Exhortation; Truth Trees; FOL IV: Layered Quantification and Measuring Intelligence Using This Phenomenon

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Intro to (Formal) Logic
2/26/2024

RAIR
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Exhortation …

Plan to make sure you're up-to-date-ish before Spring Break on HyperGrader®'s current (**Required** = Homework) Problems, due

Apr 18 2024 11am NY time.

More FOL etc problems of course forthcoming …

New Pop Problem: `FregTHEN2`, with corresponding truth-tree Exercise

…

Truth Trees vs. Truth Tables

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Violent breakage between tabular calculation and proof construction.

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LAMA®'s hypergraphs/HyperLogic® achieves
seamless unification of proofs and trees, and provides
AI oracles for their construction and *certification*.

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First very simple: truth-tree for *modus ponens* …

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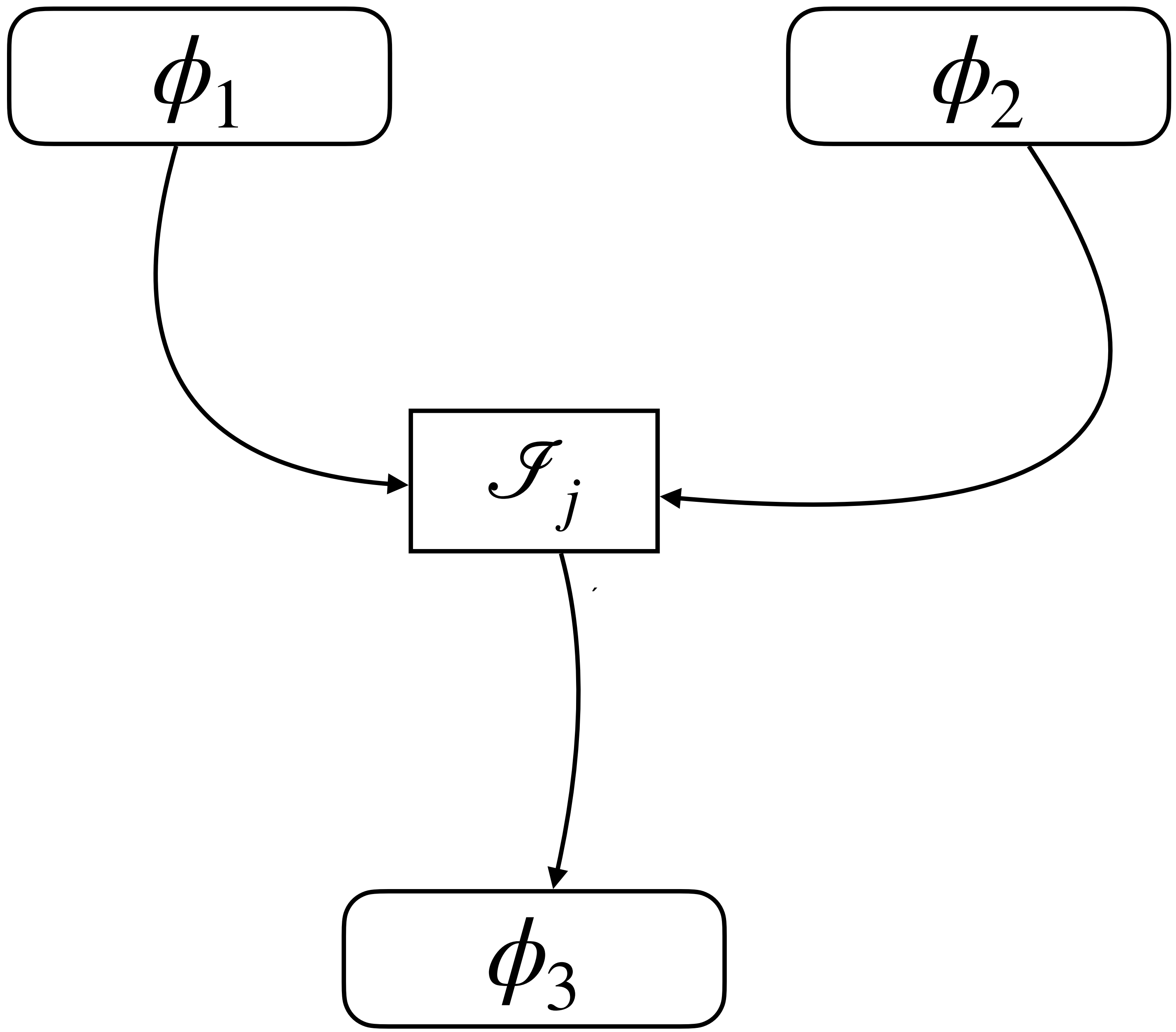
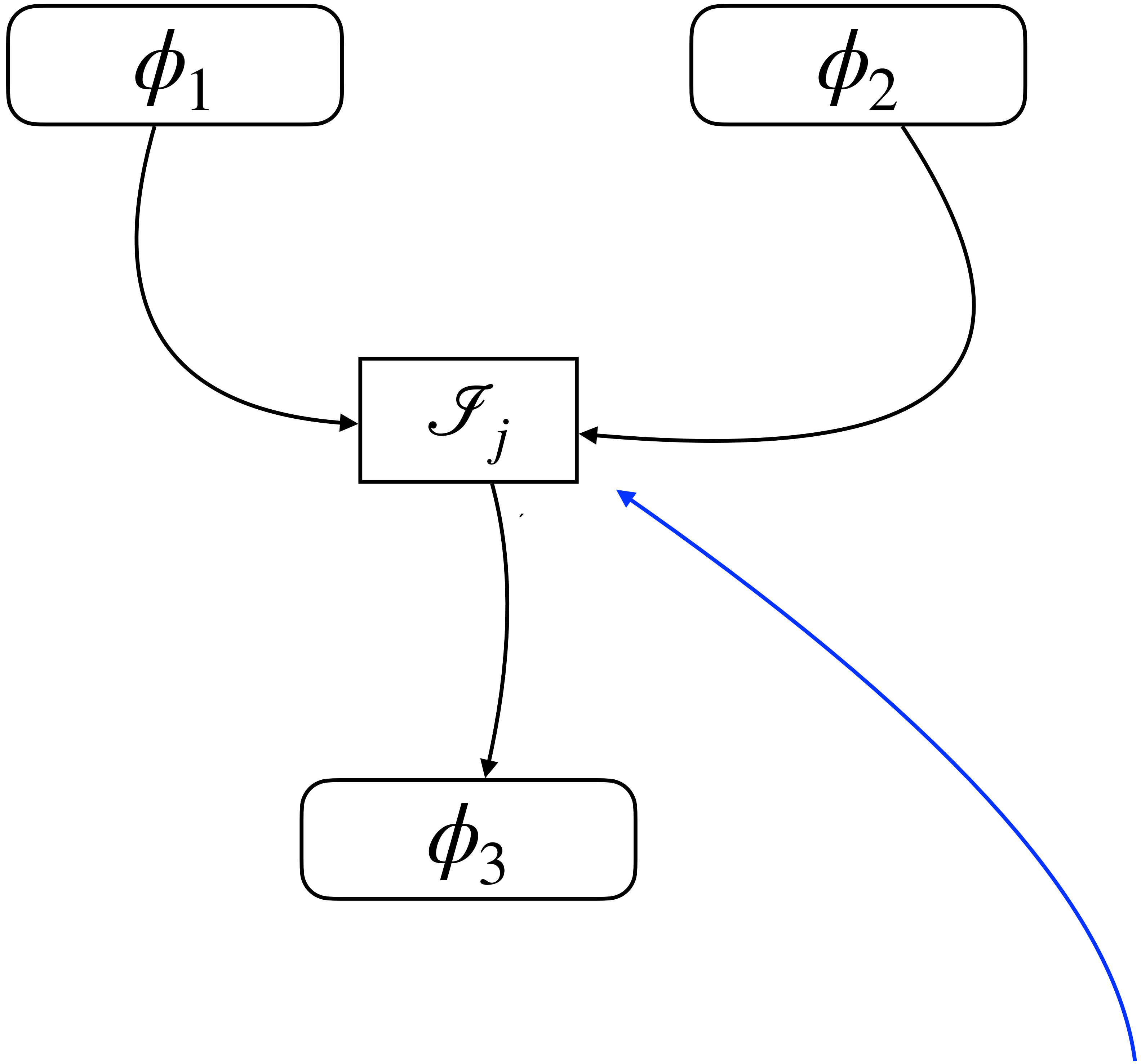
First very simple: truth-tree for *modus ponens* …



ϕ_1
ϕ_2
$\mathscr{I}_j$
ϕ_3



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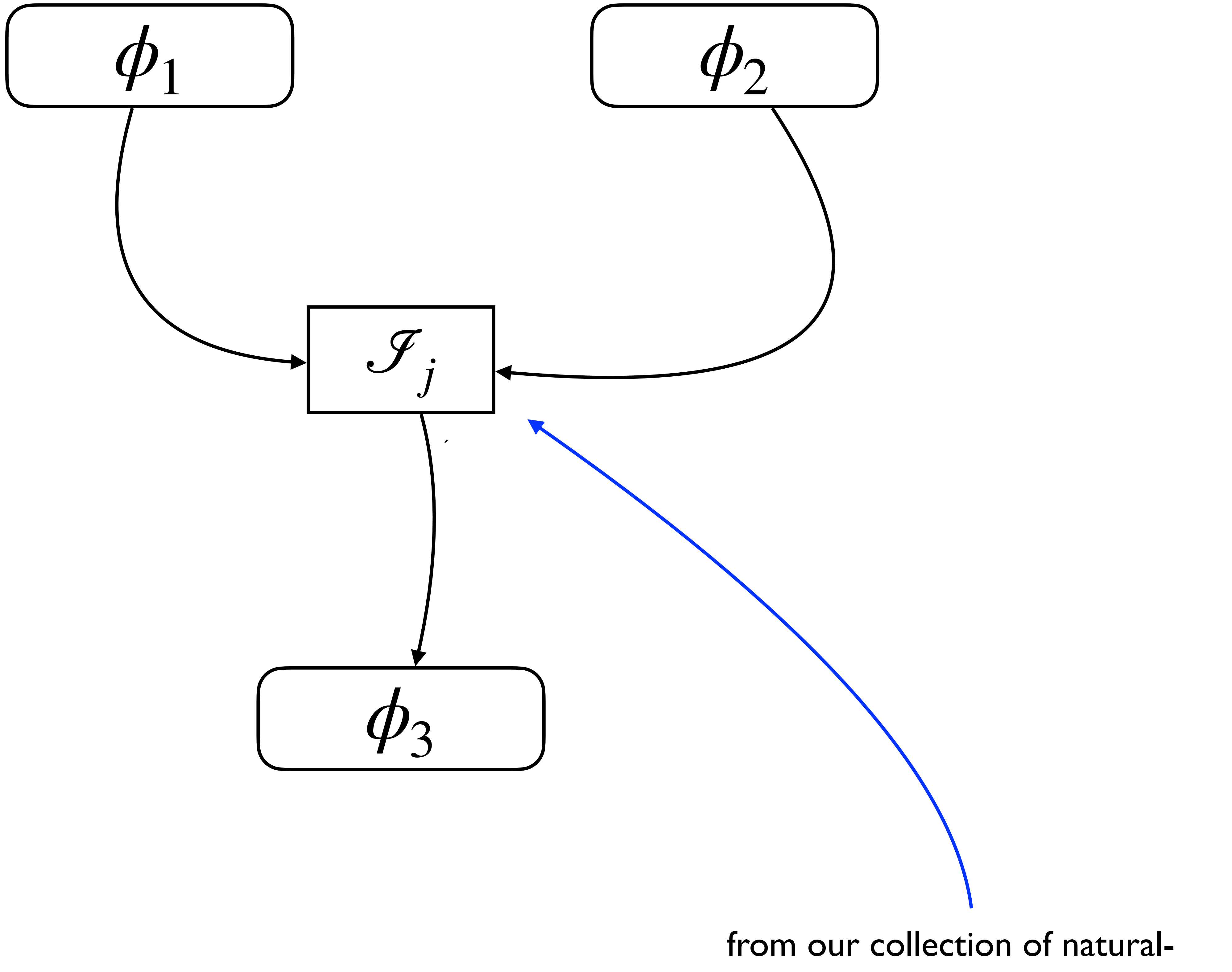
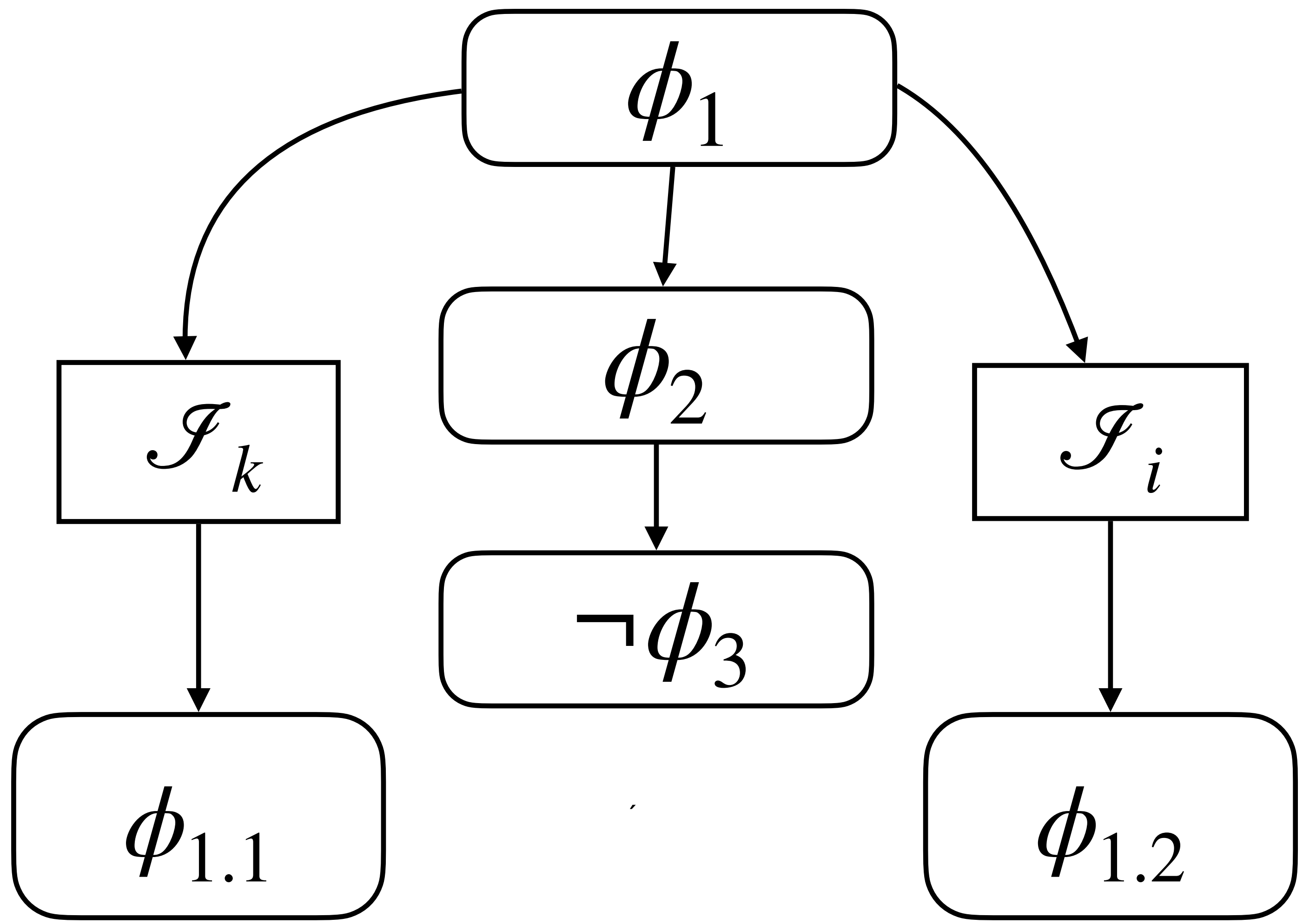

ϕ_1
ϕ_2
$\mathcal{I}_j$
ϕ_3
from our collection of natural-
deduction inference rules/schemata



