

Could AI Ever Match Gödel's Greatness?

(Part II of the Chapter; Part I is on “The Gödel Game,” for IFLAI)

Selmer Bringsjord

Intro to Formal Logic (& AI) (IFLAI)

4/21/25

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Monographic Context (yet again!)

...

Gödel's Great Theorems (OUP)

by Selmer Bringsjord

- Introduction (“The Wager”)
- Brief Preliminaries (e.g. the propositional calculus & FOL)
- The Completeness Theorem
- The First Incompleteness Theorem
- The Second Incompleteness Theorem
- The Speedup Theorem
- The Continuum-Hypothesis Theorem
- The Time-Travel Theorem
- Gödel’s “God Theorem”
- Could a Finite Machine Match Gödel’s Greatness?



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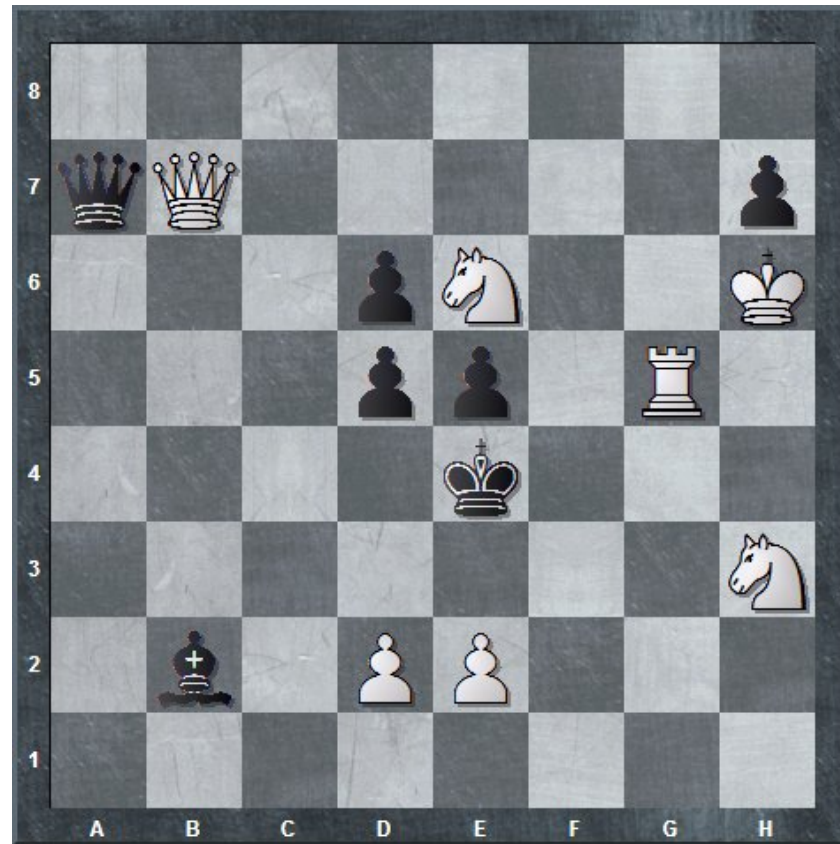
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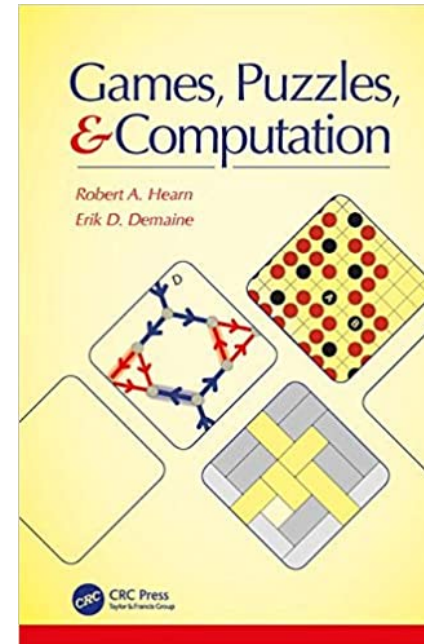
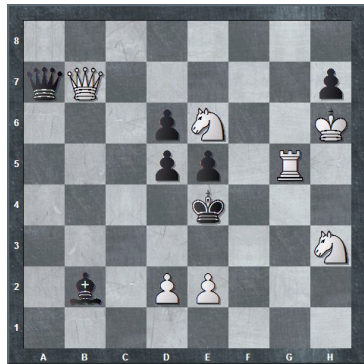
Gödel's Greatness & Games

...

Mate in 2 Problem



Mate in 2 Problem



Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Mere Calculative Cognitive Power

Entscheidungsproblem

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Super-Serious Human Cognitive Power

Serious Human Cognitive Power



Gödel

Entscheidungsproblem

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Gödel



Turing

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Entscheidungsproblem

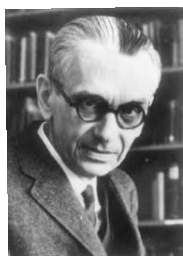
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Super-Serious Human Cognitive Power

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Podcast: The Turing Test is Dead.
Long Live the Lovelace Test.



