Astrologic; Second-Order Logic and the *k*-order Ladder; Second-Order Axiomatized Arithmetic; Gödel's "God Theorem" & Speedup Theorem

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Intro to Logic
3/18/2024

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FOL

√ FOL

√ FOL

Epistemic + FOL

$\mathbf{B}_d\mathbf{B}_v\mathbf{B}_dVv$

√ FOL

Epistemic + FOL (for coverage of "killer" robots, later)

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TOL

$\exists X[X(j) \wedge \neg X(m) \wedge S(X)]$

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Double-Minded Man

The Contemporary Craft of Creating Characters
Meets Today's Cognitive Architectures:
A Case Study in Expressivity*

Selmer Bringsjord • John Licato • Alexander Bringsjord

version of 0121161500NY

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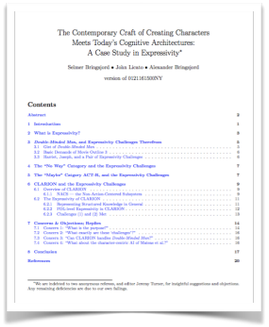

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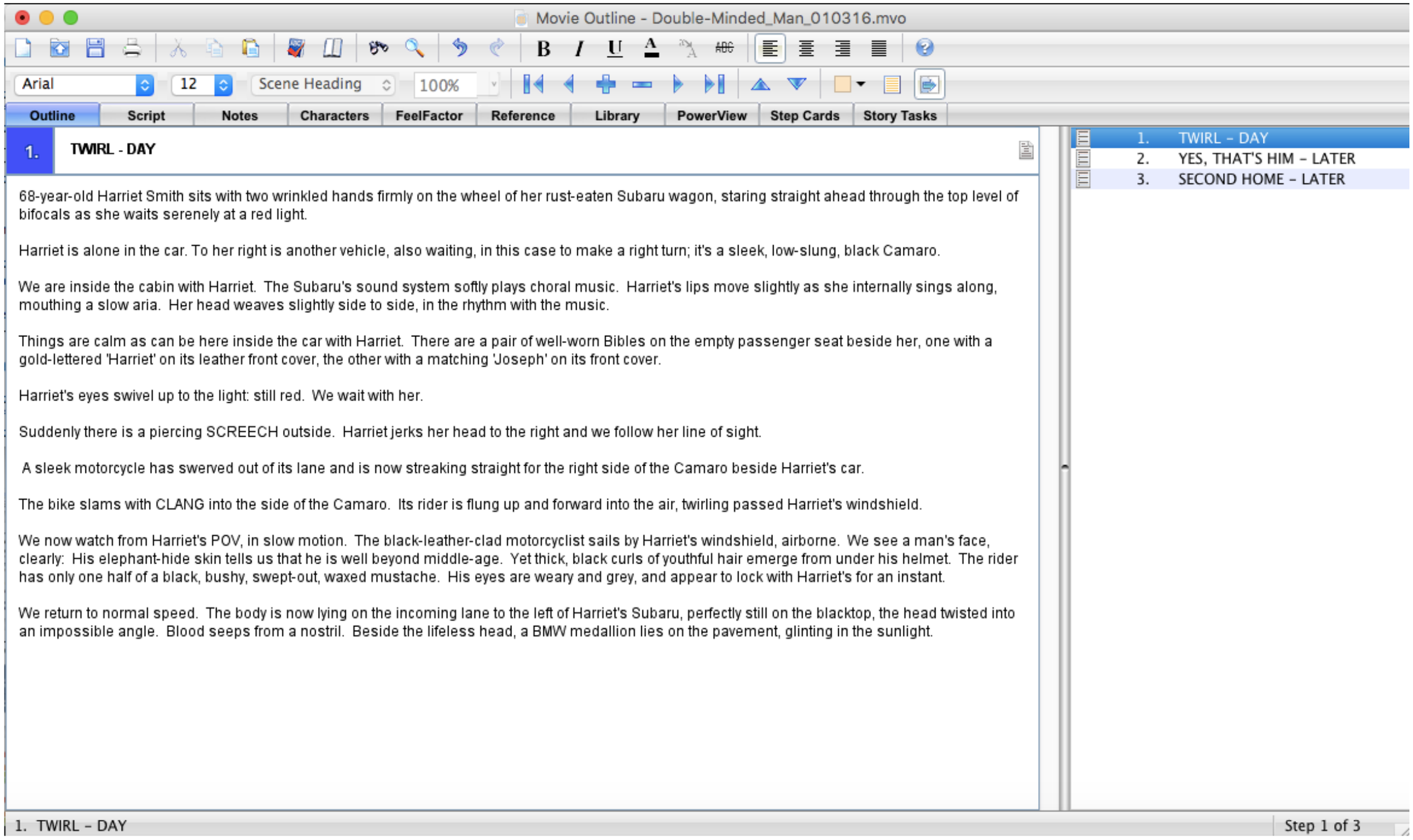

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S Bringsjord & A Bringsjord

DRAFT #5

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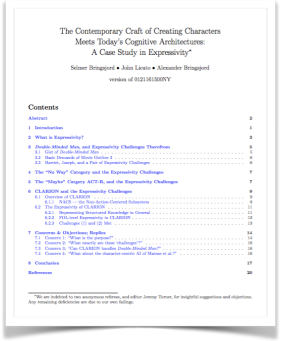

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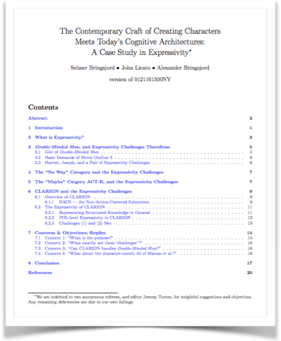
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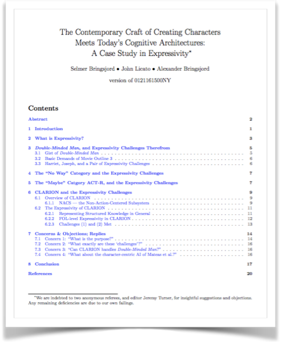