ϕ_1
$\mathcal{I}_k$
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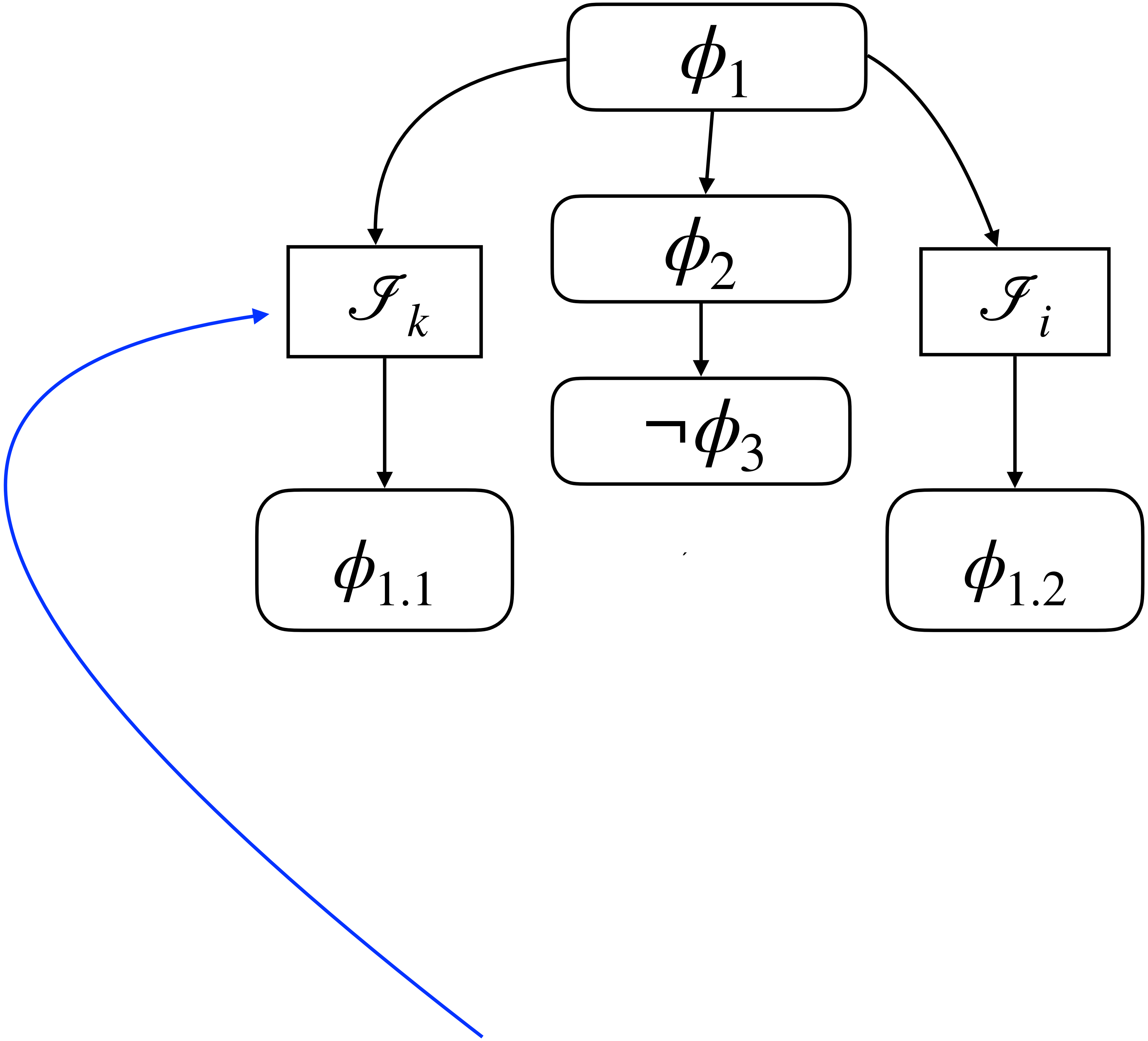
$\neg\phi_3$
$\phi_{1.1}$
$\phi_{1.2}$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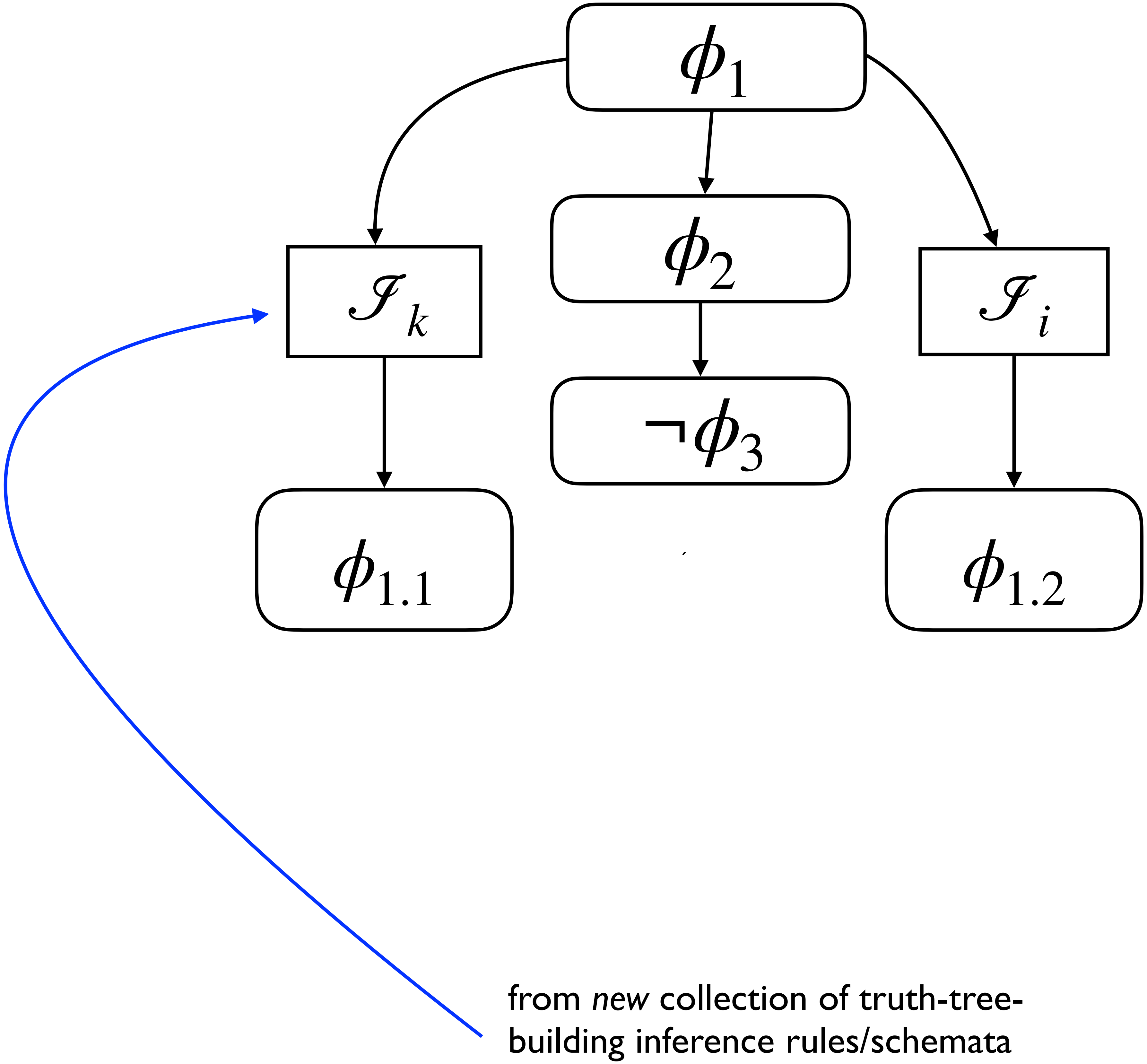

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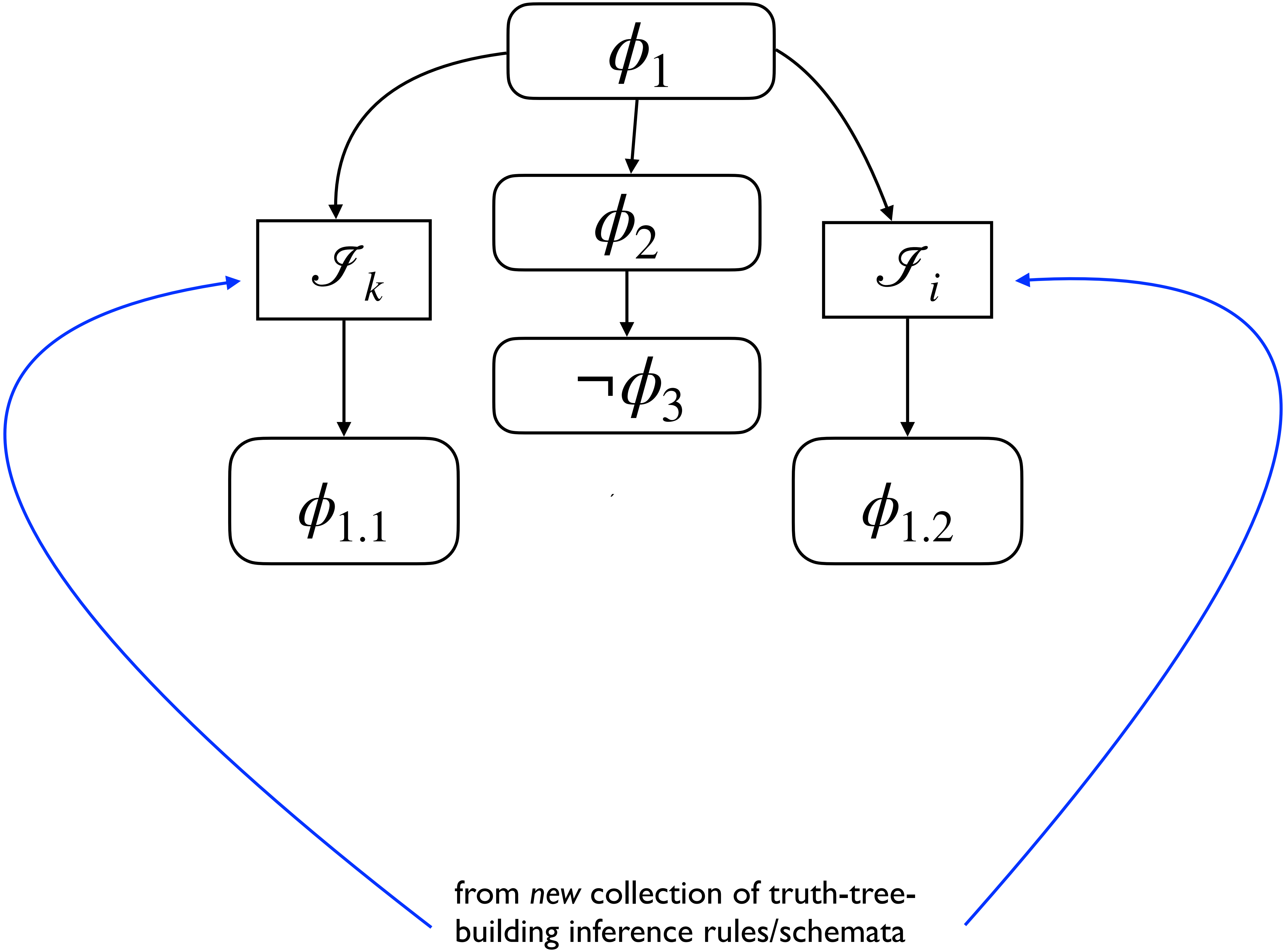
from *new* collection of truth-tree-
building inference rules/schemata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$\{P \rightarrow Q, P\} \vdash Q$