Gödel



Entscheidungsproblem

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Super-Serious Human Cognitive Power

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Gödel



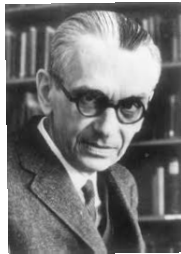
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Mere Calculative Cognitive Power

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Analytical Hierarchy

Serious Human Cognitive Power



Gödel



Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

Arithmetical Hierarchy



Gödel



Entscheidungsproblem

Mere Calculative Cognitive Power

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Analytical Hierarchy

Arithmetical Hierarchy



Gödel



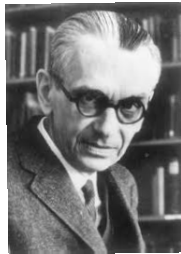
Entscheidungsproblem

Polynomial Hierarchy

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Analytical Hierarchy

Arithmetical Hierarchy



Gödel



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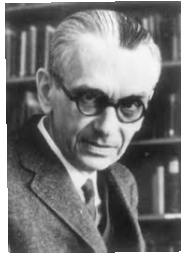
Polynomial Hierarchy

$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

Arithmetical Hierarchy



Gödel



\vdots
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Entscheidungsproblem

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Gödel



Go:AlphaGo



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Gödel



Jeopardy!:



Go: AlphaGo



⋮
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Jeopardy!

Chess: Deep Blue

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Jeopardy!:

Checkers: Chinook



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Jeopardy!:



Chess: Deep Blue
Checkers: Chinook
Go: AlphaGo



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

1994

Checkers: Tinsley vs. Chinook



Name: Marion Tinsley
Profession: Teach mathematics
Hobby: Checkers
Record: Over 42 years
loss only 2 games
of checkers
World champion for over 40
years.

Mr. Tinsley suffered his 4th and 5th losses against Chinook

1994

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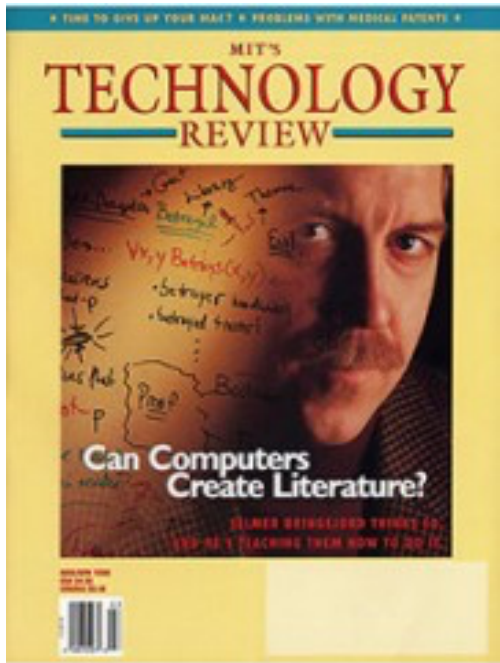
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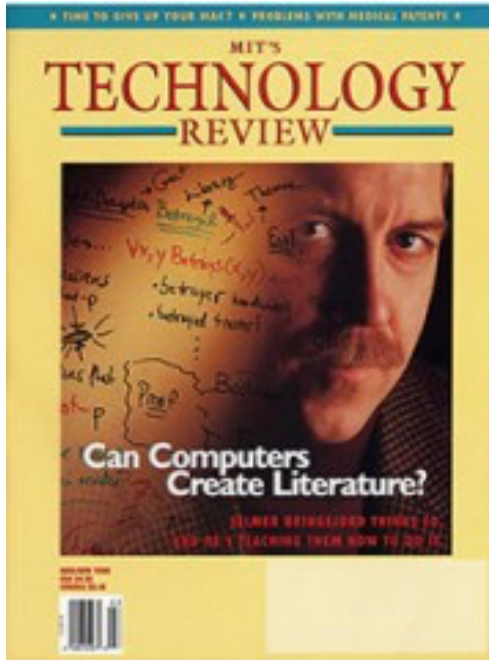
2011





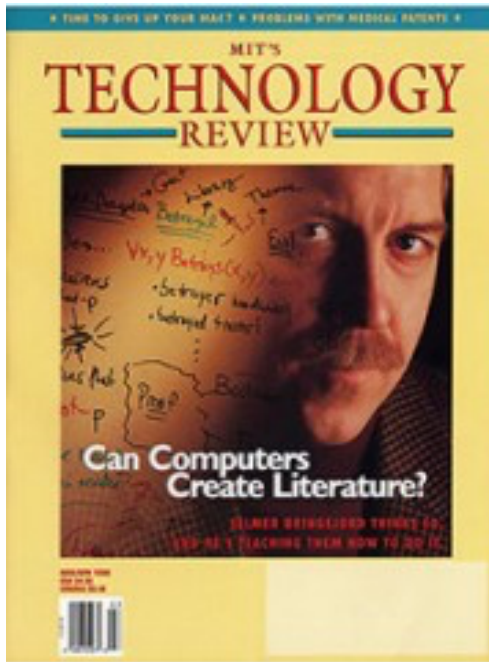
1998

“Chess is Too Easy”



1998

“Chess is Too Easy”