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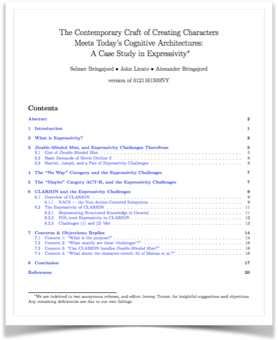
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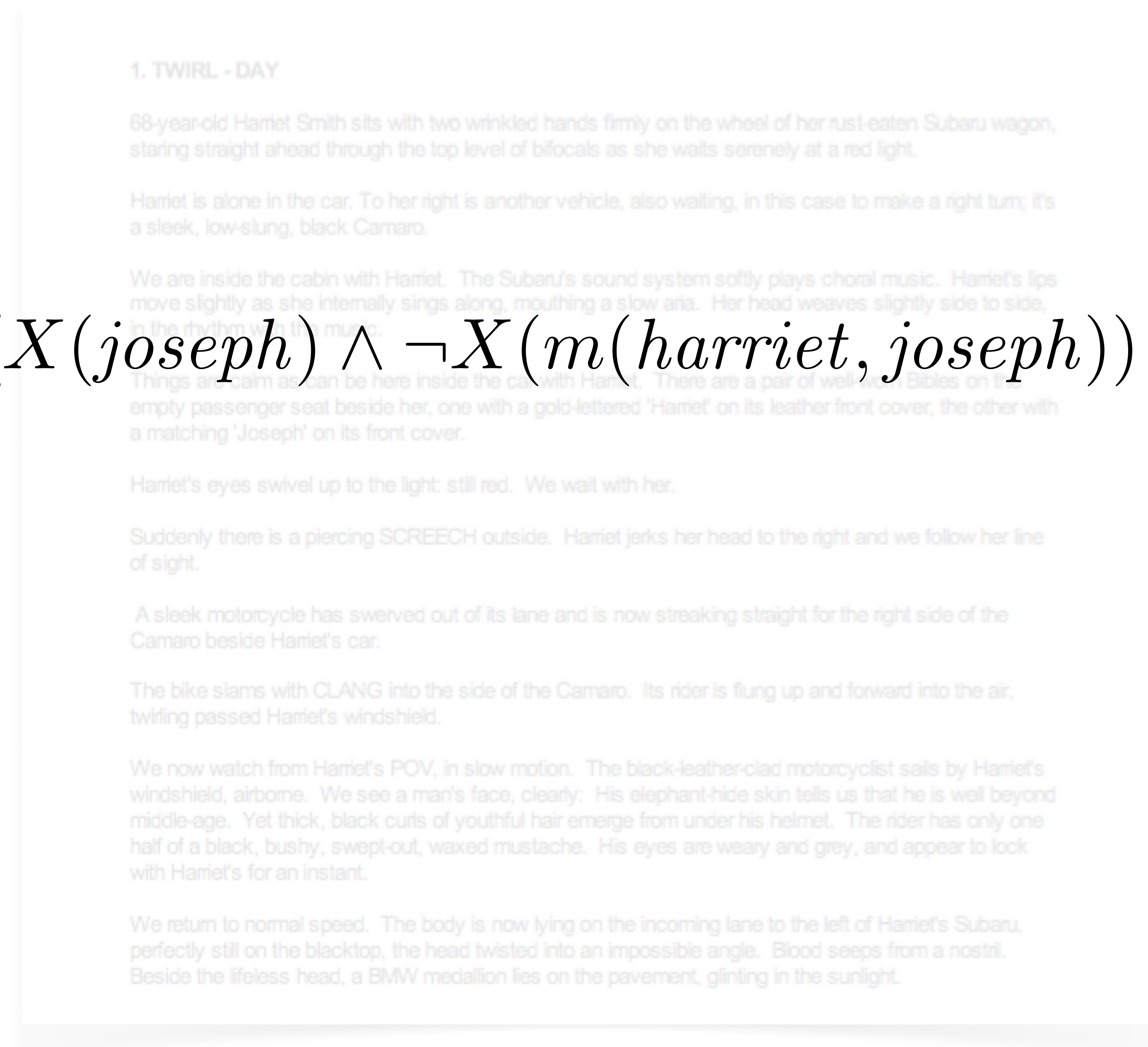
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Climbing the *k*-order Ladder

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a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

$$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$$

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There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

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Climbing the *k*-order Ladder

$$\exists x, y\ \exists R, R^2[R(x) \wedge R(y) \wedge R^2(x, y) \wedge Positive(R^2) \wedge R(fatherOf(x))]$$

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SOL $\mathscr{L}_2$

$$\exists x \exists y \exists R[R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$$

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Climbing the *k*-order Ladder

⋮

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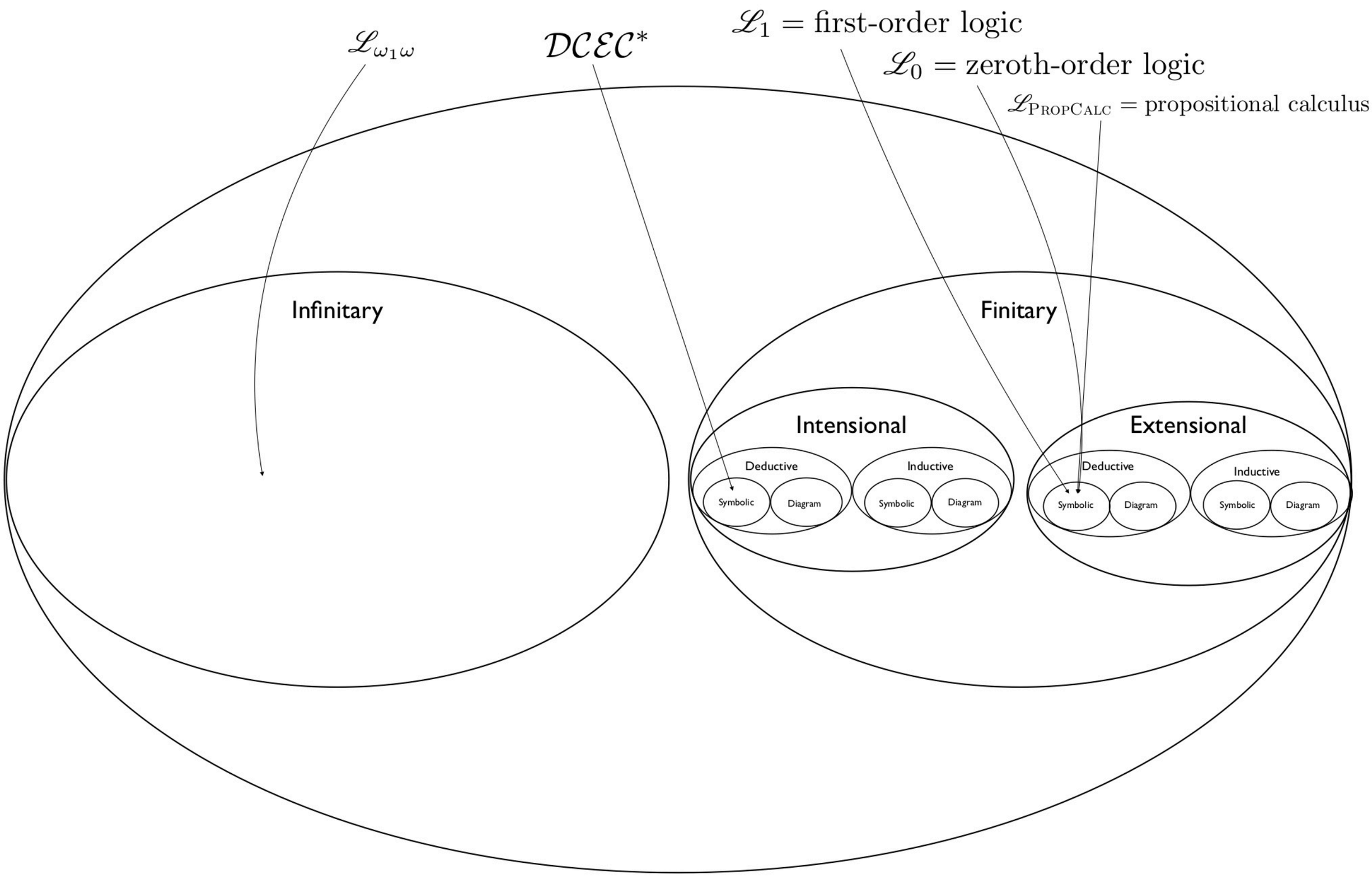
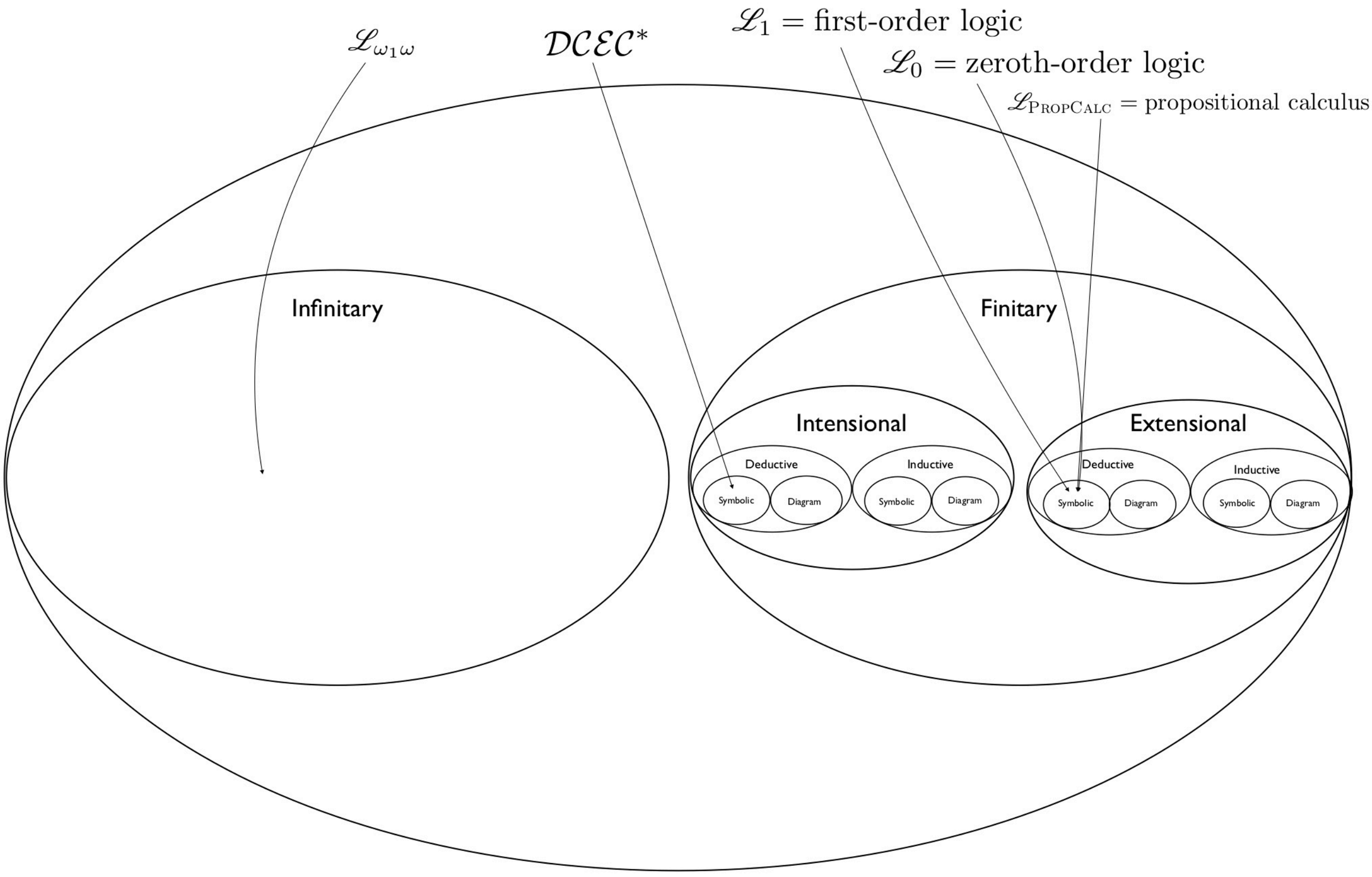
The Universe of Logics