GIVEN1. $P \rightarrow Q$

PC $\vdash$ ✗

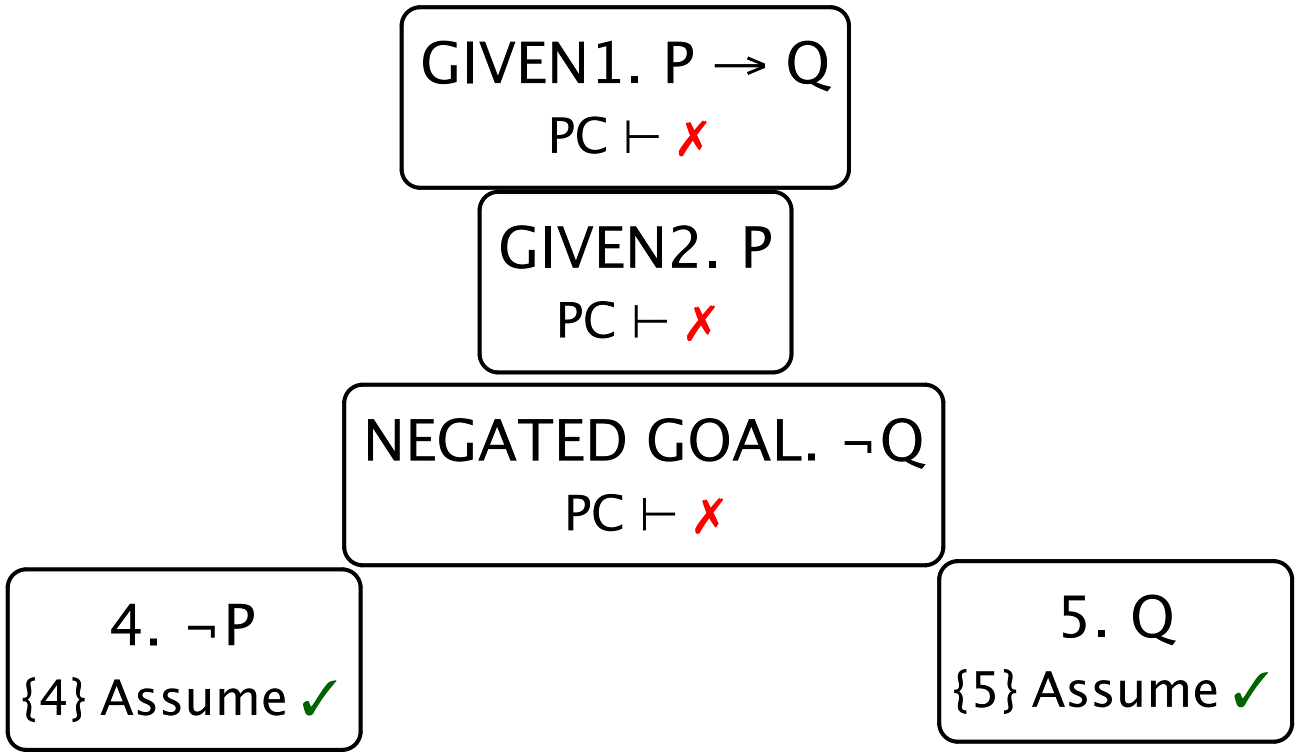
GIVEN2. P

PC $\vdash$ ✗

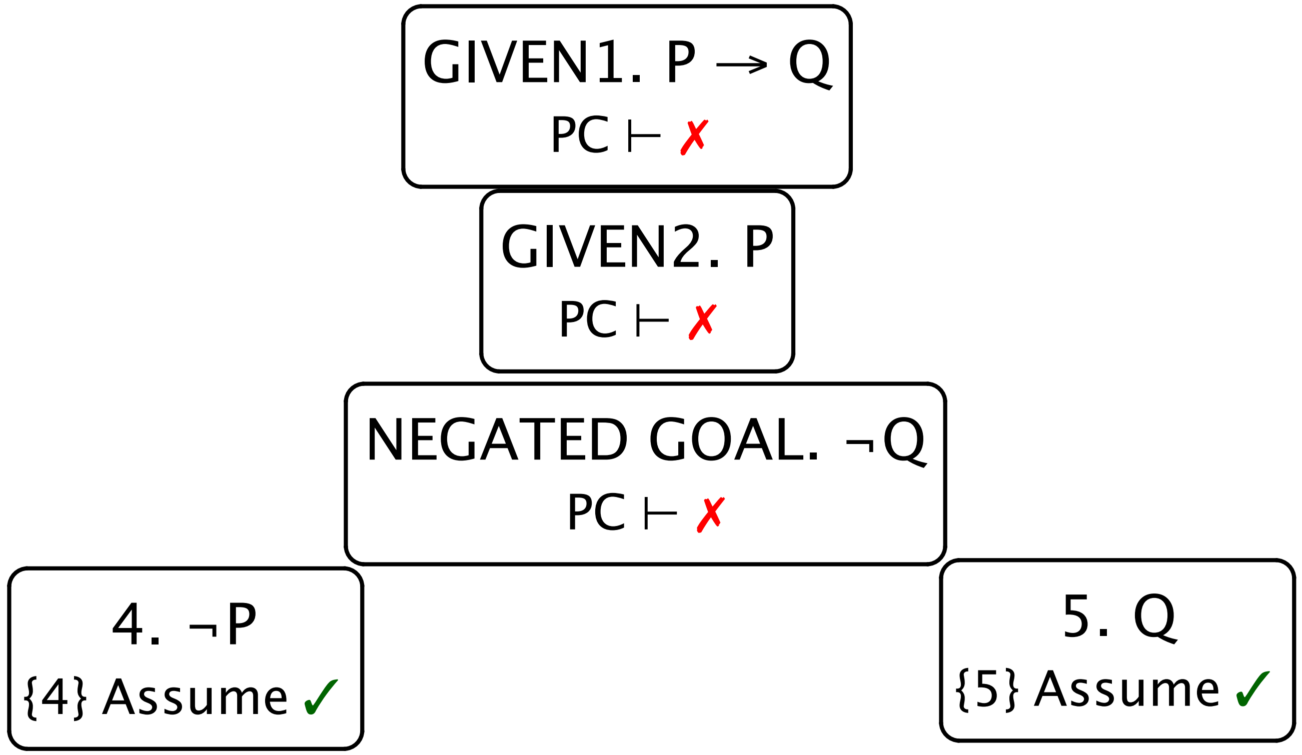
NEGATED GOAL. $\neg Q$

PC $\vdash$ ✗

$\{P \rightarrow Q, P\} \vdash Q$



$\{P \to Q, P\} \vdash Q$



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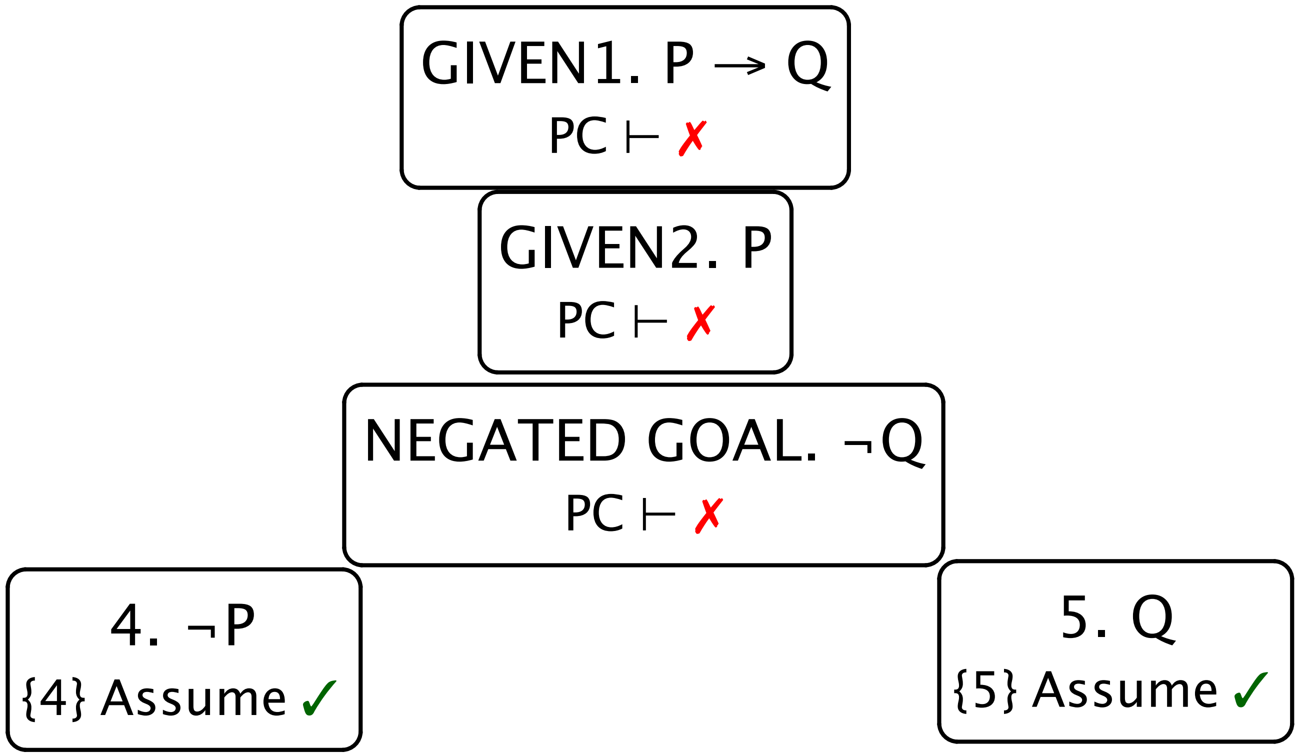


Either way, a contradiction!

$\{P \to Q, P\} \vdash Q$

ζ

GIVEN1. $P \to Q$
PC $\vdash$ ✗

GIVEN2. P
PC $\vdash$ ✗

NEGATED GOAL. $\neg Q$
PC $\vdash$ ✗

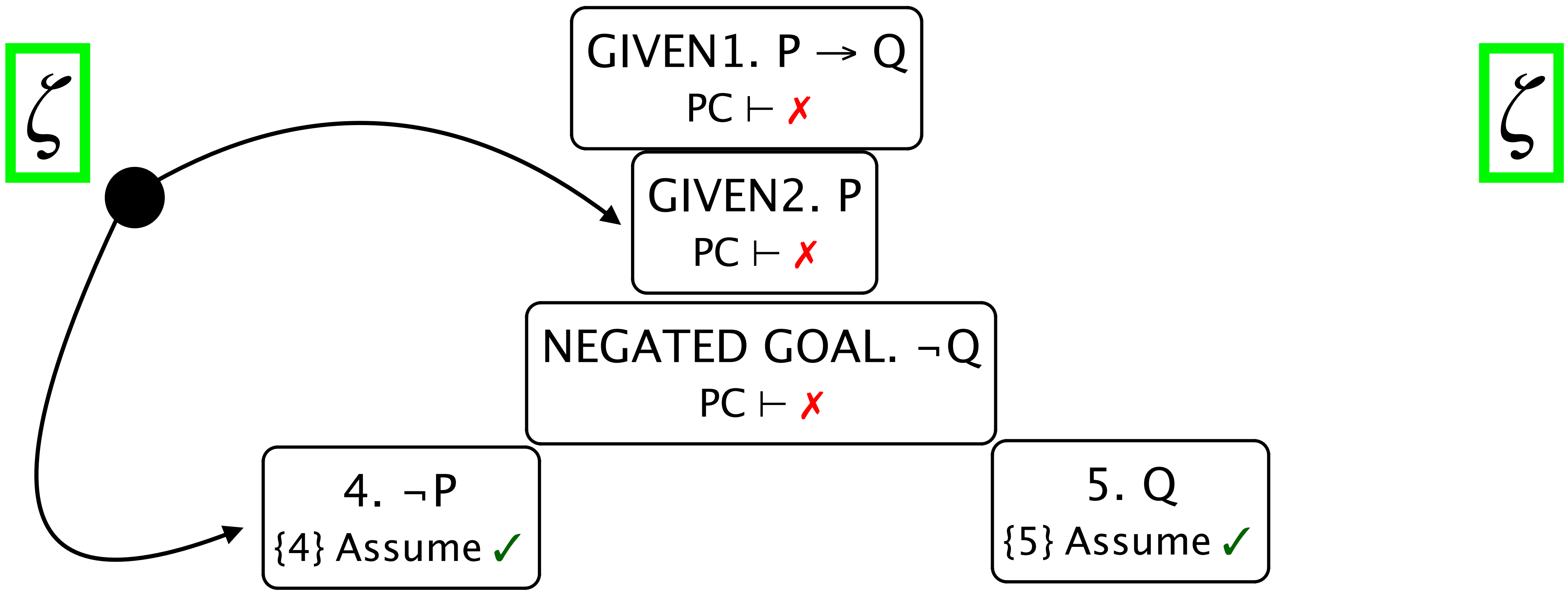
4. $\neg P$
{4} Assume ✓

5. Q
{5} Assume ✓

ζ

Either way, a contradiction!

$\{P \rightarrow Q, P\} \vdash Q$



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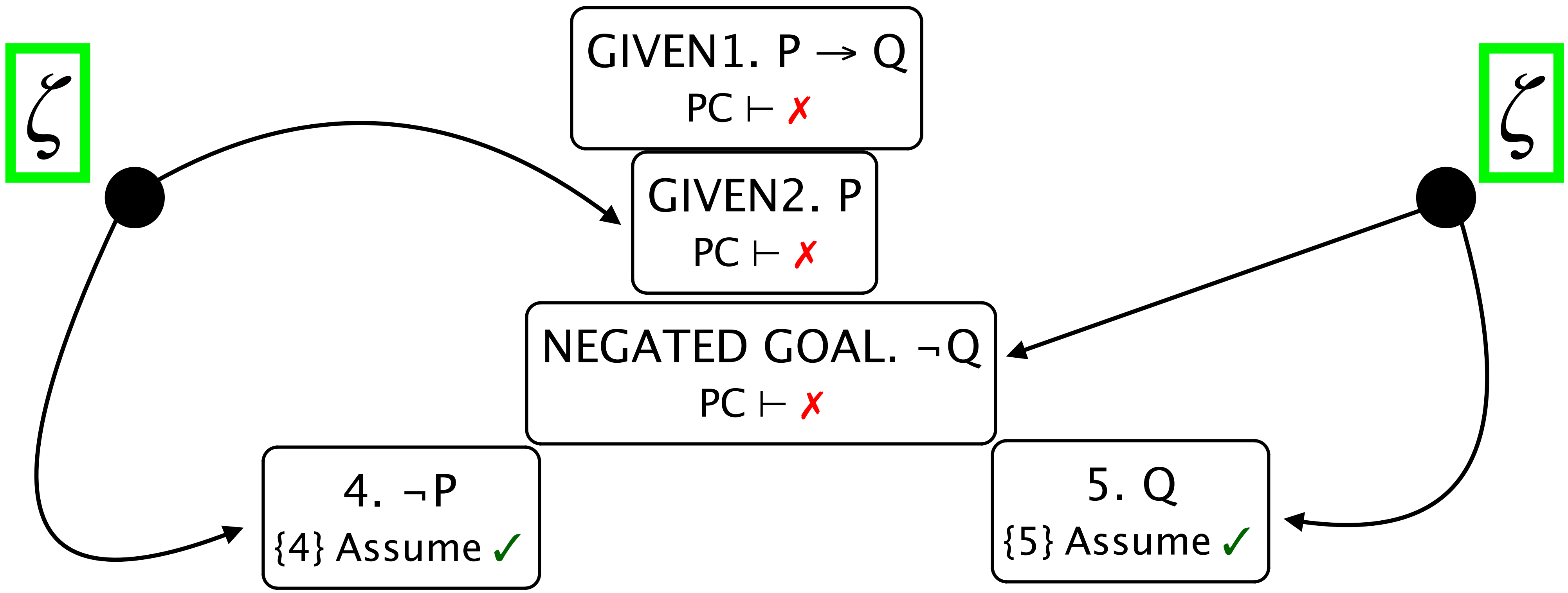

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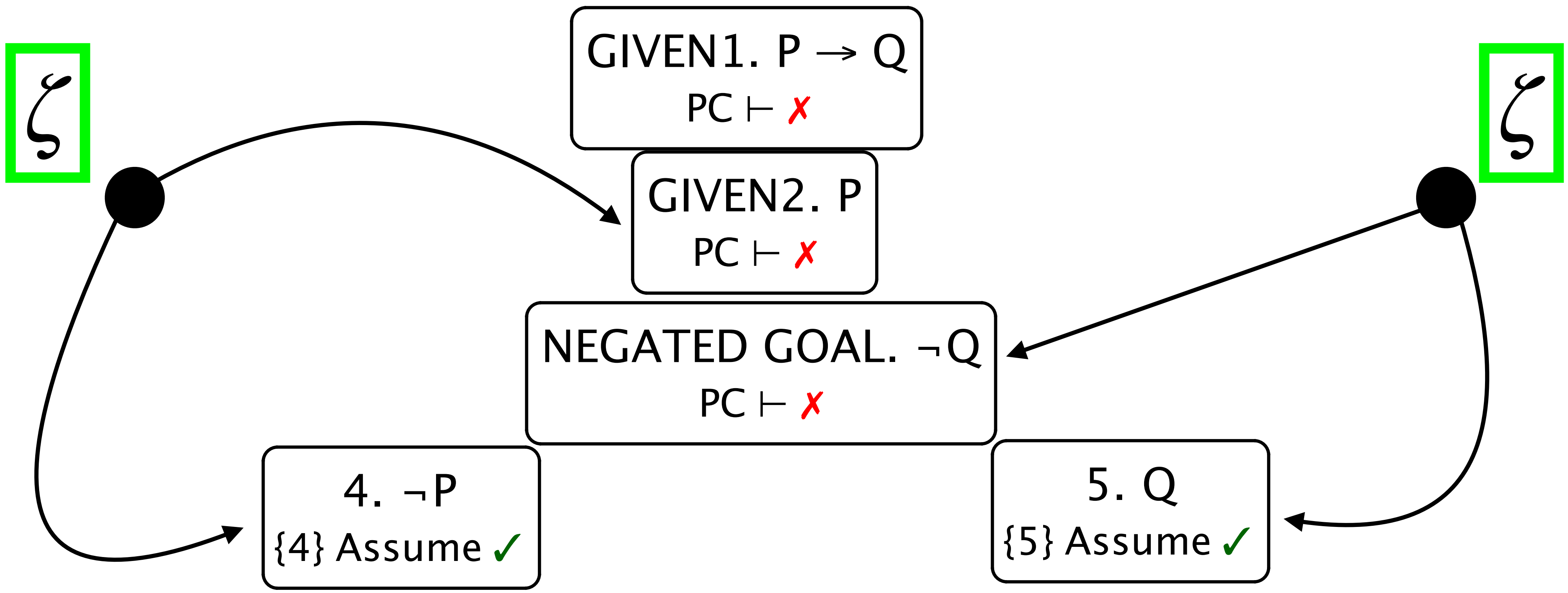

Either way, a contradiction!

Therefore the entailment holds!

Slightly Harder Truth Tree

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$$\vdash (\mathtt{P} \rightarrow (\mathtt{Q} \rightarrow \mathtt{R})) \rightarrow ((\mathtt{P} \rightarrow \mathtt{Q}) \rightarrow (\mathtt{P} \rightarrow \mathtt{R}))$$

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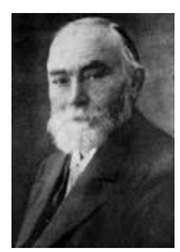
$$\vdash (\mathtt{P} \rightarrow (\mathtt{Q} \rightarrow \mathtt{R})) \rightarrow ((\mathtt{P} \rightarrow \mathtt{Q}) \rightarrow (\mathtt{P} \rightarrow \mathtt{R}))$$

(This is the axiom THEN-2 in Frege's (brutal) axiomatization of the propositional calculus.)