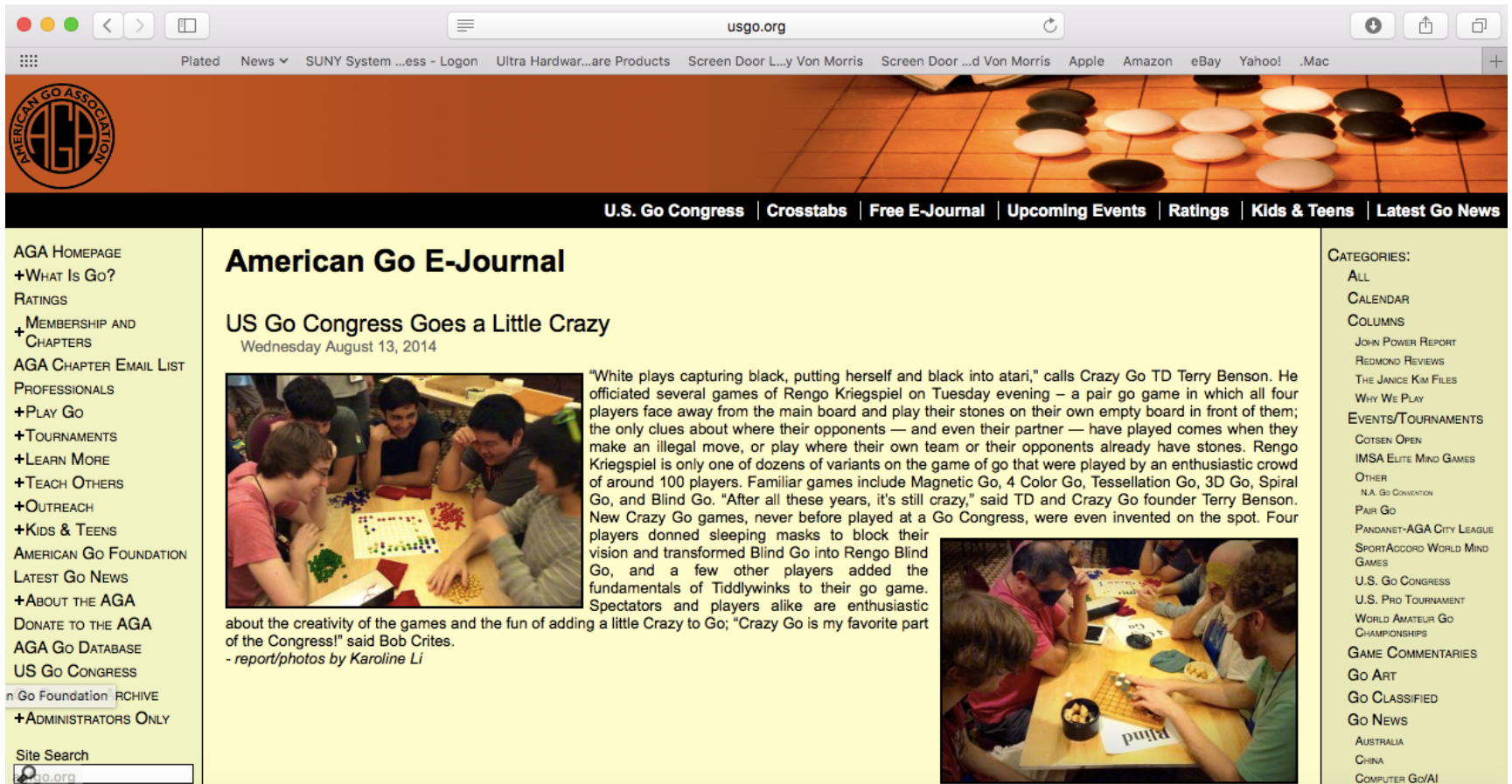


1998

Some of Gödel's great work is at the level of chess.

But to *fully* “gamify” Gödel,
we need a harder game! ...

Rengo Kriegspiel




The screenshot shows a web browser window with the URL usgo.org. The browser's address bar and tabs are visible at the top. The website header features the American Go Association logo on the left and a navigation menu with links: [U.S. Go Congress](#), [Crosstabs](#), [Free E-Journal](#), [Upcoming Events](#), [Ratings](#), [Kids & Teens](#), and [Latest Go News](#). The main content area is titled "American Go E-Journal" and features an article titled "US Go Congress Goes a Little Crazy" dated Wednesday August 13, 2014. The article includes a photograph of a group of people playing Crazy Go and a quote from Terry Benson. A sidebar on the left contains a navigation menu with links such as "AGA HOMEPAGE", "WHAT IS GO?", "RATINGS", "MEMBERSHIP AND CHAPTERS", "AGA CHAPTER EMAIL LIST", "PROFESSIONALS", "PLAY GO", "TOURNAMENTS", "LEARN MORE", "TEACH OTHERS", "OUTREACH", "KIDS & TEENS", "AMERICAN GO FOUNDATION", "LATEST GO NEWS", "ABOUT THE AGA", "DONATE TO THE AGA", "AGA GO DATABASE", "US GO CONGRESS", "Go Foundation ARCHIVE", and "ADMINISTRATORS ONLY". A "Site Search" box is located at the bottom left of the sidebar. A sidebar on the right lists "CATEGORIES:" including ALL, CALENDAR, COLUMNS, JOHN POWER REPORT, REDMOND REVIEWS, THE JANICE KIM FILES, WHY WE PLAY, EVENTS/TOURNAMENTS, COTSEN OPEN, IMSA ELITE MIND GAMES, OTHER, N.A. GO CONVENTION, PAIR GO, PANDANET-AGA CITY LEAGUE, SPORTACCORD WORLD MIND GAMES, U.S. GO CONGRESS, U.S. PRO TOURNAMENT, WORLD AMATEUR GO CHAMPIONSHIPS, GAME COMMENTARIES, GO ART, GO CLASSIFIED, GO NEWS, AUSTRALIA, CHINA, and COMPUTER GO/AI.

