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The Liar; Russell's Paradox; *Toward* Thoraf's Paradox

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

IFLAI1
1/29/2024

RAIR
Rensselaer AI and Reasoning Lab

Logistics …

Test 1 grades now appear in your HG® account, in “My Progression” (but not Test 2 and Test 3).

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If you wish to attempt "resurrection" from a Test-1 grade of C (or nothing), pls email me.

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If you wish to attempt “resurrection” from a Test-1 grade of C (or nothing), pls email me.

Test 2 will start one class later than the syllabus’ schedule. March 11 and 14 in the syllabus will be switched, so that immediately after break you don’t face a test, and James will lecture on March 11.

Types of Paradoxes

- Deductive Paradoxes
- Inductive Paradoxes — coming (e.g. The Lottery Paradox & The St Petersburg Paradox)

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First:

- Deductive Paradoxes
- Inductive Paradoxes — coming (e.g. The Lottery Paradox & The St Petersburg Paradox)

The Liar (Paradox) …

The (Economical) Liar

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L: This sentence is false.

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If T(**L)** then ¬T(**L**)

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T(**L**) iff (i.e., if & only if) ¬T(L)

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Contradiction!

The (Verbose) Liar — With a Twist

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Theorem: 2+2 = 5.

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Proof: Set:

The (Verbose) Liar — With a Twist

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The (Verbose) Liar — With a Twist

Theorem: 2+2 = 5.

Proof: Set:

L: This sentence is false.

L is either true or false. Suppose that it's true. Then since what it says is that it's false, it *is* false; i.e., **L** is false, on this supposition. So we've proved that if **L** is true, L is false. Now suppose instead that **L** is false. Then since it says that it's false, it's true; i.e., **L** is true, on our current supposition. We have thus proved that if **L** is false, **L** is true. Combining the conditionals we've proved yields this: **L** is true if and only if **L** is false, which is a contradiction. (*P* if and only if ¬*P* is logically equivalent to *P* and ¬*P*.) By inference schema *explosion*, it follows that 2+2 = 5. **QED**

Outlawing Self-Referential Sentences Isn't the Answer!

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  - `This sentence is a sentence.`

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  - `This sentence is a sentence.`
  - `This sentence contains the letter 'r'.`

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  - `This sentence has more than three letters in it.`

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  - `This sentence has more than three letters in it.`
  - `This sentence ends with a period, starts with a capital 'T', and has more than two words.`

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  - ...

Outlawing Self-Referential Sentences Isn't the Answer!

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Box 1

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1

Box 2

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1	Box 2
The sentence in Box 2 is true.	The sentence in Box 1 is false.

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1		Box 2
The sentence in Box 2 is true.	Neither sentence is self-referential.	The sentence in Box 1 is false.

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1

The sentence in Box 2 is true.

Neither sentence is self-referential.

Box 2

The sentence in Box 1 is false.

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1		Box 2
The sentence in Box 2 is true.	Neither sentence is self-referential.	The sentence in Box 1 is false.

Suppose that the sentence in Box 1 is true. Then the sentence in Box 2 is true (because the sentence in Box 1 says that that sentence is true). But then the sentence in Box 1 is false (because the sentence in Box 2 says that that sentence is false). So, if the sentence in Box 1 is true, it's false. On the other hand, by parallel deduction, if the sentence in Box 1 is false, the sentence in Box 1 is true. (Make sure you work out and verify the reasoning that establishes the previous sentence.) We thus have again a contradiction: The sentence in Box 1 is true if and only if it's not true.

Well do you have a solution, Selmer?