Frege

Slightly Harder Truth Tree

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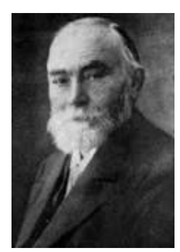

Frege

https://en.wikipedia.org/wiki/Frege%27s_propositional_calculus

Slightly Harder Truth Tree

$$\vdash (\mathtt{P} \to (\mathtt{Q} \to \mathtt{R})) \to ((\mathtt{P} \to \mathtt{Q}) \to (\mathtt{P} \to \mathtt{R}))$$

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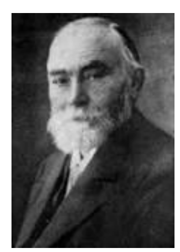

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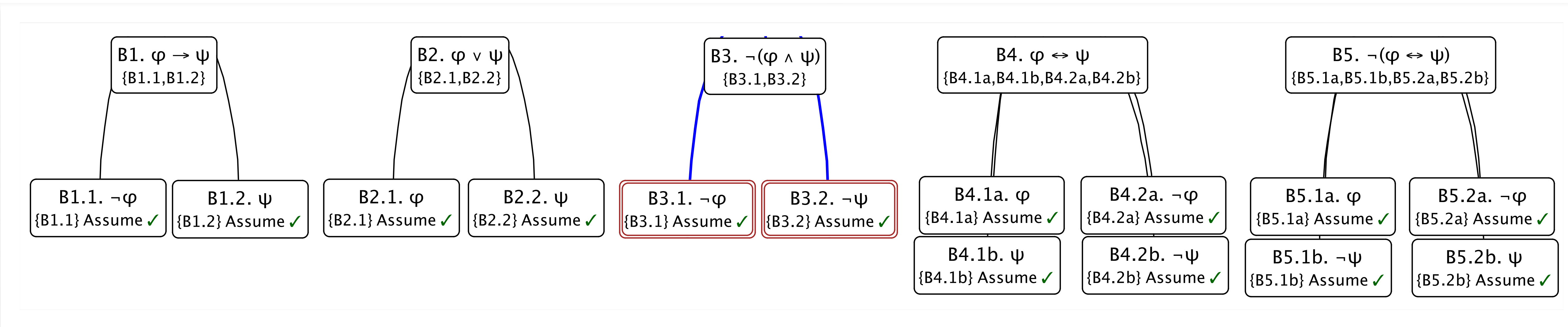

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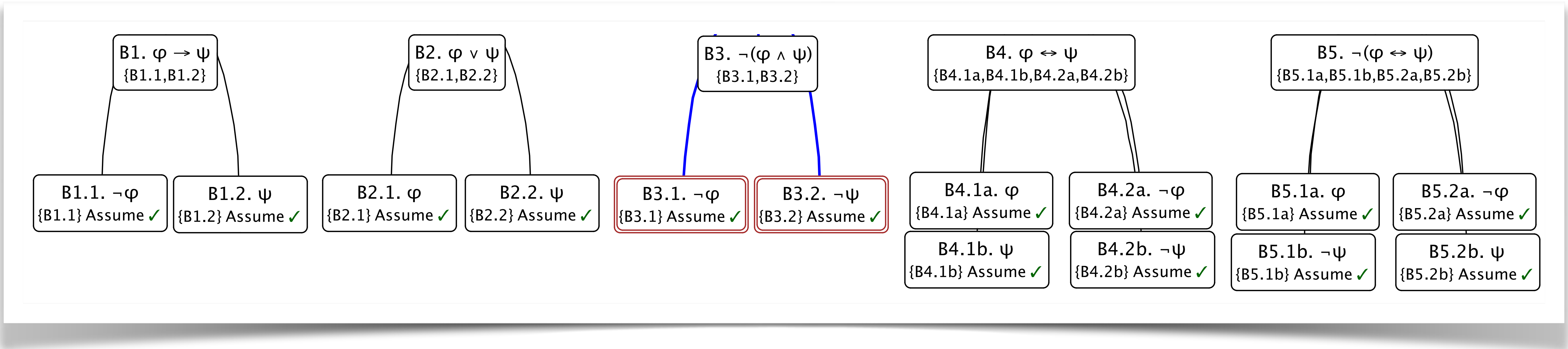
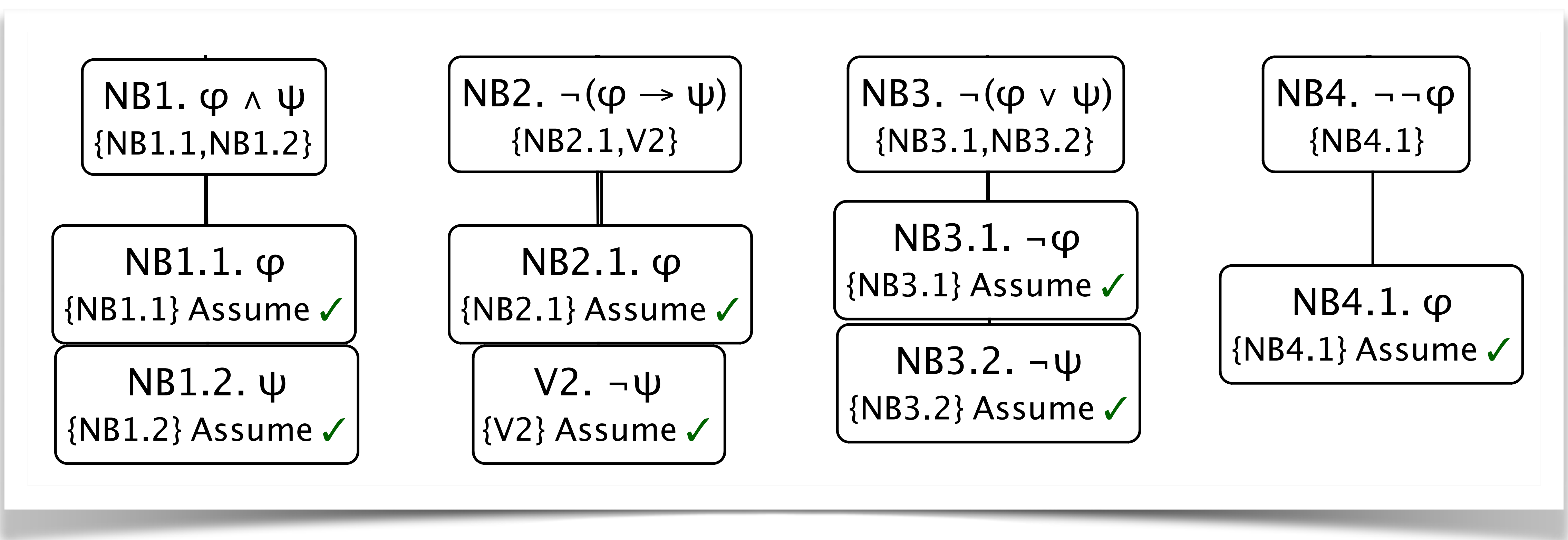




The Rules of the Game!

The Rules of the Game!

Questions?

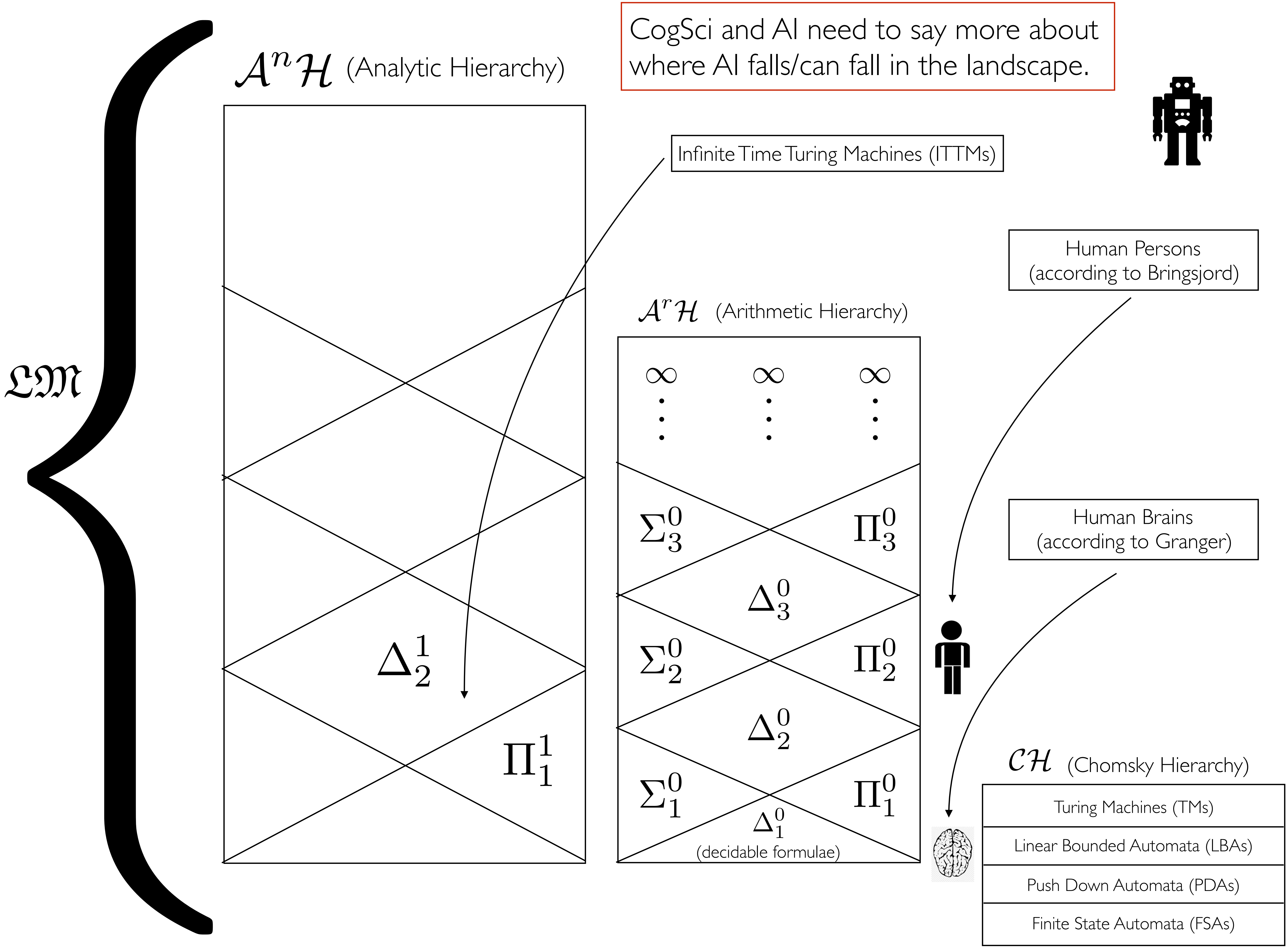
Theorem:

Let ϕ be a theorem in the propositional calculus $= \mathcal{L}_{\text{PC}}$.
Then the truth-tree algorithm will lead to no open branches.

On Measuring The Intelligence of Agents

…

using quantification …



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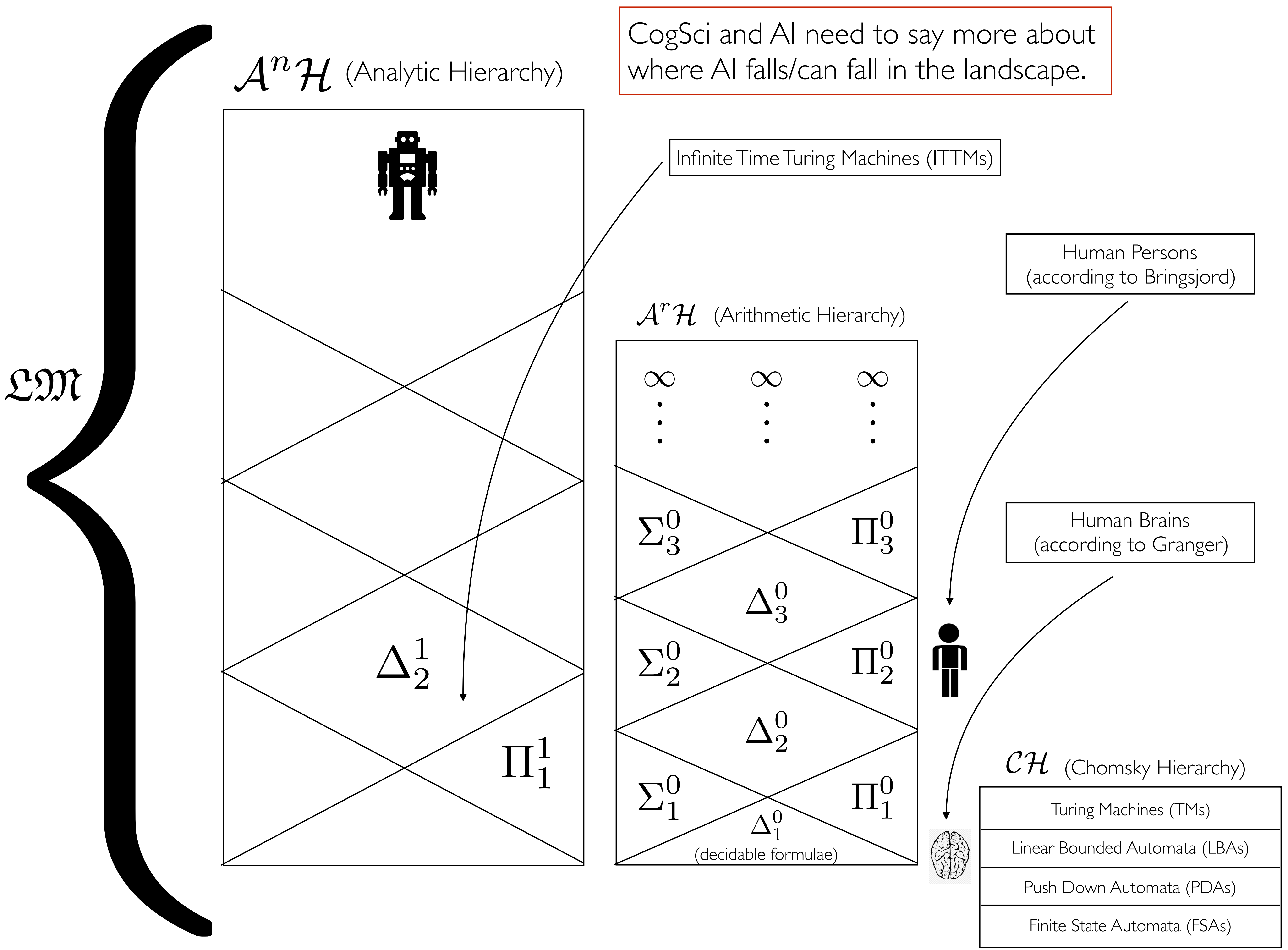

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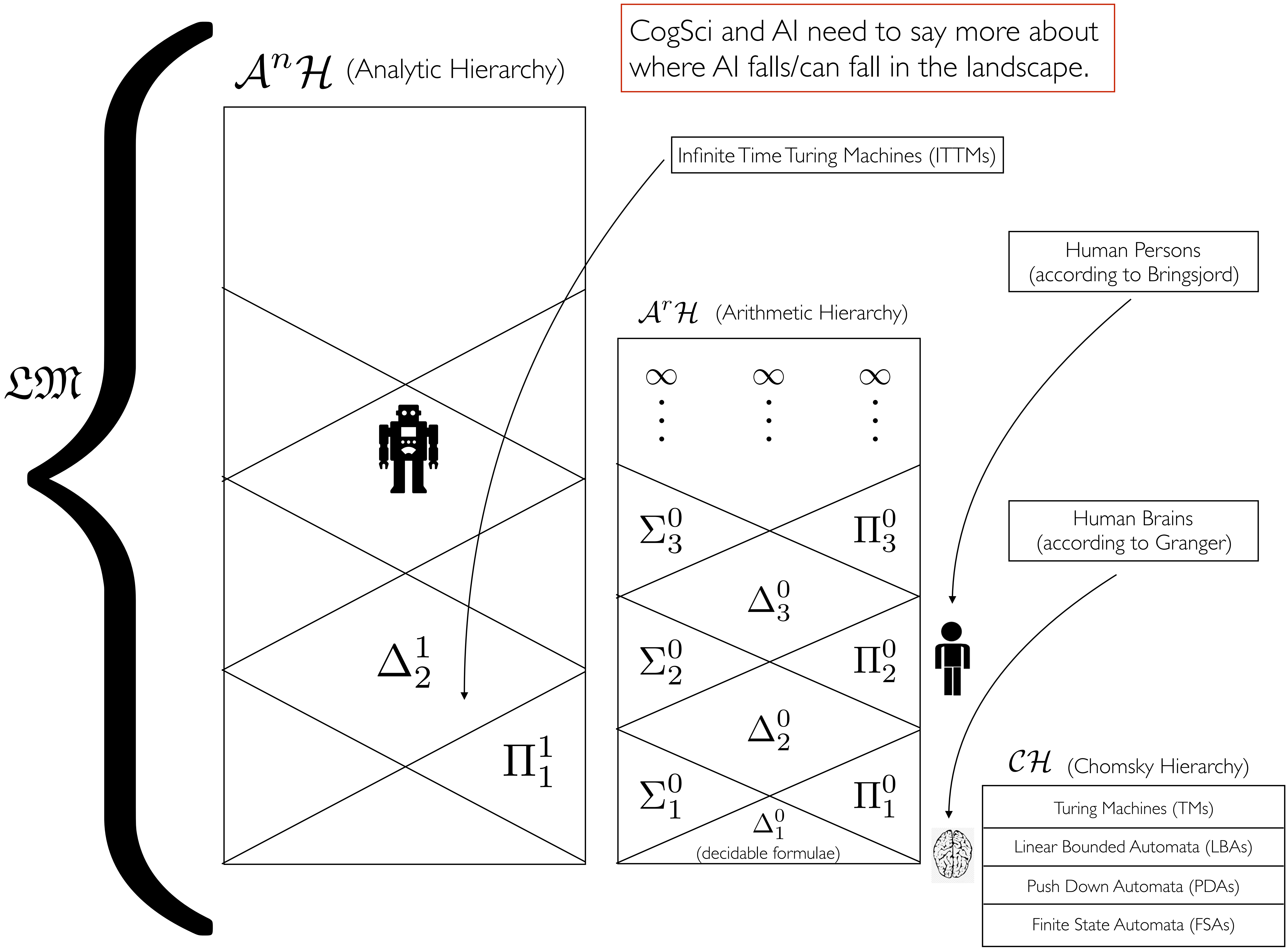