AGA HOMEPAGE
+WHAT IS GO?
RATINGS
+MEMBERSHIP AND CHAPTERS
AGA CHAPTER EMAIL LIST
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American Go E-Journal

US Go Congress Goes a Little Crazy

Wednesday August 13, 2014



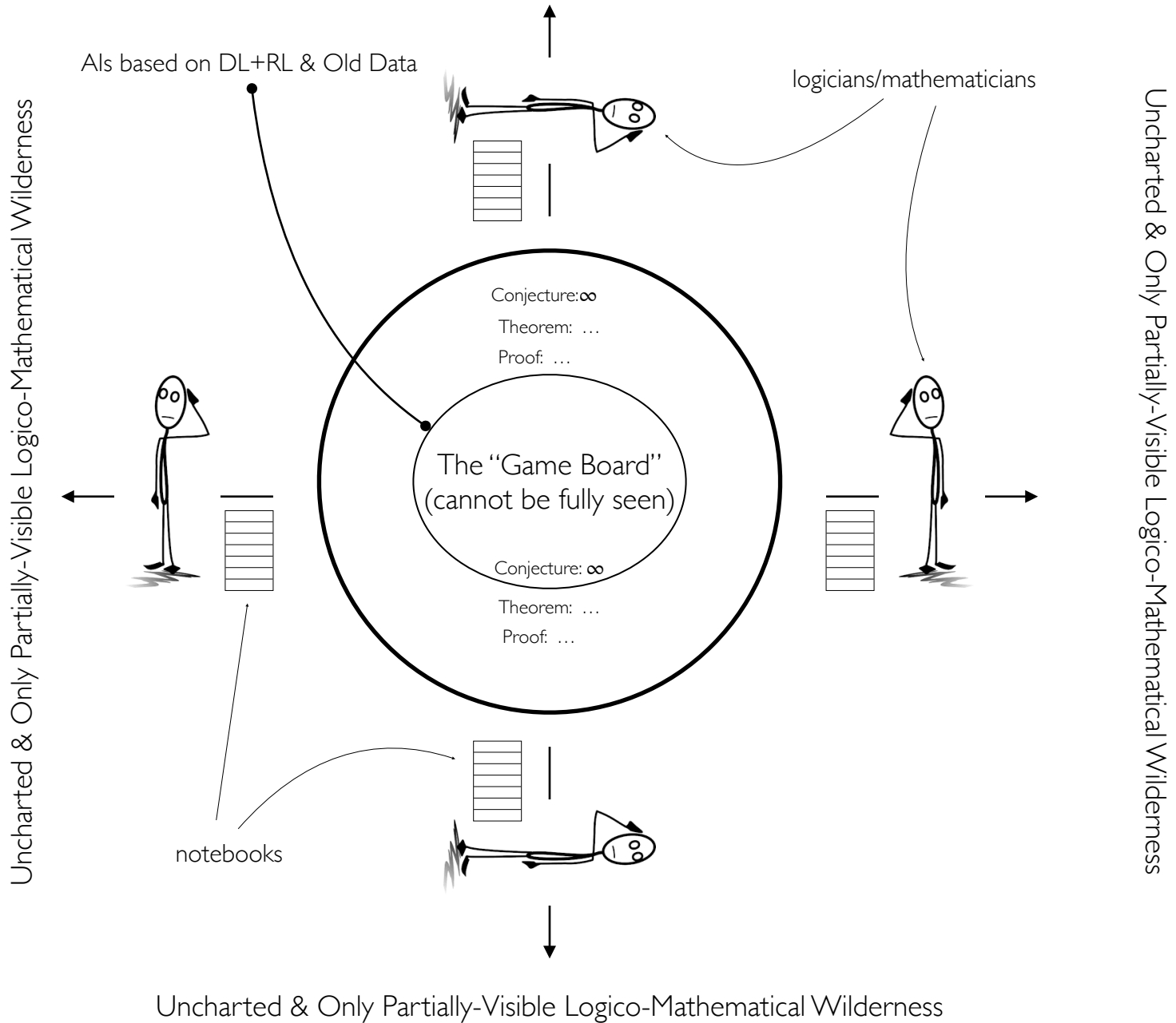
"White plays capturing black, putting herself and black into atari," calls Crazy Go TD Terry Benson. He officiated several games of Rengo Kriegspiel on Tuesday evening – a pair go game in which all four players face away from the main board and play their stones on their own empty board in front of them; the only clues about where their opponents — and even their partner — have played comes when they make an illegal move, or play where their own team or their opponents already have stones. Rengo Kriegspiel is only one of dozens of variants on the game of go that were played by an enthusiastic crowd of around 100 players. Familiar games include Magnetic Go, 4 Color Go, Tessellation Go, 3D Go, Spiral Go, and Blind Go. "After all these years, it's still crazy," said TD and Crazy Go founder Terry Benson. New Crazy Go games, never before played at a Go Congress, were even invented on the spot. Four players donned sleeping masks to block their vision and transformed Blind Go into Rengo Blind Go, and a few other players added the fundamentals of Tiddlywinks to their go game. Spectators and players alike are enthusiastic about the creativity of the games and the fun of adding a little Crazy to Go; "Crazy Go is my favorite part of the Congress!" said Bob Crites.
- report/photos by Karoline Li