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Of course :). But …

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Further Reading …

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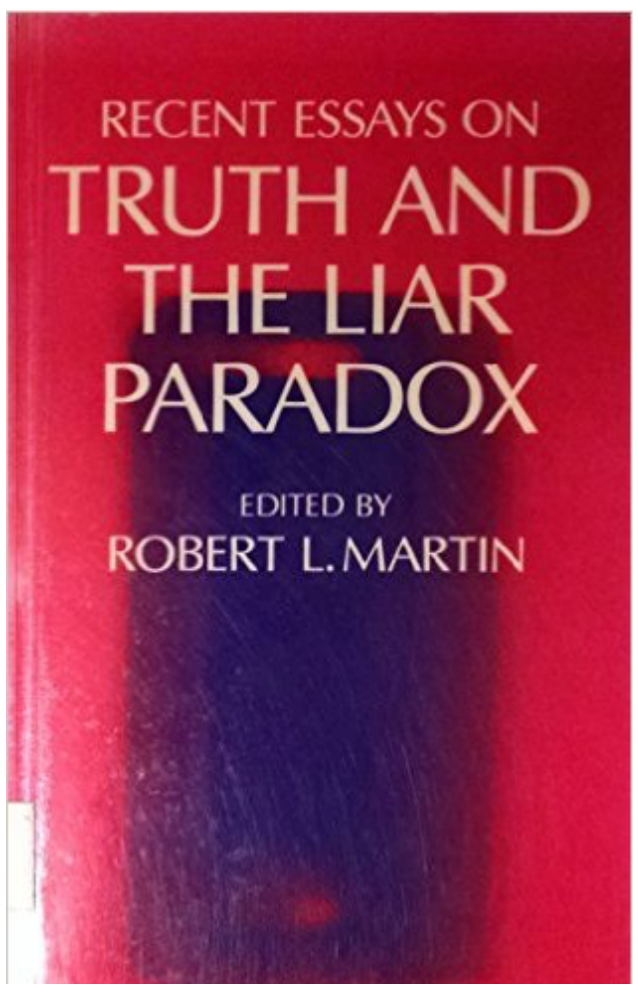

