Propositional Calculus II:

Two more Rules of Inference/Inference Schemata

(`conditional elim` = *modus ponens;* *proof by cases* = *disjunction elimination*), Applying Them to Additional Motivating Problems

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Logic
1/29/2024

R A I R
Rensselaer AI and Reasoning Lab

Logic-and-AI in the news

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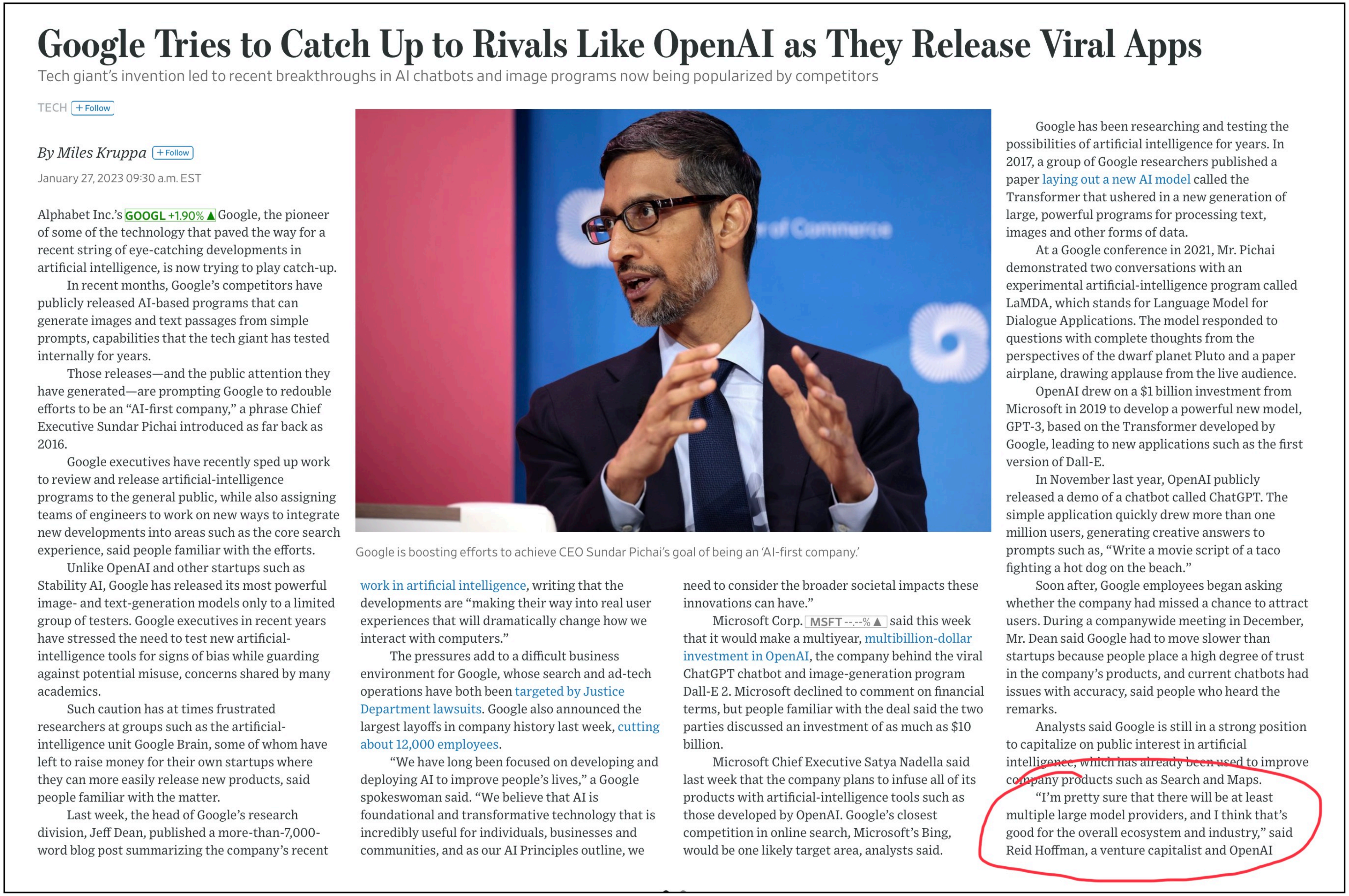


Google Tries to Catch Up to Rivals Like OpenAI as They Release Viral Apps

Tech giant's invention led to recent breakthroughs in AI chatbots and image programs now being popularized by competitors

TECH + Follow

By Miles Kruppa + Follow

January 27, 2023 09:30 a.m. EST

Alphabet Inc.'s GOOGL +1.90% ▲ Google, the pioneer of some of the technology that paved the way for a recent string of eye-catching developments in artificial intelligence, is now trying to play catch-up.

In recent months, Google's competitors have publicly released AI-based programs that can generate images and text passages from simple prompts, capabilities that the tech giant has tested internally for years.

Those releases—and the public attention they have generated—are prompting Google to redouble efforts to be an "AI-first company," a phrase Chief Executive Sundar Pichai introduced as far back as 2016.

Google executives have recently sped up work to review and release artificial-intelligence programs to the general public, while also assigning teams of engineers to work on new ways to integrate new developments into areas such as the core search experience, said people familiar with the efforts.

Unlike OpenAI and other startups such as Stability AI, Google has released its most powerful image- and text-generation models only to a limited group of testers. Google executives in recent years have stressed the need to test new artificial-intelligence tools for signs of bias while guarding against potential misuse, concerns shared by many academics.

Such caution has at times frustrated researchers at groups such as the artificial-intelligence unit Google Brain, some of whom have left to raise money for their own startups where they can more easily release new products, said people familiar with the matter.

Last week, the head of Google's research division, Jeff Dean, published a more-than-7,000-word blog post summarizing the company's recent work in artificial intelligence, writing that the developments are "making their way into real user experiences that will dramatically change how we interact with computers."

Google is boosting efforts to achieve CEO Sundar Pichai's goal of being an 'AI-first company.'

The pressures add to a difficult business environment for Google, whose search and ad-tech operations have both been targeted by Justice Department lawsuits. Google also announced the largest layoffs in company history last week, cutting about 12,000 employees.

"We have long been focused on developing and deploying AI to improve people's lives," a Google spokeswoman said. "We believe that AI is foundational and transformative technology that is incredibly useful for individuals, businesses and communities, and as our AI Principles outline, we need to consider the broader societal impacts these innovations can have."

Microsoft Corp. MSFT --.--% ▲ said this week that it would make a multiyear, multibillion-dollar investment in OpenAI, the company behind the viral ChatGPT chatbot and image-generation program Dall-E 2. Microsoft declined to comment on financial terms, but people familiar with the deal said the two parties discussed an investment of as much as $10 billion.

Microsoft Chief Executive Satya Nadella said last week that the company plans to infuse all of its products with artificial-intelligence tools such as those developed by OpenAI. Google's closest competition in online search, Microsoft's Bing, would be one likely target area, analysts said.

Google has been researching and testing the possibilities of artificial intelligence for years. In 2017, a group of Google researchers published a paper laying out a new AI model called the Transformer that ushered in a new generation of large, powerful programs for processing text, images and other forms of data.

At a Google conference in 2021, Mr. Pichai demonstrated two conversations with an experimental artificial-intelligence program called LaMDA, which stands for Language Model for Dialogue Applications. The model responded to questions with complete thoughts from the perspectives of the dwarf planet Pluto and a paper airplane, drawing applause from the live audience.

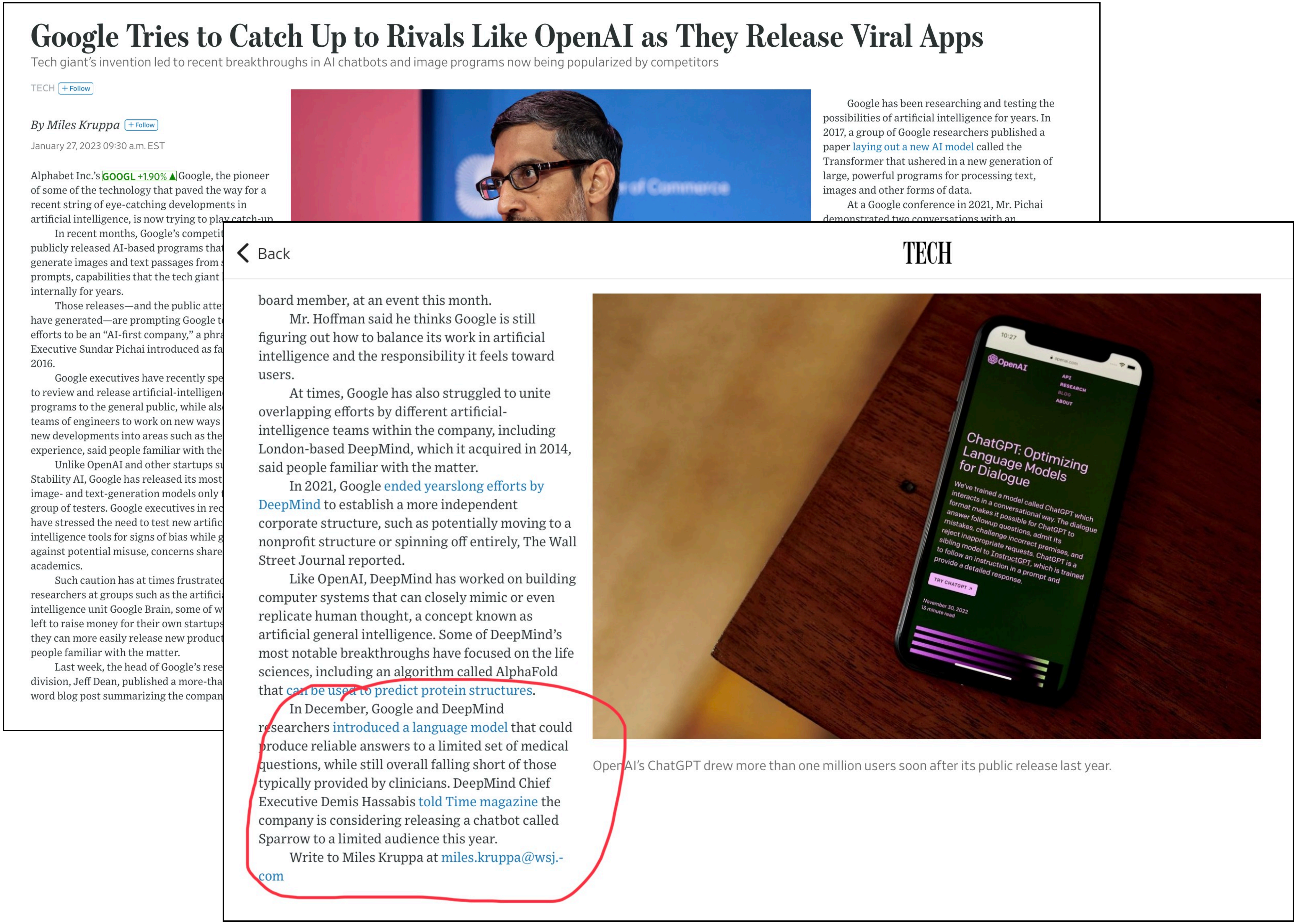
OpenAI drew on a $1 billion investment from Microsoft in 2019 to develop a powerful new model, GPT-3, based on the Transformer developed by Google, leading to new applications such as the first version of Dall-E.

In November last year, OpenAI publicly released a demo of a chatbot called ChatGPT. The simple application quickly drew more than one million users, generating creative answers to prompts such as, "Write a movie script of a taco fighting a hot dog on the beach."

Soon after, Google employees began asking whether the company had missed a chance to attract users. During a companywide meeting in December, Mr. Dean said Google had to move slower than startups because people place a high degree of trust in the company's products, and current chatbots had issues with accuracy, said people who heard the remarks.

Analysts said Google is still in a strong position to capitalize on public interest in artificial intelligence, which has already been used to improve company products such as Search and Maps.

"I'm pretty sure that there will be at least multiple large model providers, and I think that's good for the overall ecosystem and industry," said Reid Hoffman, a venture capitalist and OpenAI



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---

board member, at an event this month.

Mr. Hoffman said he thinks Google is still figuring out how to balance its work in artificial intelligence and the responsibility it feels toward users.

At times, Google has also struggled to unite overlapping efforts by different artificial-intelligence teams within the company, including London-based DeepMind, which it acquired in 2014, said people familiar with the matter.

In 2021, Google ended yearslong efforts by DeepMind to establish a more independent corporate structure, such as potentially moving to a nonprofit structure or spinning off entirely, The Wall Street Journal reported.

Like OpenAI, DeepMind has worked on building computer systems that can closely mimic or even replicate human thought, a concept known as artificial general intelligence. Some of DeepMind's most notable breakthroughs have focused on the life sciences, including an algorithm called AlphaFold that can be used to predict protein structures.

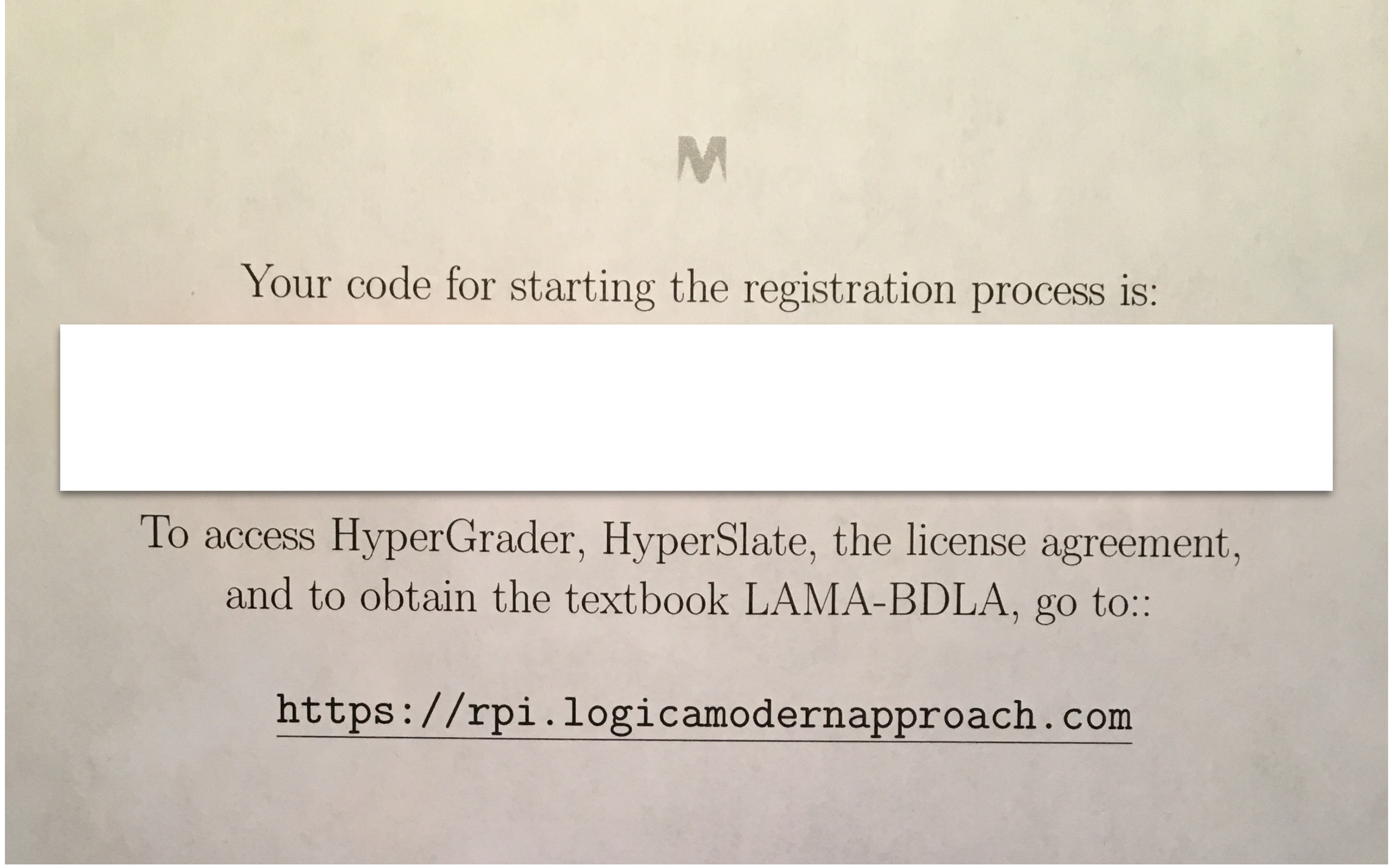
In December, Google and DeepMind researchers introduced a language model that could produce reliable answers to a limited set of medical questions, while still overall falling short of those typically provided by clinicians. DeepMind Chief Executive Demis Hassabis told Time magazine the company is considering releasing a chatbot called Sparrow to a limited audience this year.

Write to Miles Kruppa at miles.kruppa@wsj.com

OpenAI's ChatGPT drew more than one million users soon after its public release last year.

