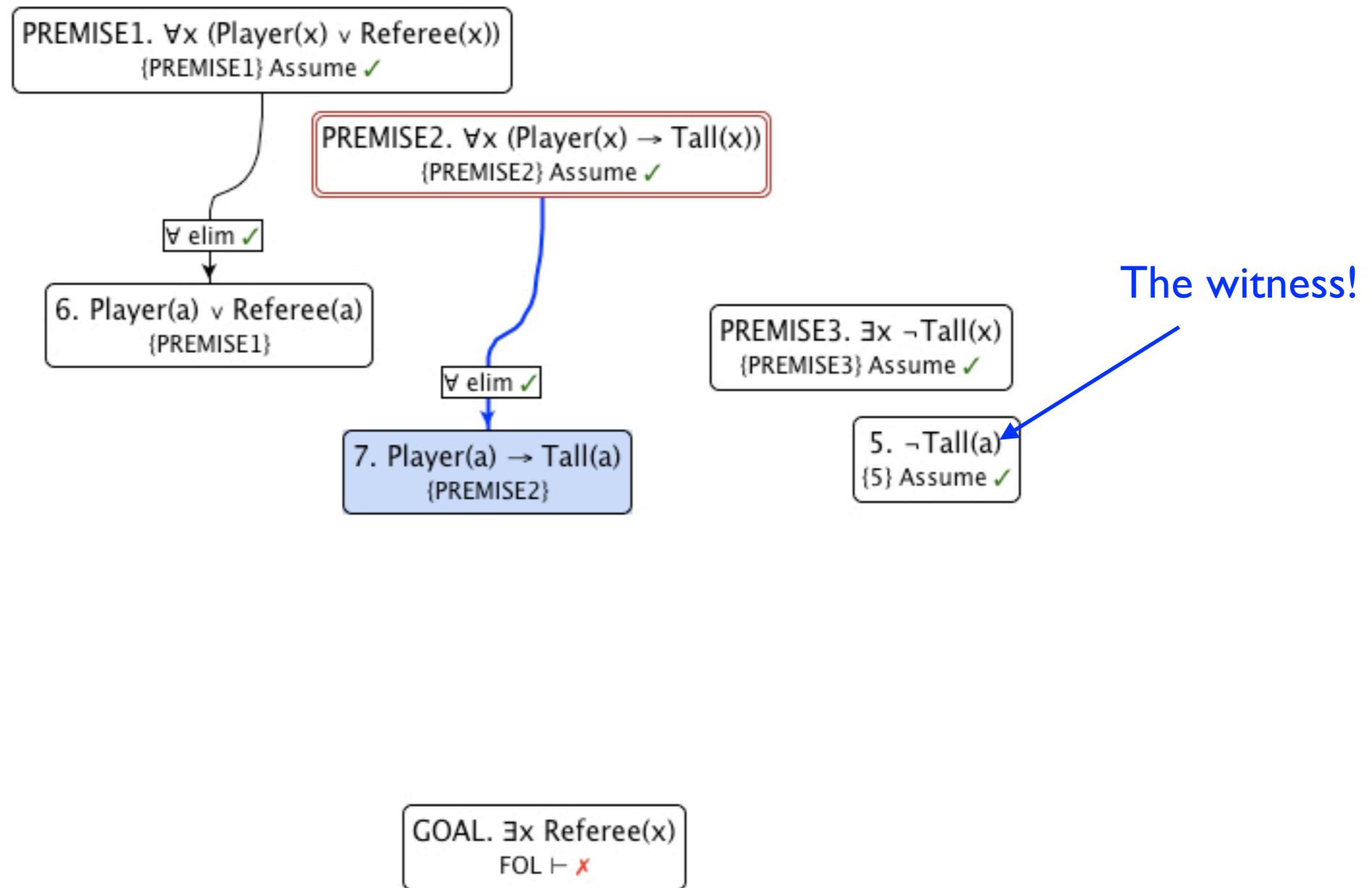


GOAL. $\exists x \text{Referee}(x)$
FOL ⊢ ✗

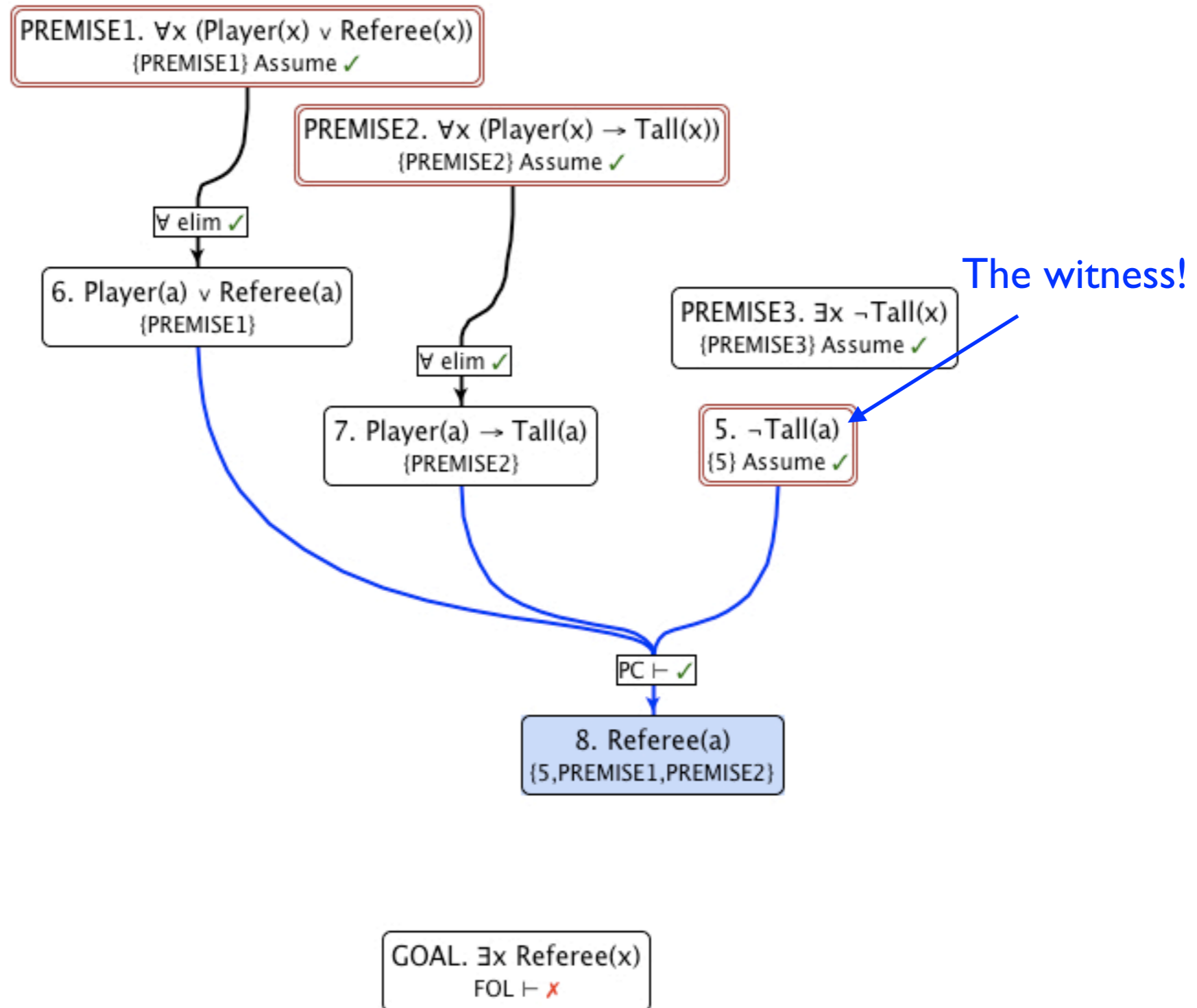
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