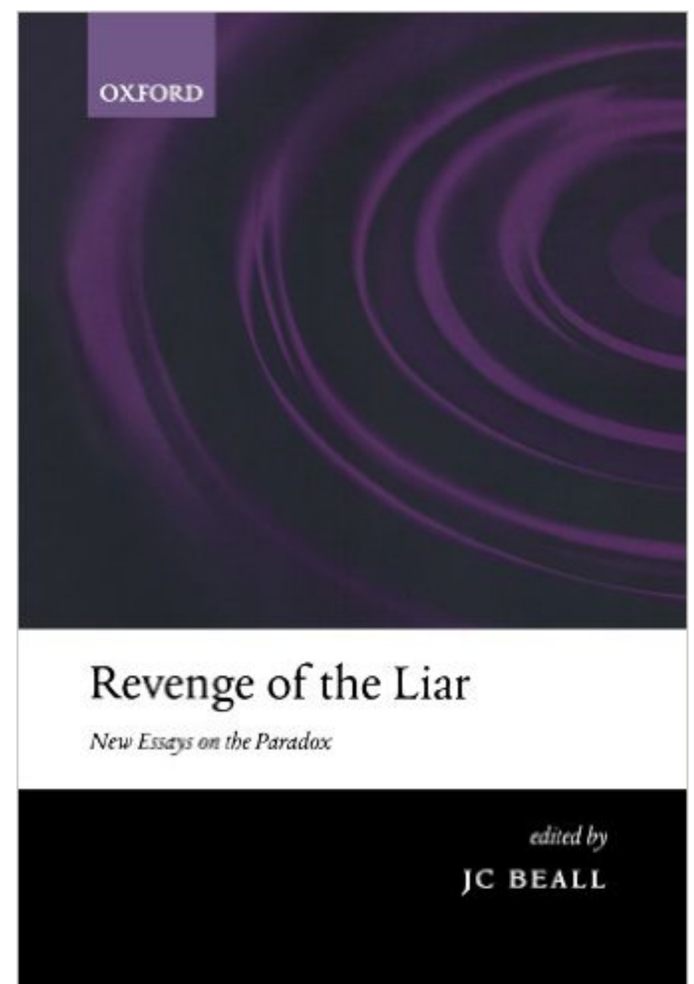

Russell's Paradox …

Friday's Hill, Haslemere, 16 June 1902

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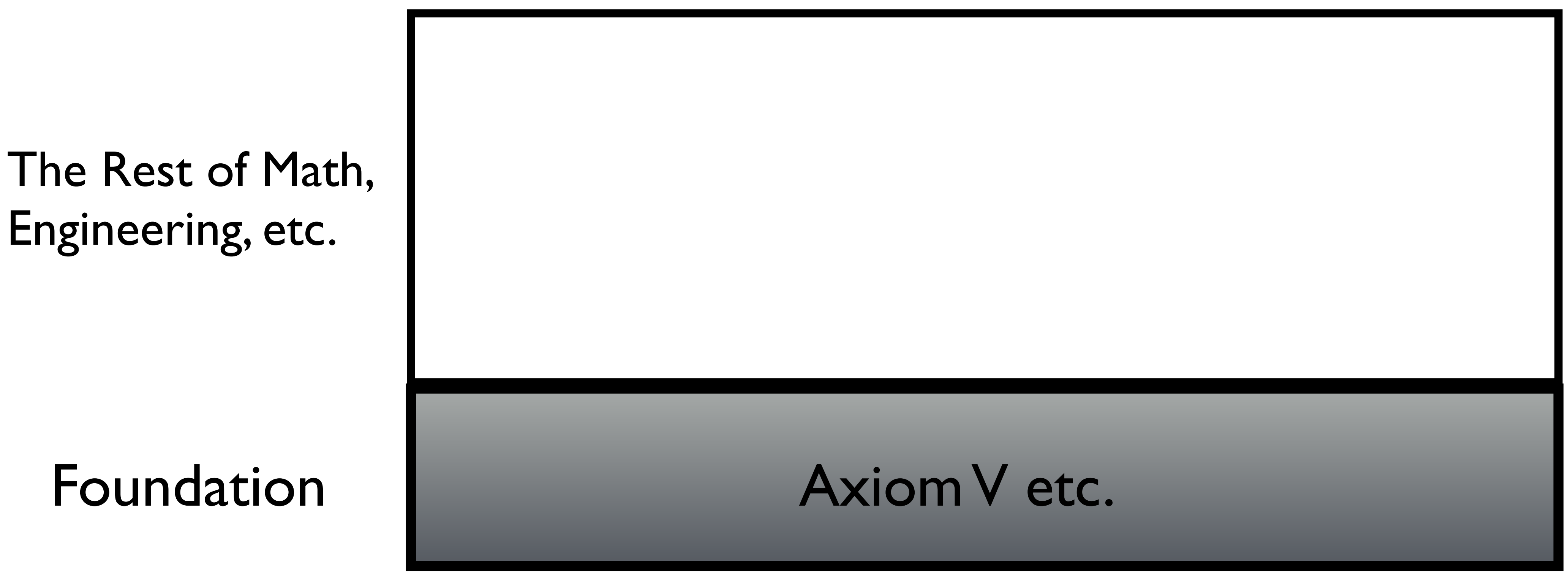
The Foundation Crumbles