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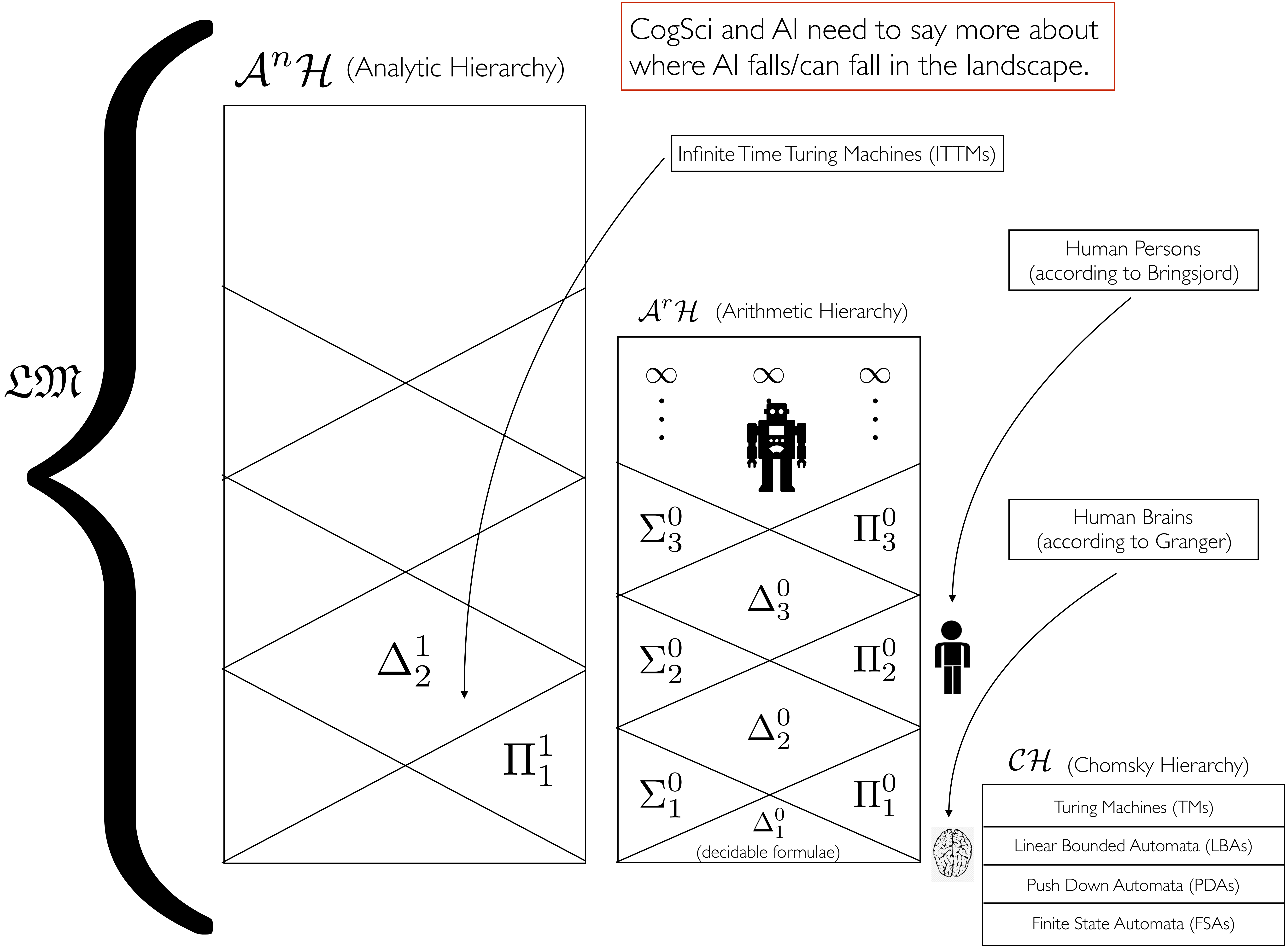

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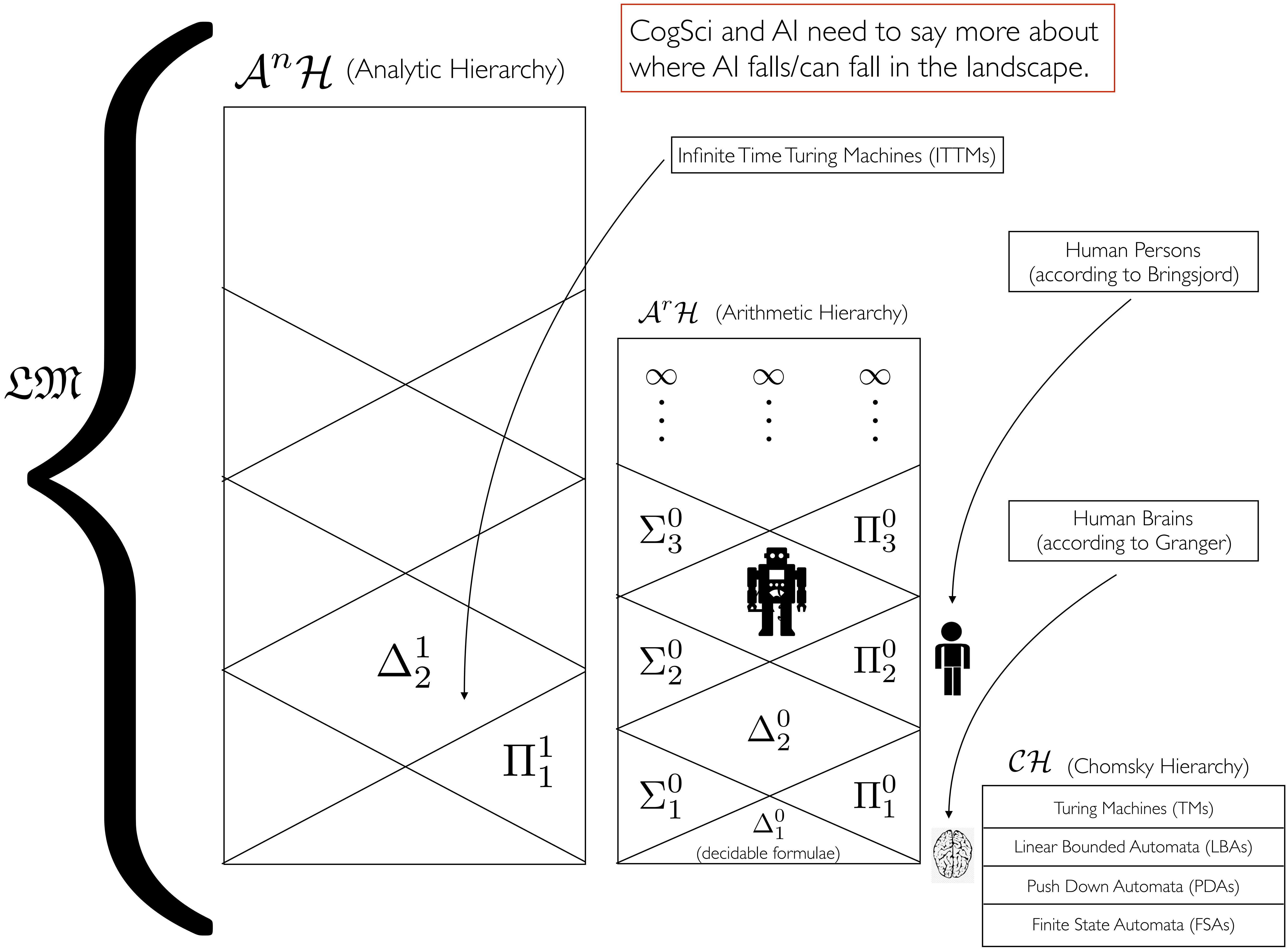

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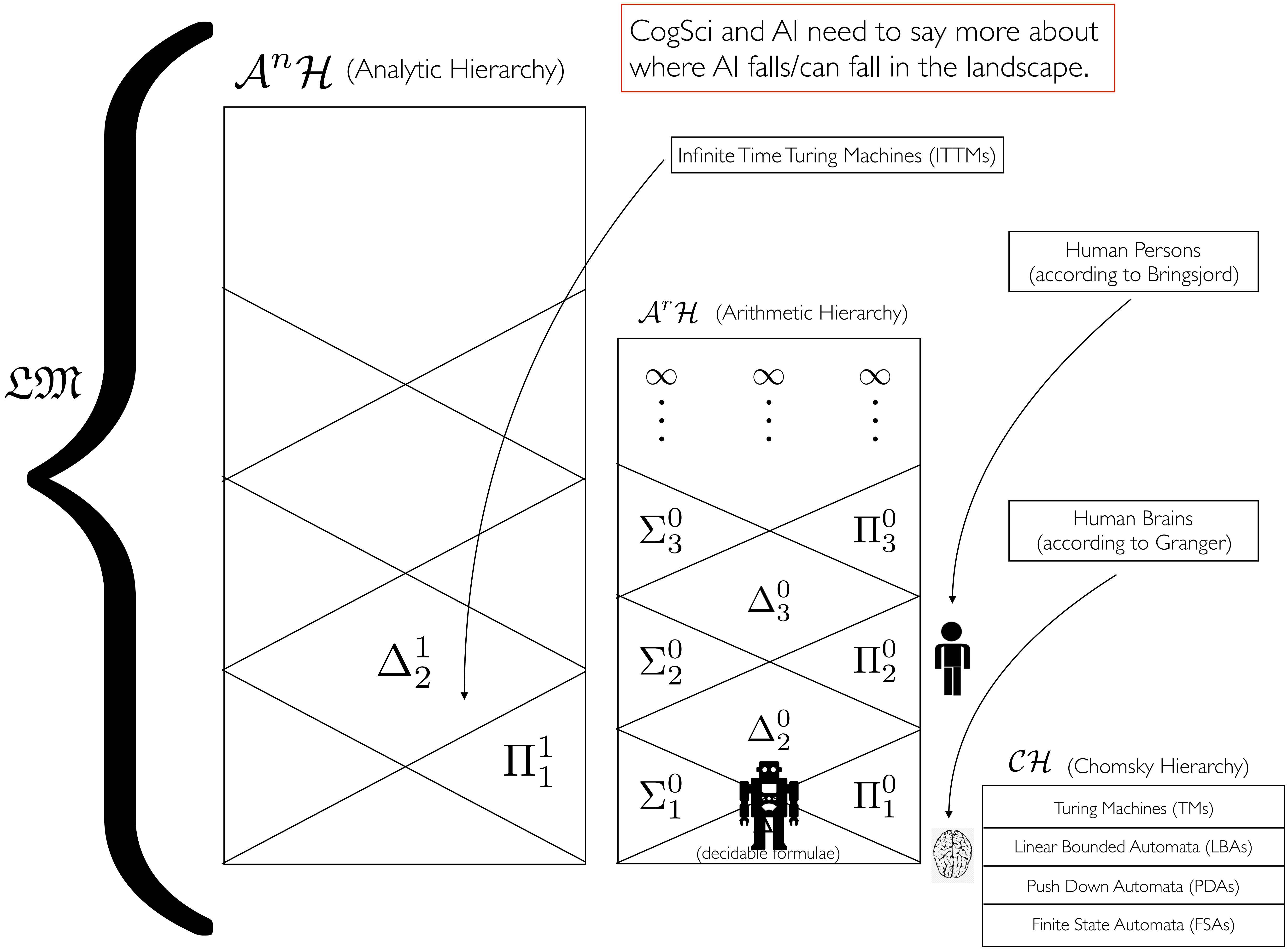
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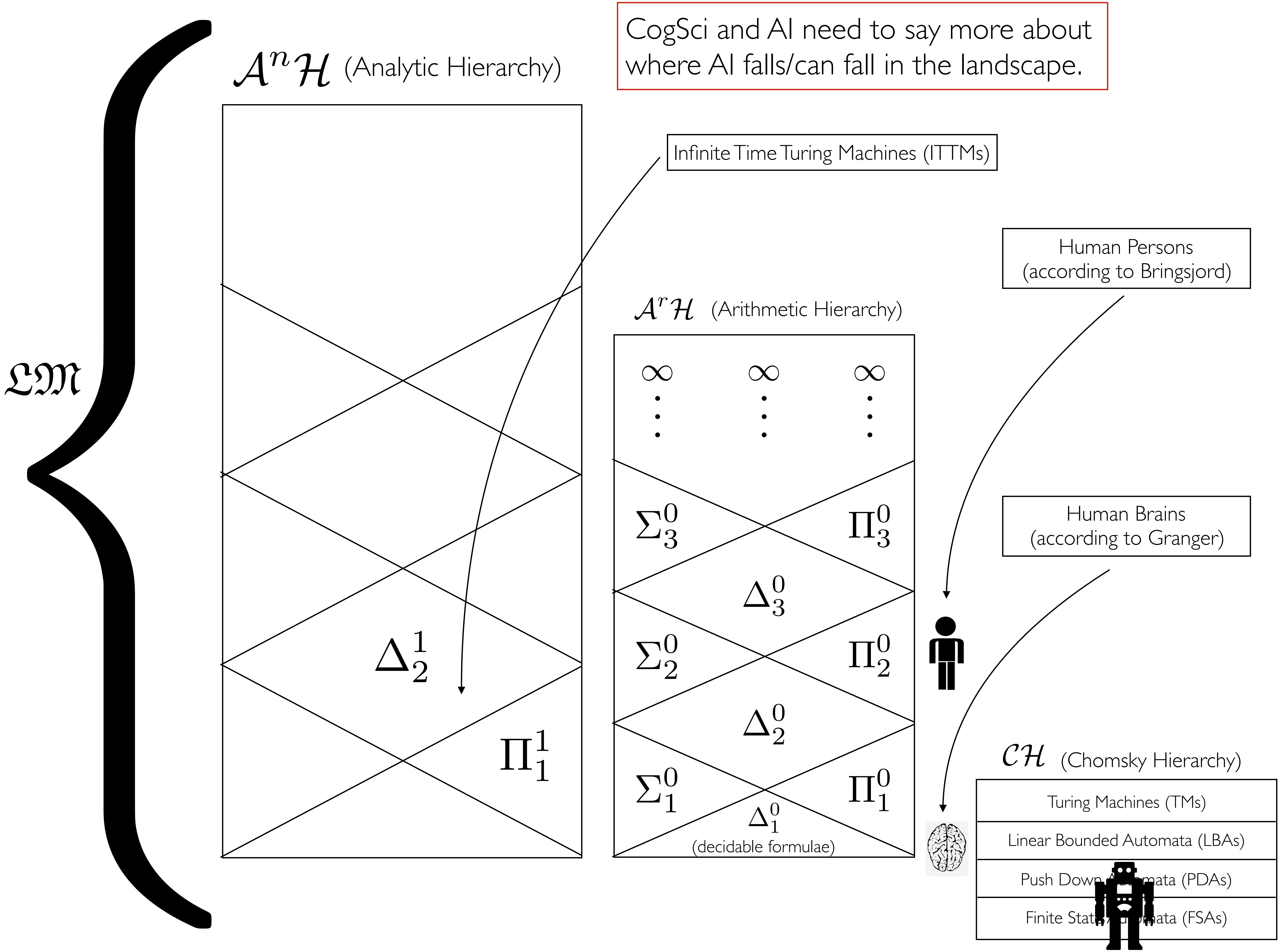
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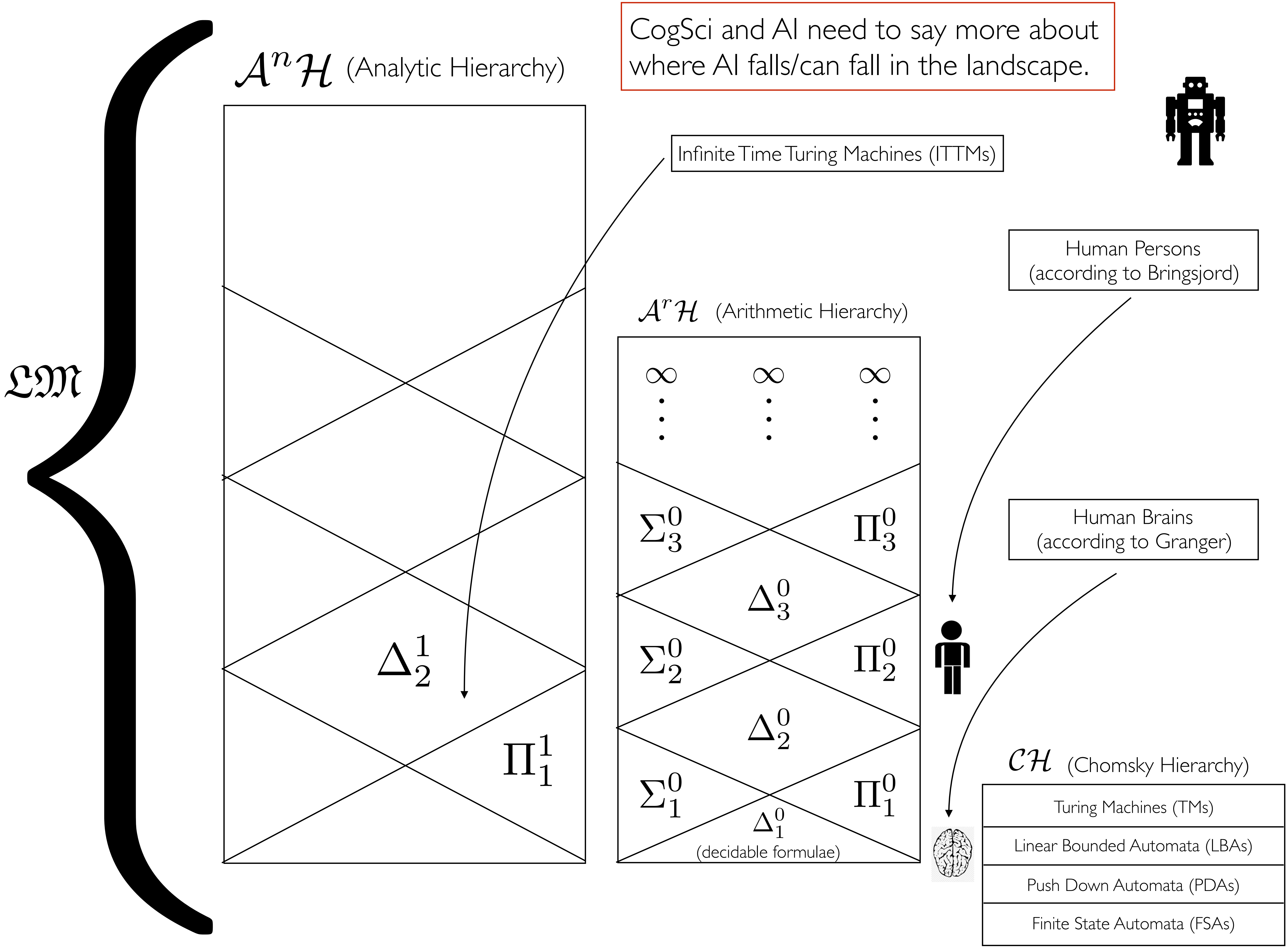