Logistics …

The Starting Code to Purchase in Bookstore



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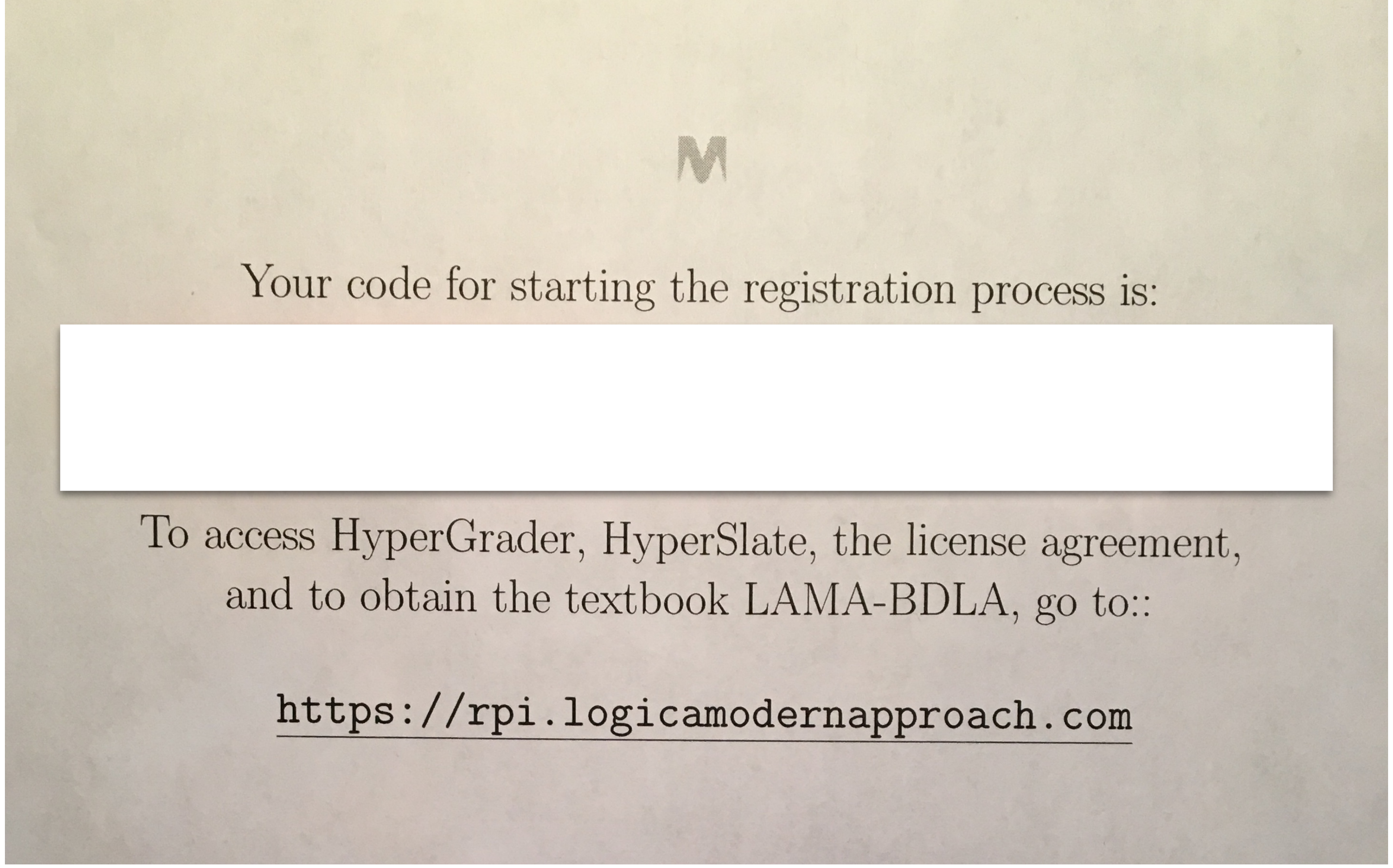

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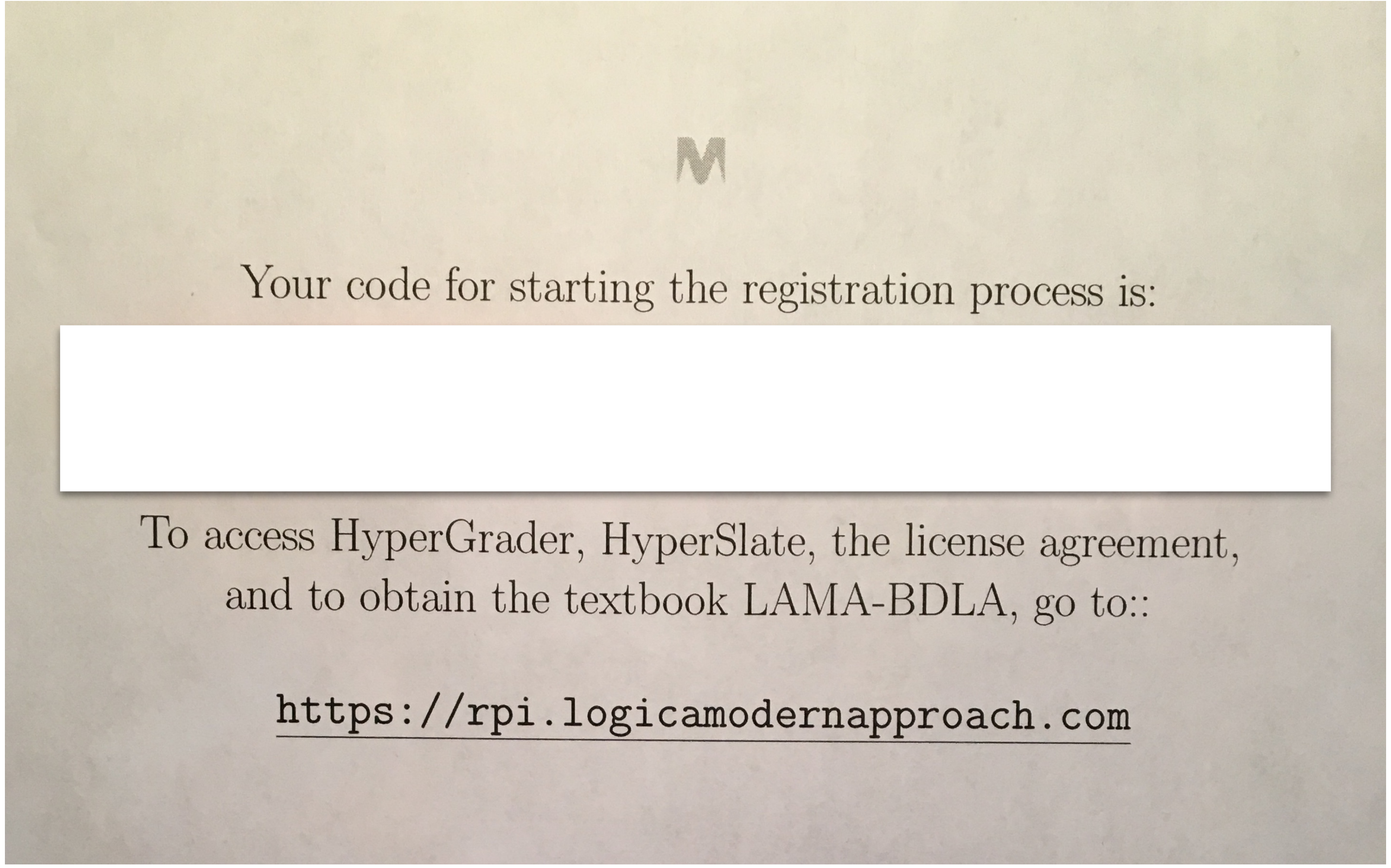

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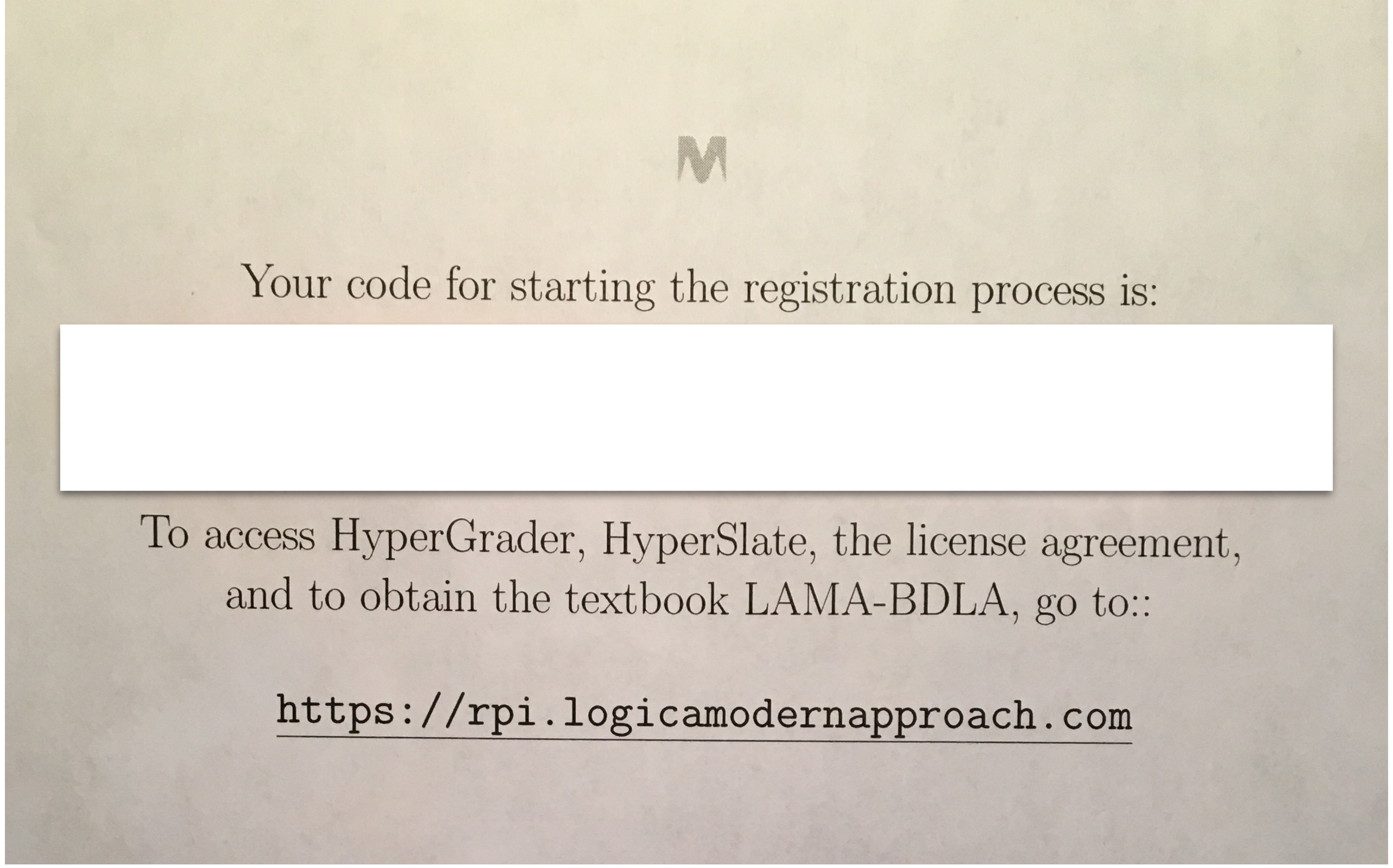

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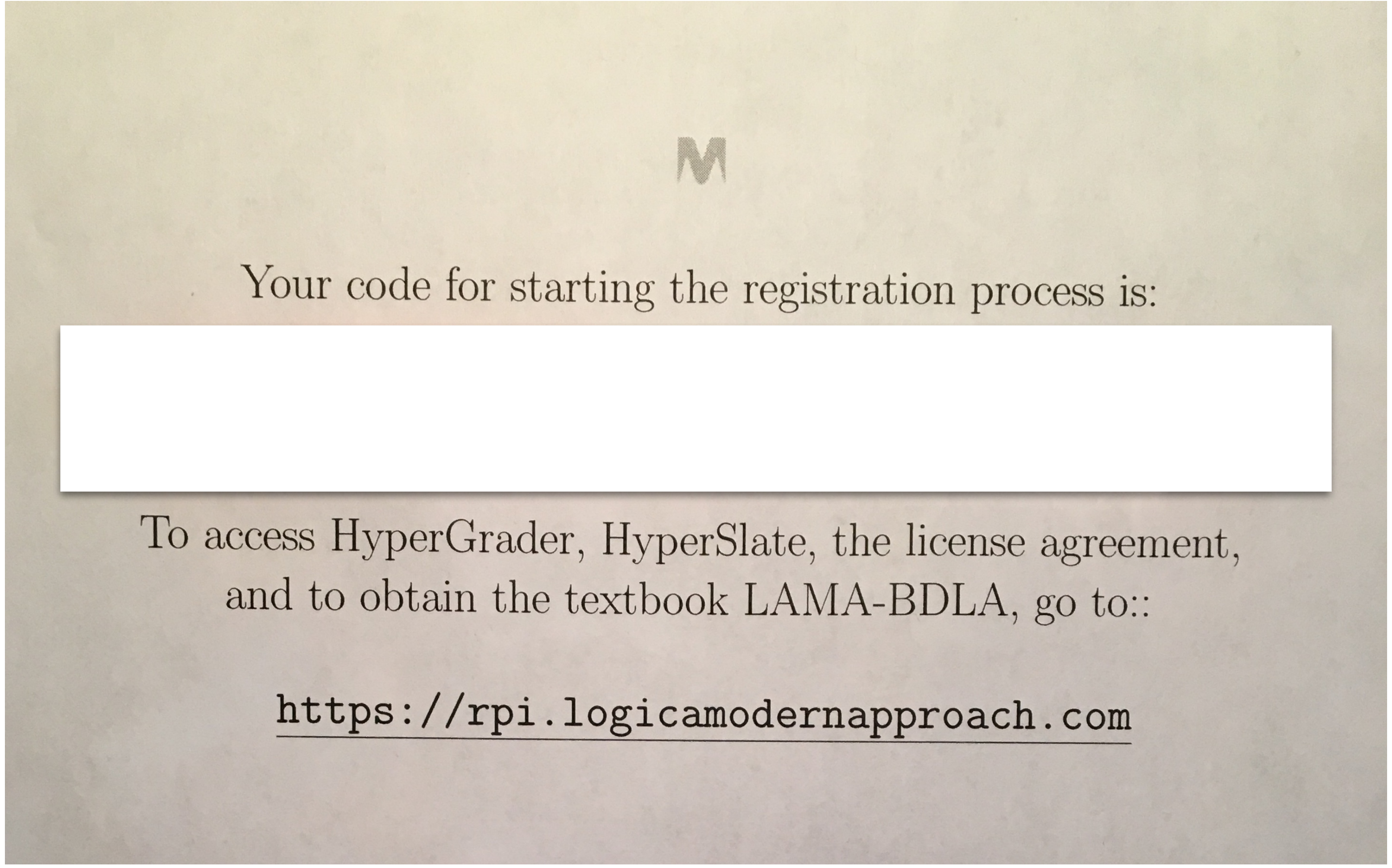
Your OS and browser must be fully up-to-date; Chrome is the best choice, browser-wise (though I use Safari).

The Starting Code Purchased in Bookstore Should By Now’ve Been Used to Register & Subsequently Sign In

First prop. calc. (Exercise) Problems:

`switching_conjuncts_fine`, `switching_disjuncts_fine`



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E-Housekeeping Pts (again)

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- Make sure OS fully up-to-date.
- Make sure browser fully up-to-date.
- Chrome best (but I use Safari).
- Always work in the same browser window with multiple tabs; must do this with email and HyperGrader® & HyperSlate®.

Propositional Calculus II:

Two more Rules of Inference/Inference Schemata
(`conditional elim` = *modus ponens;*
proof by cases = *disjunction elimination*),
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Intro to Logic
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Last time we introduced and and lauded the power of **oracles**, and questions … and now … picking up where we left off …

“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above

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“NYS 3” Revisited

Given the statements

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After last class, should have explored if you are registered … Show in HyperSlate® as I did that each of the first four options can be proved using the PC entailment (= provability) oracle.

which one of the following statements must also be true?

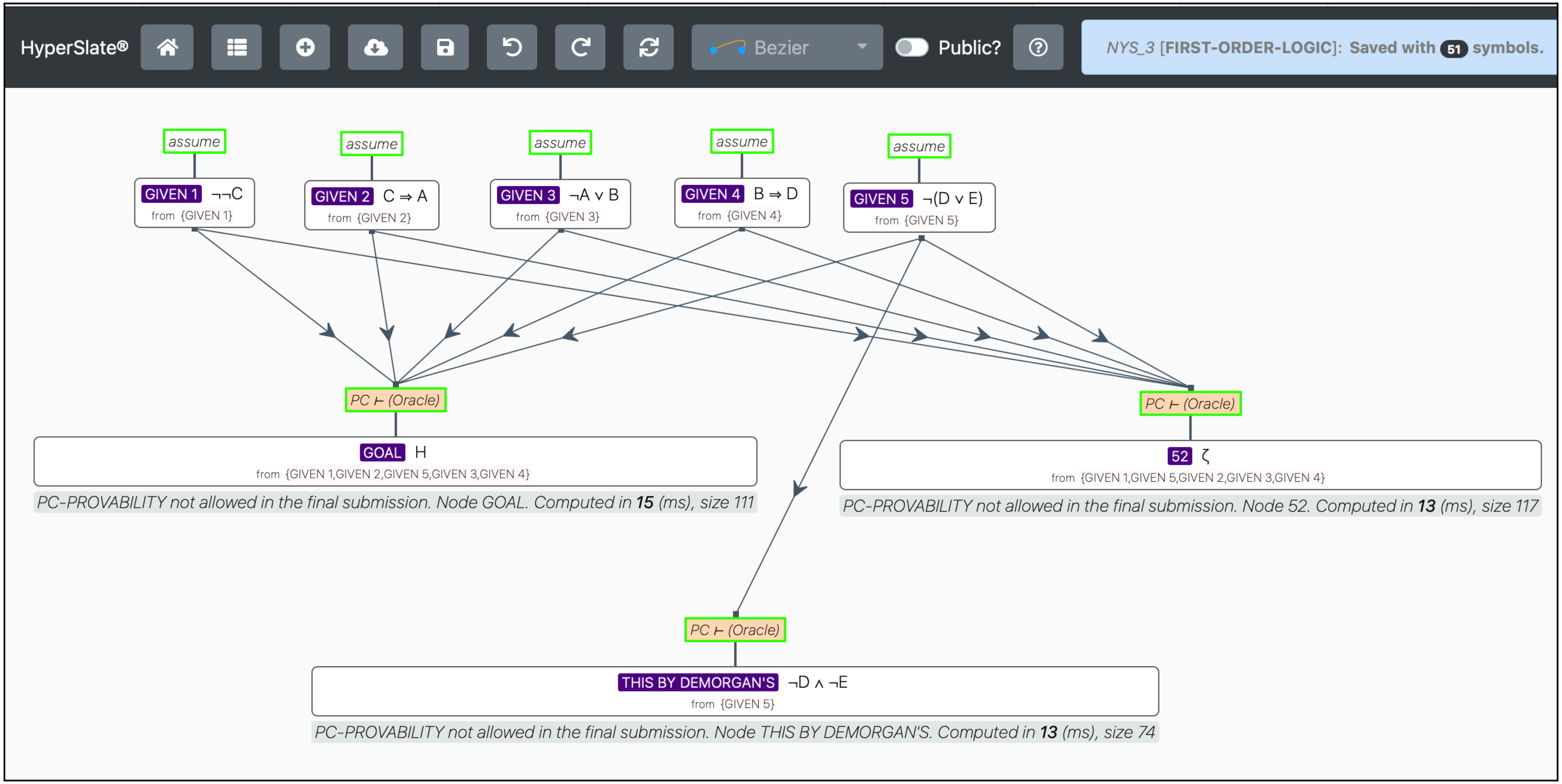
$\neg c$

e

h

$\neg a$

all of the above



NYS_3 [**FIRST-ORDER-LOGIC**]: **Saved with** 51 **symbols.**

PC-PROVABILITY not allowed in the final submission. Node GOAL. Computed in ***15*** *(ms), size 111*