CATEGORIES:
ALL
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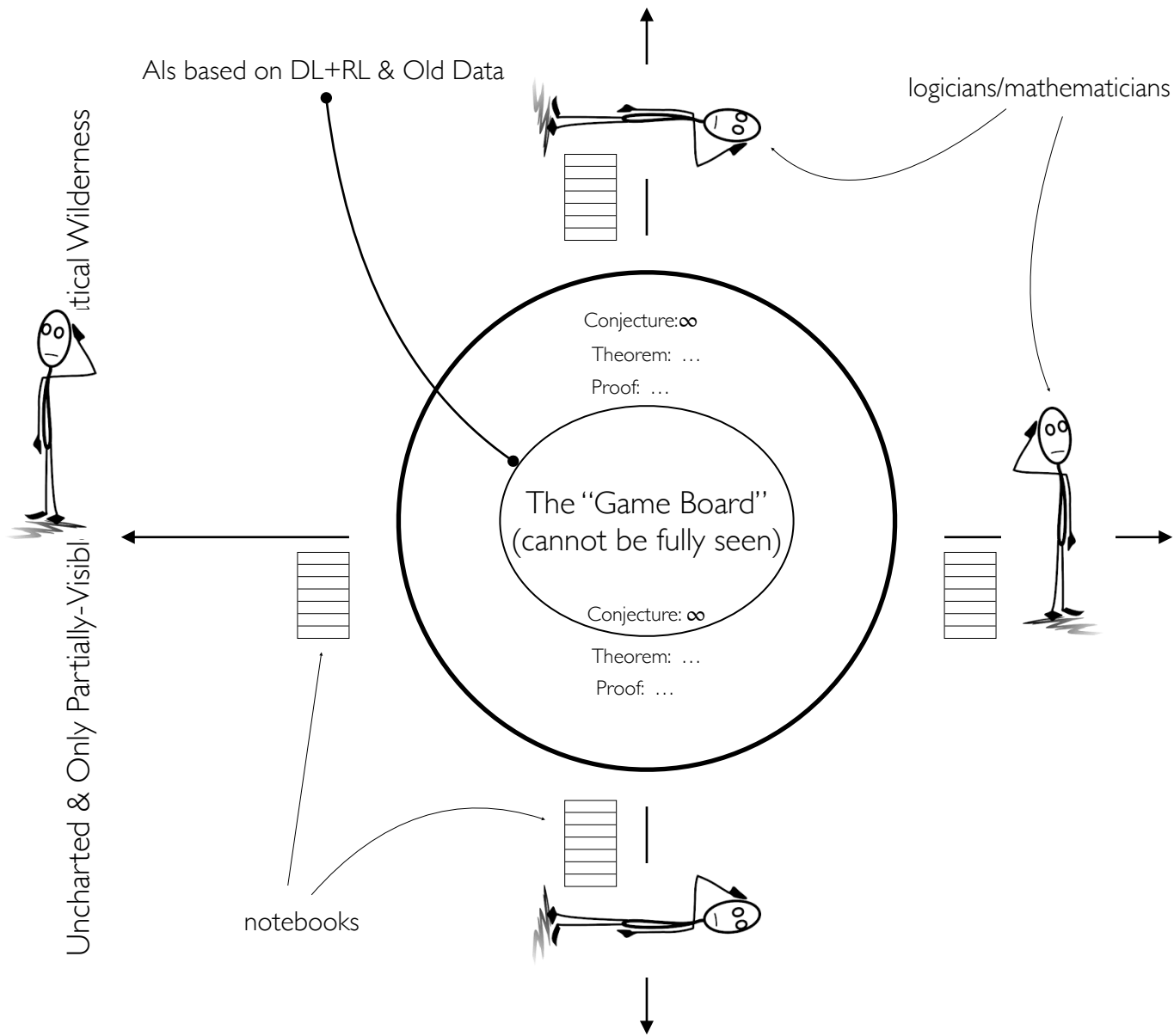
The Gödel Game