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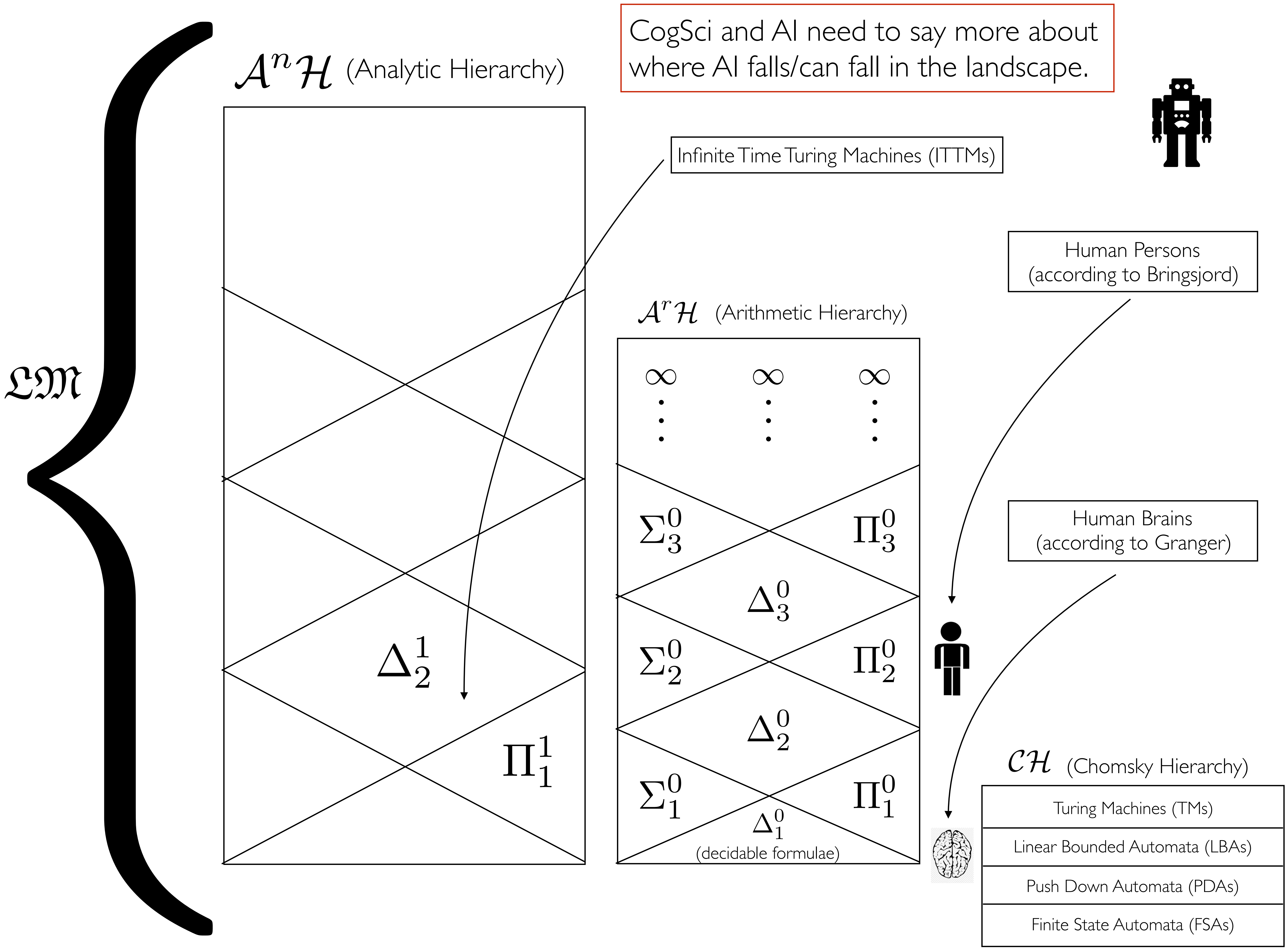
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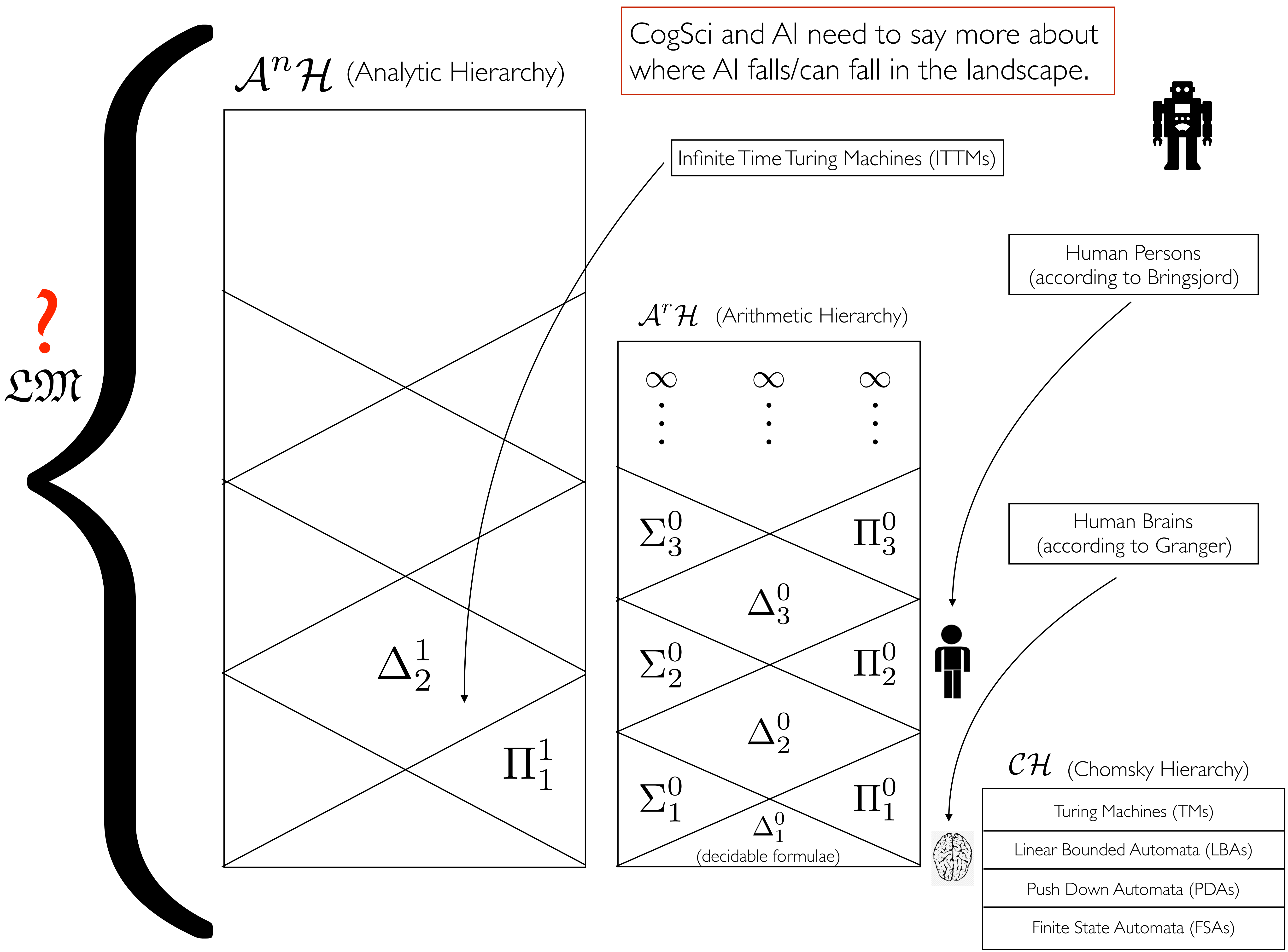
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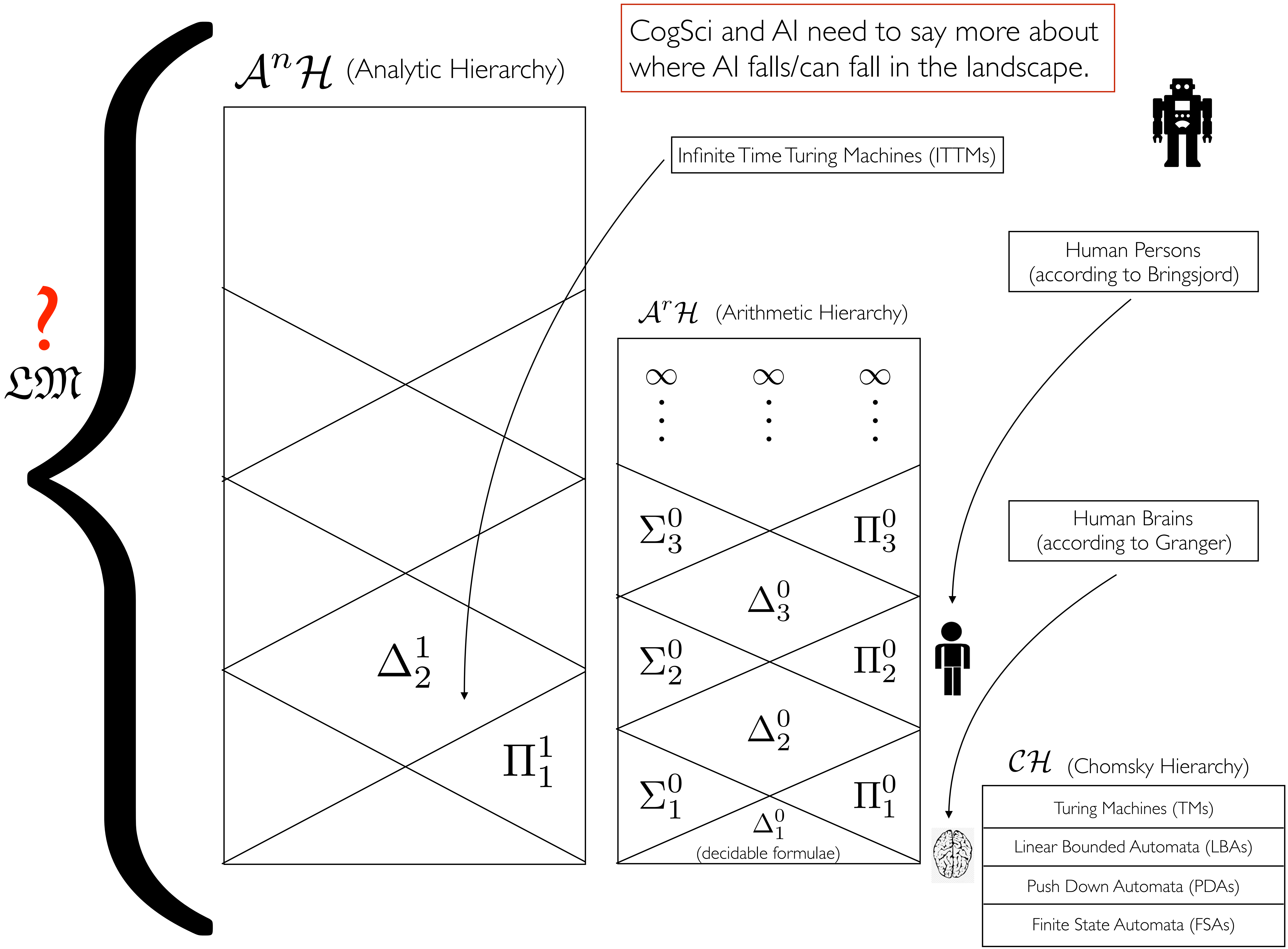