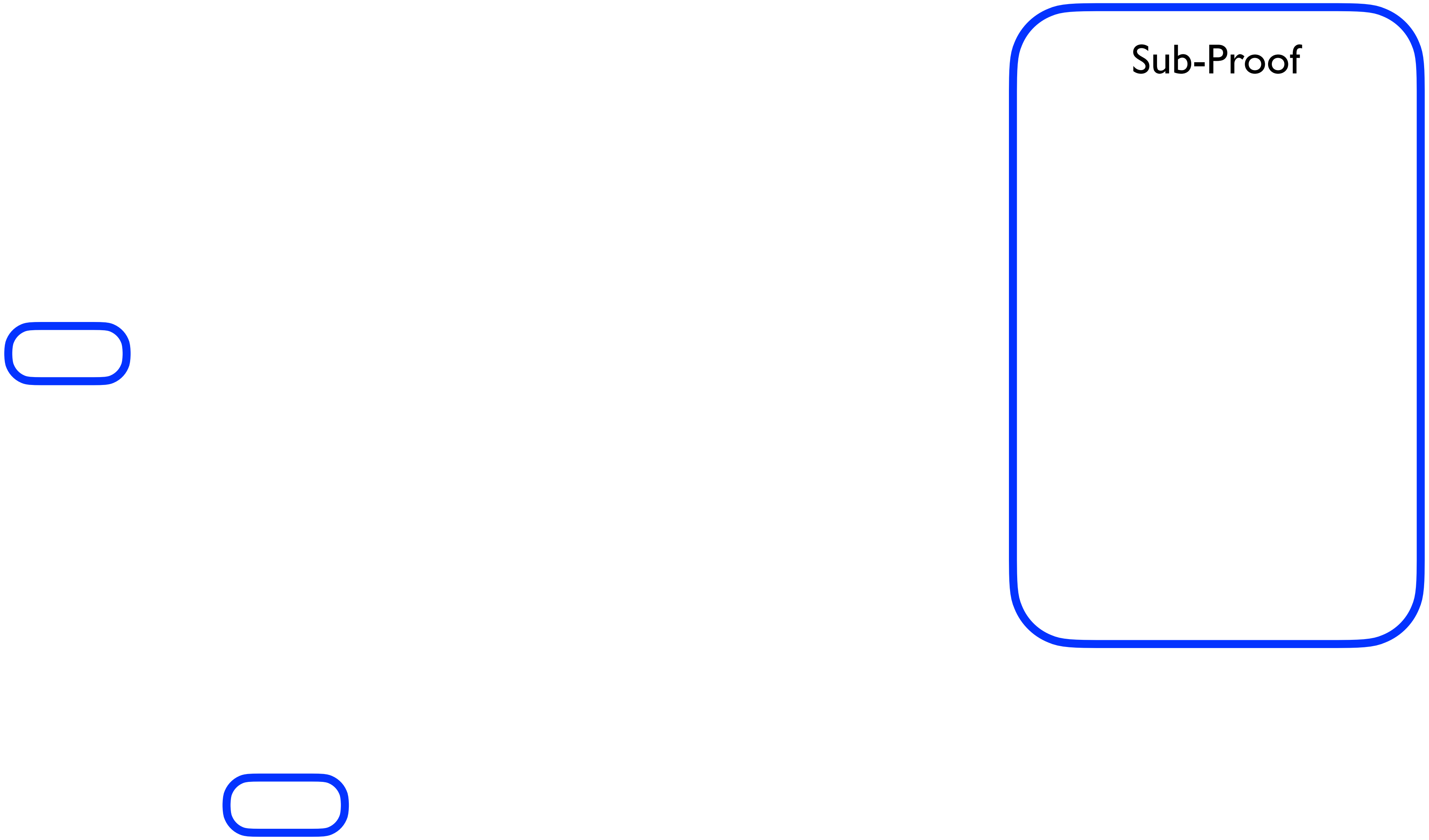
PC-PROVABILITY not allowed in the final submission. Node 52. Computed in ***13*** *(ms), size 117*

PC-PROVABILITY not allowed in the final submission. Node THIS BY DEMORGAN'S. Computed in ***13*** *(ms), size 74*

Proof Plan …

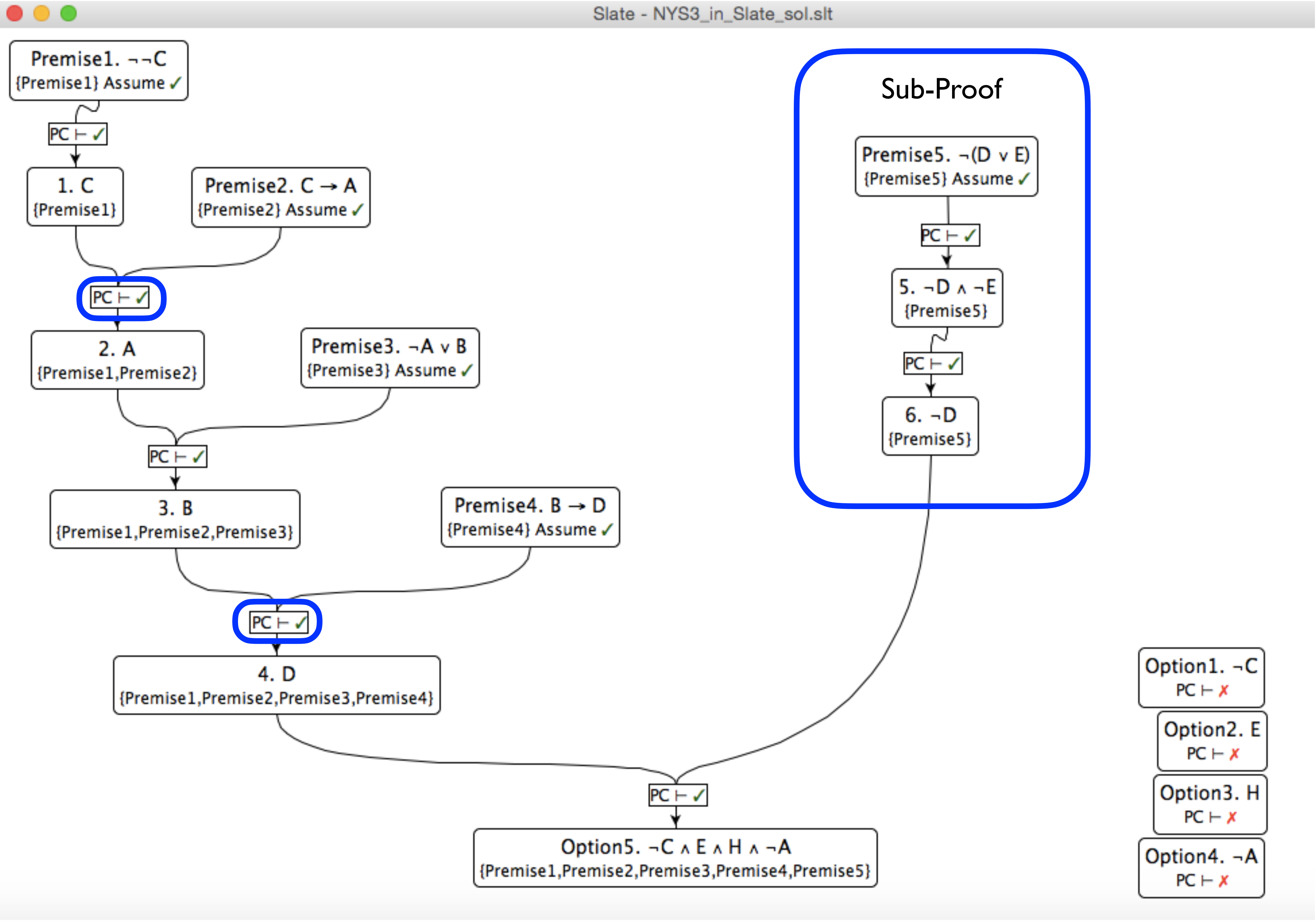
Proof Plan …

Proof Plan …



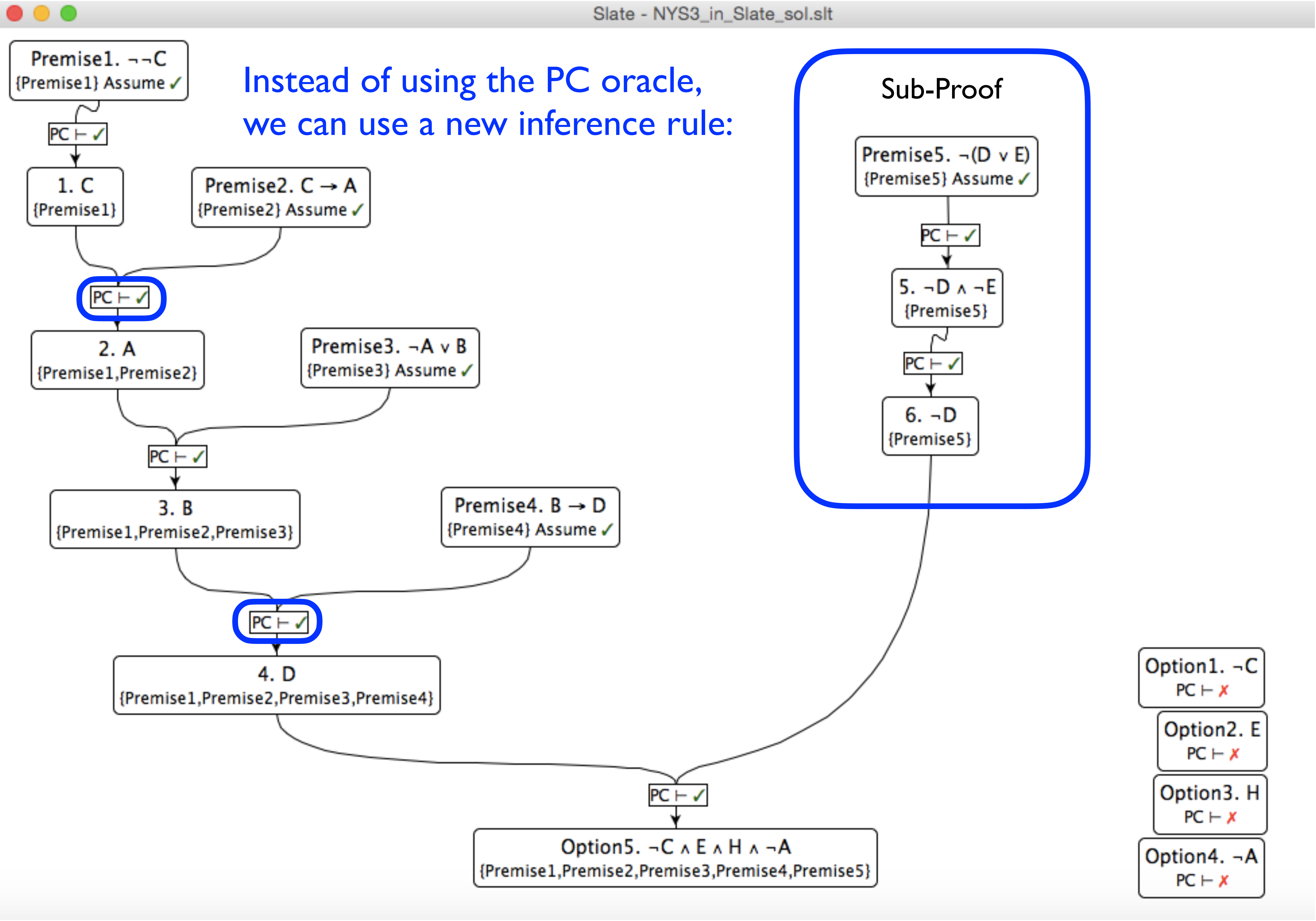
Sub-Proof

Proof Plan …


Slate - NYS3_in_Slate_sol.slt
Premise1. ¬¬C
{Premise1} Assume ✓
PC ⊢ ✓
1. C
{Premise1}
Premise2. C → A
{Premise2} Assume ✓
PC ⊢ ✓
2. A
{Premise1,Premise2}
Premise3. ¬A ∨ B
{Premise3} Assume ✓
PC ⊢ ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Sub-Proof
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
PC ⊢ ✓
5. ¬D ∧ ¬E
{Premise5}
PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
Option4. ¬A
PC ⊢ ✗

Proof Plan …

Instead of using the PC oracle, we can use a new inference rule:



Slate - NYS3_in_Slate_sol.slt

Premise1. ¬¬C
{Premise1} Assume ✓

PC ⊢ ✓

1. C
{Premise1}

Premise2. C → A
{Premise2} Assume ✓

PC ⊢ ✓

2. A
{Premise1,Premise2}

Premise3. ¬A ∨ B
{Premise3} Assume ✓

PC ⊢ ✓

3. B
{Premise1,Premise2,Premise3}

Premise4. B → D
{Premise4} Assume ✓

PC ⊢ ✓

4. D
{Premise1,Premise2,Premise3,Premise4}

Sub-Proof

Premise5. ¬(D ∨ E)
{Premise5} Assume ✓

PC ⊢ ✓

5. ¬D ∧ ¬E
{Premise5}

PC ⊢ ✓

6. ¬D
{Premise5}

PC ⊢ ✓

Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}

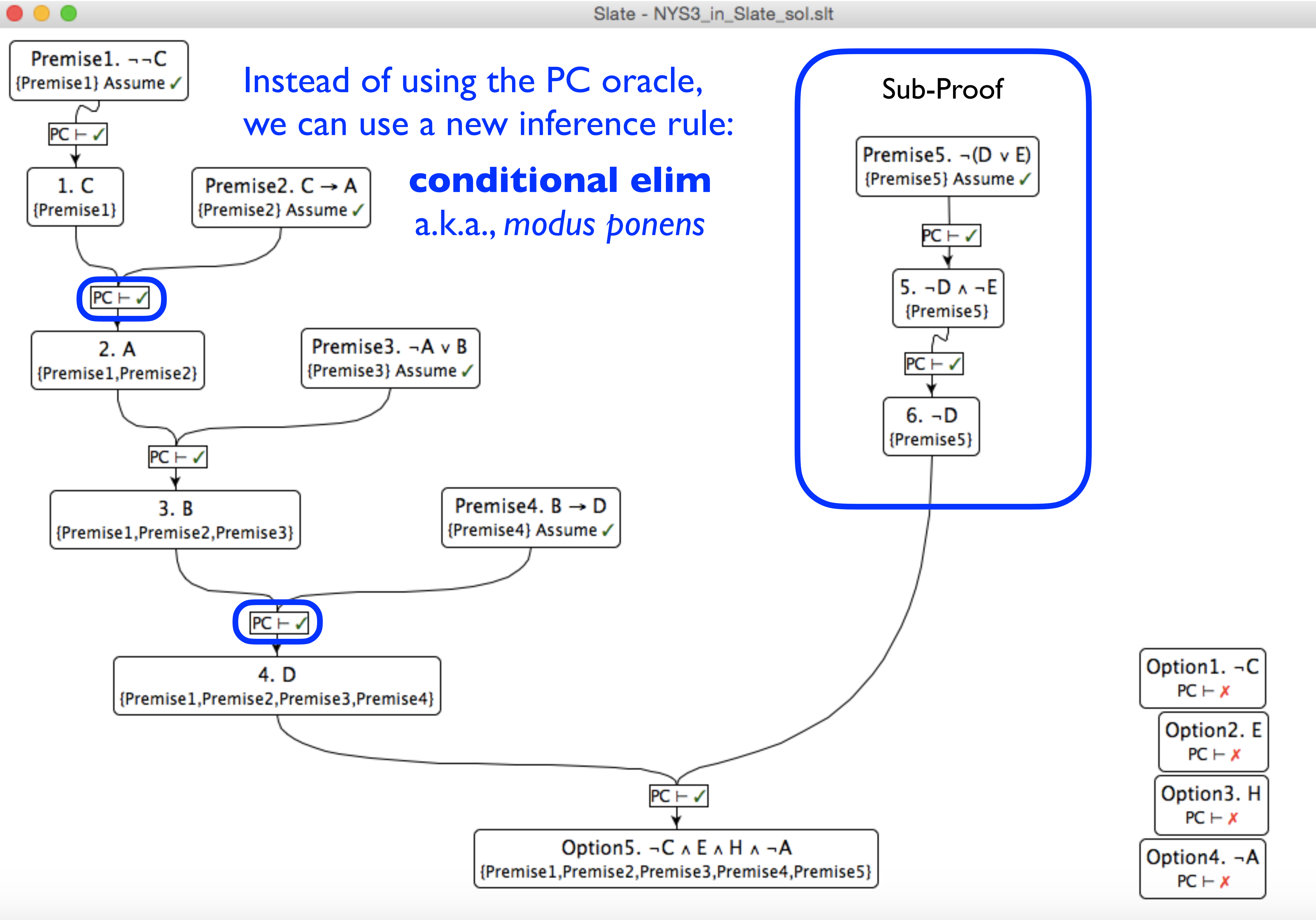
Option1. ¬C
PC ⊢ ✗

Option2. E
PC ⊢ ✗

Option3. H
PC ⊢ ✗

Option4. ¬A
PC ⊢ ✗

Proof Plan …



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a.k.a., *modus ponens*

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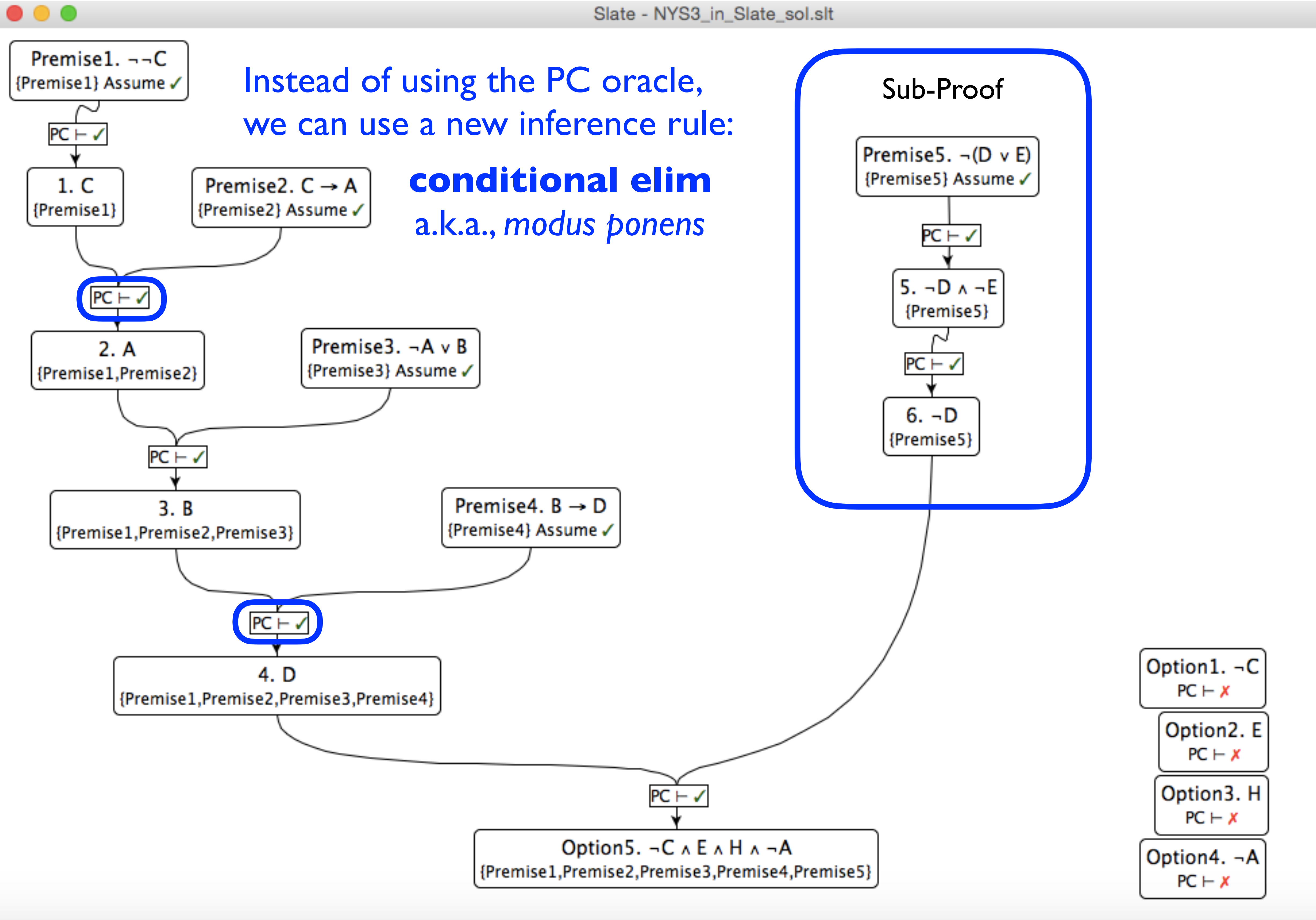