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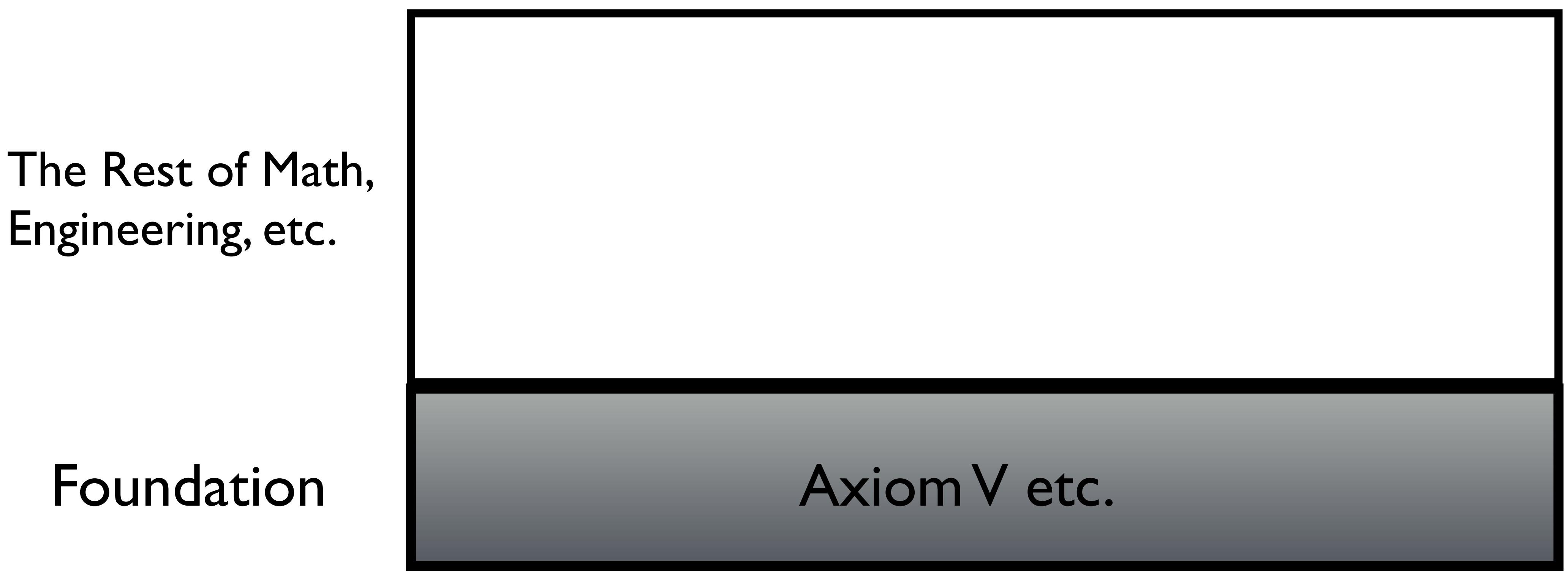

$$\text{Axiom V} \quad \exists x \forall y [y \in x \leftrightarrow \phi(y)]$$

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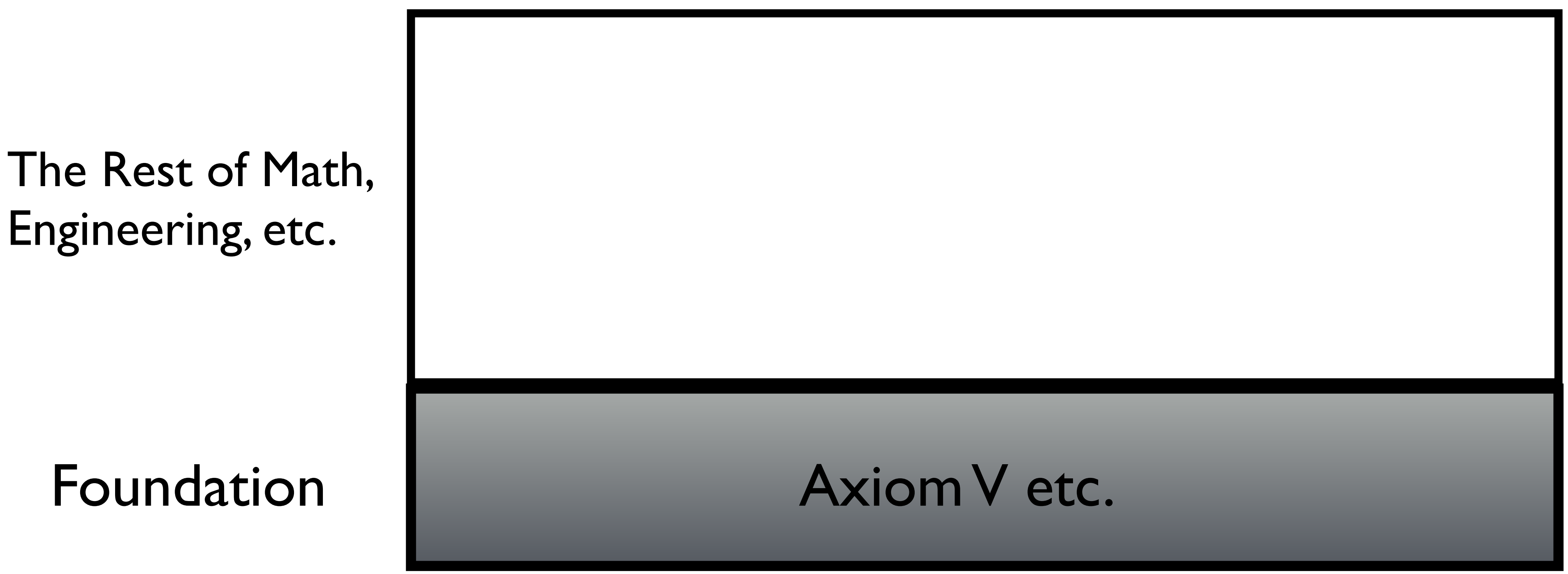