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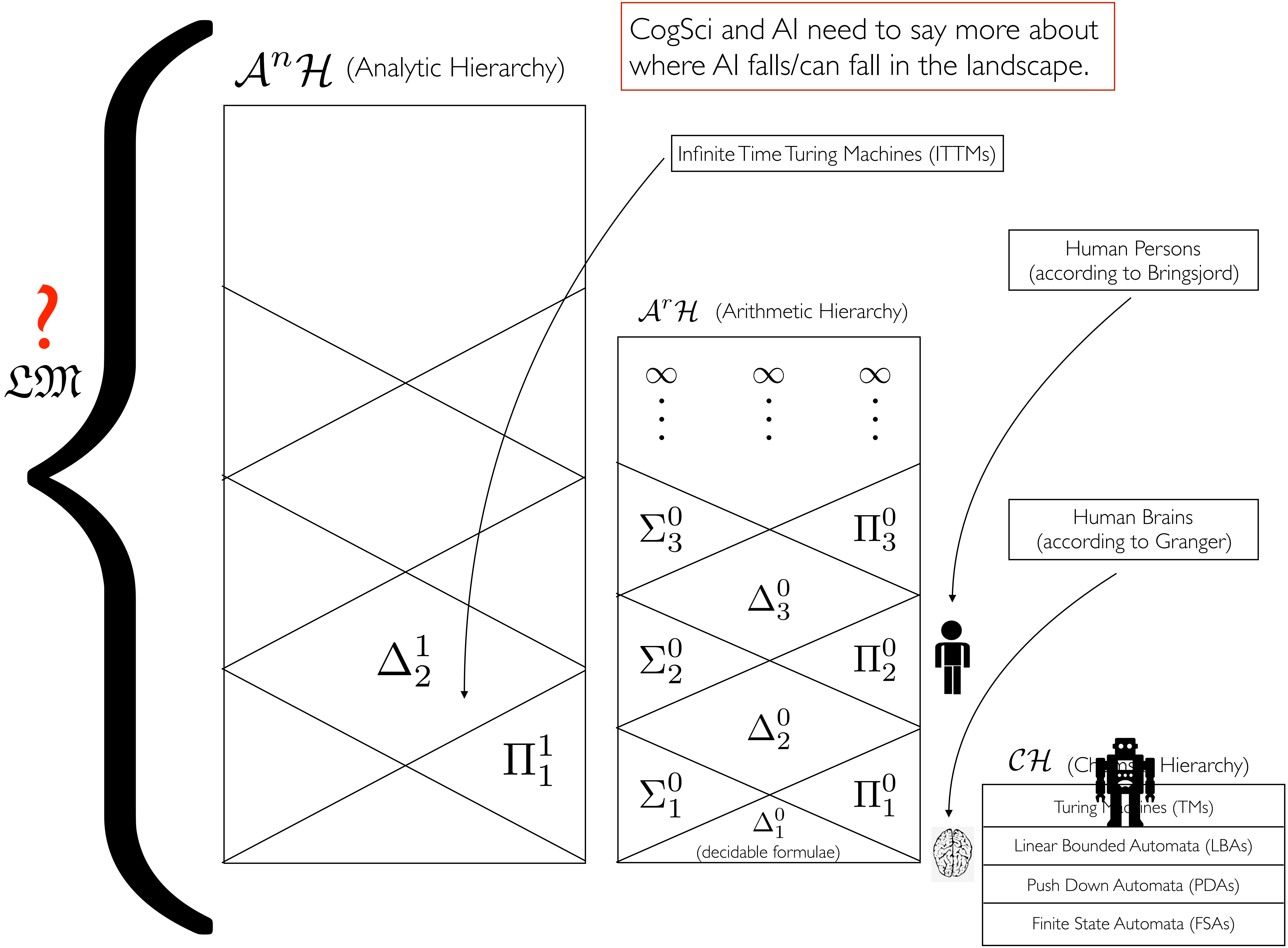
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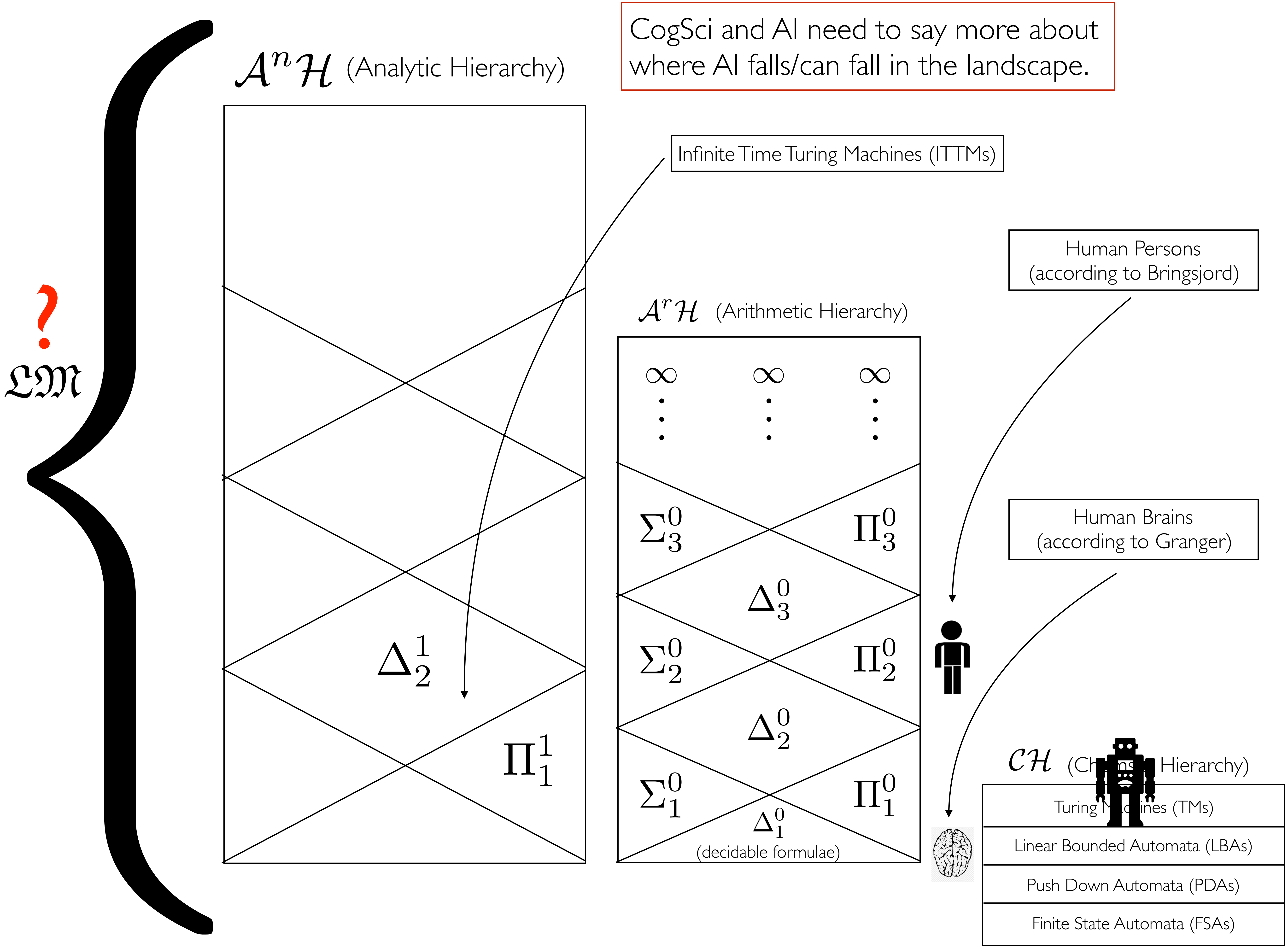
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Π^1_1
Infinite Time Turing Machines (ITTMs)
$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)
∞ ∞ ∞
Σ^0_3
Π^0_3
Δ^0_3
Σ^0_2
Π^0_2
Δ^0_2
Σ^0_1
Π^0_1
Δ^0_1
(decidable formulae)
Human Persons (according to Bringsjord)
Human Brains (according to Granger)
$\mathcal{CH}$ Hierarchy)
(TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)

Measuring Intelligence & AI/The Singularity

Measuring Intelligence & AI/The Singularity

The Singularity (superhuman machine intelligence) is near!!

Measuring Intelligence & AI/The Singularity

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Is that so? And how are you measuring intelligence, pray tell?

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Polynomial Hierarchy

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Measuring Intelligence & AI/The Singularity

Checkers:Chinook

Polynomial Hierarchy

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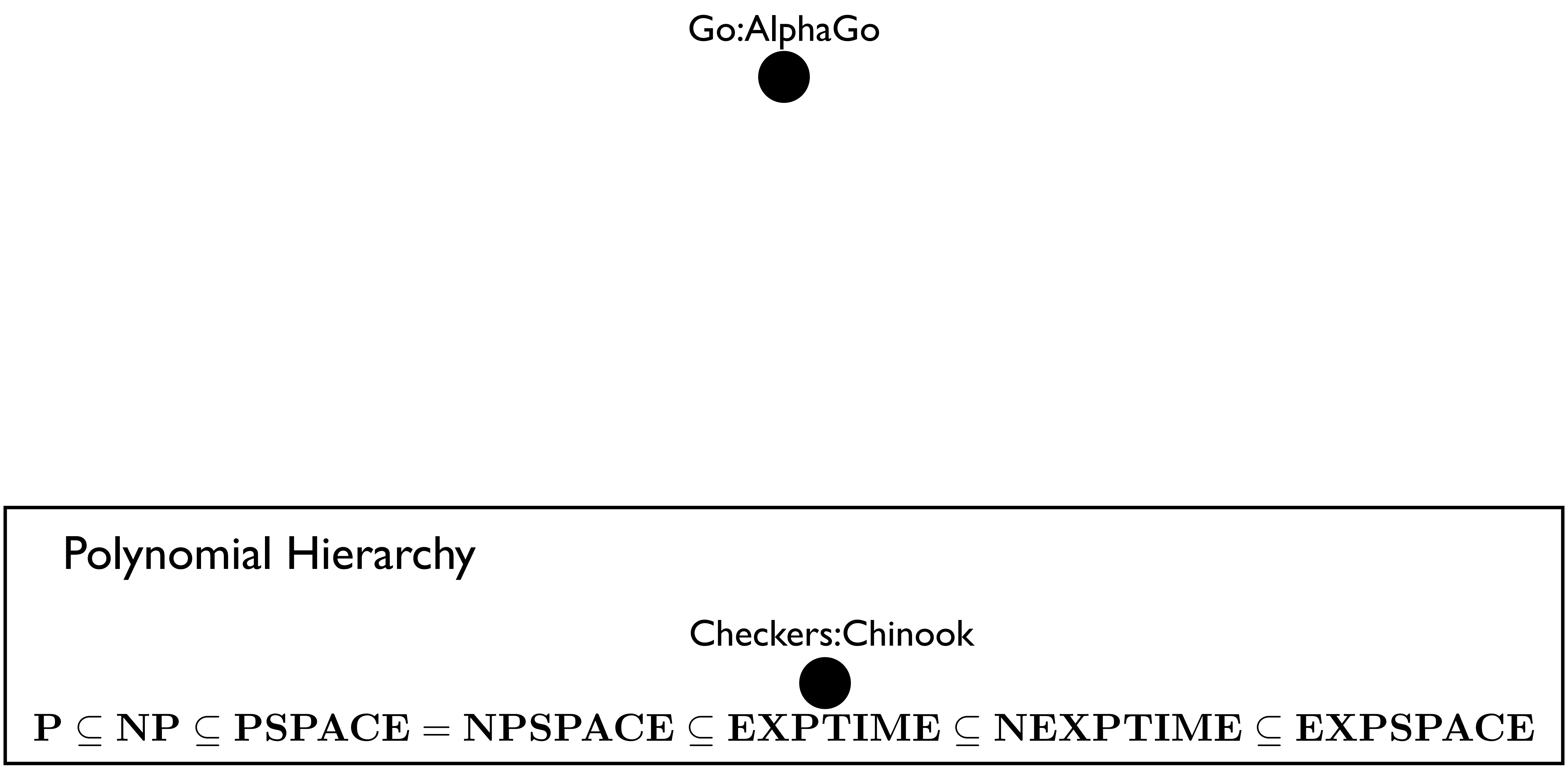
Measuring Intelligence & AI/The Singularity



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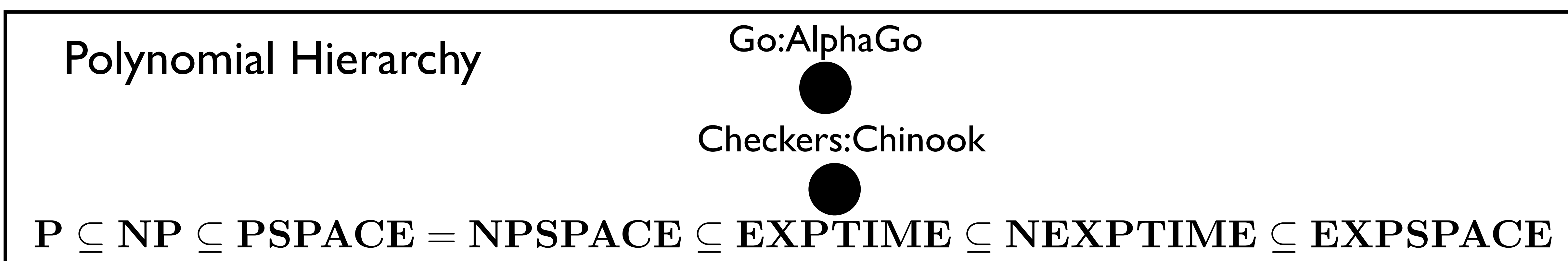

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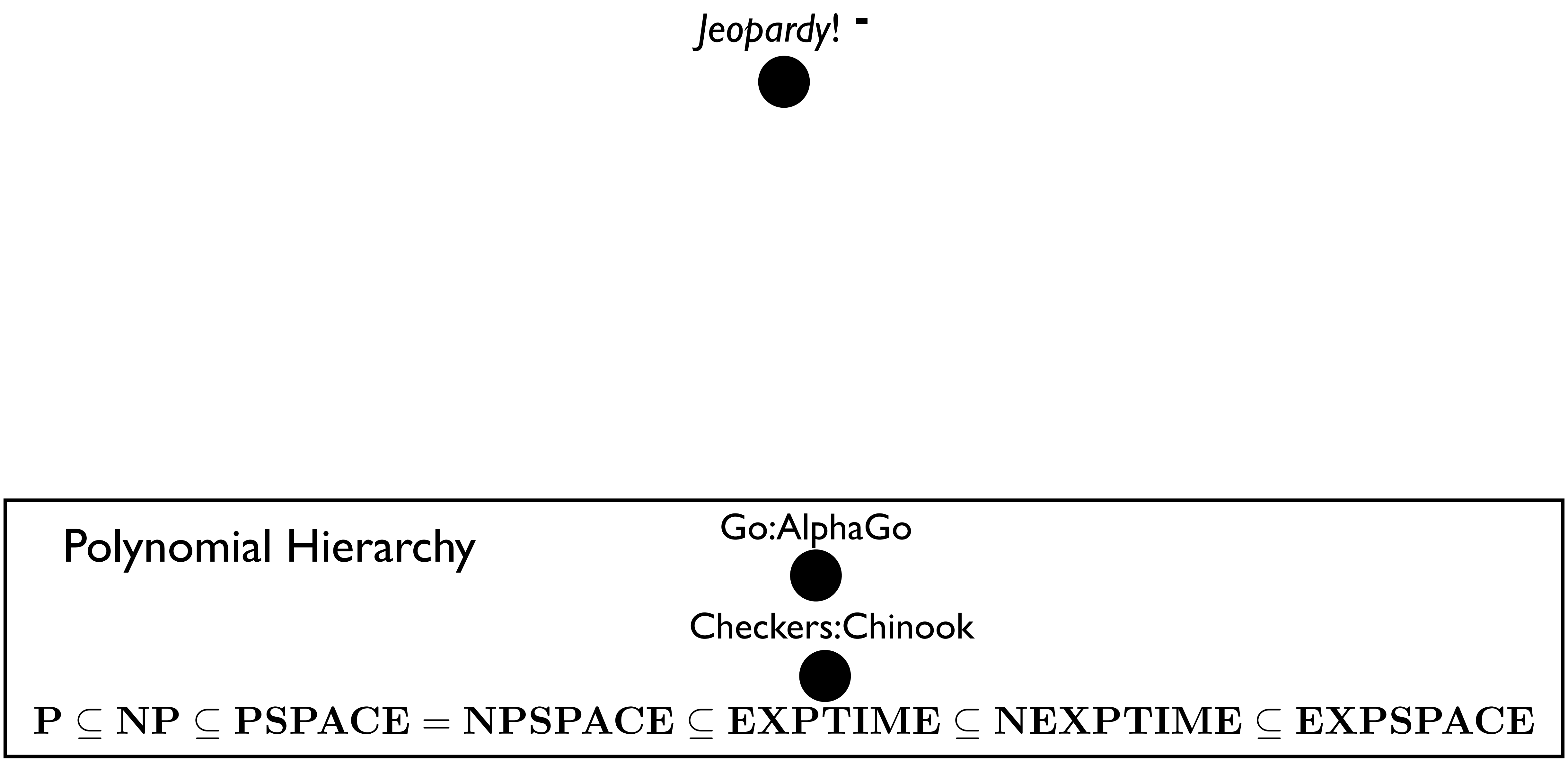

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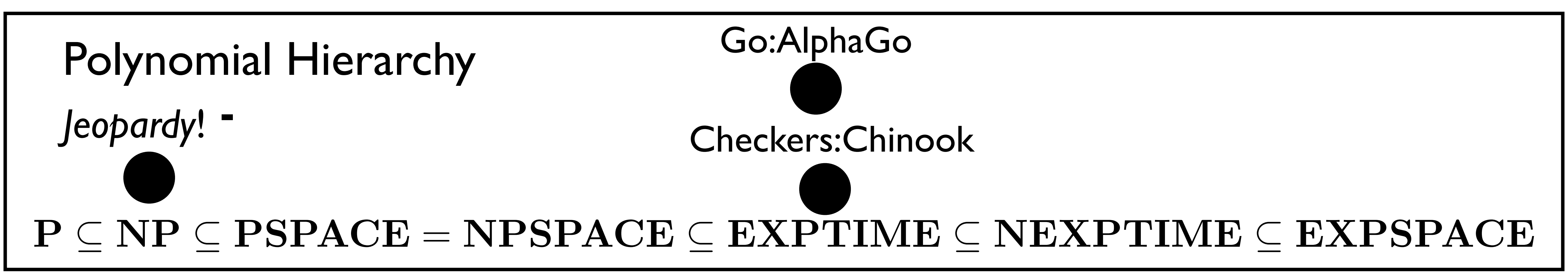

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

Go:AlphaGo

Jeopardy! -

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

Go:AlphaGo

Jeopardy! -

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$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Measuring Intelligence & AI/The Singularity

 

Polynomial Hierarchy

Jeopardy! -

Go:AlphaGo

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy

Polynomial Hierarchy

Jeopardy! -

Go:AlphaGo

Checkers:Chinook

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Polynomial Hierarchy

Go:AlphaGo

Jeopardy!

Checkers:Chinook

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy

$\vdots$

Π_2

"Hey, do these two Java programs compute the very same function?"