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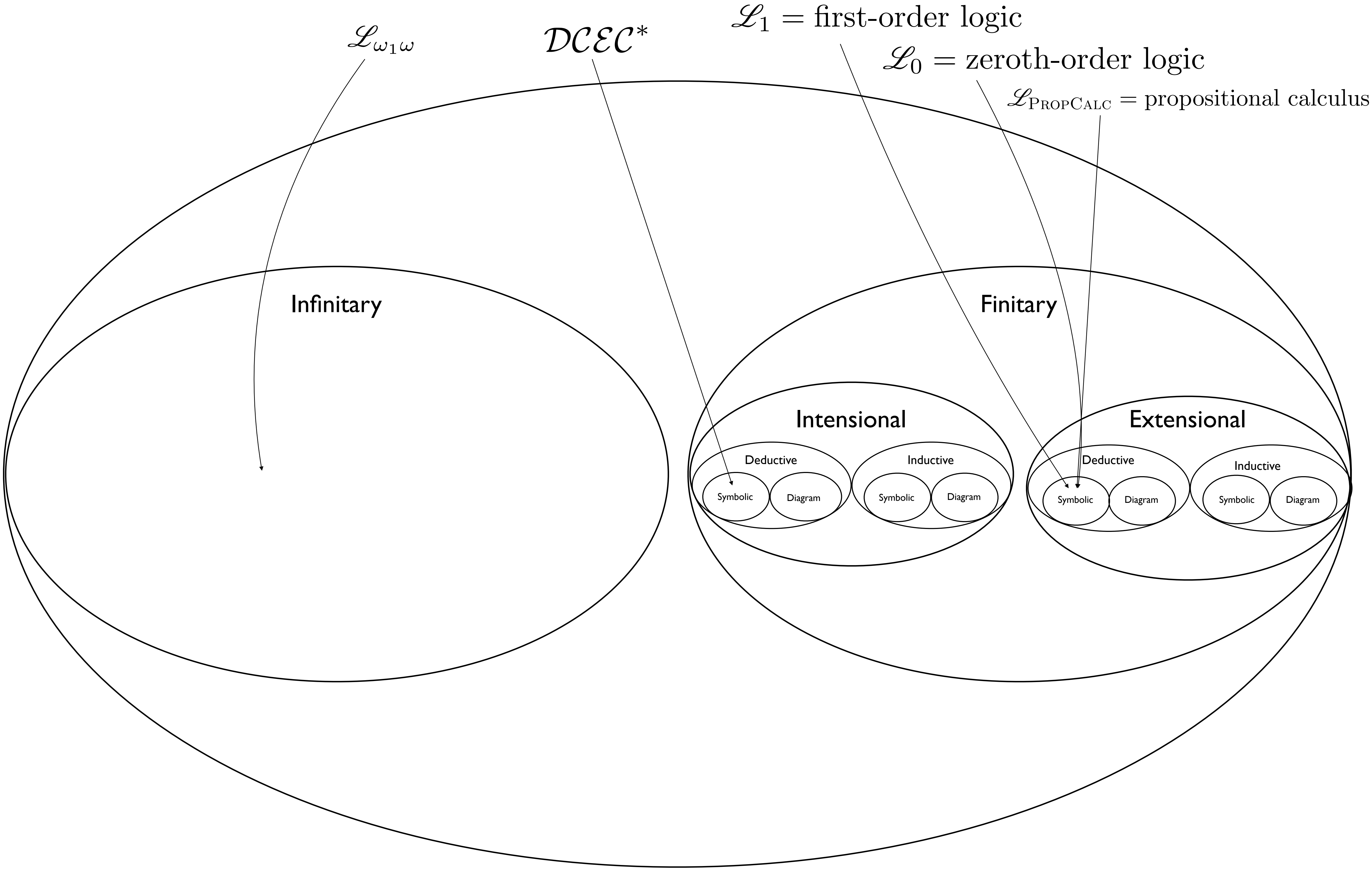
The Universe of Logics



$\mathscr{L}_{\omega_1\omega}$

$\mathcal{DCEC}^*$

$\mathscr{L}_1 =$ first-order logic

$\mathscr{L}_0 =$ zeroth-order logic

$\mathscr{L}_{\textsc{PropCalc}} =$ propositional calculus

Infinitary

Finitary

Intensional

Deductive

Symbolic

Diagram

Inductive

Symbolic

Diagram

Extensional

Deductive

Symbolic

Diagram

Inductive

Symbolic

Diagram

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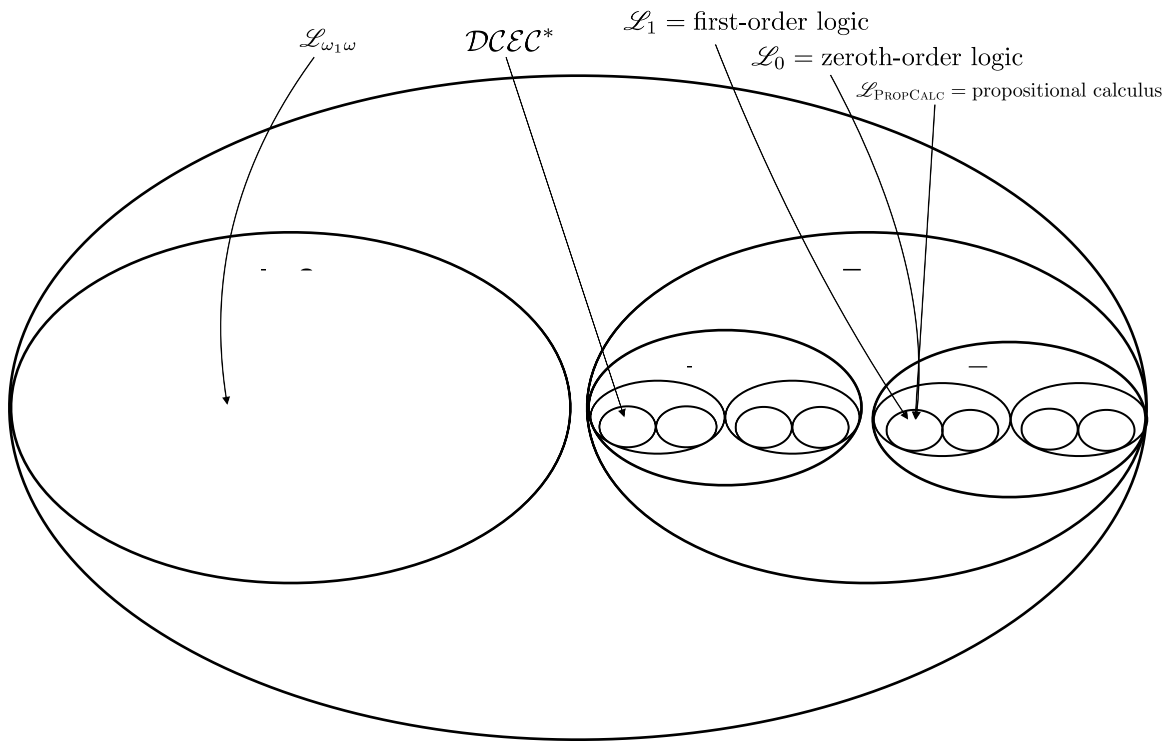

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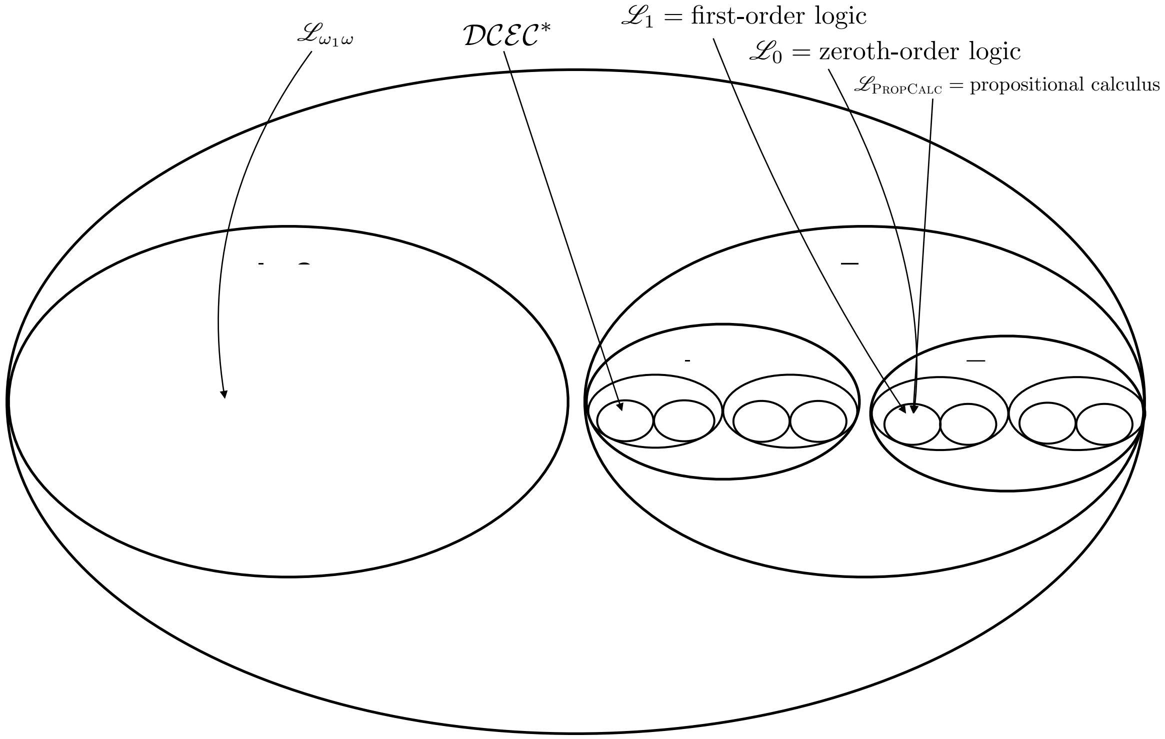
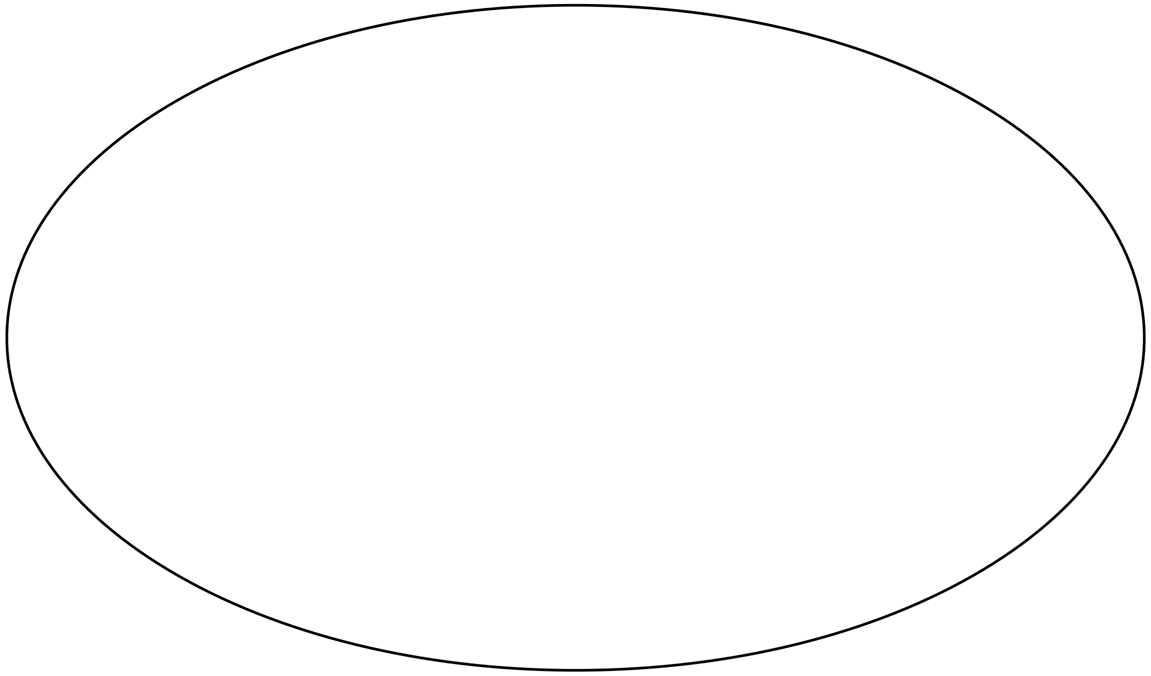


The Physical Universe



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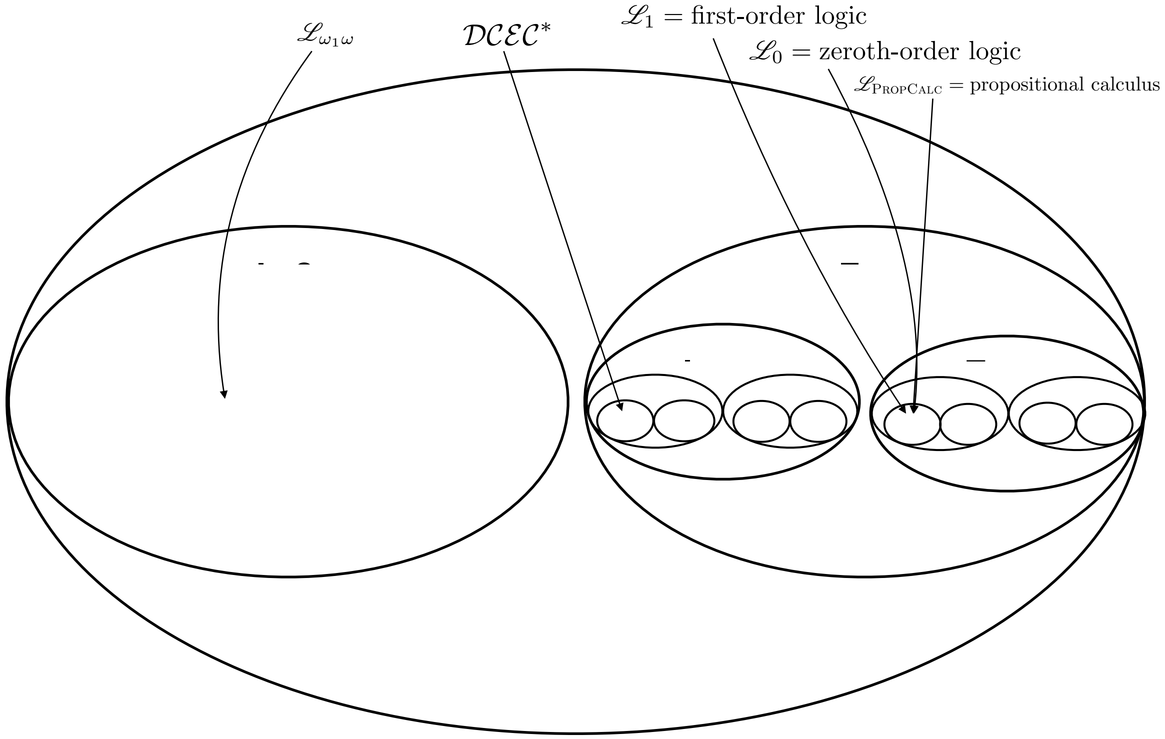