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a formula of arbitrary size in which the variable *y* is free; this formula ascribes a property to *y*

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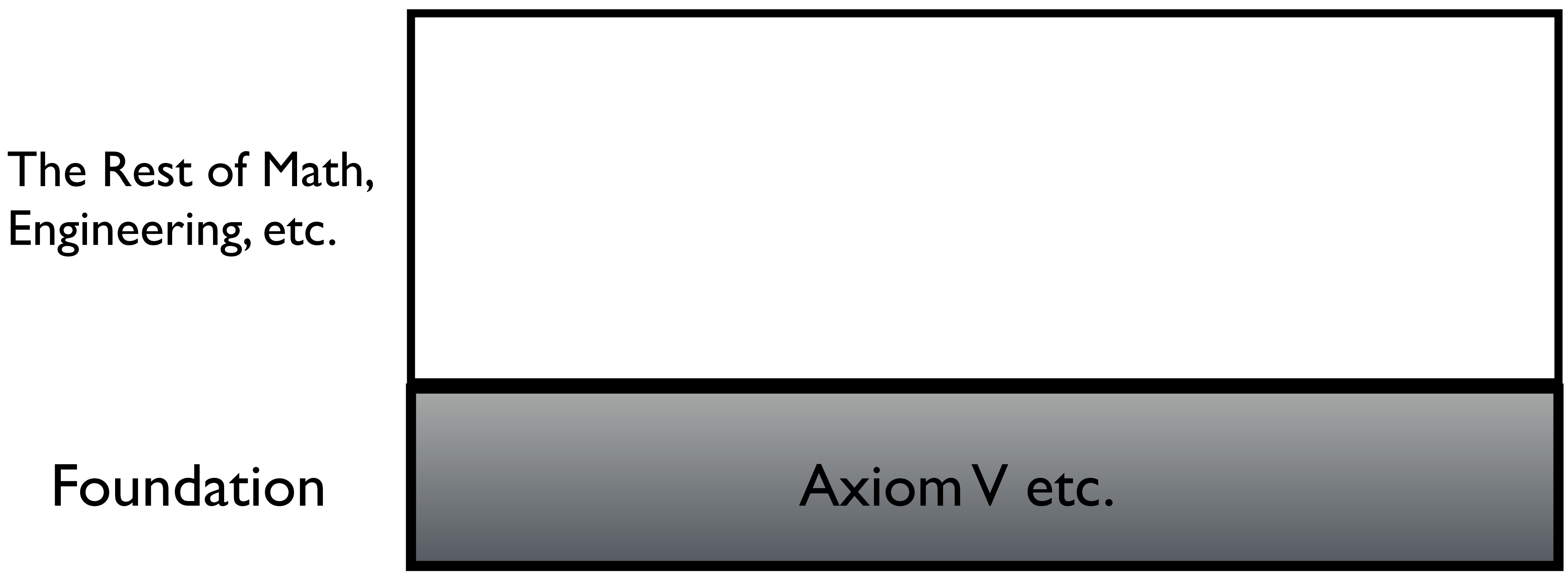