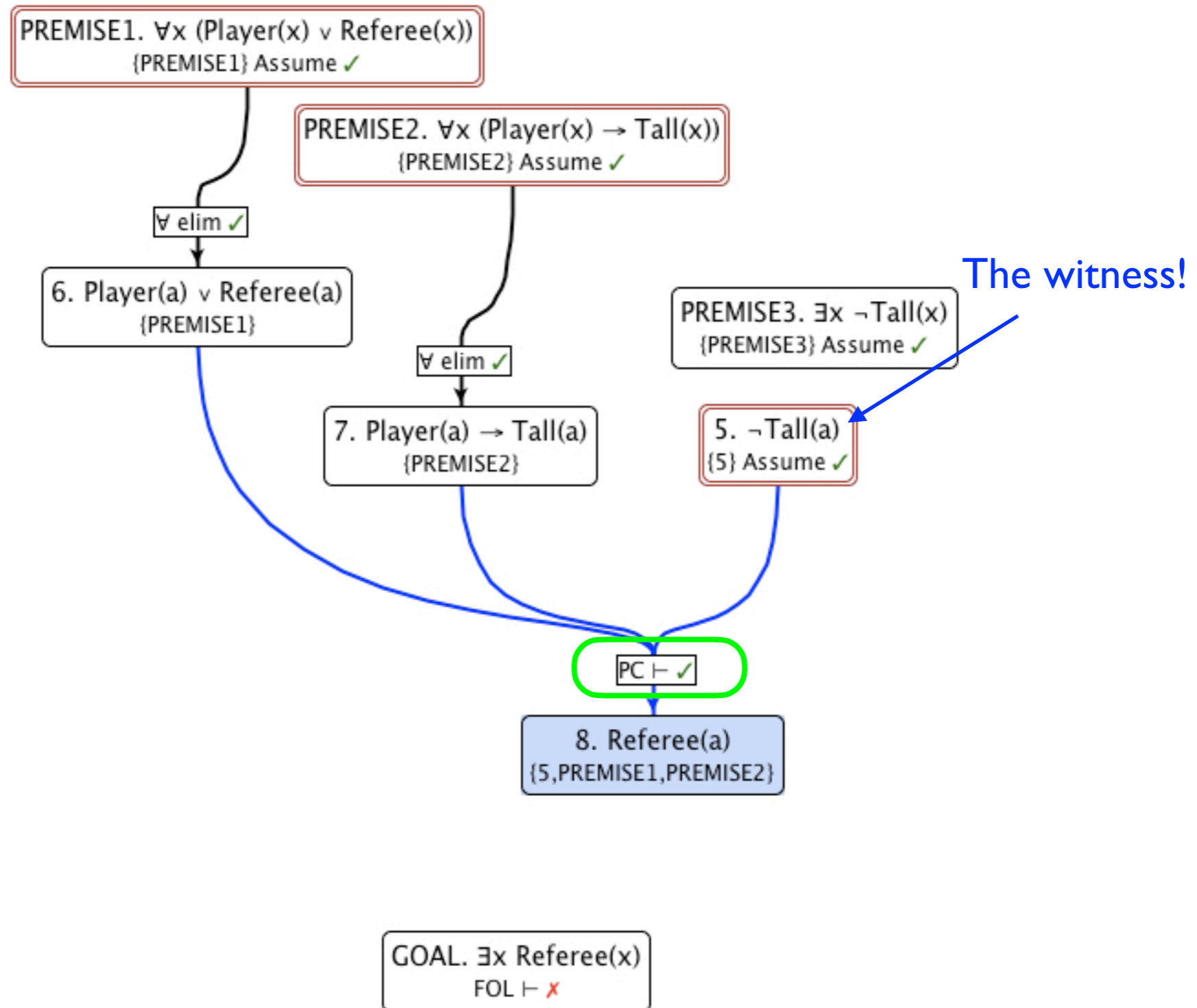
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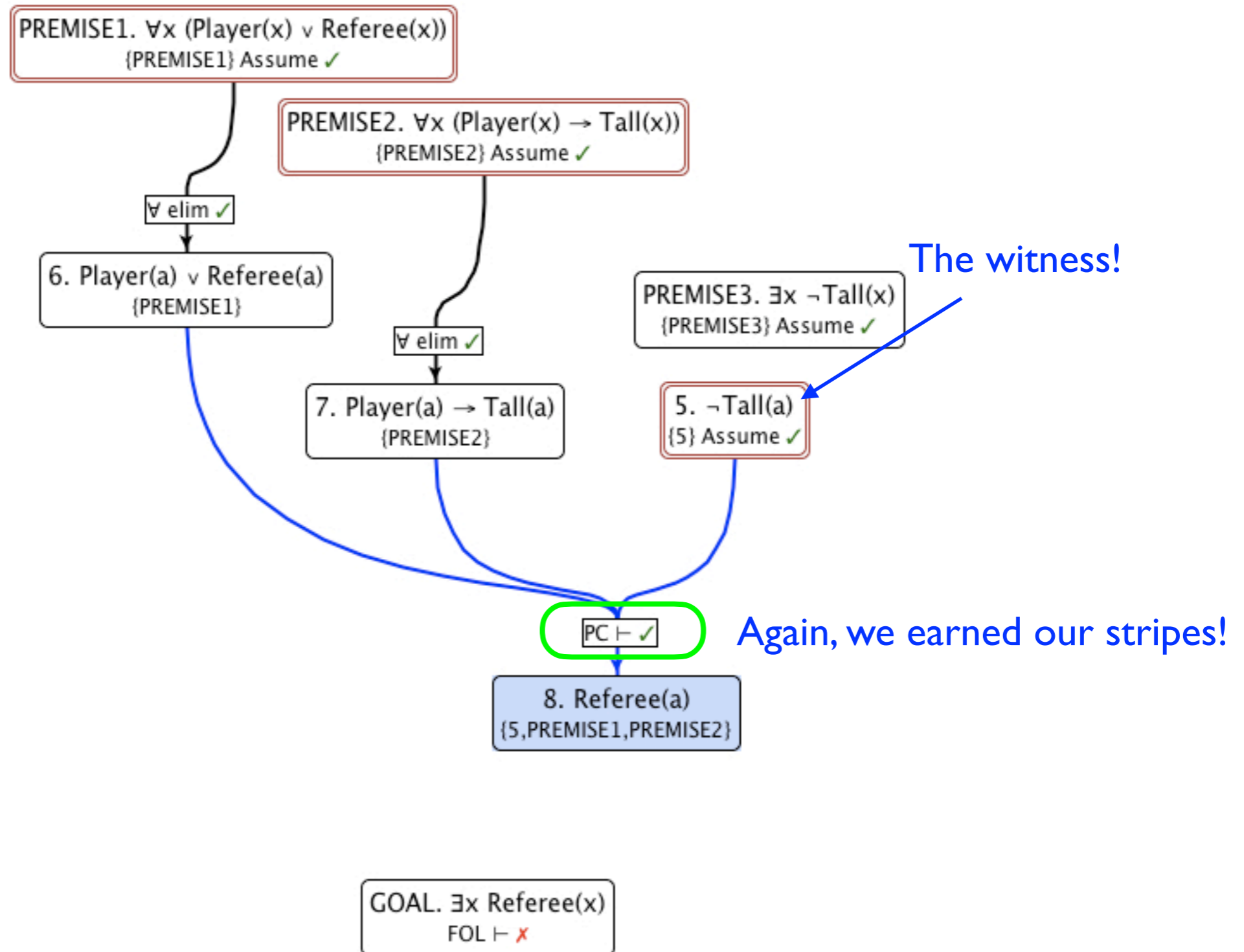