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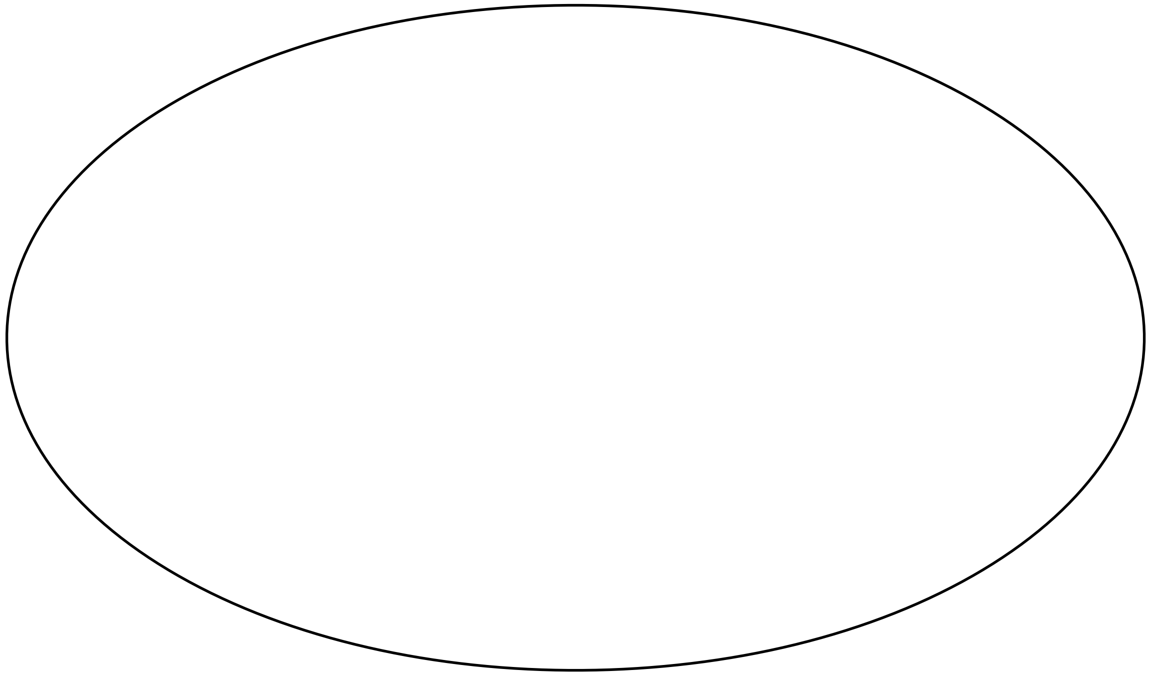
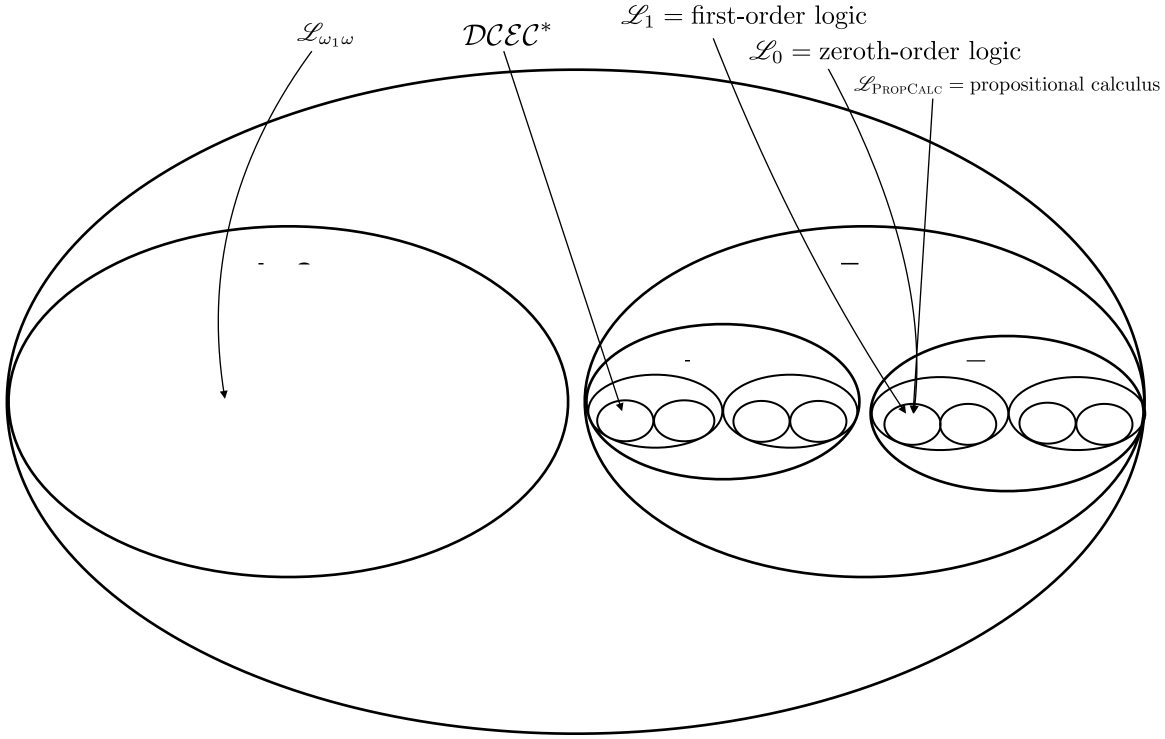
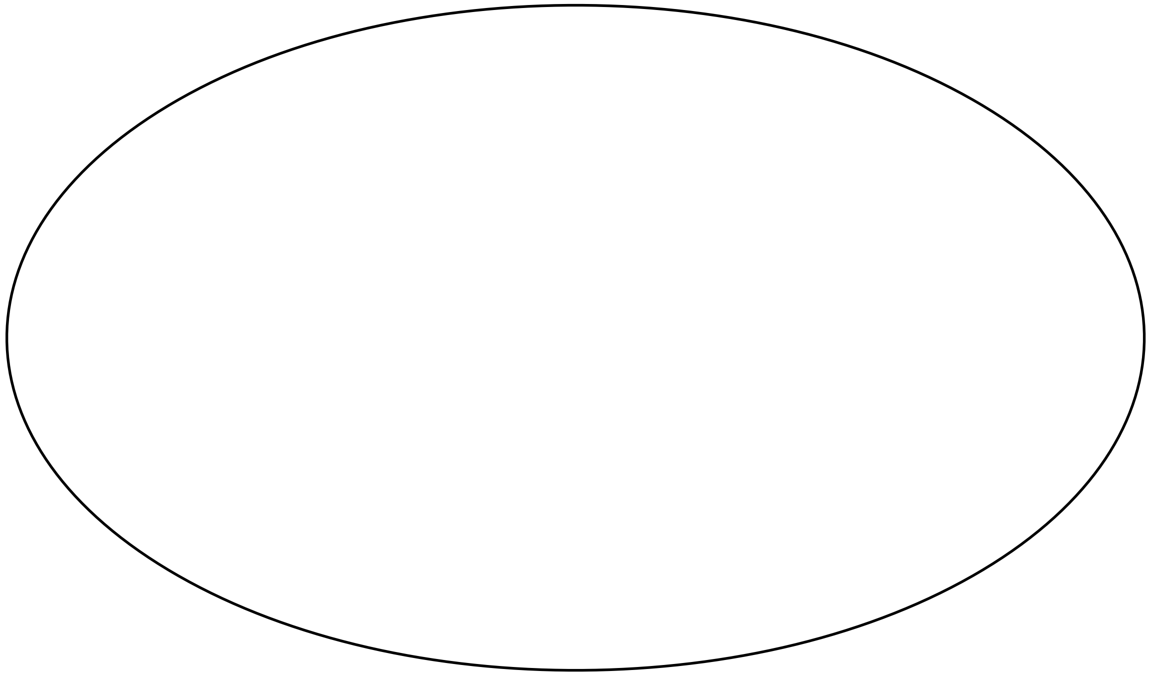


Non-Physical

The Universe of Logics

The Physical Universe





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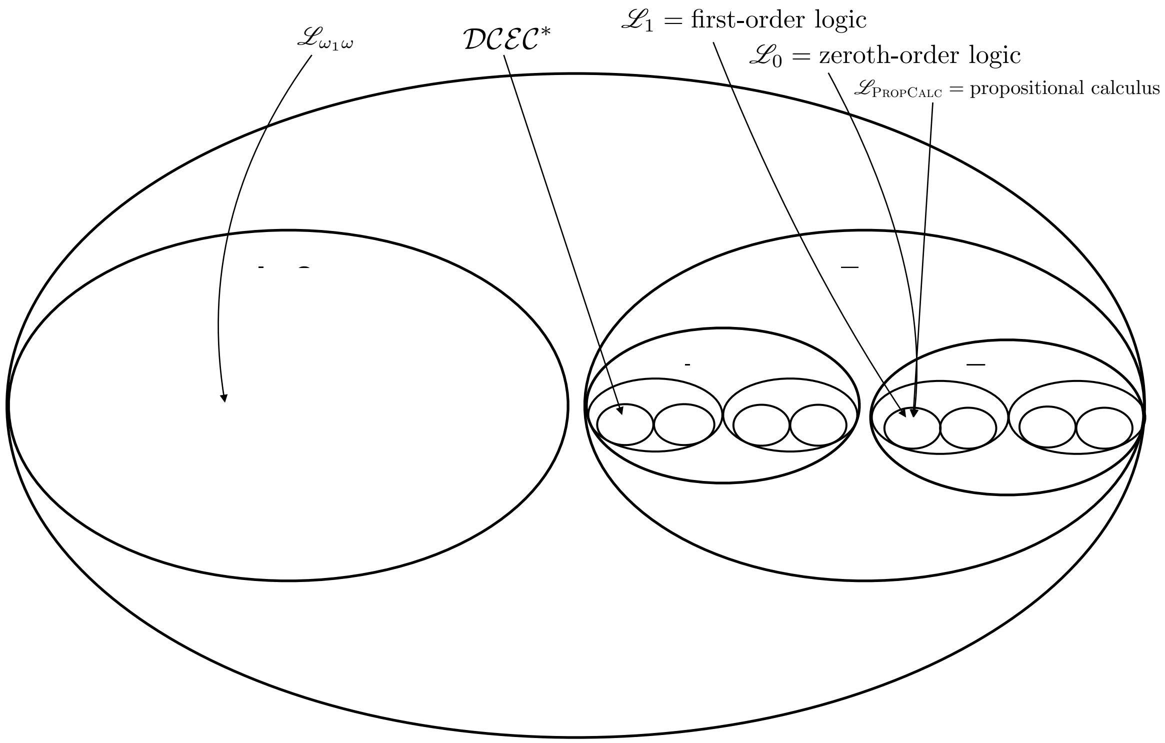
The Universe of Logics



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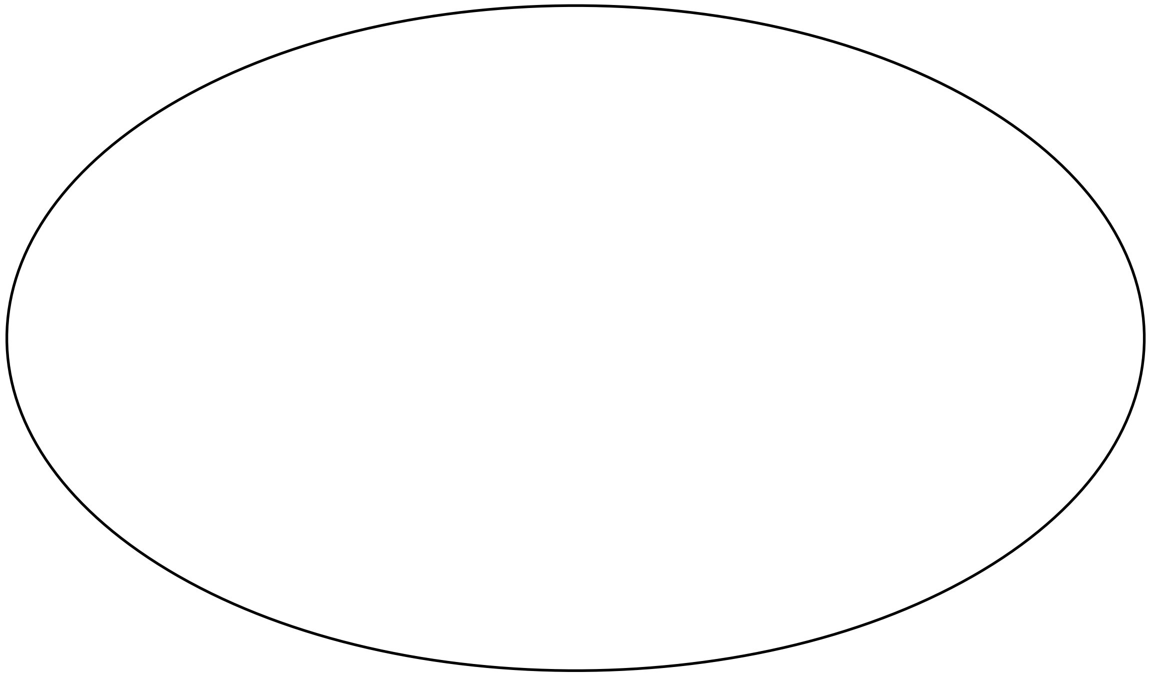

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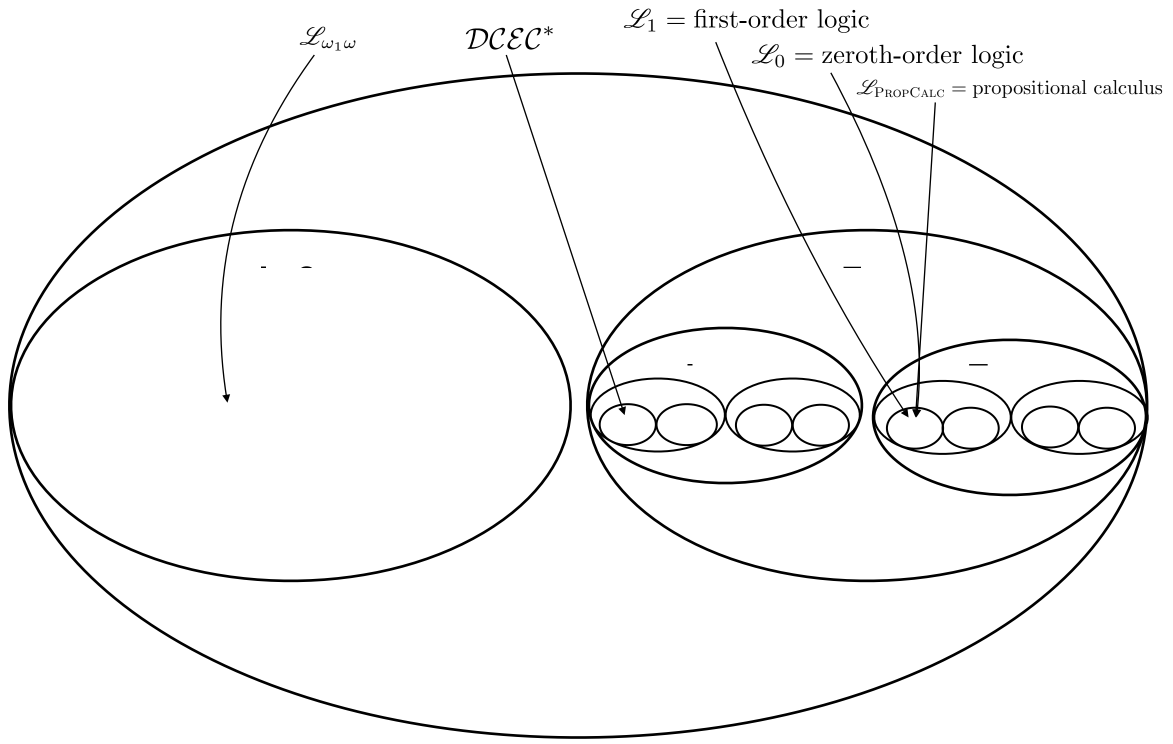

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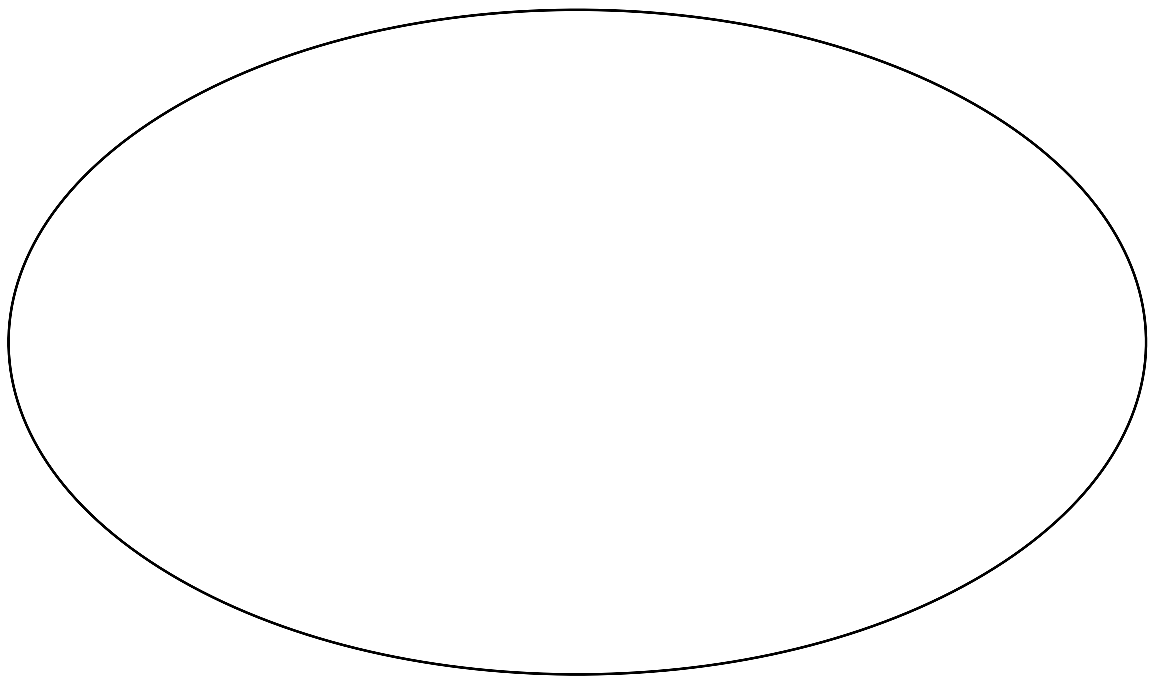

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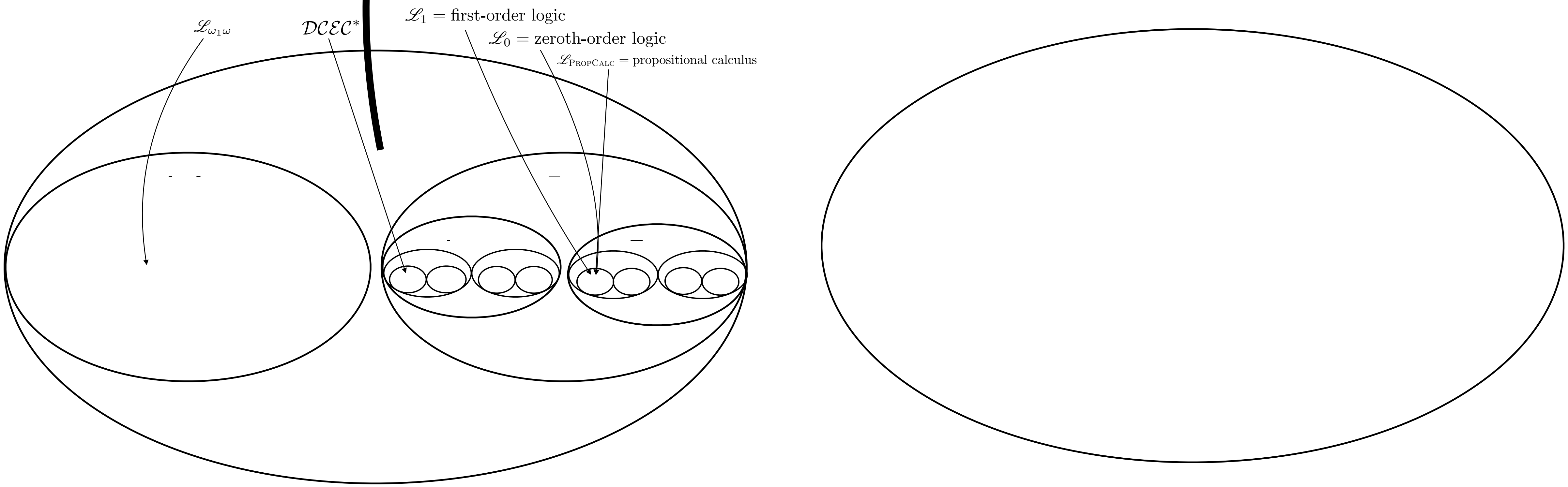