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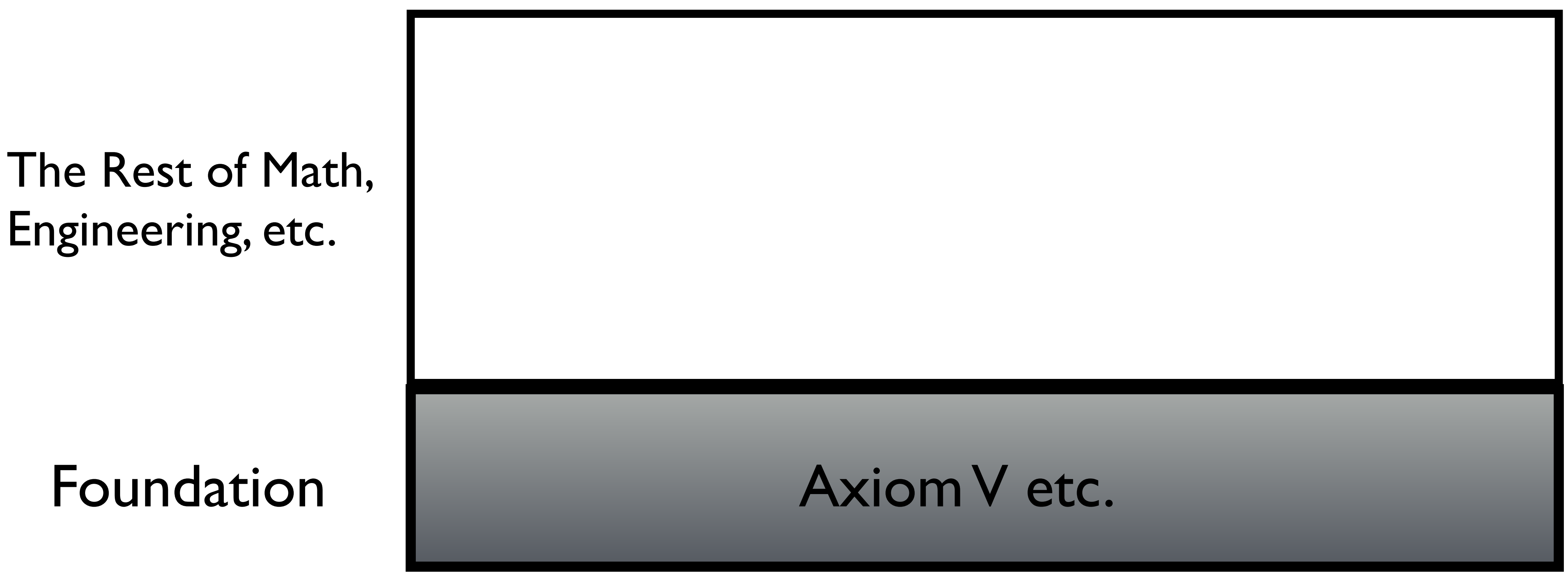