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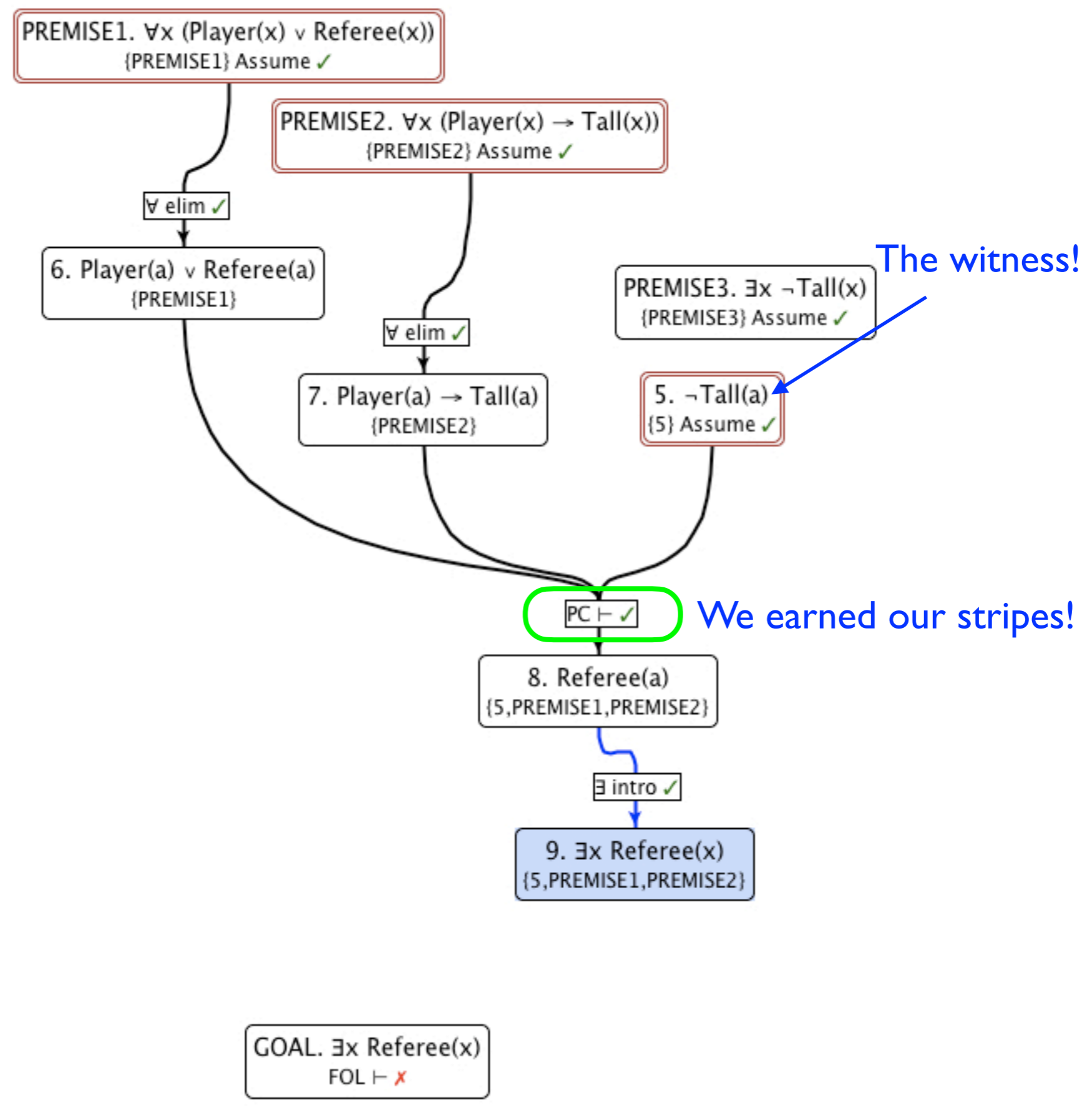
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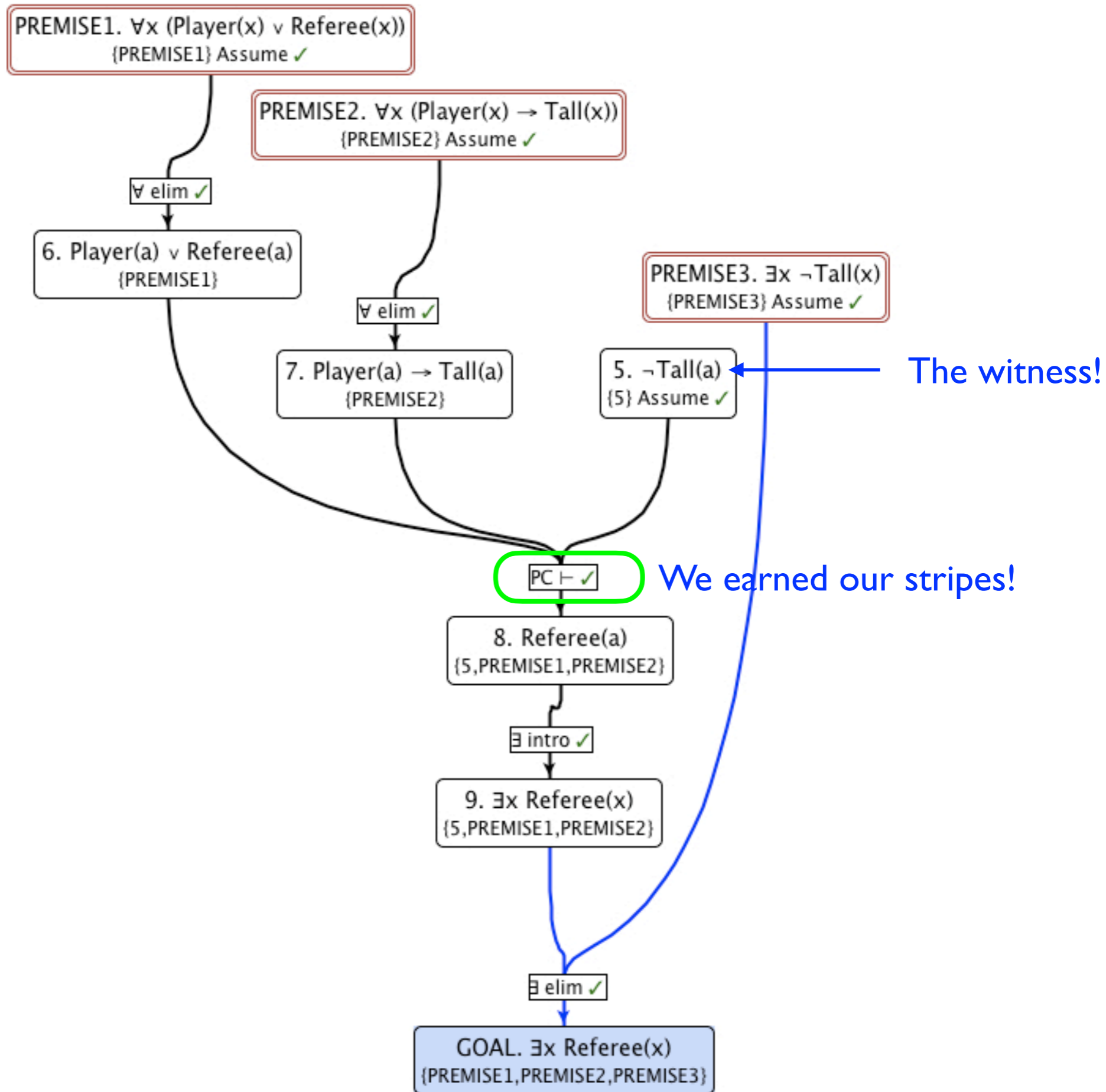