$\mathscr{L}_3$
$\mathscr{L}_2$

The Universe of Logics



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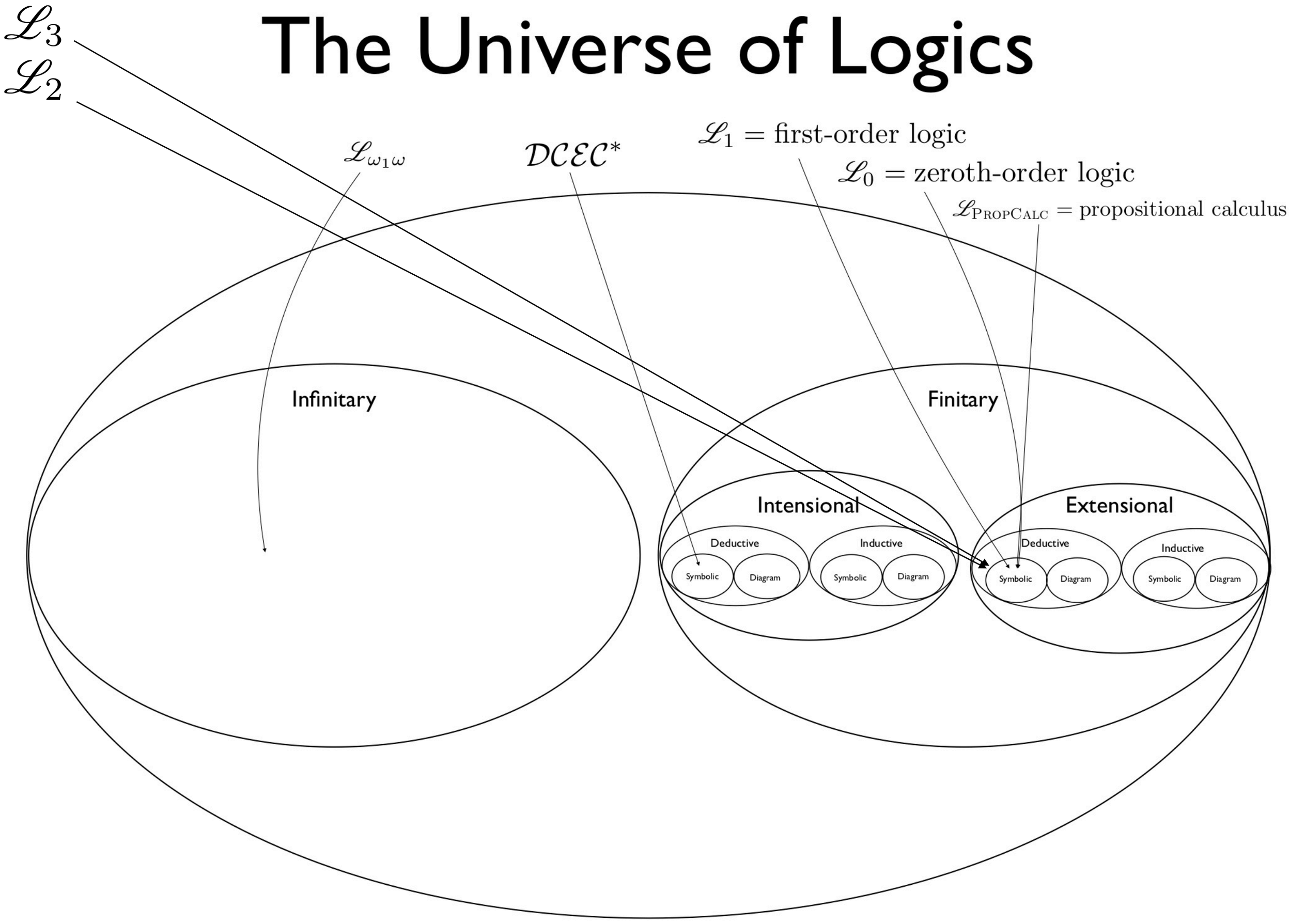

Climbing the *k*-order Ladder

⋮

TOL $\mathscr{L}_3$

$$\exists x, y\ \exists R, R^2[R(x) \wedge R(y) \wedge R^2(x, y) \wedge Positive(R^2) \wedge R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property; and, *x* R^2s *y*, where R^2 is a positive property.

SOL $\mathscr{L}_2$

$$\exists x \exists y \exists R[R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property (and *x* likes *y*).

FOL $\mathscr{L}_1$

$$\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$$

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $\mathscr{L}_0$

$$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

"Leibniz was right that Descartes was right that … : God exists, necessarily."