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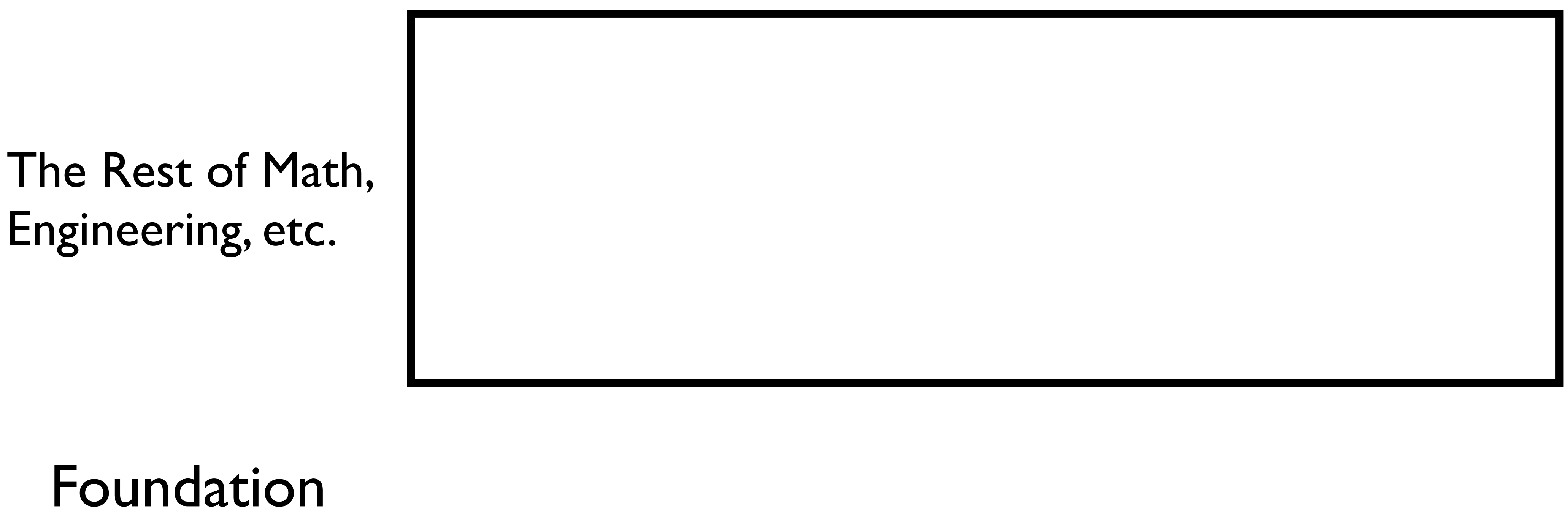

The Foundation Crumbles

The Rest of Math,
Engineering, etc.

Foundation

There was once a small town in Norway in which there was a barber who shaved all and only the men residing in the town who didn't shave themselves.

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There was once a small town in Norway in which there resided a male barber who shaved all and only the men residing in the town who didn't shave themselves.

Such a situation is impossible!

Such a situation is impossible!

Proof: Let's assume for the sake of argument that such a situation can be. Without loss of generality, let the town be Lyngdal and the male Lyngdalian barber be Olaf. Either Olaf shaves himself or he doesn't. But either case leads straight to a contradiction. Therefore the situation is in fact impossible. Here we go …

Suppose Olaf shaves himself. Then it follows that he doesn't shave himself. Suppose on the other hand that Olaf doesn't shave himself. Then is follows that he does shave himself. Hence, Olaf shaves himself if and only if he doesn't shave himself, which is a contradiction. **QED**

Axiom V $\quad \exists x \forall y[y \in x \leftrightarrow \phi(y)]$

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Russell's Theorem:

Axiom V $\exists x \forall y[y \in x \leftrightarrow \phi(y)]$

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$$\vdash \neg \exists x \forall y(y \in x \leftrightarrow y \notin y)$$

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(Take that, Frege!)

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http://plato.stanford.edu/entries/russell-paradox/#HOTP

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What about Thoraf’s paradox?

(Skolem’s Paradox)

(For a nice overview of Skolem’s Paradox, see https://plato.stanford.edu/entries/paradox-skolem.)

What about Thoraf's paradox?