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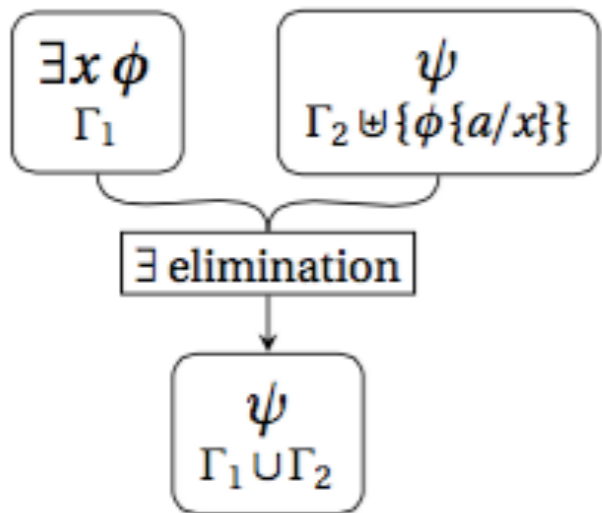
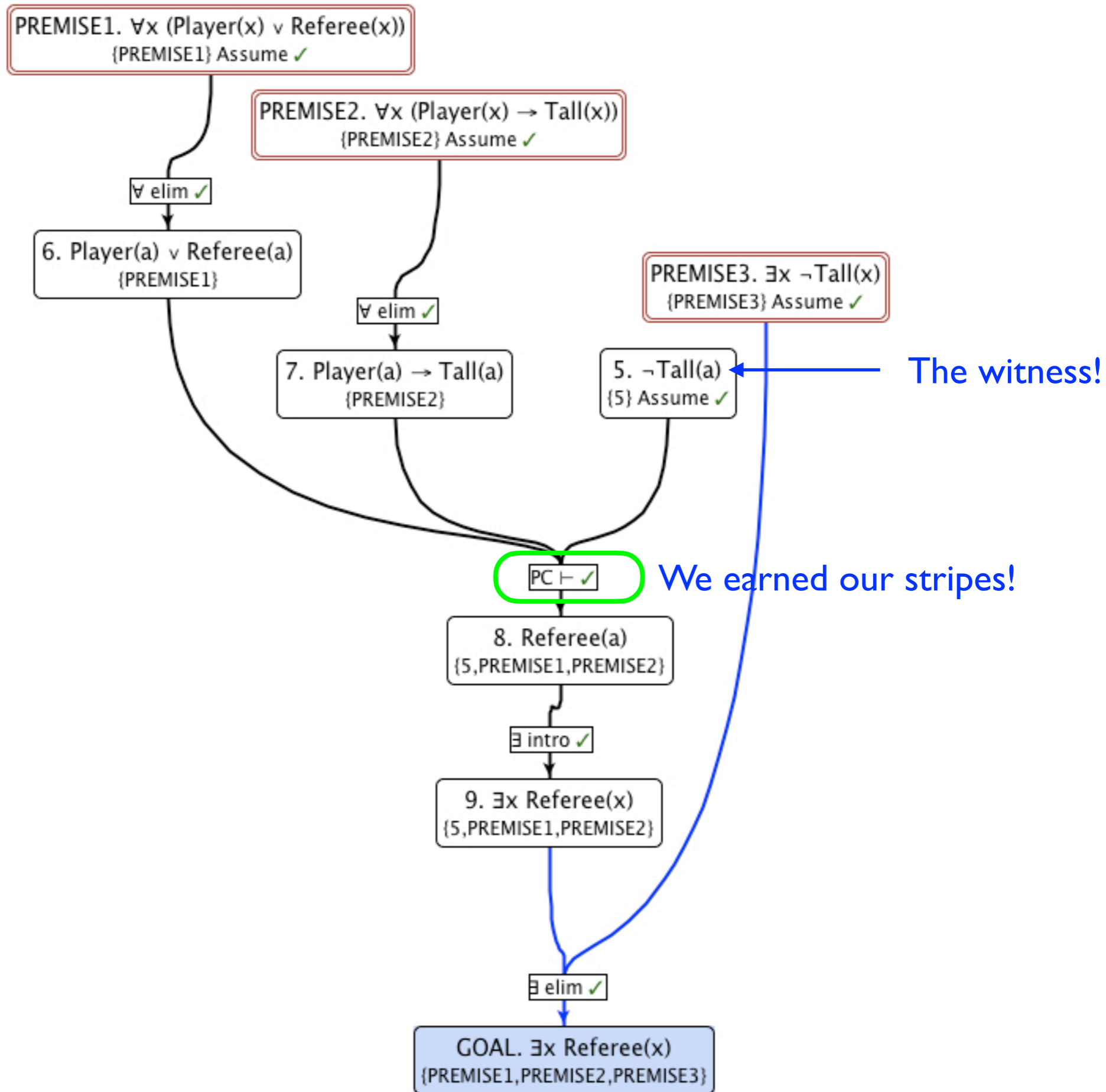
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