Gödel's "God Theorem"

		Part I	
	(1′)	The absence of a positive property is not positive.	premise
	(1)	The absence of a positive property is not positive; and if a property isn't positive, the absence of that property is.	premise
	(2)	Any property entailed by a positive property is itself positive.	premise
∴	(3)	Every positive property P is possibly possessed by something.	(1), (2)
		Part II	
	(4)	Anything that is God has all positive properties.	definition
	(5)	The property of being God is itself a positive property.	premise
∴	(6)	It's possible that God exists.	(3), (5)
		Part III	
	(7)	Positive properties are necessarily positive.	premise
	(8)	A thing x has an essence E if and only if (i) E is a property x has; and (ii) for any property P that x has, x's having this property P is necessarily implied by x's having essence E.	definition
∴	(9)	The property of being God is an essence of any thing that has this property.	(8), (7), (4), (1)
∴	(9)	The property of *being God* ($= G$) is an essential property of any thing that has G.	(8), (7), (4), (1)
		Part IV	
	(10)	A thing has necessary existence if and only if all the essences that thing has imply that something exists and has all those essences.	definition
	(10)	A thing has necessary existence if and only if all the essential properties that thing has imply that something exists and has all those essential properties.	definition
	(11)	Necessary existence is a positive property.	premise
∴	(12)	Necessarily, God exists.	(6), (9), (10), (11) **QED**

Gödel's "God Theorem" (formalized, machine verified)

	(1)	$\forall P\ [Pos(\neg P) \leftrightarrow \neg Pos(P)]$	premise
	(2)	$\forall P_1\ \forall P_2\ \{Pos(P_1) \wedge \Box\forall x\ [P_1(x) \rightarrow P_2(x)] \rightarrow Pos(P_2)\}$	premise
$\therefore$	(3)	$\forall P\ [Pos(P) \rightarrow \Diamond\exists x\ P(x)]$	theorem
	(4)	$\forall x\ [G(x) \leftrightarrow \forall P\ [Pos(P) \rightarrow P(x)]$	definition
	(5)	$Pos(G)$	premise
$\therefore$	(6)	$\Diamond\exists x\ G(x)$	corollary
	(7)	$\forall P\ [Pos(P) \rightarrow \Box Pos(P)]$	premise
	(8)	$\forall x \forall P\ \{Ess(P,x) \leftrightarrow [P(x) \wedge \forall P'\ (P'(x) \rightarrow \Box\forall y(P(y) \rightarrow P'(y))\}$	definition
$\therefore$	(9)	$\forall x\ [G(x) \rightarrow Ess(G,x)]$	theorem
	(10)	$\forall x\ \{NE(x) \leftrightarrow \forall P\ [Ess(P,x) \rightarrow \Box\exists y\ P(y)]\}$	definition
	(11)	$Pos(NE)$	premise
$\therefore$	(12)	$\Box\exists x\ G(x)$ (a.k.a. "Necessarily, God exists.)	theorem

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$PA_I = Z_I$

$$
\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
\text{A2} & \forall x \forall y(s(x) = s(y) \rightarrow x = y) \\
\text{A3} & \forall x(x \neq 0 \rightarrow \exists y(x = s(y)) \\
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\text{A7} & \forall x \forall y(x \times s(y) = (x \times y) + x)
\end{array}
$$

And, every sentence that is the universal closure of an instance of

$$([\phi(0) \wedge \forall x(\phi(x) \rightarrow \phi(s(x))] \rightarrow \forall x \phi(x))$$

where $\phi(x)$ is open wff with variable x, and perhaps others, free.

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$\mathsf{PA}_2 = \mathsf{Z}_2$

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(if we drop any restriction C, we have full second-order arithmetic.)

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Induction Axiom $\quad \forall X([X(0) \wedge \forall x(X(x) \rightarrow X(s(x))] \rightarrow \forall x X(x))$

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Induction Axiom $\forall X([X(0) \wedge \forall x(X(x) \rightarrow X(s(x))] \rightarrow \forall x X(x))$

Comprehension Schema $\exists X(\forall x X(x) \leftrightarrow \phi(x)) \qquad \text{where } \phi(x) \in C$

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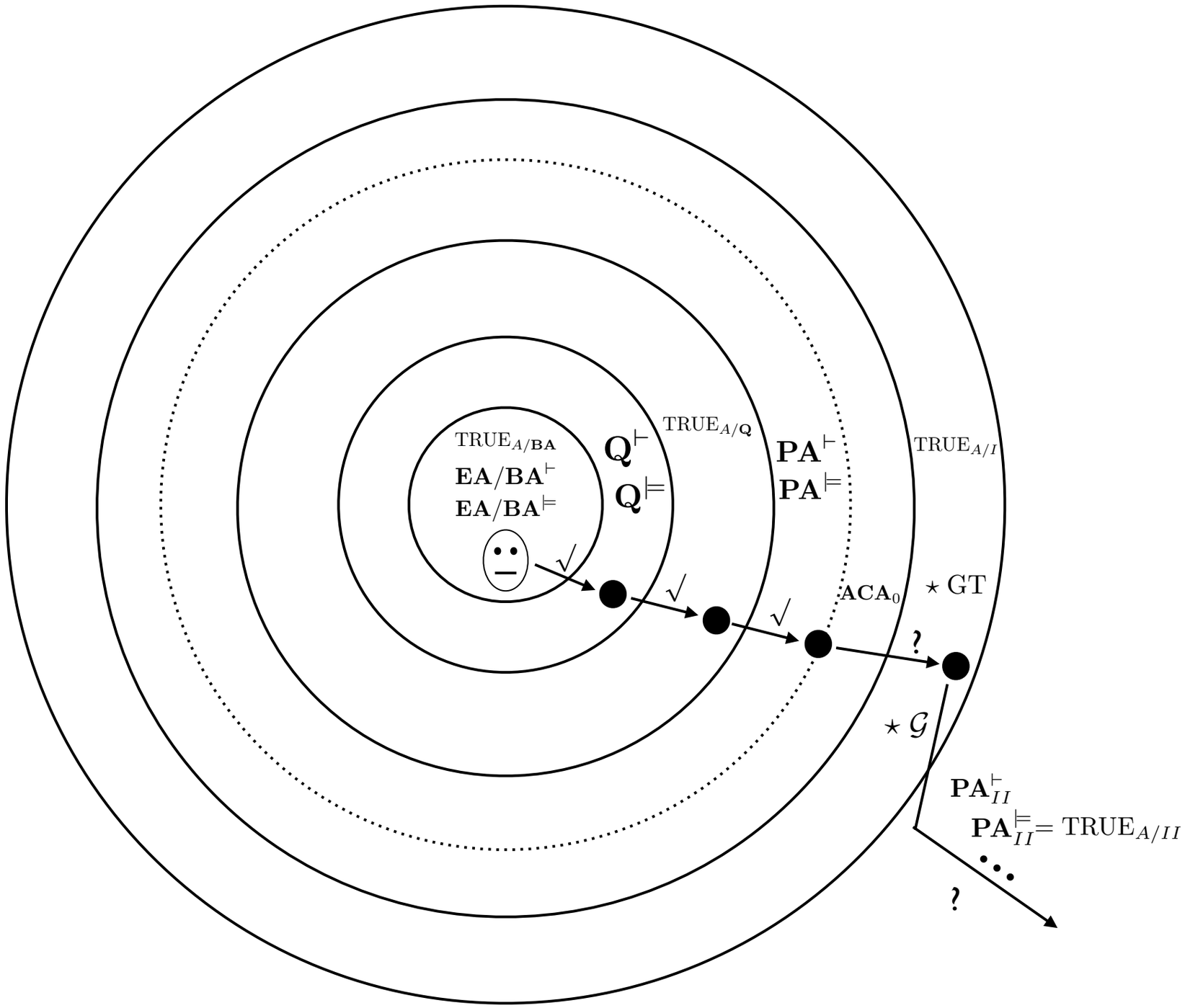
Astrologic