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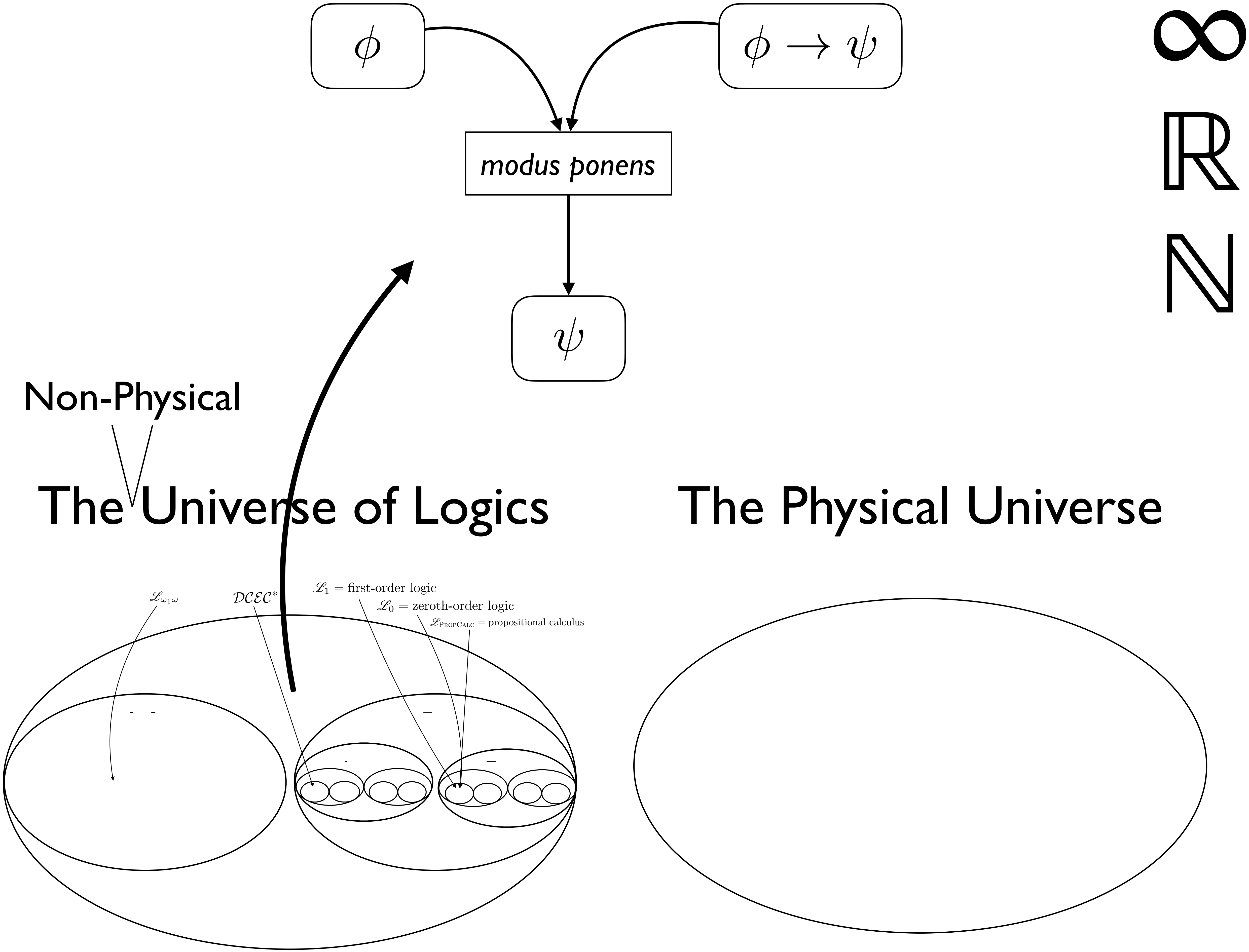

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$\phi \rightarrow \psi$

modus ponens

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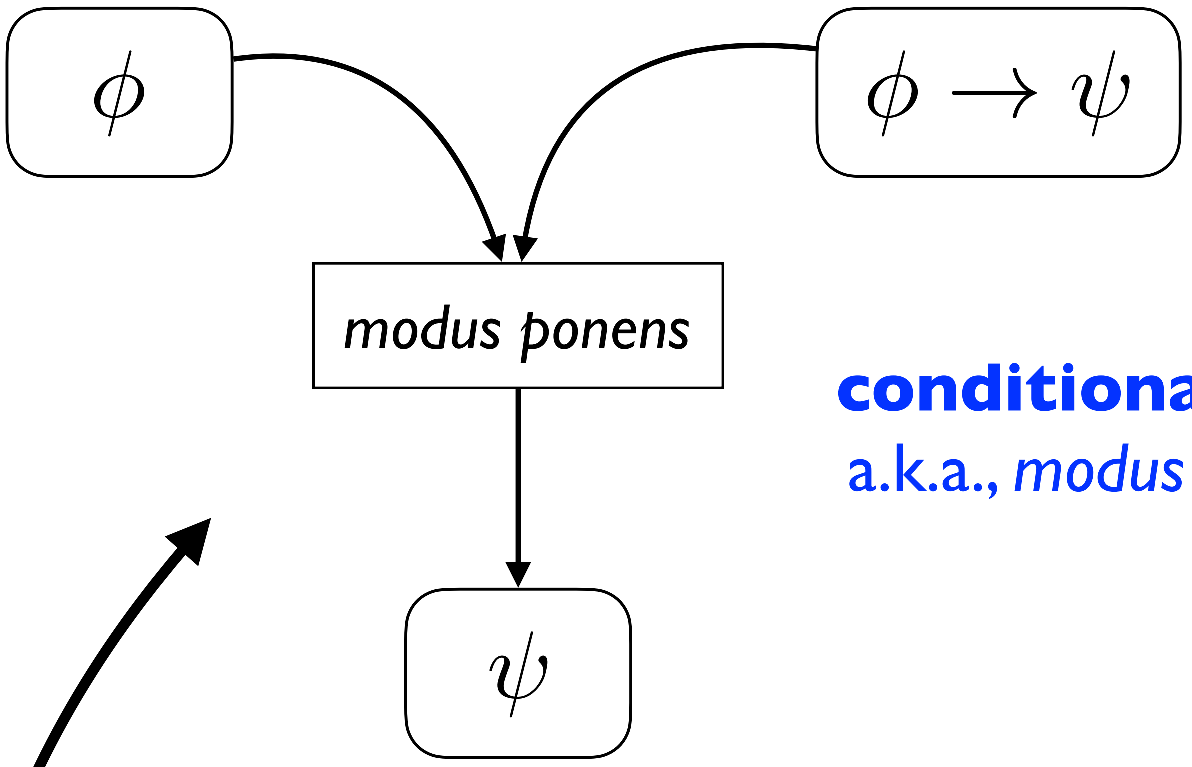

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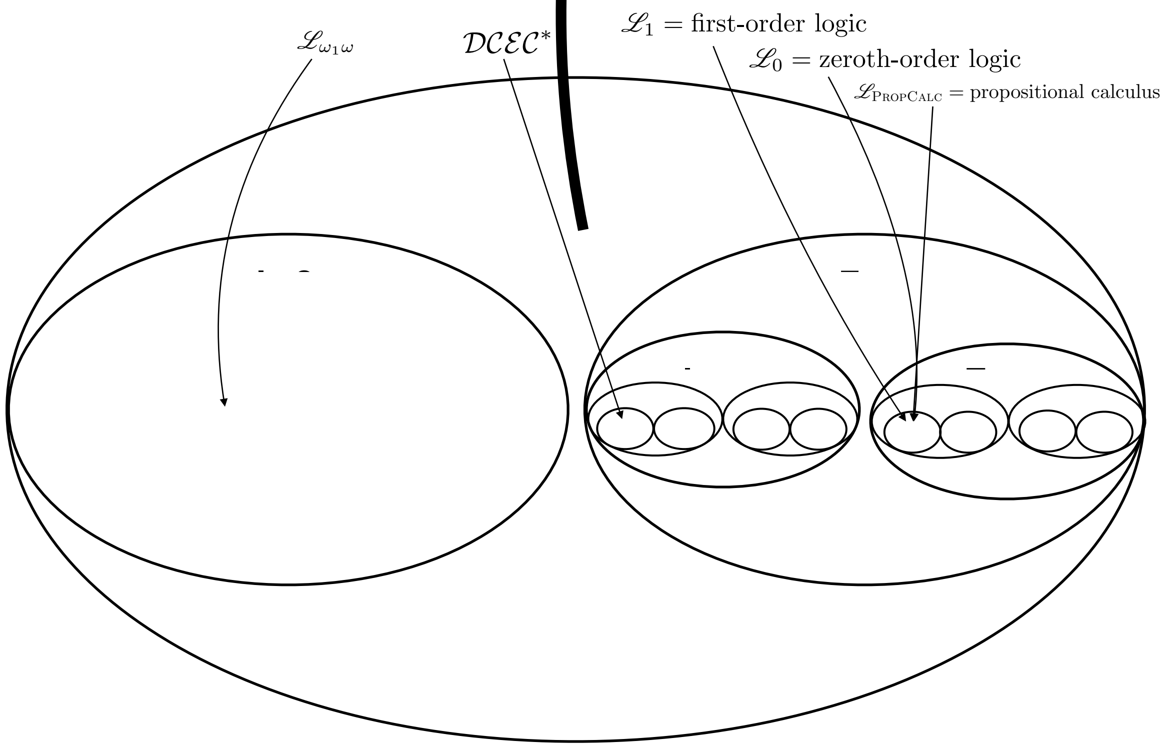
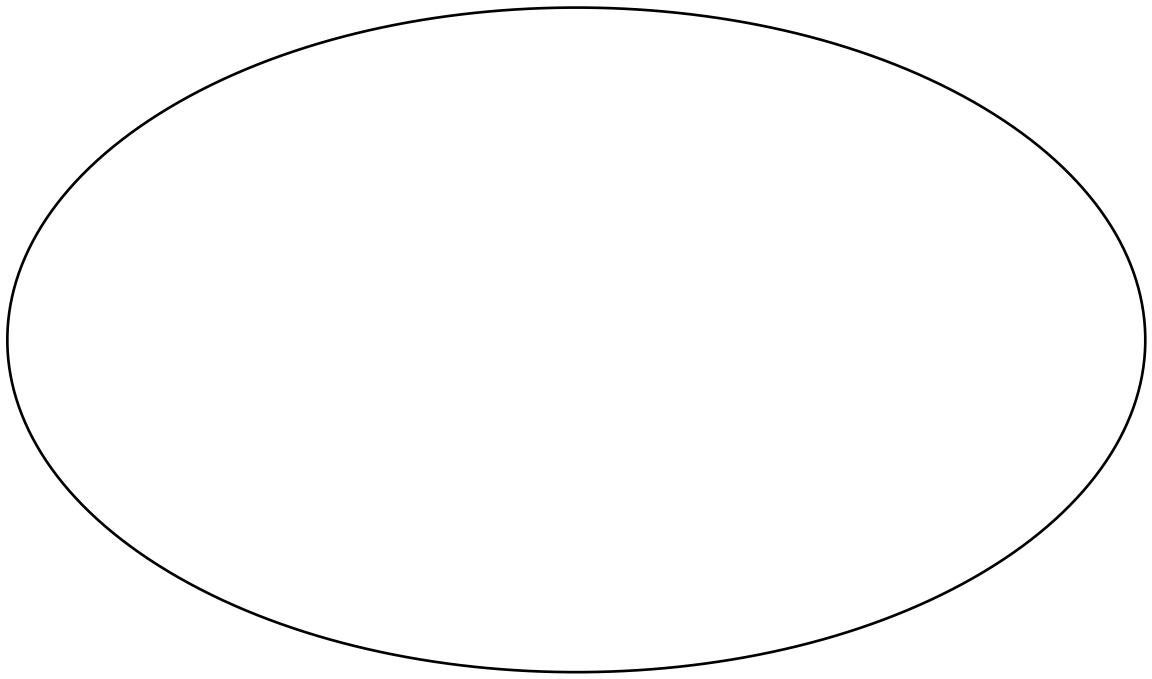
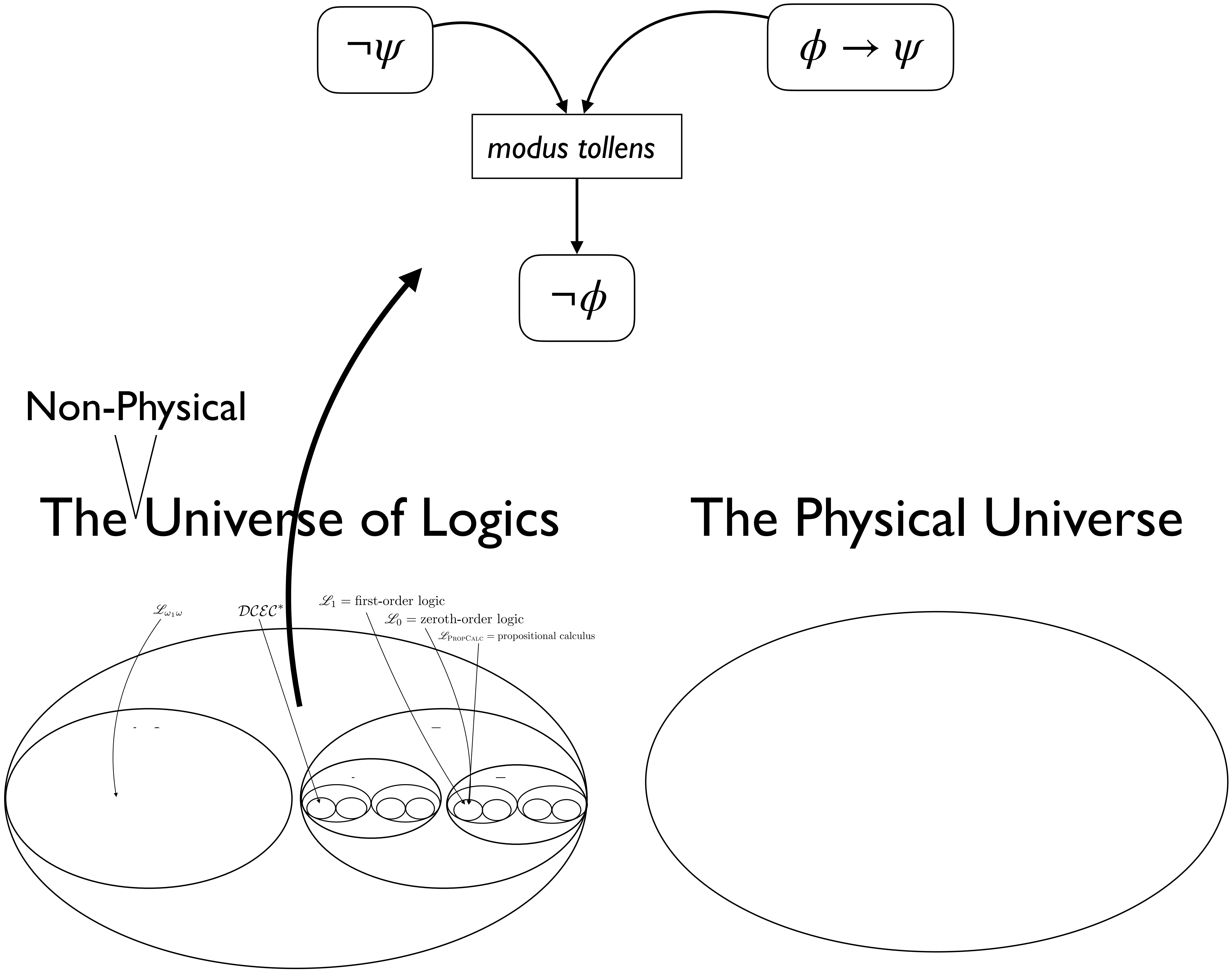
Non-Physical

The Universe of Logics

The Physical Universe





$\neg\psi$
$\phi \rightarrow \psi$
modus tollens
$\neg\phi$
Non-Physical
The Universe of Logics
The Physical Universe
$\mathscr{L}_{\omega_1\omega}$
$\mathcal{DCEC}^*$
$\mathscr{L}_1$ = first-order logic
$\mathscr{L}_0$ = zeroth-order logic
$\mathscr{L}_{\text{PropCalc}}$ = propositional calculus

Mathematical Objects Are Non-Physical, So We Are Too

Selmer Bringsjord *&* **Naveen Sundar Govindarajulu**

`version 0125221730NY`

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itself is at bottom a simple recursive one. (There are now numerous variants, but we ignore this for efficiency.) The algorithm is to receive an array of ordered objects, for example

$$\left\langle \boxed{5}\ \ \boxed{9}\ \ \boxed{10}\ \ \boxed{7}\ \ \boxed{4}\ \ \boxed{3}\ \ \boxed{11}\ \ \boxed{8}\ \ \boxed{6} \right\rangle,$$

and to then produce as output the sorted version of this input, which in this case is:

$$\left\langle \boxed{3}\ \ \boxed{4}\ \ \boxed{5}\ \ \boxed{6}\ \ \boxed{7}\ \ \boxed{8}\ \ \boxed{9}\ \ \boxed{10}\ \ \boxed{11} \right\rangle.$$

So, what's the algorithm? In order to answer this question, we can't avoid resorting to what we can call *embodiments* or *tokens* of the general and abstract *type* $\mathcal{Q}$.[9] This terminology, and the associated concrete practice, is easy to grasp. For an example, we give one high-level embodiment/token $\hat{\mathcal{Q}}_1$ of $\mathcal{Q}$ that views the algorithm as a three-stage one.[10] Before supplying the example in question, we draw the reader's attention to what we just did with a bit of suggestive notation: We used "hat" $\hat{O}$ to indicate that what is being referred to an an embodiment of the thing O (in this case, of course, an algorithm). Hence, the hat in '$\hat{\mathcal{Q}}_1$' says that we have here an embodiment of the algorithm $\mathcal{Q}$ itself. Very well, and now to the embodiment in question itself:

I Pick the rightmost element in the array as the *pivot*.
II Partition the array so that all elements in the array less than the pivot are before it, and all elements greater than the pivot are placed after it.
III Recursively apply both I and II to the sub-array now before the pivot, as well as to the sub-array now after the pivot.