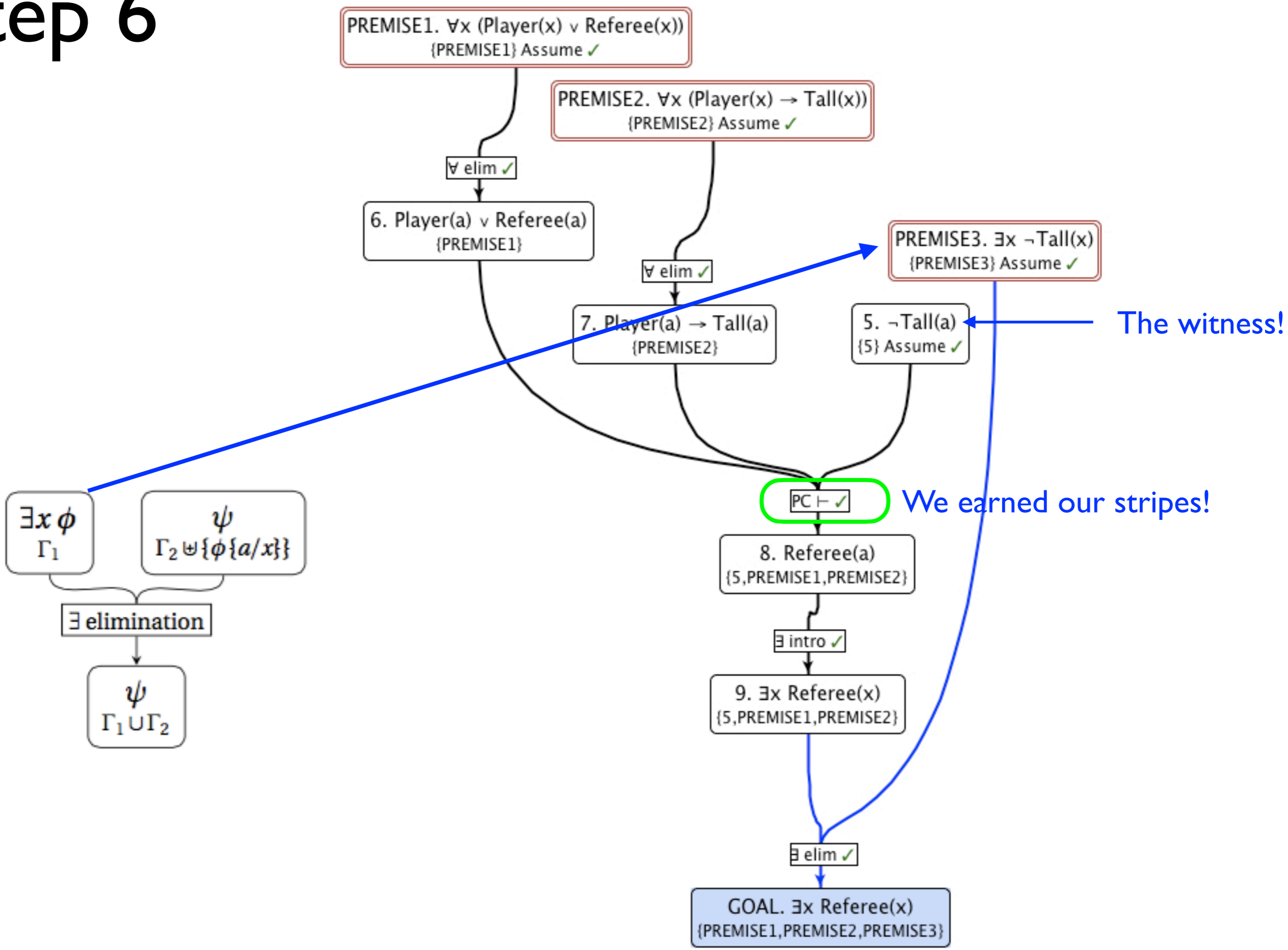
Step 6



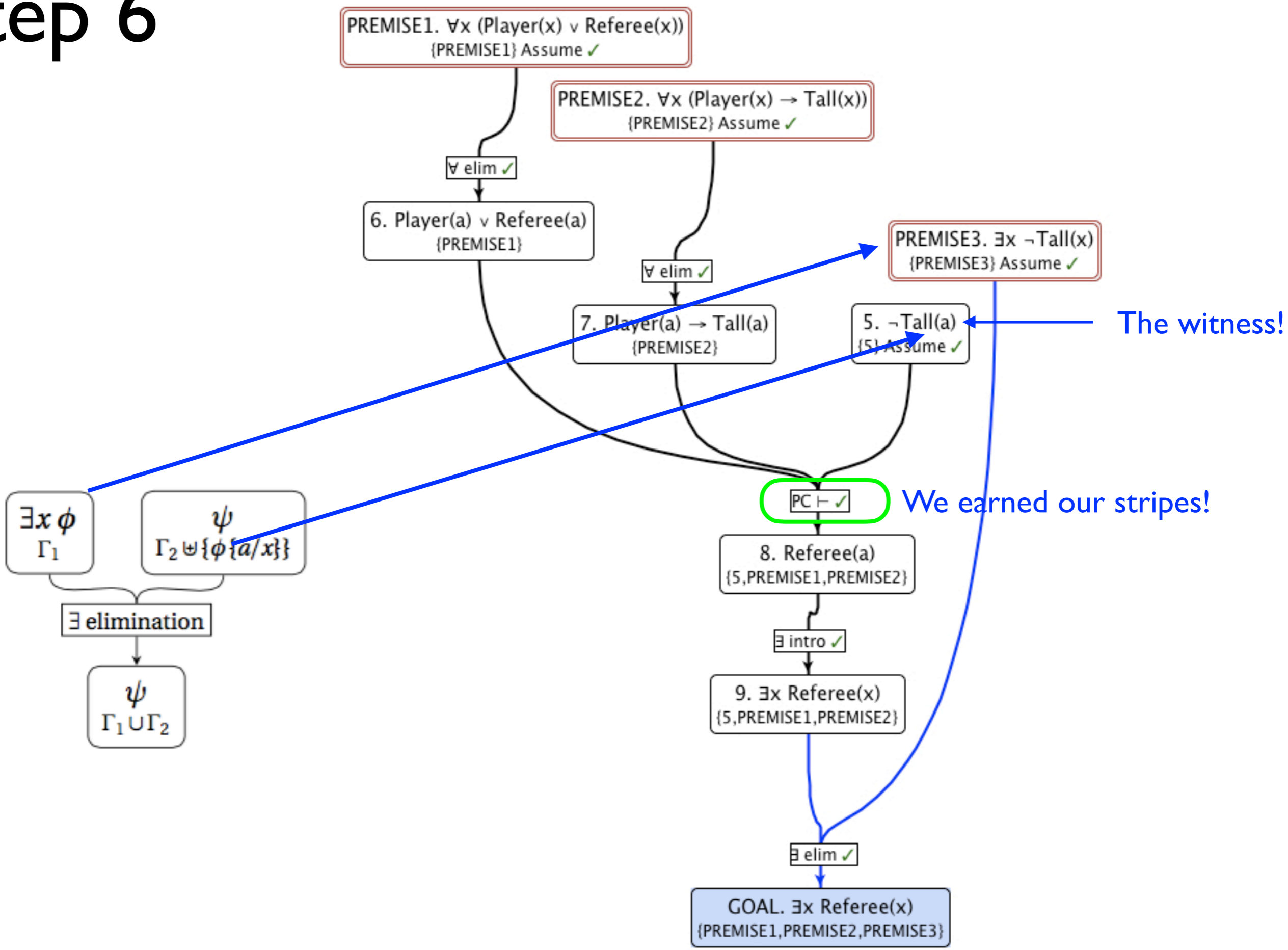
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