(Aliens & Angels on the Same "Race Track")



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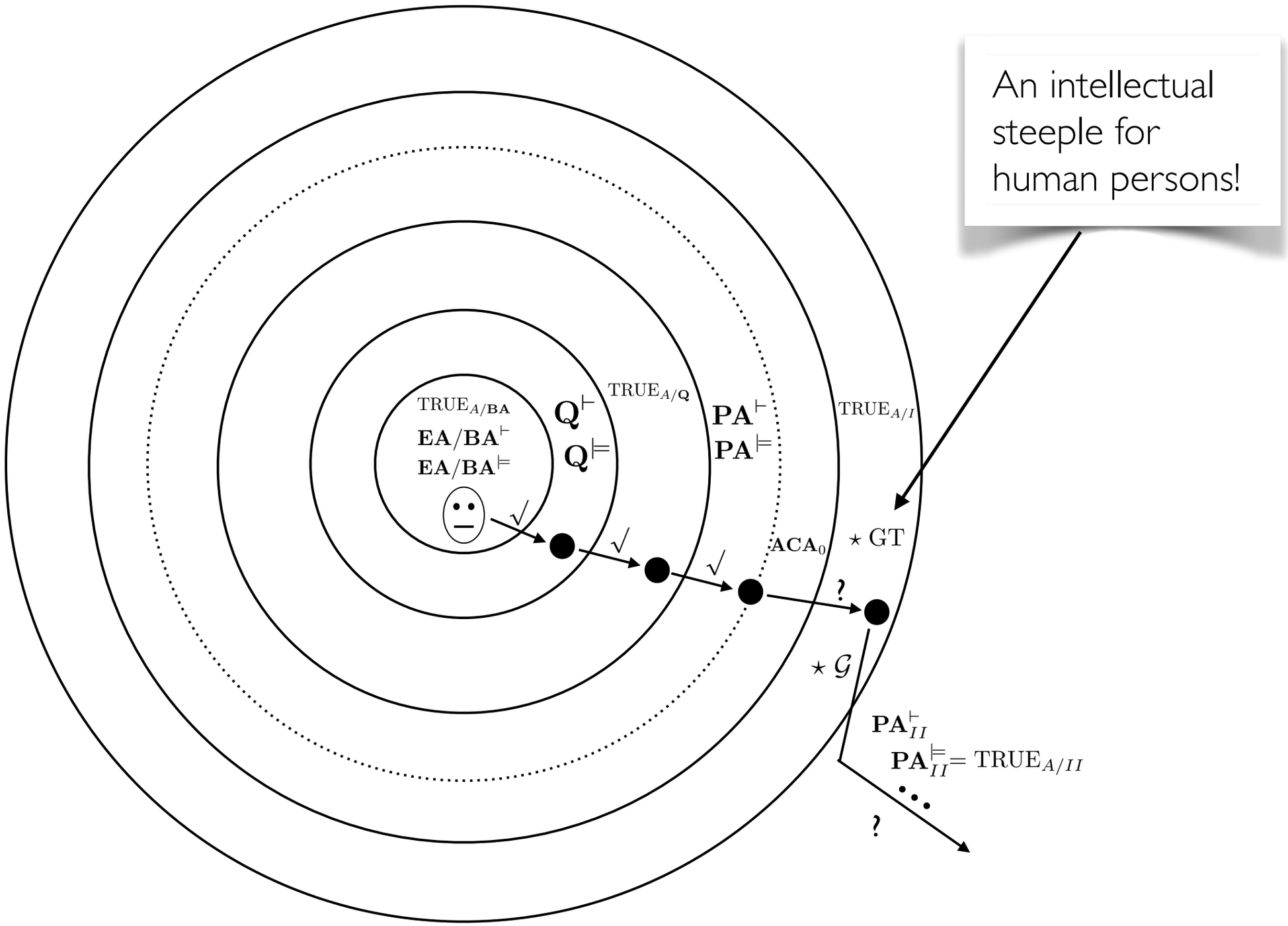

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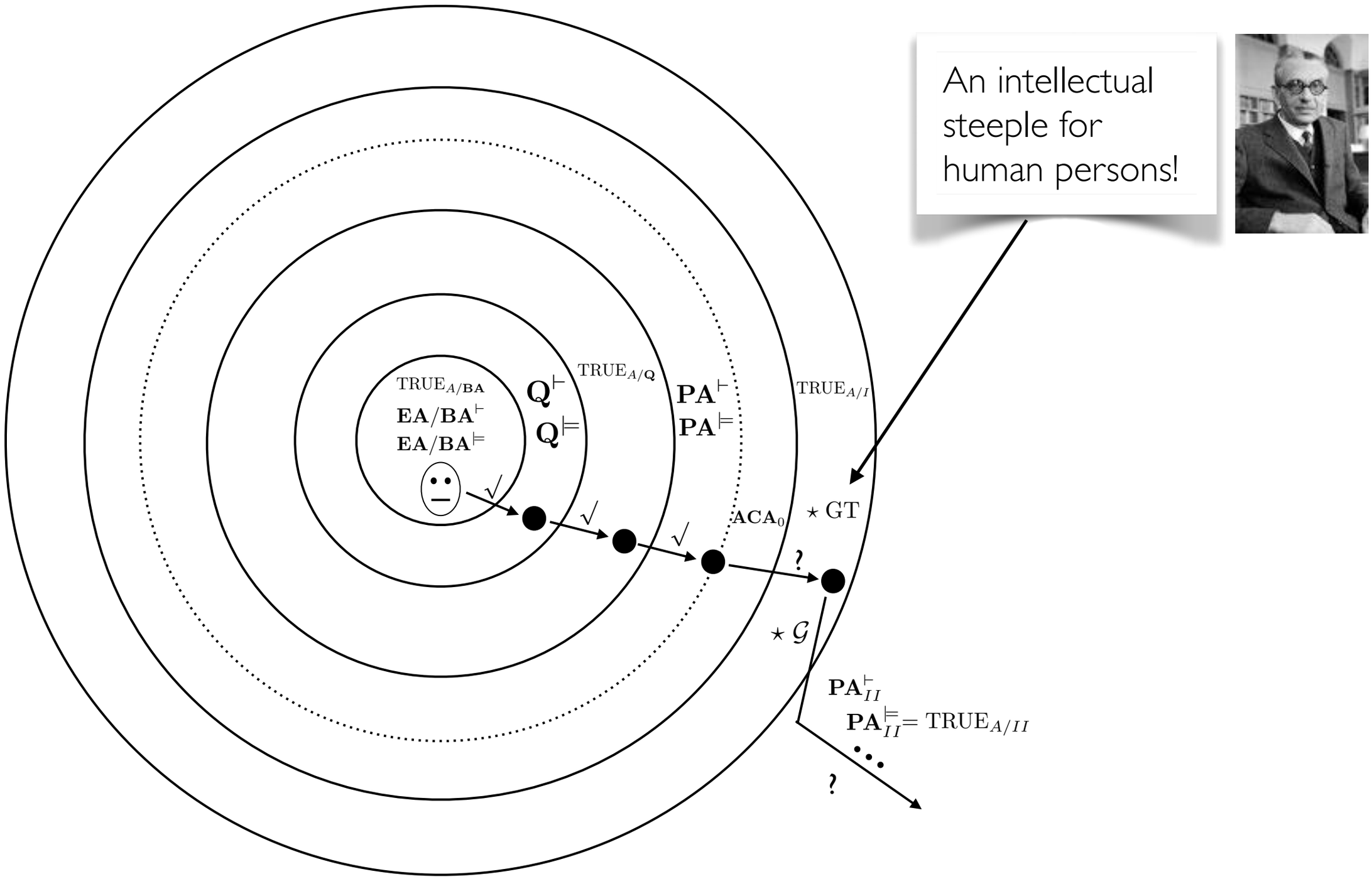