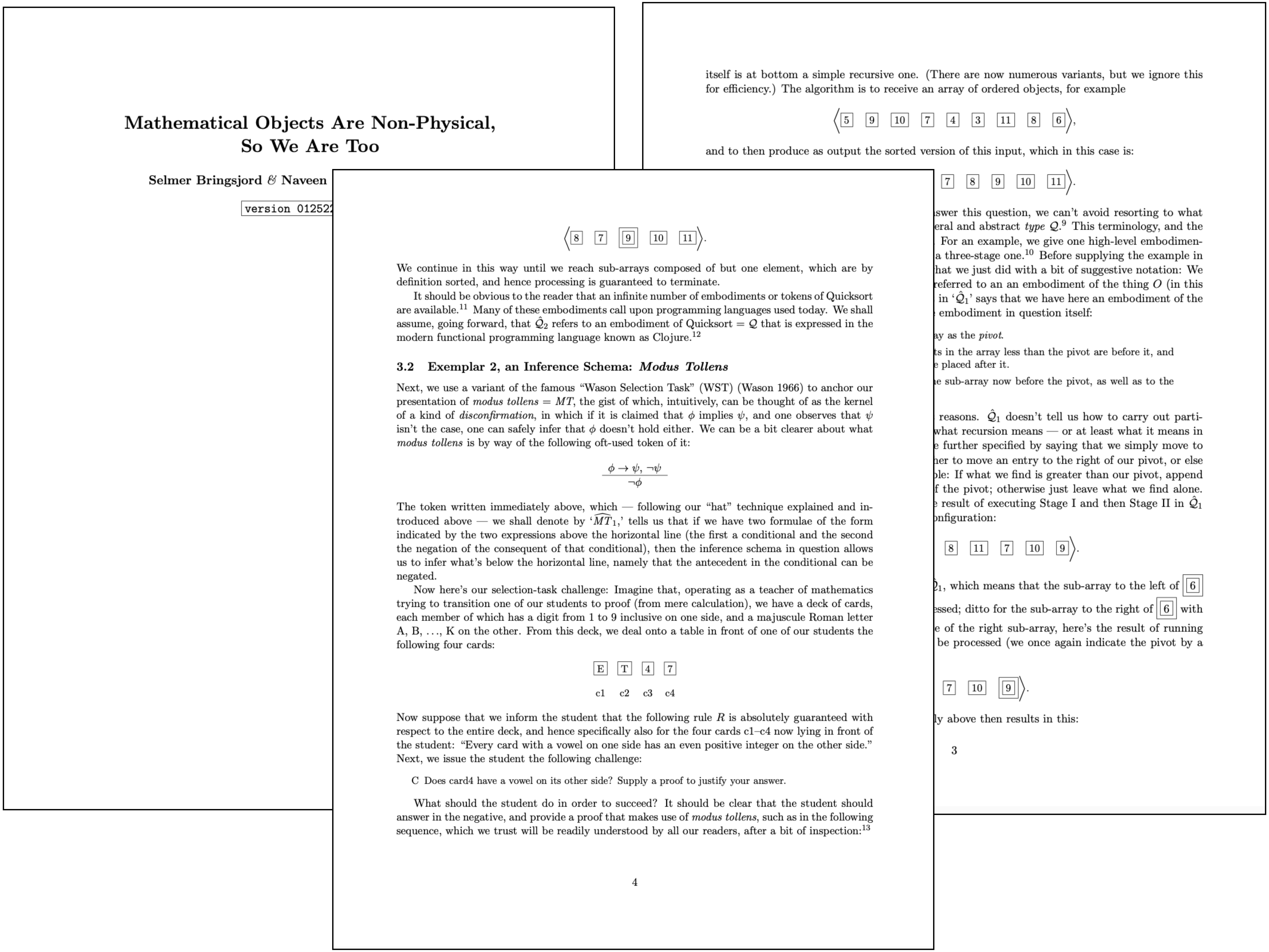
This is said to be 'high-level' for obvious reasons. $\hat{\mathcal{Q}}_1$ doesn't tell us how to carry out partitioning, and it relies on an understanding of what recursion means — or at least what it means in this context. But no worries: Stage II can be further specified by saying that we simply move to the left one entry at a time, and decide whether to move an entry to the right of our pivot, or else leave it where it is. And how to decide? Simple: If what we find is greater than our pivot, append it to whatever sub-sequence is to the right of the pivot; otherwise just leave what we find alone. Using a double-box to indicate our pivot, the result of executing Stage I and then Stage II in $\hat{\mathcal{Q}}_1$ on the initial input array will result in this configuration:

$$\left\langle \boxed{5}\ \ \boxed{4}\ \ \boxed{3}\ \ \boxed{\boxed{6}}\ \ \boxed{8}\ \ \boxed{11}\ \ \boxed{7}\ \ \boxed{10}\ \ \boxed{9} \right\rangle.$$

Now the algorithm calls for Stage III in $\hat{\mathcal{Q}}_1$, which means that the sub-array to the left of $\boxed{\boxed{6}}$ with $\boxed{3}$ as the pivot of this sub-array is processed; ditto for the sub-array to the right of $\boxed{\boxed{6}}$ with $\boxed{9}$ as the pivot of this sub-array. In the case of the right sub-array, here's the result of running Stage I, which is to be passed to Stage II to be processed (we once again indicate the pivot by a double-box):

$$\left\langle \boxed{8}\ \ \boxed{11}\ \ \boxed{7}\ \ \boxed{10}\ \ \boxed{\boxed{9}} \right\rangle.$$

Stage II applied to the input to it immediately above then results in this:



Mathematical Objects Are Non-Physical,
So We Are Too

Selmer Bringsjord & Naveen

`version 01252`

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and to then produce as output the sorted version of this input, which in this case is:

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$$\left\langle \boxed{8}\ \ \boxed{7}\ \ \boxed{\boxed{9}}\ \ \boxed{10}\ \ \boxed{11} \right\rangle.$$

We continue in this way until we reach sub-arrays composed of but one element, which are by definition sorted, and hence processing is guaranteed to terminate.

It should be obvious to the reader that an infinite number of embodiments or tokens of Quicksort are available.[11] Many of these embodiments call upon programming languages used today. We shall assume, going forward, that $\hat{\mathcal{Q}}_2$ refers to an embodiment of Quicksort $= \mathcal{Q}$ that is expressed in the modern functional programming language known as Clojure.[12]

3.2 Exemplar 2, an Inference Schema: *Modus Tollens*

Next, we use a variant of the famous "Wason Selection Task" (WST) (Wason 1966) to anchor our presentation of *modus tollens* $= MT$, the gist of which, intuitively, can be thought of as the kernel of a kind of *disconfirmation*, in which if it is claimed that ϕ implies ψ, and one observes that ψ isn't the case, one can safely infer that ϕ doesn't hold either. We can be a bit clearer about what *modus tollens* is by way of the following oft-used token of it:

$$\frac{\phi \rightarrow \psi,\ \neg\psi}{\neg\phi}$$

The token written immediately above, which — following our "hat" technique explained and introduced above — we shall denote by '$\widehat{MT}_1$,' tells us that if we have two formulae of the form indicated by the two expressions above the horizontal line (the first a conditional and the second the negation of the consequent of that conditional), then the inference schema in question allows us to infer what's below the horizontal line, namely that the antecedent in the conditional can be negated.

Now here's our selection-task challenge: Imagine that, operating as a teacher of mathematics trying to transition one of our students to proof (from mere calculation), we have a deck of cards, each member of which has a digit from 1 to 9 inclusive on one side, and a majuscule Roman letter A, B, …, K on the other. From this deck, we deal onto a table in front of one of our students the following four cards:

E	T	4	7
c1	c2	c3	c4

Now suppose that we inform the student that the following rule R is absolutely guaranteed with respect to the entire deck, and hence specifically also for the four cards c1–c4 now lying in front of the student: "Every card with a vowel on one side has an even positive integer on the other side." Next, we issue the student the following challenge:

C Does card4 have a vowel on its other side? Supply a proof to justify your answer.

What should the student do in order to succeed? It should be clear that the student should answer in the negative, and provide a proof that makes use of *modus tollens*, such as in the following sequence, which we trust will be readily understood by all our readers, after a bit of inspection:[13]

Next problem (King-Ace) …

King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

King-Ace 2

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If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

There is an ace in the hand.

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What can you infer from this premise?

NO! ~~There is an ace in the hand.~~

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Suppose that the following premise is true:

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What can you infer from this premise?

NO! ~~There is an ace in the hand.~~ NO!

In fact, what you *can* infer is that there *isn't* an ace in the hand!

King-Ace Solved

Proposition: There is *not* an ace in the hand.

Proof: We know that at least one of the if-thens (i.e., at least one of the **conditionals**) is false. So we have two cases to consider, viz., that *K* => *A* is false, and that ¬*K* => *A* is false. Take first the first case; accordingly, suppose that *K* => *A* is false. Then it follows that *K* is true (since when a conditional is false, its antecedent holds but its consequent doesn't), and *A* is false. Now consider the second case, which consists in ¬*K* => *A* being false. Here, in a direct parallel, we know ¬*K* and, once again, ¬*A*. In both of our two cases, which are exhaustive, there is no ace in the hand. The proposition is established. **QED**

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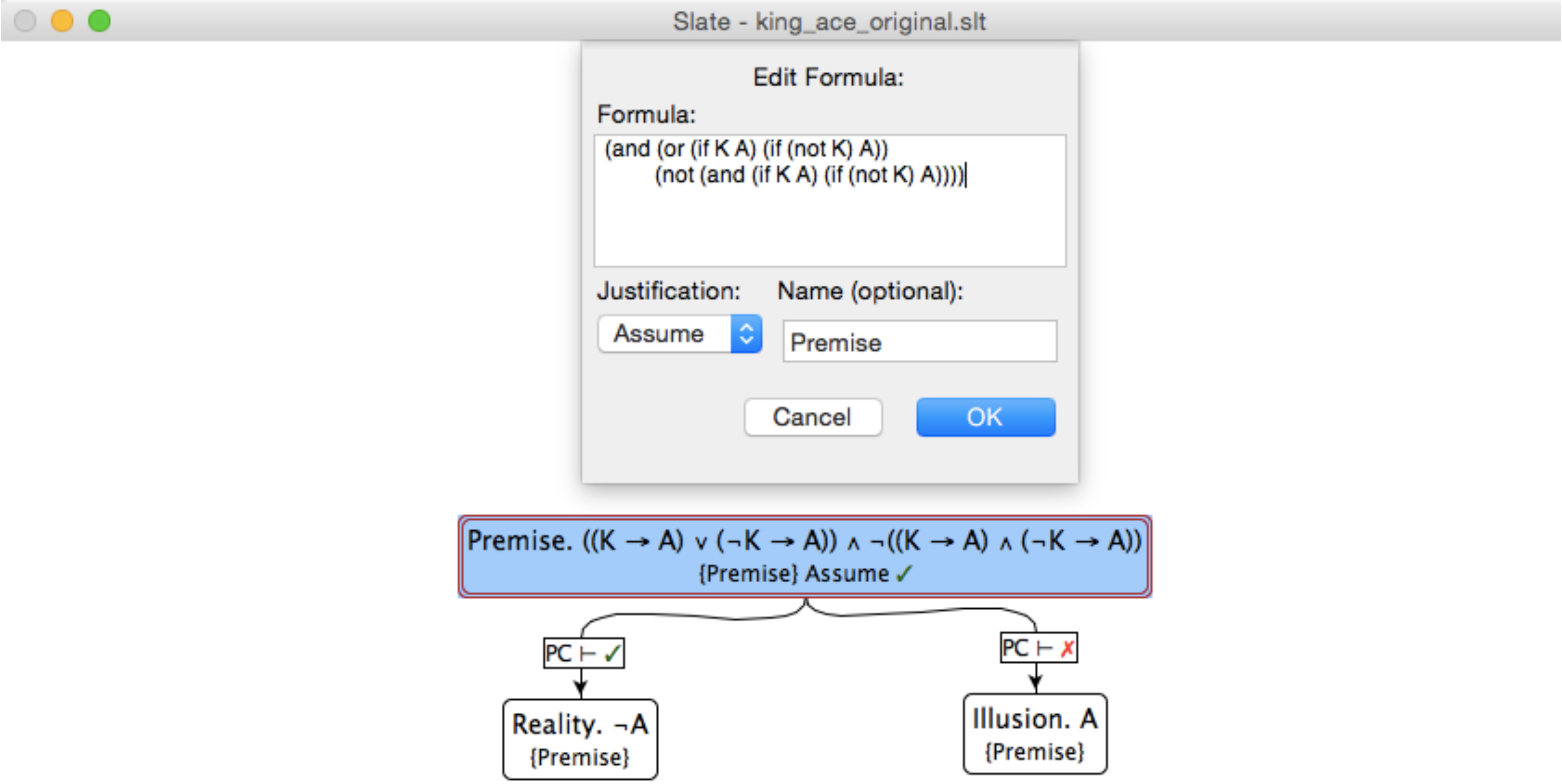


Formula:

```
(and (or (if K A) (if (not K) A))
      (not (and (if K A) (if (not K) A))))
```

Justification: Assume

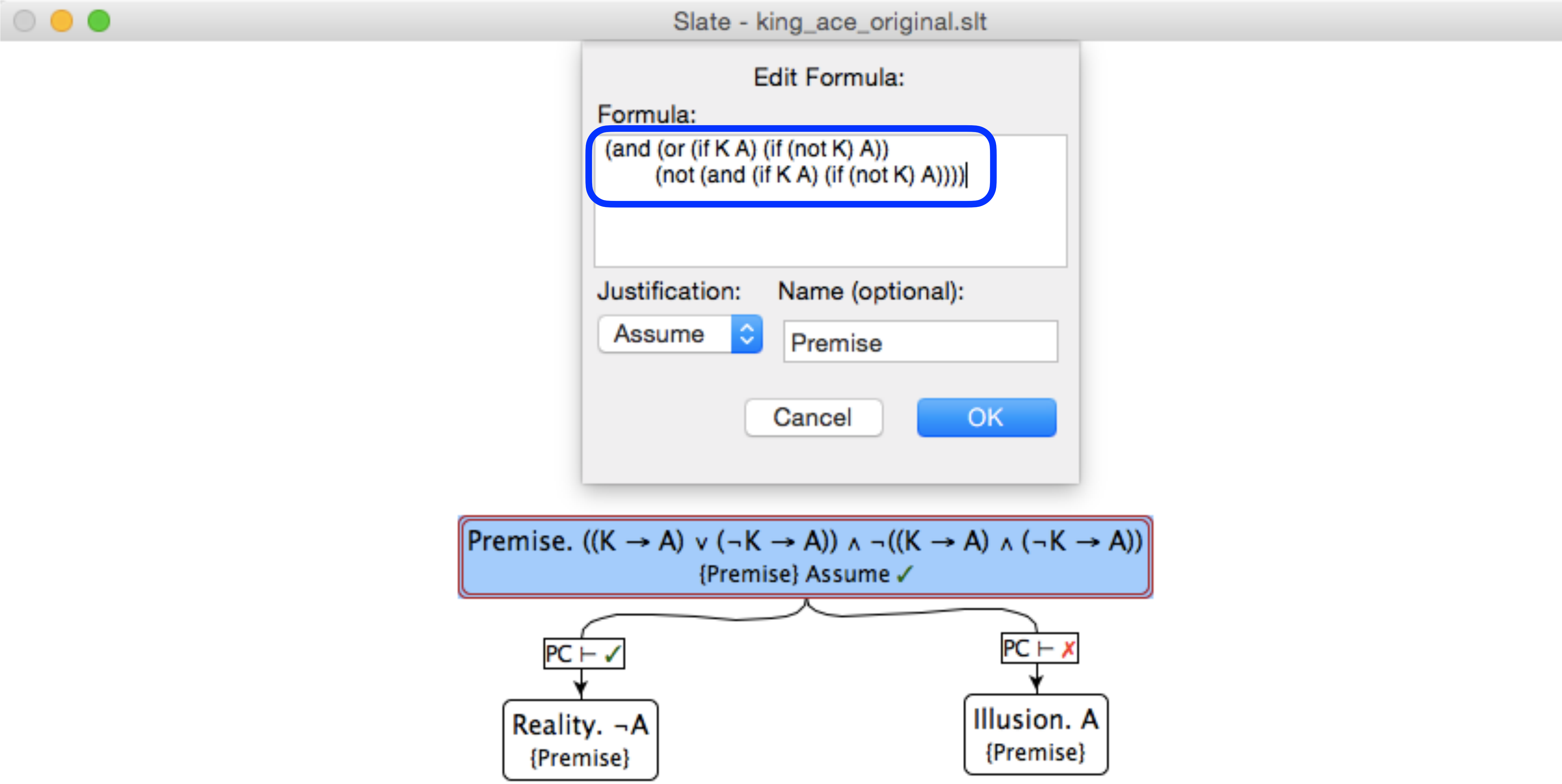
Name (optional): Premise

Premise. $((K \to A) \lor (\neg K \to A)) \land \neg((K \to A) \land (\neg K \to A))$
{Premise} Assume ✓

PC ⊢ ✓ → Reality. $\neg A$
{Premise}

PC ⊢ ✗ → Illusion. A
{Premise}

Study the S-expression



Slate - king_ace_original.slt

Edit Formula:

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```
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Justification: Assume

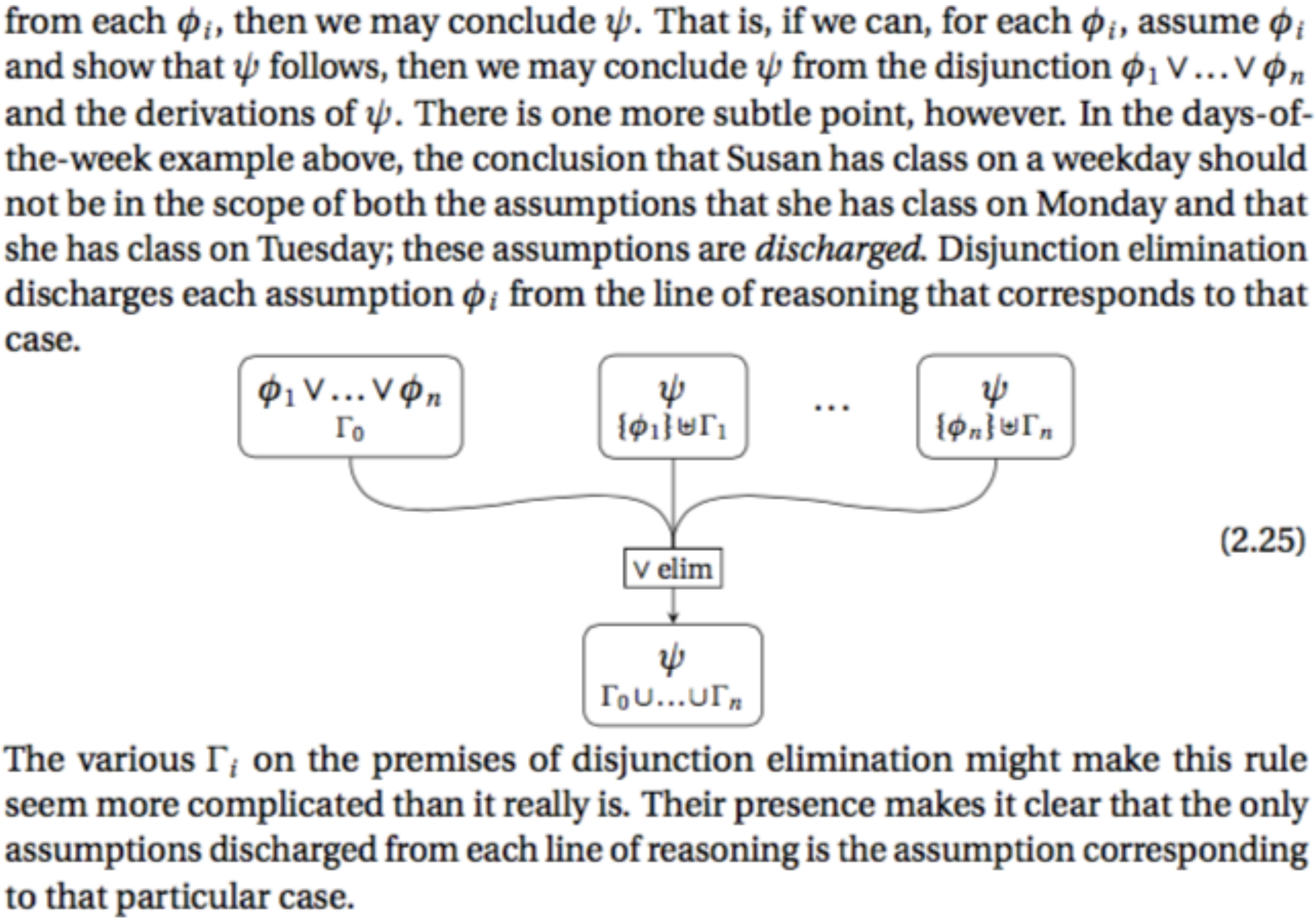
Name (optional): Premise

We need another rule of inference to crack this problem … …

We need another rule of inference to crack this problem … …

`disjunction elimination`

From ~ p. 54 in LAMA-BDLA



from each ϕ_i, then we may conclude ψ. That is, if we can, for each ϕ_i, assume ϕ_i and show that ψ follows, then we may conclude ψ from the disjunction $\phi_1 \vee \ldots \vee \phi_n$ and the derivations of ψ. There is one more subtle point, however. In the days-of-the-week example above, the conclusion that Susan has class on a weekday should not be in the scope of both the assumptions that she has class on Monday and that she has class on Tuesday; these assumptions are *discharged*. Disjunction elimination discharges each assumption ϕ_i from the line of reasoning that corresponds to that case.

(2.25)

The various Γ_i on the premises of disjunction elimination might make this rule seem more complicated than it really is. Their presence makes it clear that the only assumptions discharged from each line of reasoning is the assumption corresponding to that particular case.