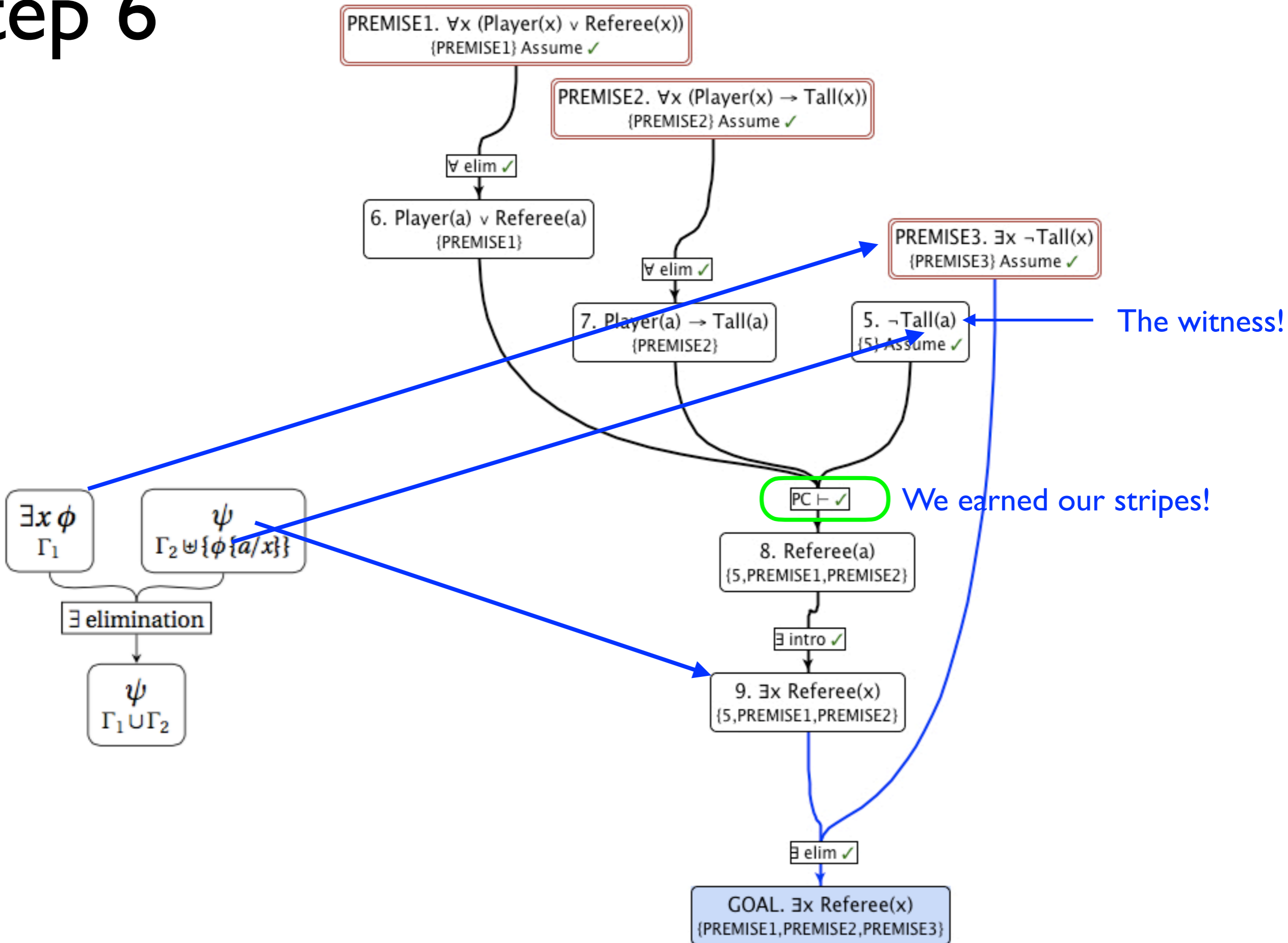
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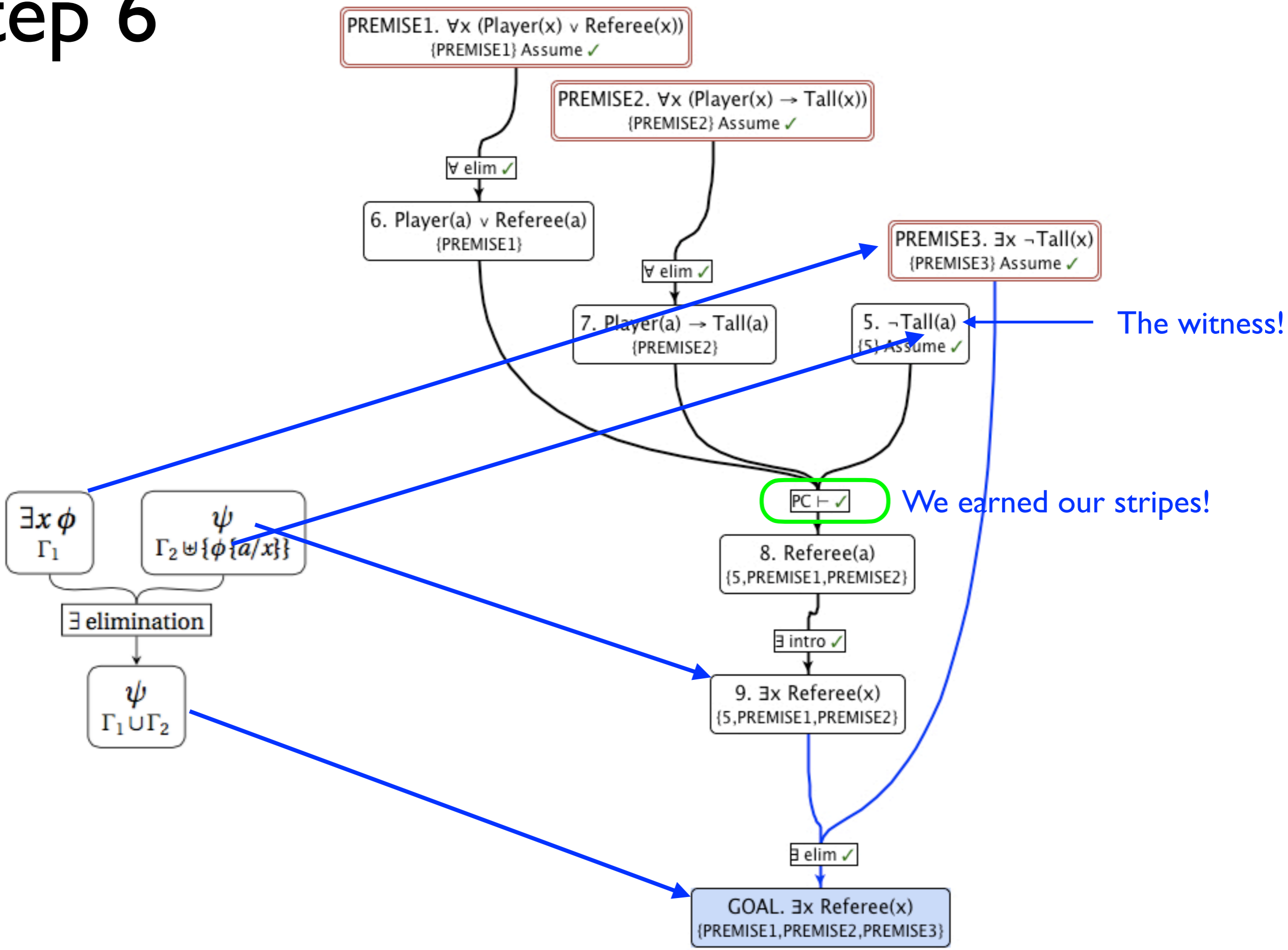
Step 6



Step 6



Step 6



$\{\forall x(\text{Scared}(x) \leftrightarrow \text{Small}(x)), \exists x \neg \text{Scared}(x)\} \vdash \exists x \neg \text{Small}(x)$

$\{\exists x, y \text{Contiguous}(x, y), \forall x, y(\text{Contiguous}(x, y) \rightarrow \neg \text{SameCountry}(x, y))\} \vdash \exists x, y \neg \text{SameCountry}(x, y)$

*Hvis du forstår det, kan
du bevise det.*