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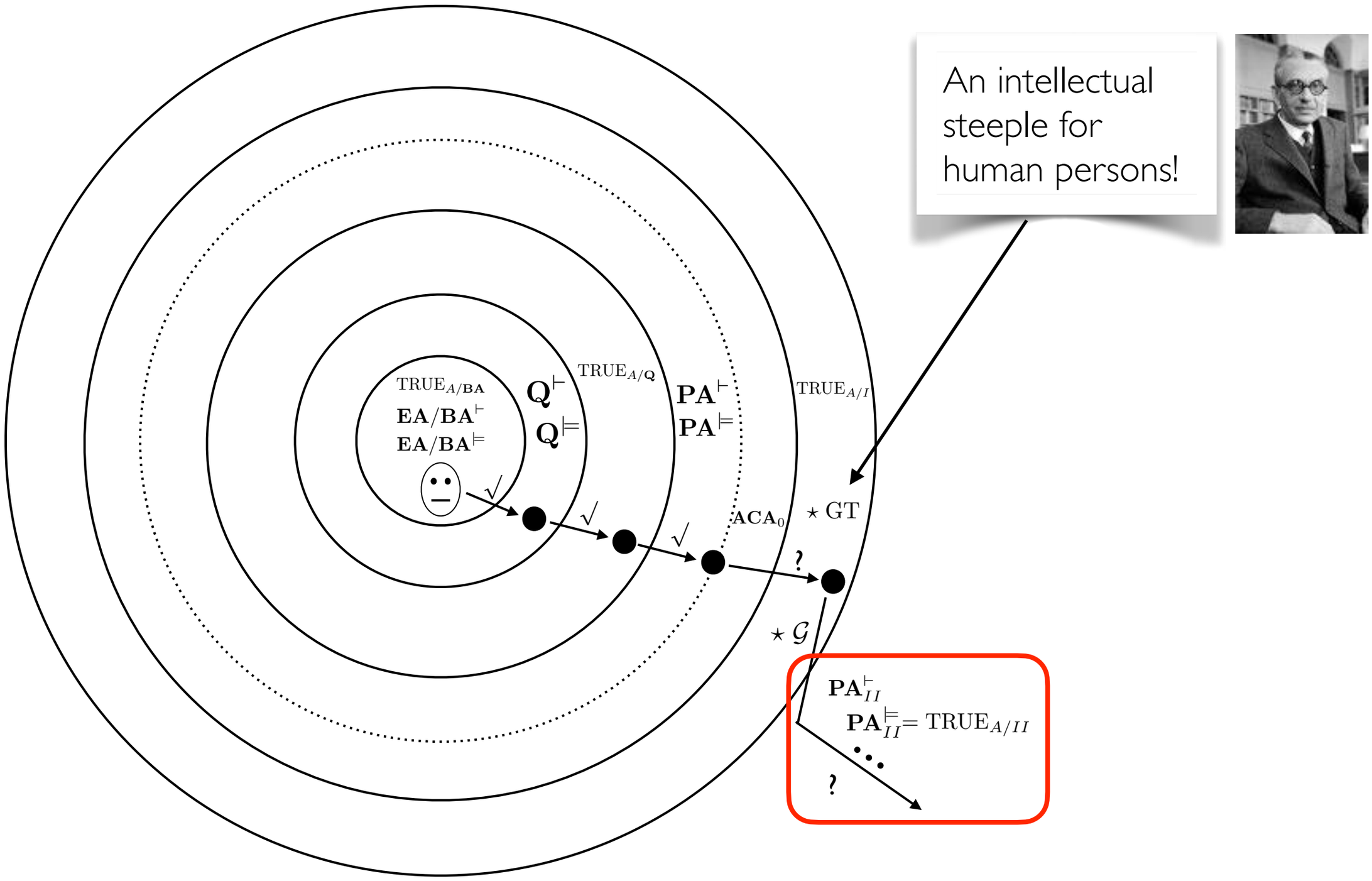

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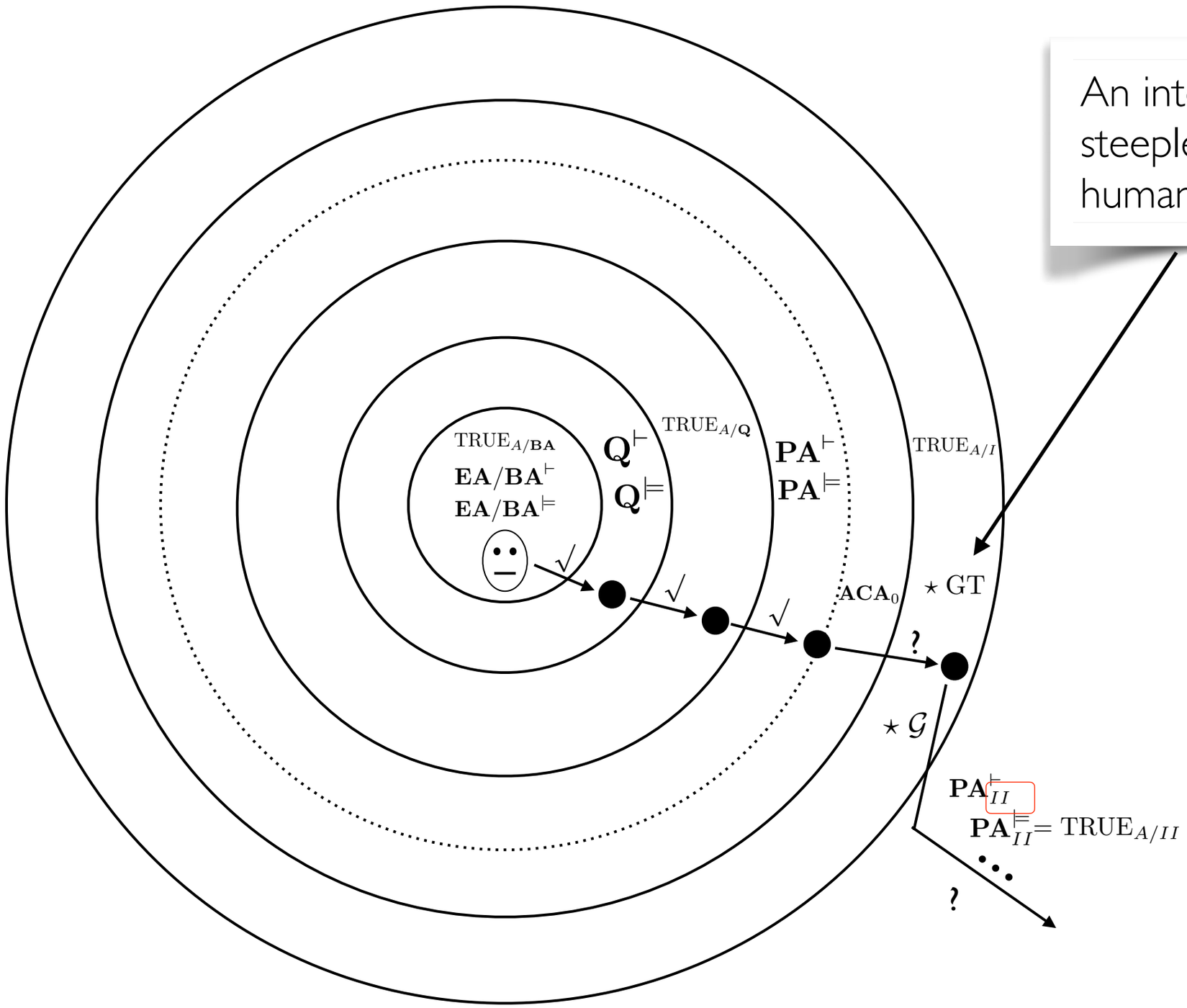

An intellectual steeple for human persons!