(Skolem's Paradox)

3.8.4.1 Can First-Order Logic Capture Infinitude and Finitude?

Does the machinery introduced in the previous section enable us to show that the concepts of finitude and infinitude can be captured by suitable use of first-order logic? If so, how? We should first immediately sharpen this question, which as it stands is somewhat unclear. Let's first target the capturing of infinitude in FOL. Then our initial sharpening move is to stipulate that we are interested specifically in figuring out how we might use FOL to express that a set is countably infinite. (Recall that we defined what it is for a set to be countably infinite in §1.5.3.) In further sharpening of the intuitively expressed question that kicked off the present section, what shall be looking for is how to specify a set Φ that is such that a given interpretation

$$\mathfrak{I} \models \Phi \text{ iff domain } \mathscr{D} \text{ in } \mathfrak{I} \text{ is countably infinite}$$

where the set Φ contains only formulae in FOL. If we can somehow obtain such a set Φ, then we will have found a way to capture countable infinitude because the domain $\mathscr{D}$ here must be countably infinite. Can you meet this challenge, by drawing upon what was done in the previous section?

Now, what about finitude? Can it be captured by formulae in FOL? The question here can be taken to consist in the challenge to find a set Ψ such that a given interpretation

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3.8.4.1 Can First-Order Logic Capture Infinitude and Finitude?

Does the machinery introduced in the previous section enable us to show that the concepts of finitude and infinitude can be captured by suitable use of first-order logic? If so, how? We should first immediately sharpen this question, which as it stands is somewhat unclear. Let's first target the capturing of infinitude in FOL. Then our initial sharpening move is to stipulate that we are interested specifically in figuring out how we might use FOL to express that a set is countably infinite. (Recall that we defined what it is for a set to be countably infinite in §1.5.3.) In further sharpening of the intuitively expressed question that kicked off the present section, what shall be looking for is how to specify a set Φ that is such that a given interpretation

$$\mathfrak{I} \models \Phi \text{ iff domain } \mathscr{D} \text{ in } \mathfrak{I} \text{ is countably infinite}$$ $20!

where the set Φ contains only formulae in FOL. If we can somehow obtain such a set Φ, then we will have found a way to capture countable infinitude because the domain $\mathscr{D}$ here must be countably infinite. Can you meet this challenge, by drawing upon what was done in the previous section?

Now, what about finitude? Can it be captured by formulae in FOL? The question here can be taken to consist in the challenge to find a set Ψ such that a given interpretation

$$\mathfrak{I} \models \Psi \text{ iff domain } \mathscr{D} \text{ in } \mathfrak{I} \text{ is finite}$$

where, again, the set in question nce again contains only formulae in FOL.

$$\Phi := \{\underline{\phi_k} : k \in \mathbb{Z}^+\}$$

Remember from last time: This formula uses quantifiers to say there are at at least *k* objects.

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Any interpretation that renders all members of this set true must have a countably infinite domain, and *vice versa*!

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Prove it!

Remember from last time: This formula uses quantifiers to say there are at at least *k* objects.

$$\Phi := \{\underline{\phi_k} : k \in \mathbb{Z}^+\}$$

Any interpretation that renders all members of this set true must have a countably infinite domain, and *vice versa*!

Prove it!

Now, can you find a set of formulae s.t. any interpretation that renders all members of it true must have a *finite* domain, and *vice versa*?

Remember from last time: This formula uses quantifiers to say there are at at least *k* objects.

$$\Phi := \{\underline{\phi_k} : k \in \mathbb{Z}^+\}$$

Any interpretation that renders all members of this set true must have a countably infinite domain, and *vice versa*!

Prove it!

Now, can you find a set of formulae s.t. any interpretation that renders all members of it true must have a *finite* domain, and *vice versa*?

$1000!

Remember from last time: This formula uses quantifiers to say there are at at least *k* objects.

$$\Phi := \{\underline{\phi_k} : k \in \mathbb{Z}^+\}$$

Any interpretation that renders all members of this set true must have a countably infinite domain, and *vice versa*!

Prove it!

Now, can you find a set of formulae s.t. any interpretation that renders all members of it true must have a *finite* domain, and *vice versa*?

$1000!

Selmer.Bringsjord@gmail.com

Hvis du forstår det, kan du bevise det.

Part I: *Slutten — for i dag.*

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Part II: Hands-on Q&A & Review …