Uncharted & Only Partially-Visible Logico-Mathematical Wilderness



The Gödel Game

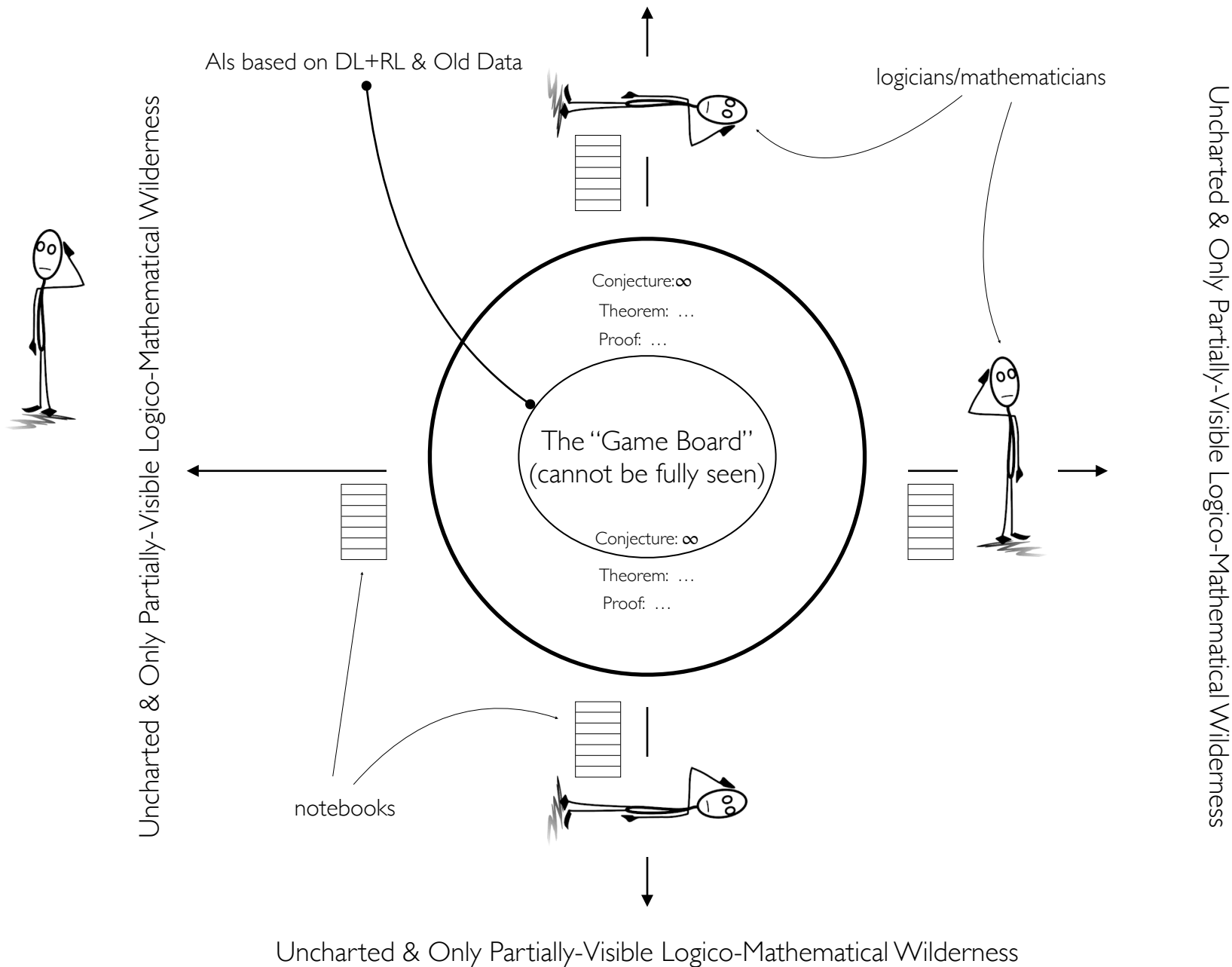
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The Gödel Game

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Uncharted & Only Partially-Visible Logico-Mathematical Wilderness

Gödel's Either/Or ...

The Question

Q* Is the human mind more powerful than the class of standard computing machines?

The Question

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(= finite machines)



The Question

Q* Is the human mind more powerful than the class of standard computing machines?

(= finite machines)

(= Turing machines)

(= register machines)

(= KU machines)

...

Gödel's Either/Or

“[E]ither ... the human mind (even within the realm of pure mathematics) infinitely surpasses the power of any finite machine, or else there exist absolutely [humanly?] unsolvable diophantine problems.”

— Gödel, 1951, Providence RI

Gödel's Either/Or

“[E]ither ... the human mind (even within the realm of pure mathematics) infinitely surpasses the power of any finite machine, or else there exist absolutely [humanly?] unsolvable diophantine problems.”
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More precisely, what does this mean?

PT as a Diophantine Equation

Equations of this sort were introduced to you in middle-school, when you were asked to find the hypotenuse of a right triangle when you knew its sides; the familiar equation, the famous Pythagorean Theorem that most adults will remember at least echoes of into their old age, is:

$$(PT) \quad a^2 + b^2 = c^2,$$

and this is of course equivalent to

$$(PT') \quad a^2 + b^2 - c^2 = 0,$$

which is a Diophantine equation. Such equations have at least two unknowns (here, we of course have three: a, b, c), and the equation is solved when positive integers for the unknowns are found that render the equation true. Three positive integers that render (PT') true are

$$a = 4, b = 3, c = 5.$$

It is *mathematically impossible* that there is a finite computing machine capable of solving any Diophantine equation given to it as a challenge (!).