New!

ACA_0

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Comprehension Schema

$\exists X[\forall x\ X(x) \leftrightarrow \phi(x)]$ where $\phi(x)$ has no second-order quantifiers

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ACA_0

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An "Advanced" Word on Quantum Computing & Higher-Order Logic

Rensselaer Polytechnic Institute Begins Installation of First-Ever IBM Quantum System One on a University Campus

Groundbreaking celebrated new educational, workforce training, and research opportunities in quantum once system is installed in January 2024

October 13, 2023

On October 13, 2023, Rensselaer Polytechnic Institute (RPI) and IBM held a ceremonial groundbreaking to celebrate the first-ever deployment of an IBM Quantum System One on a university campus. The event, held at the RPI's Curtis R. Priem Experimental Media and Performing Arts Center (EMPAC), featured a grand reveal of the IBM Quantum System One chandelier, the intricately wired golden structure containing the quantum processor chip.

"We are celebrating a new era at RPI," said RPI President Marty A. Schmidt '81, Ph.D. "Today's groundbreaking is an enormous win, not just for RPI, but for the region. It is part of a surge of regional strength in all aspects of computing. Today we are headed even deeper into the future. New York's Hudson River Valley has the potential to become Quantum Valley."

Schmidt; Curtis R. Priem '82, vice chair of RPI's Board of Trustees; Darío Gil, Ph.D., Senior Vice President and Director of IBM Research; and John E. Kelly, '78G, '80 Ph.D., D.H.L. (Hon.), Chair of the Rensselaer Board of Trustees, were all featured speakers at the event. Buck Bobbin, represented the Office of U.S. Senator Kirsten Gillibrand and Congressman Paul D. Tonko (NY-20) provided remarks via video. Students, alumni, faculty, staff,

From left, John E. Kelly III, RPI Board of Trustees chair and retired IBM executive vice president, Curtis R. Priem, co-founder of NVIDIA and RPI Board of Trustees vice chair, Darío Gil, senior vice president and director of IBM Research, and Martin A. Schmidt, president of Rensselaer Polytechnic Institute take part in a ground breaking ceremony for the installation of a IBM Quantum System One quantum computing system on Friday, Oct. 13, 2023 in Troy, N.Y. (Hans Pennink/AP Images for Rensselaer Polytechnic Institute)

An "Advanced" Word on Quantum Computing & Higher-Order Logic

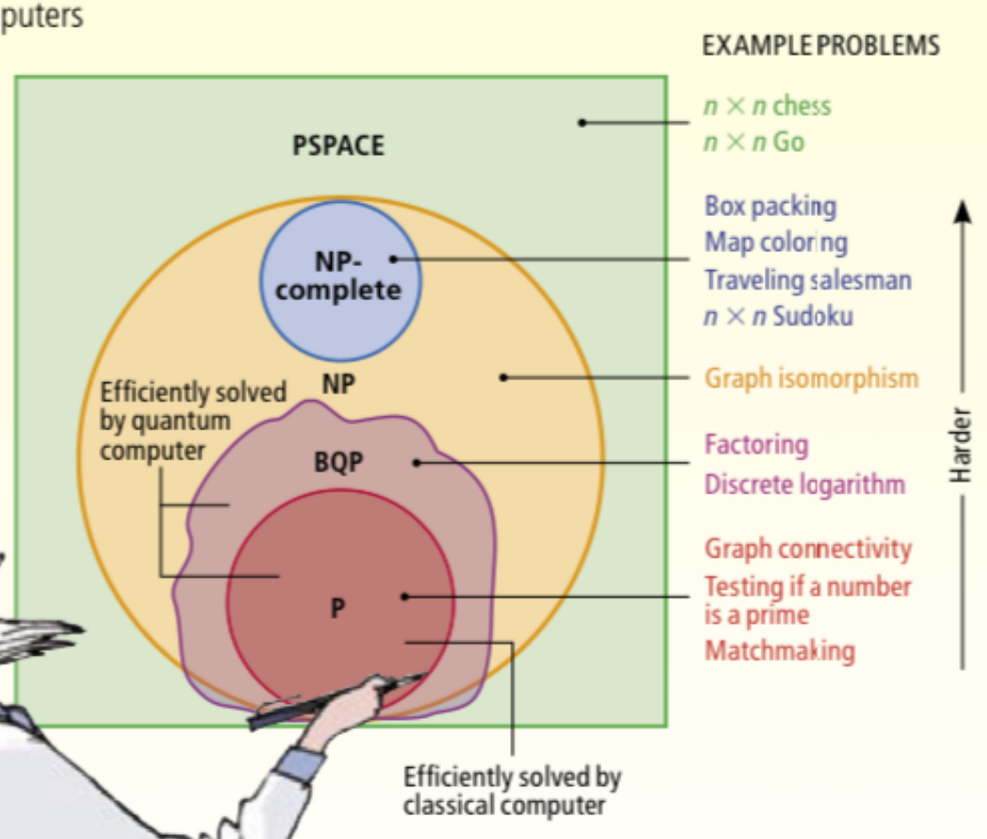
Where Quantum Computers Fit In

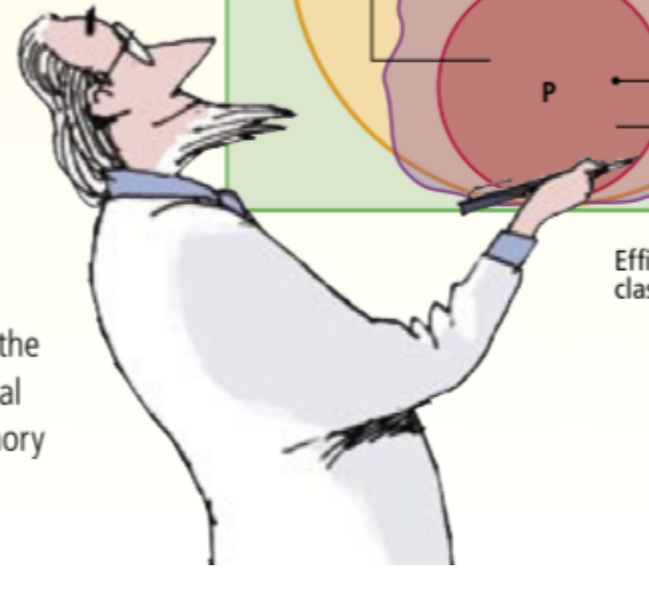
The map at the right depicts how the class of problems that quantum computers would solve efficiently (BQP) might relate to other fundamental classes of computational problems. (The irregular border signifies that BQP does not seem to fit neatly with the other classes.)

The BQP class (the letters stand for *b*ounded-error, *q*uantum, *p*olynomial time) includes all the P problems and also a few other NP problems, such as factoring and the so-called discrete logarithm problem. Most other NP and all NP-complete problems are believed to be outside BQP, meaning that even a quantum computer would require more than a polynomial number of steps to solve them.

In addition, BQP might protrude beyond NP, meaning that quantum computers could solve certain problems faster than classical computers could even check the answer. (Recall that a conventional computer can efficiently verify the answer of an NP problem but can efficiently solve only the P problems.) To date, however, no convincing example of such a problem is known.

Computer scientists do know that BQP cannot extend outside the class known as PSPACE, which also contains all the NP problems. PSPACE problems are those that a conventional computer can solve using only a polynomial amount of memory but possibly requiring an exponential number of steps.