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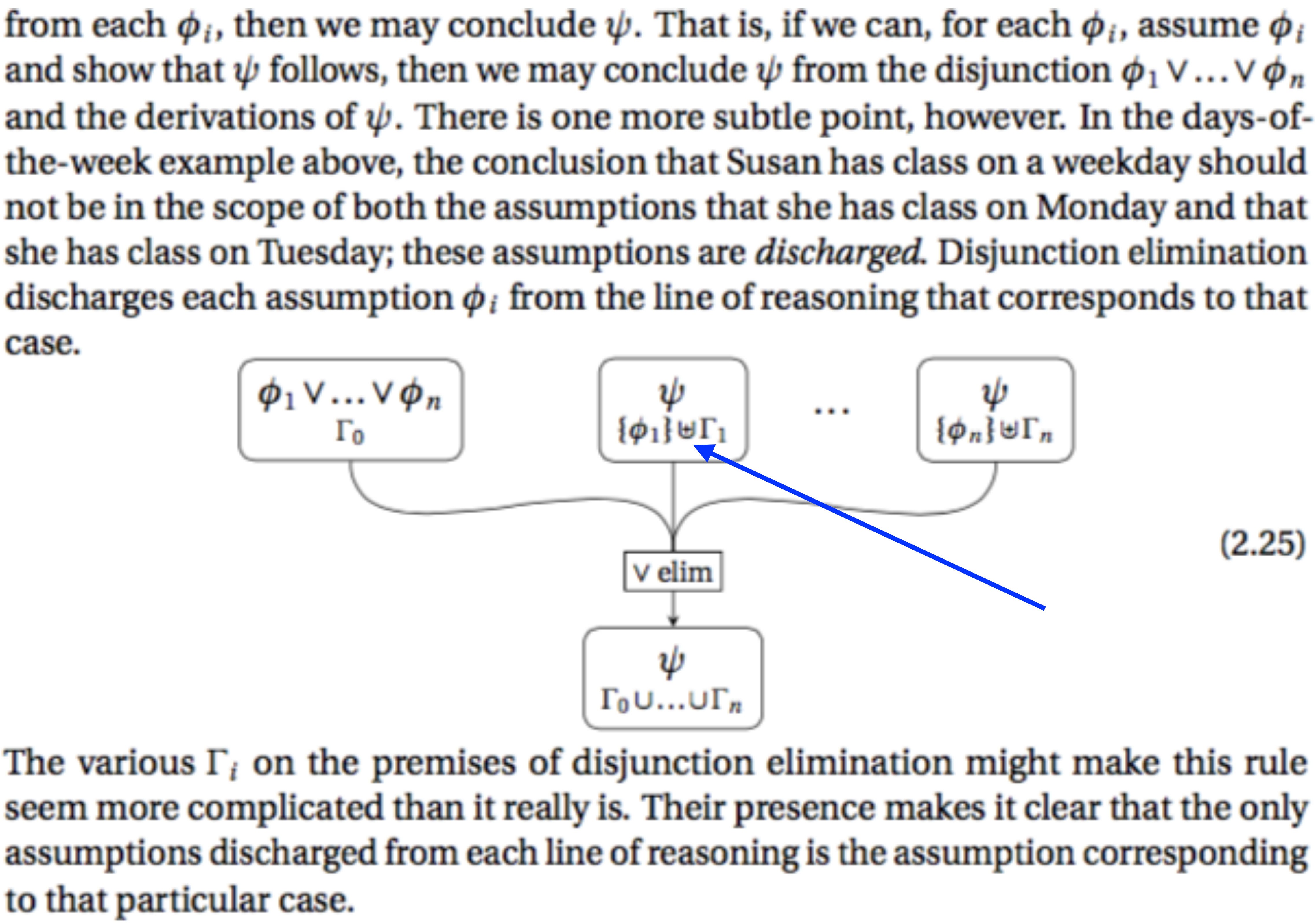

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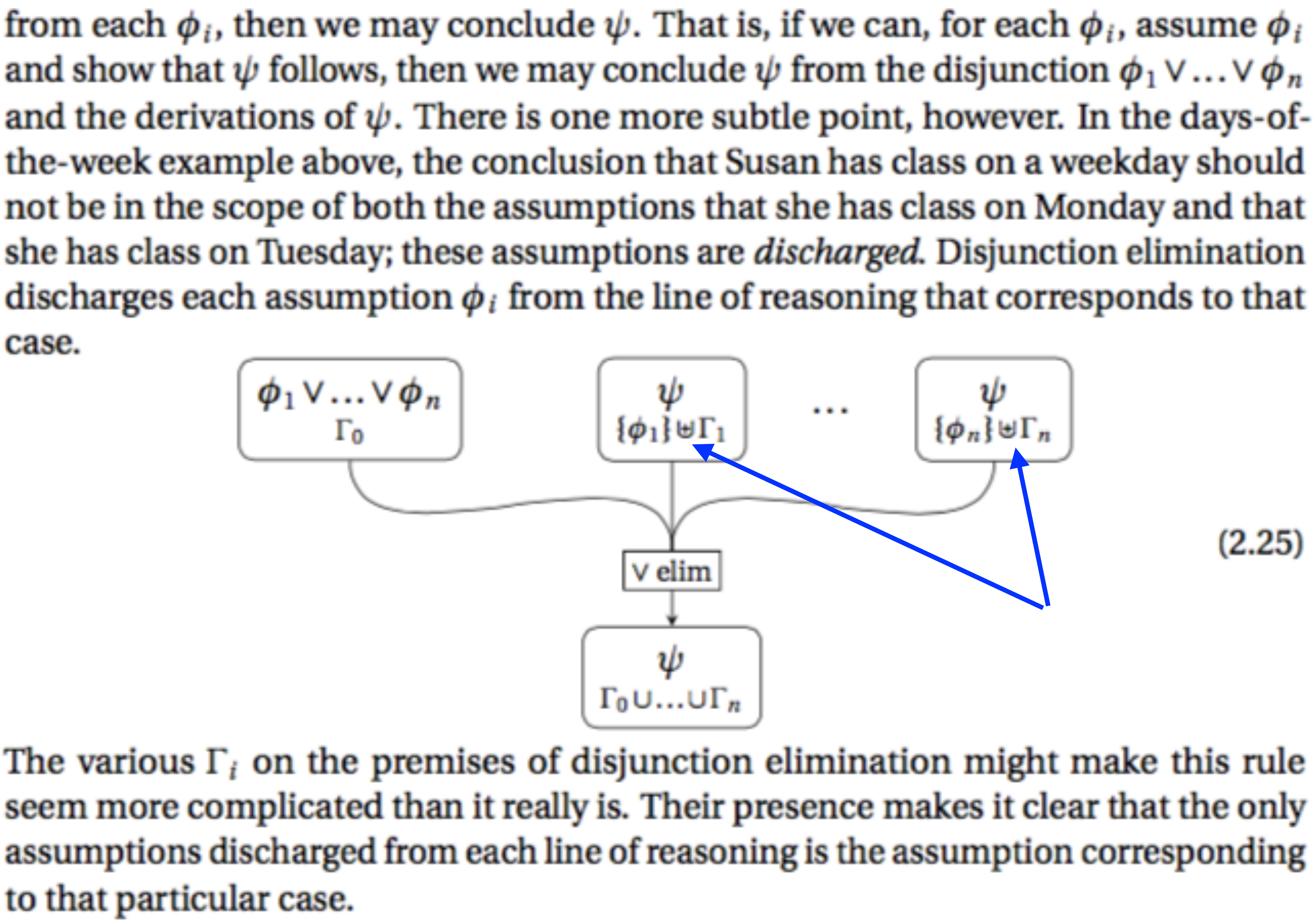

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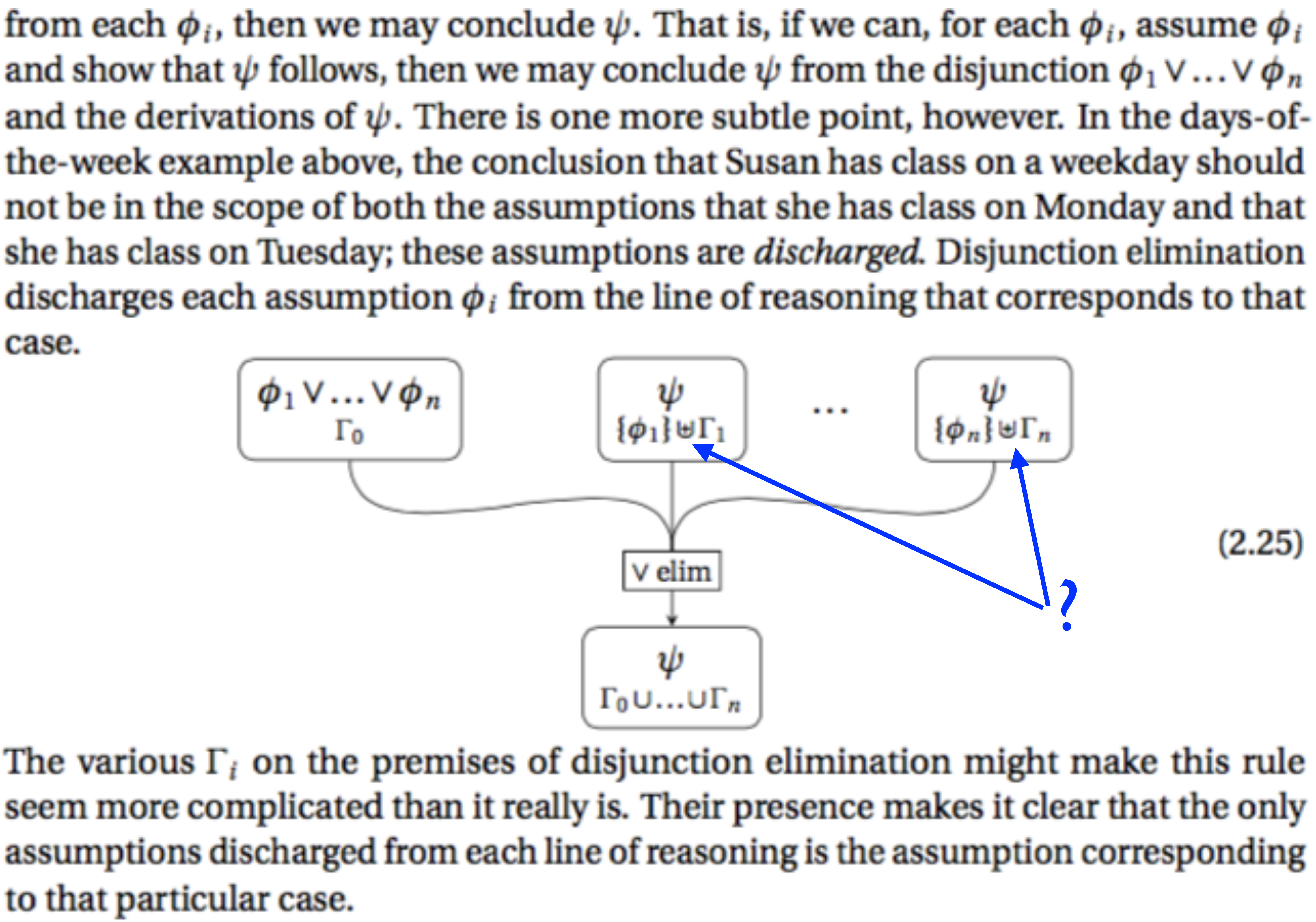

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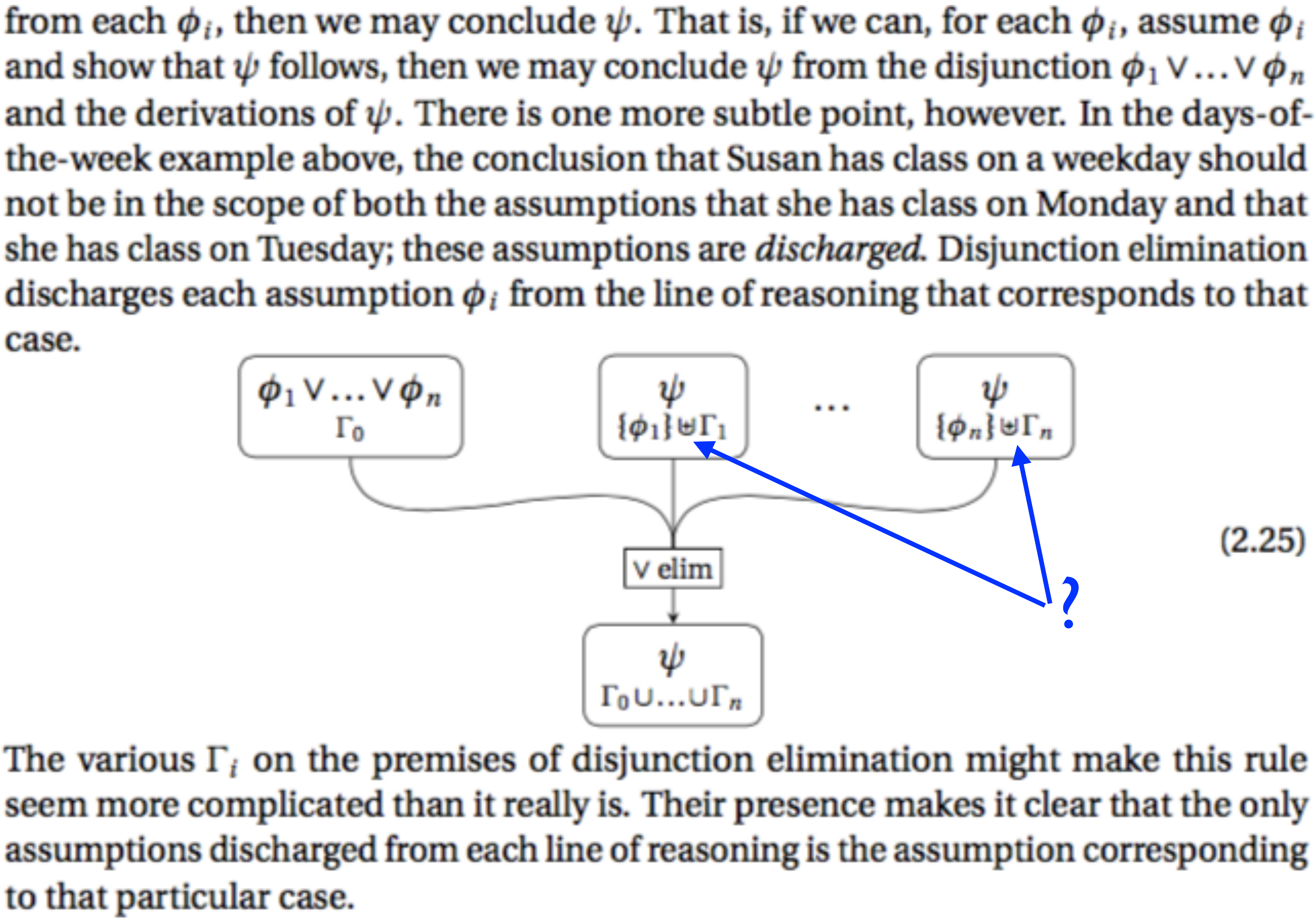

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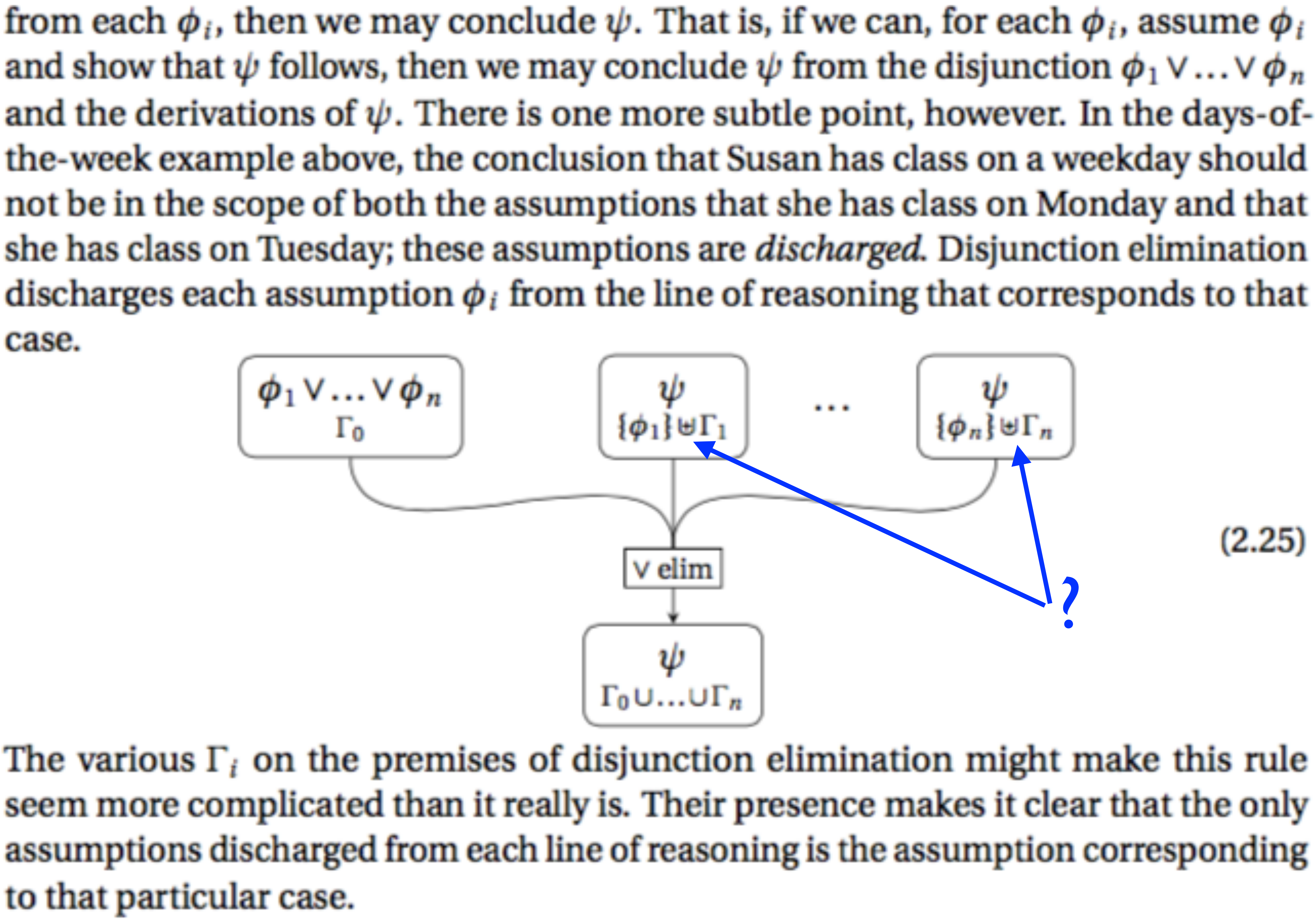

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King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

NO! ~~There is an ace in the hand.~~ NO!

In fact, what you *can* infer is that there *isn't* an ace in the hand!

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the ha en
there i
statem

Future Required problem (on HyperGrader®): You will need to finish the proof in HyperSlate® — with no remaining use of an oracle.

What can you infer from this premise?

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Part II of Class