... which means that the 10th of Hilbert's Problems is settled:

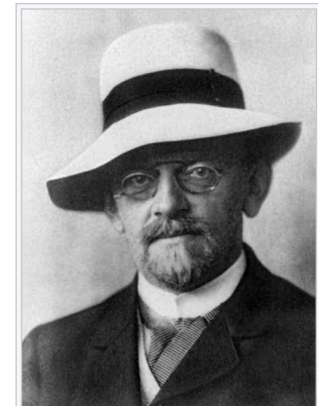
Hilbert's problems

From Wikipedia, the free encyclopedia

Hilbert's problems are twenty-three problems in [mathematics](#) published by German mathematician [David Hilbert](#) in 1900. The problems were all unsolved at the time, and several of them were very influential for 20th-century mathematics. Hilbert presented ten of the problems (1, 2, 6, 7, 8, 13, 16, 19, 21, and 22) at the [Paris](#) conference of the [International Congress of Mathematicians](#), speaking on August 8 in the [Sorbonne](#). The complete list of 23 problems was published later, most notably in English translation in 1902 by [Mary Frances Winston Newson](#) in the *[Bulletin of the American Mathematical Society](#)*.^[1]

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David Hilbert



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10th	Find an algorithm to determine whether a given polynomial Diophantine equation with integer coefficients has an integer solution.	Resolved. Result: Impossible; Matiyasevich's theorem implies that there is no such algorithm.	1970
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Julia **R**obinson

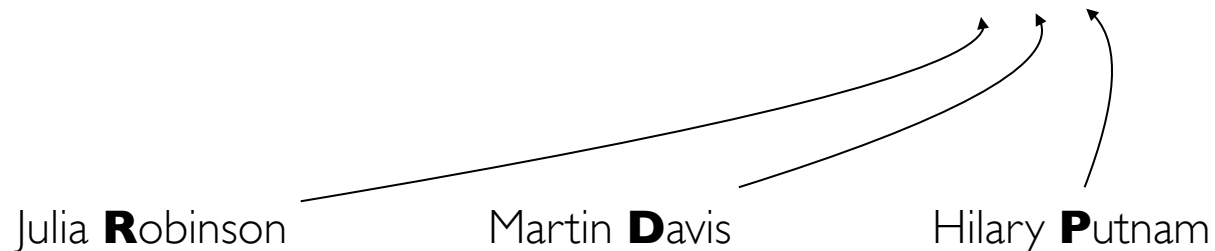
Martin **D**avis

Hilary **P**utnam

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Background

problem?⁷ In his lecture, Gödel precisely defines diophantine problems, but we don't need to bother with all of the details here; we only need to appreciate the general structure of such a problem, and that can be achieved quickly as follows, given what was introduced in Chapter 2.

Each diophantine problem has at its core a polynomial \mathcal{P} whose variables are comprised by two lists, x_1, x_2, \dots, x_n and y_1, y_2, \dots, y_m ; all variables must be integers, and the same for subscripts n and m . To represent a polynomial in a manner that announces its variables, we can write

$$\mathcal{P}(x_1, x_2, \dots, x_k, y_1, y_2, \dots, y_j).$$

But Gödel was specifically interested in whether, for all integers that can be set to the variables x_i , there are integers that can be set to the y_j , such that the polynomial equals 0. To make this clearer, first, here are two particular, simple equations that employ polynomials that are both instances of the needed form:

$$\text{E1} \quad 3x - 2y = 0$$

$$\text{E2} \quad 2x^2 - y = 0$$

All we need to do now is prefix these equations with quantifiers in the pattern Gödel gave. This pattern is quite simple: universally quantify over each x_i variable (using the now-familiar \forall), after which we existentially quantify over each y_i variable (using the also-now-familiar \exists). Thus, here are the two diophantine problems that correspond to the pair E1 and E2 from just above:

$$\text{P1} \quad \text{Is it true that } \forall x \exists y (3x - 2y = 0)?$$

$$\text{P2} \quad \text{Is it true that } \forall x \exists y (2x^2 - y = 0)?$$

Great Paper!



Hilbert's Tenth Problem is Unsolvable

Author(s): Martin Davis

Source: *The American Mathematical Monthly*, Vol. 80, No. 3 (Mar., 1973), pp. 233-269

Published by: [Mathematical Association of America](#)

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Great Paper!



Hilbert's Tenth Problem is Unsolvable

Author(s): Martin Davis

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Great Paper!

1. Diophantine Sets. In this article the usual problem of Diophantine equations will be inverted. Instead of being given an equation and seeking its solutions, one will begin with the set of “solutions” and seek a corresponding Diophantine equation. More precisely:

DEFINITION. A set S of ordered n -tuples of positive integers is called **Diophantine** if there is a polynomial $P(x_1, \dots, x_n, y_1, \dots, y_m)$, where $m \geq 0$, with integer coefficients such that a given n -tuple $\langle x_1, \dots, x_n \rangle$ belongs to S if and only if there exist positive integers y_1, \dots, y_m for which

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1973] HILBERT'S TENTH PROBLEM IS UNSOLVABLE 235

$$P(x_1, \dots, x_n, y_1, \dots, y_m) = 0.$$

Borrowing from logic the symbols “ \exists ” for “there exists” and “ \Leftrightarrow ” for “if and only if”, the relation between the set S and the polynomial P can be written succinctly as:

$$\langle x_1, \dots, x_n \rangle \in S \Leftrightarrow (\exists y_1, \dots, y_m) [P(x_1, \dots, x_n, y_1, \dots, y_m) = 0],$$

or equivalently:

$$S = \{ \langle x_1, \dots, x_n \rangle \mid (\exists y_1, \dots, y_m) [P(x_1, \dots, x_n, y_1, \dots, y_m) = 0] \}.$$

Note that P may (and in non-trivial cases always will) have negative coefficients. The word “**polynomial**” should always be so construed in the article except where the contrary is explicitly stated. Also all numbers in this article are positive integers unless the contrary is stated.