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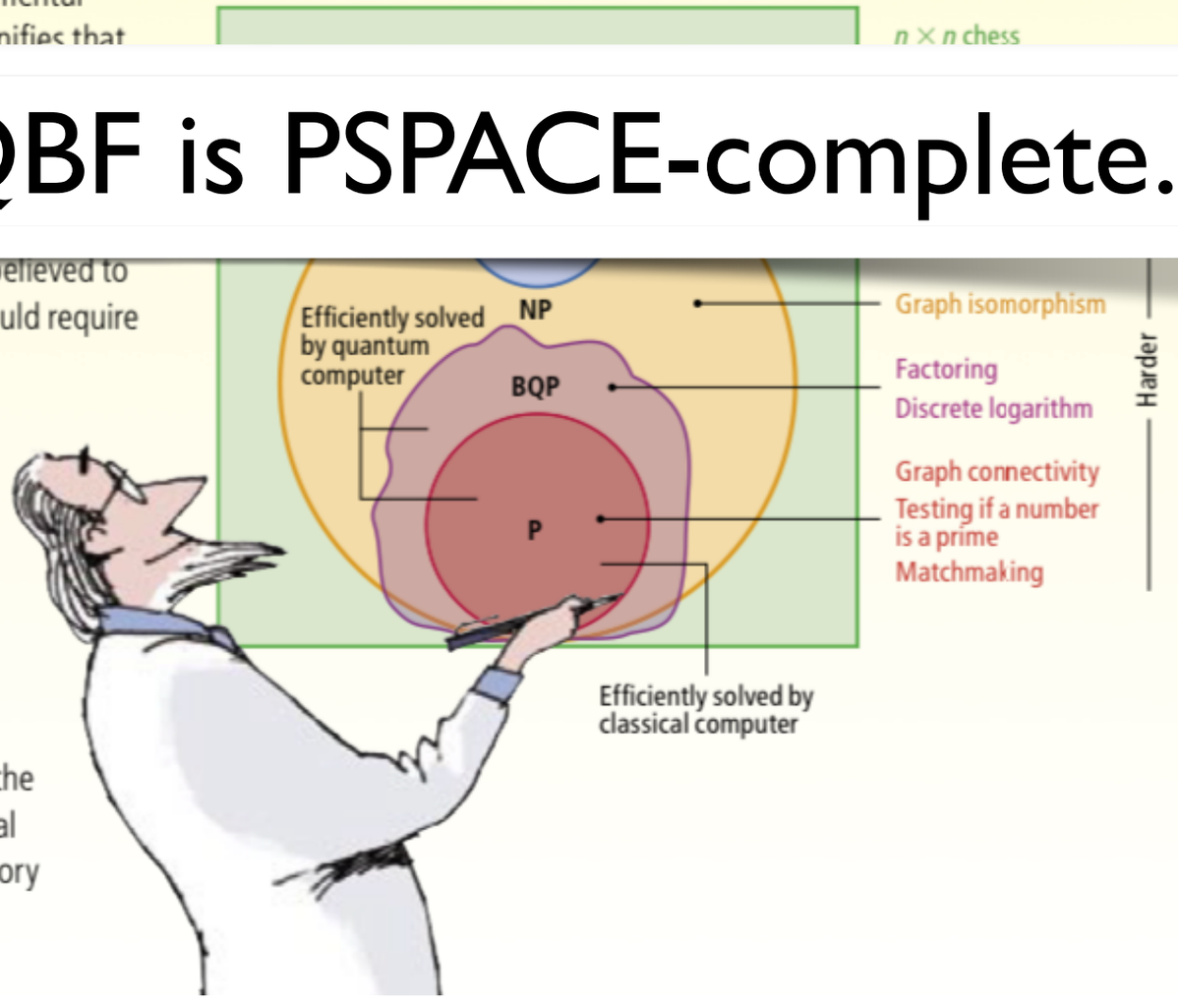

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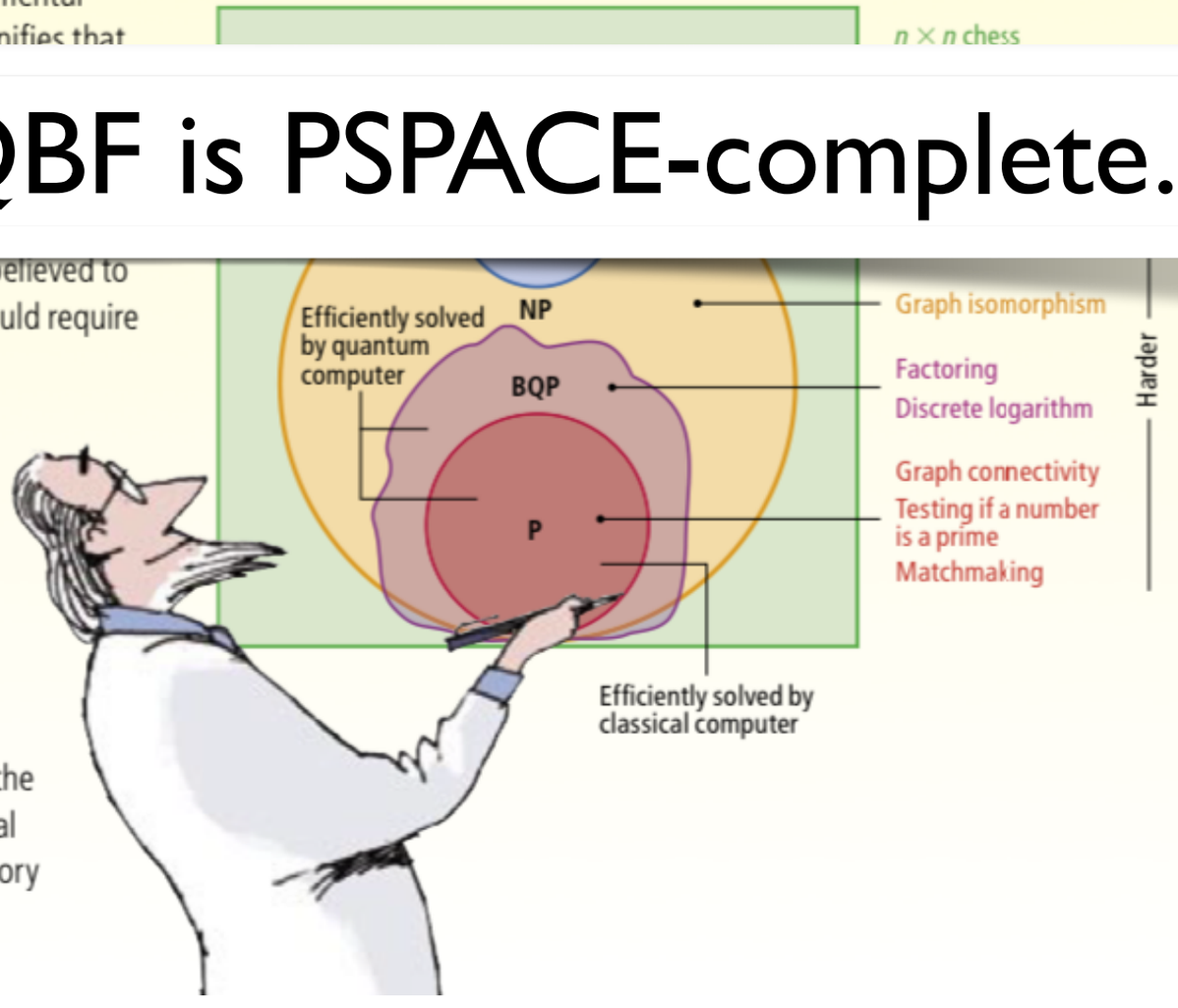

Slutten