Σ_2

Π_1

Σ_1

Σ_0

Polynomial Hierarchy

Go:AlphaGo

Jeopardy! -

Checkers:Chinook

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Measuring Intelligence & AI/The Singularity

Analytical Hierarchy

Arithmetical Hierarchy

$\vdots$

"Hey, do these two Java programs compute the very same function?" Π_2

Σ_2

Π_1


Σ_1

Σ_0

Polynomial Hierarchy

Go:AlphaGo

Jeopardy! -

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Analytical Hierarchy

Arithmetical Hierarchy

This, all of this, is derived from consideration of first-order logic and second-order logic, with an emphasis on *quantification* and proof.

$\vdots$

"Hey, do these two Java programs compute the very same function?"

Π_2

Σ_2

Π_1

Σ_1

Σ_0


Polynomial Hierarchy

Go:AlphaGo

Jeopardy!

Checkers:Chinook

$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$

An “Advanced” Topic for Measuring Intelligence …

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- FOL formulae that (only) enforce domain size:

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$$\exists x \exists y (x \neq y)$$

An "Advanced" Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

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An "Advanced" Topic for Measuring Intelligence …

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$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

An "Advanced" Topic for Measuring Intelligence …

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$$\exists x \exists y (x \neq y) \text{ at least two things}$$

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$$\vdots$$

An "Advanced" Topic for Measuring Intelligence ...

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$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

$$\vdots$$

$$\underline{\phi_n}$$

An "Advanced" Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

$$\exists x \exists y (x \neq y) \quad \text{at least two things}$$

$$\exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z) \quad \text{at least three things}$$

$$\vdots$$

$$\underline{\phi_n} \qquad \text{domain of at least } n \text{ things}$$

An "Advanced" Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

$$\exists x \exists y (x \neq y)$$ at least two things

$$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$$ at least three things

$$\vdots$$

$$\underline{\phi_n}$$ domain of at least *n* things

$$\exists x \forall y (y = x)$$

An "Advanced" Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

$$\exists x \exists y (x \neq y)$$ at least two things

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$\vdots$

$\underline{\phi_n}$ domain of at least *n* things

$\exists x \forall y (y = x)$ at most one thing

$\exists x \exists y \forall z (z = x \vee z = y)$

An "Advanced" Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

$$\exists x \exists y (x \neq y)$$ at least two things

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$$\vdots$$

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An "Advanced" Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z)$ at least three things

$$\vdots$$

$\underline{\phi_n}$ domain of at least *n* things

$\exists x \forall y (y = x)$ at most one thing

$\exists x \exists y \forall z (z = x \lor z = y)$ at most two things

$\exists x_1 \exists x_2 \exists x_3 \forall y (y = x_1 \lor y = x_2 \lor y = x_3)$

An "Advanced" Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

$\vdots$

$\underline{\phi_n}$ domain of at least *n* things

$\exists x \forall y (y = x)$ at most one thing

$\exists x \exists y \forall z (z = x \vee z = y)$ at most two things

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An "Advanced" Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

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$\vdots$

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$\exists x \forall y (y = x)$ at most one thing

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$\vdots$

$\underline{\phi_n}$ domain of at least *n* things

$\exists x \forall y (y = x)$ at most one thing

$\exists x \exists y \forall z (z = x \lor z = y)$ at most two things

$\exists x_1 \exists x_2 \exists x_3 \forall y (y = x_1 \lor y = x_2 \lor y = x_3)$ at most three things

$\vdots$

$\overline{\phi_n}$

For now, let’s settle for a focus on quantification. Then …

Measuring AI Intelligence via (in part) Logic:Quantification

Toby Walsh: “The Singularity May Never Be Near”
(http://arxiv.org/pdf/1602.06462v1.pdf)

Measuring AI Intelligence via (in part) Logic:Quantification

Toby Walsh: "The Singularity May Never Be Near"
(http://arxiv.org/pdf/1602.06462v1.pdf)

"I will not tackle here head on what we mean by measuring the intelligence of machines (or of humans). I will simply suppose there is such a property as intelligence, that it can be measured and compared, and that the technological singularity is when this measure increases exponentially fast in an appropriate and reasonable scale." (p. 1)

But logico-mathematical definitions of intelligence for animals, humans, machines, aliens, gods … *are* possible; recall our consideration of the *Entscheidungsproblem*. We can specifically challenge today's AI on the basis of simple quantification and simple deduction …

First, need some numerical quantifiers:

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$$\exists x \forall y (y = x \wedge \phi(x)) \text{ will be } \exists^{=1} x \phi(x)$$

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$$\exists x \forall y (y = x \wedge \phi(x)) \text{ will be } \exists^{=1} x \phi(x)$$

$$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z)) \text{ will be } \exists^{\geq 3} x \phi(x)$$

First, need some numerical quantifiers:

$\exists x \forall y (y = x \wedge \phi(x))$ will be $\exists^{=1} x \phi(x)$

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z))$ will be $\exists^{\geq 3} x \phi(x)$

How do we define formulae of this type: $\exists^{=k} x \psi(x)$

First, need some numerical quantifiers:

$\exists x \forall y (y = x \wedge \phi(x))$ will be $\exists^{=1} x \phi(x)$

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z))$ will be $\exists^{\geq 3} x \phi(x)$

How do we define formulae of this type: $\exists^{=k} x \psi(x)$

How do we define formulae of this type: $\exists^{\leq n} x \psi(x)$

First, need some numerical quantifiers:

$$\exists x \forall y (y = x \land \phi(x)) \text{ will be } \exists^{=1} x \phi(x)$$

$$\exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z \land \phi(x, y, z)) \text{ will be } \exists^{\geq 3} x \phi(x)$$

How do we define formulae of this type: $\exists^{=k} x \psi(x)$

How do we define formulae of this type: $\exists^{\leq n} x \psi(x)$

$$\vdots$$

First, need some numerical quantifiers:

$$\exists x \forall y (y = x \wedge \phi(x)) \text{ will be } \exists^{=1} x \phi(x)$$

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$$\text{How do we define formulae of this type: } \exists^{=k} x \psi(x)$$

$$\text{How do we define formulae of this type: } \exists^{\leq n} x \psi(x)$$

$$\vdots$$

Okay, now AI:

First, need some numerical quantifiers:

$$\exists x \forall y (y = x \wedge \phi(x)) \text{ will be } \exists^{=1} x \phi(x)$$

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$$\text{How do we define formulae of this type: } \exists^{=k} x \psi(x)$$

$$\text{How do we define formulae of this type: } \exists^{\leq n} x \psi(x)$$

$$\vdots$$

Okay, now AI:

At least seven kenspeckle blookers are red.

First, need some numerical quantifiers:

$$\exists x \forall y (y = x \wedge \phi(x)) \text{ will be } \exists^{=1} x \phi(x)$$

$$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z)) \text{ will be } \exists^{\geq 3} x \phi(x)$$

$$\text{How do we define formulae of this type: } \exists^{=k} x \psi(x)$$

$$\text{How do we define formulae of this type: } \exists^{\leq n} x \psi(x)$$

$$\vdots$$

Okay, now AI:

At least seven kenspeckle blookers are red.

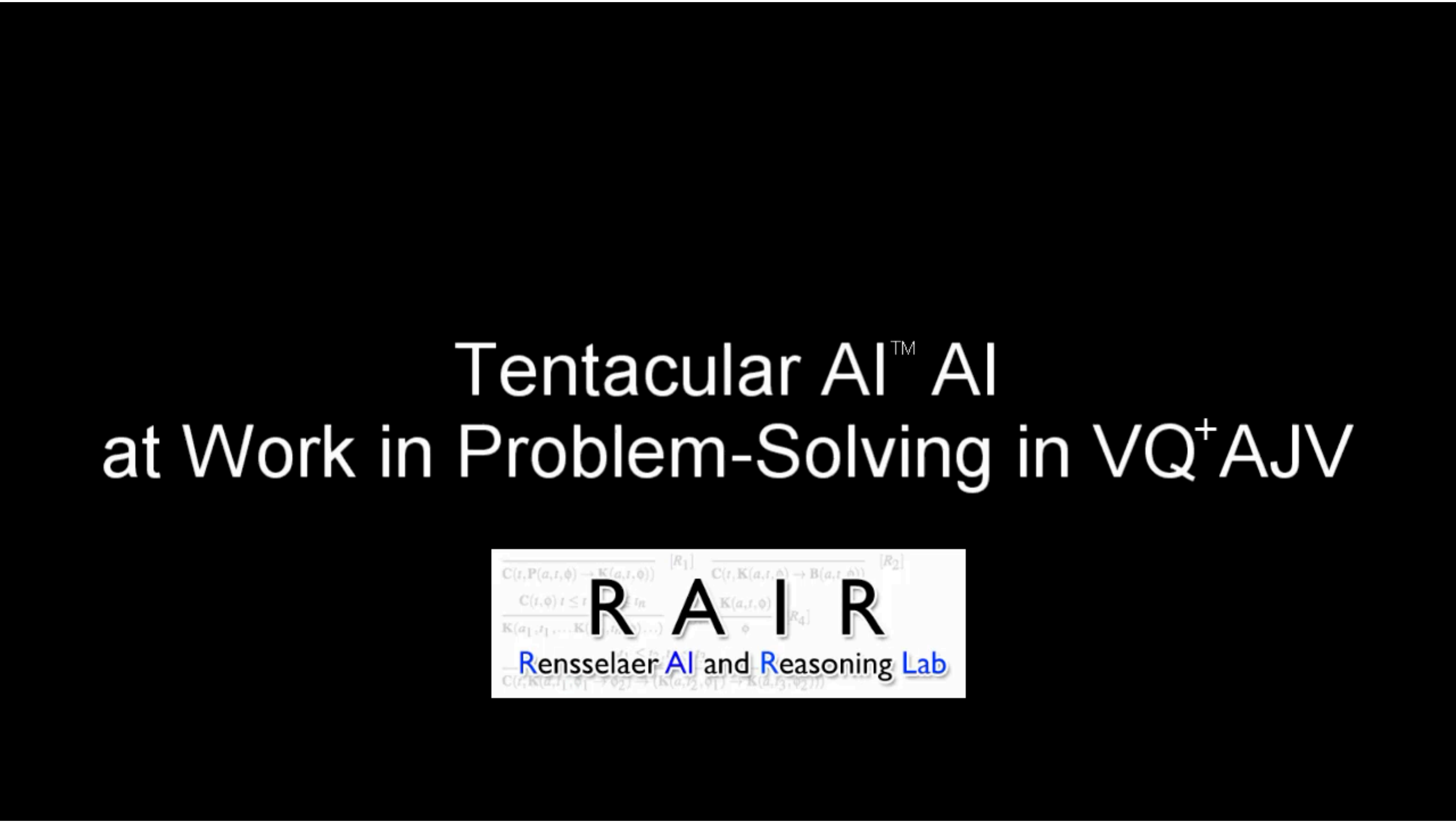
Given this, is it true that there are two red blookers? Why, exactly?

$$
\begin{aligned}
&\forall x \forall y \forall z \; \{[x \neq y \wedge y \neq z \wedge x \neq z \wedge Cx \wedge Cy \wedge Cz \; \wedge \\
&Tz' \wedge \\
&\exists w_1 \exists w_2 \; (w_1 \neq w_2 \wedge Aw_1 \wedge Aw_2 \wedge Gz'w_1 \wedge Gz'w_2) \; \wedge \\
&\forall u_1 \forall u_2 \forall u_3 \; ((Gz'u_1 \wedge Gz'u_2 \wedge Gz'u_3 \wedge C^b u_1 \wedge C^b u_2 \wedge C^b u_3) \rightarrow \\
&\forall v((Gz'v \wedge C^b v) \rightarrow (v = u_1 \vee v = u_2 \vee v = u_3)))] \\
&\rightarrow \\
&(Gxz' \wedge Gyz' \wedge Gzz')\}
\end{aligned}
$$

Every three cylinders glower at any triangular prism that glowers at at least two arches and at at most three cubes.

$$\forall x \forall y \forall z \forall z' \left\{ \begin{array}{c} \left[\begin{array}{c} x \neq y \wedge y \neq z \wedge x \neq z \\ \wedge \\ Cx \wedge Cy \wedge Cz \\ \wedge \\ Tz' \\ \wedge \\ \exists w_1 \exists w_2 \ (w_1 \neq w_2 \wedge Aw_1 \wedge Aw_2 \wedge Gz'w_1 \wedge Gz'w_2) \\ \wedge \\ \forall u_1 \forall u_2 \forall u_3 \quad \left(\begin{array}{c} \left[Gz'u_1 \wedge Gz'u_2 \wedge Gz'u_3 \wedge C^b u_1 \wedge C^b u_2 \wedge C^b u_3 \right] \\ \rightarrow \\ \forall v \left[(Gz'v \wedge C^b v) \rightarrow (v = u_1 \vee v = u_2 \vee v = u_3) \right] \end{array} \right) \end{array} \right] \\ \rightarrow \\ (Gxz' \wedge Gyz' \wedge Gzz') \end{array} \right\}$$

Intelligent Artificial Multi-Agent



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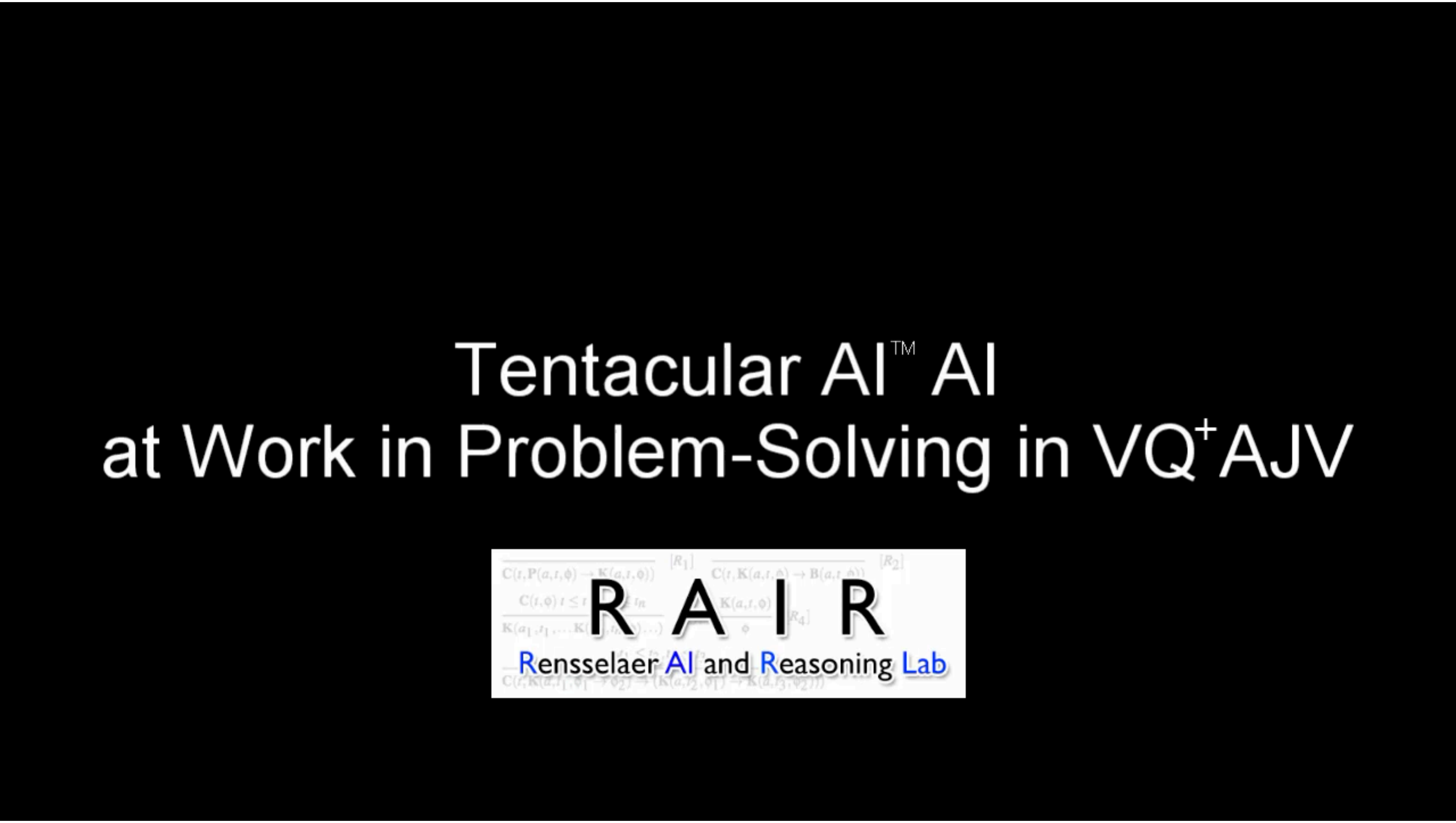

Part I: *Slutten — for i dag.*

Part I: *Slutten — for i dag.*

Part II: Hands-on Q&A & Review …