Hilbert's Tenth
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233-269

on a wide range of
facilitate new forms

Great Paper!



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Notice that this is a perfect fit with how we used formal logic to present and understand the Polynomial Hierarchy and the Arithmetic Hierarchy.

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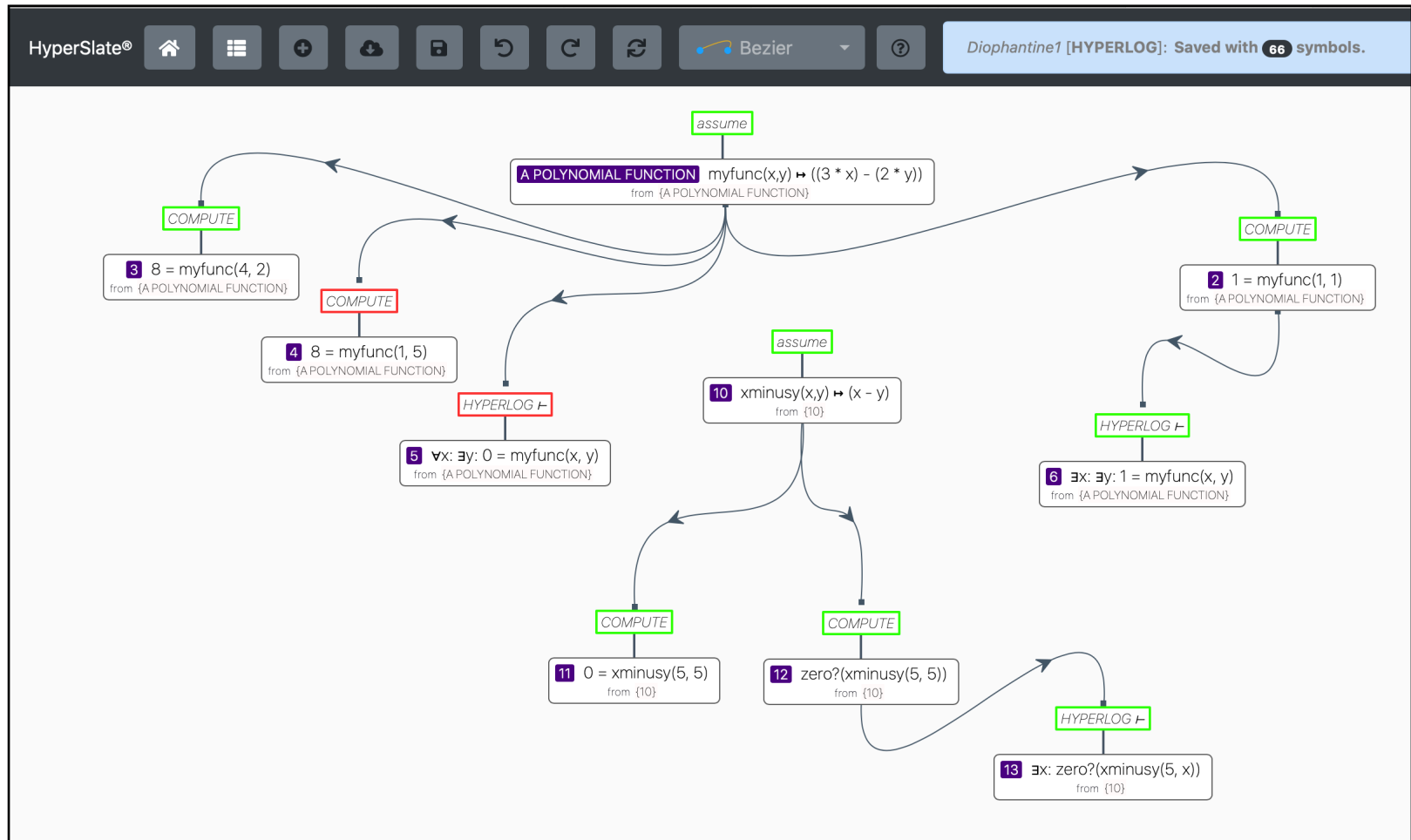
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Diophantine “Threat” in the Programming Language Hyperlog[®]



Where have we seen this before from Dr Gödel? \$20

The Crux

$\exists \mathcal{P}$ s.t. no human mind could ever decide $\forall x_1 \forall x_2 \cdots \forall x_k \exists y_1 \exists y_2 \cdots \exists y_j (\mathcal{P}(x_1, x_2, \dots, x_k, y_1, y_2, \dots, y_j))$?

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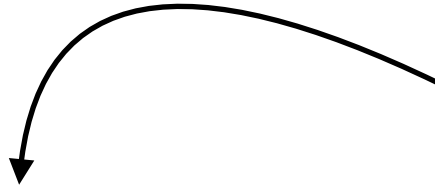
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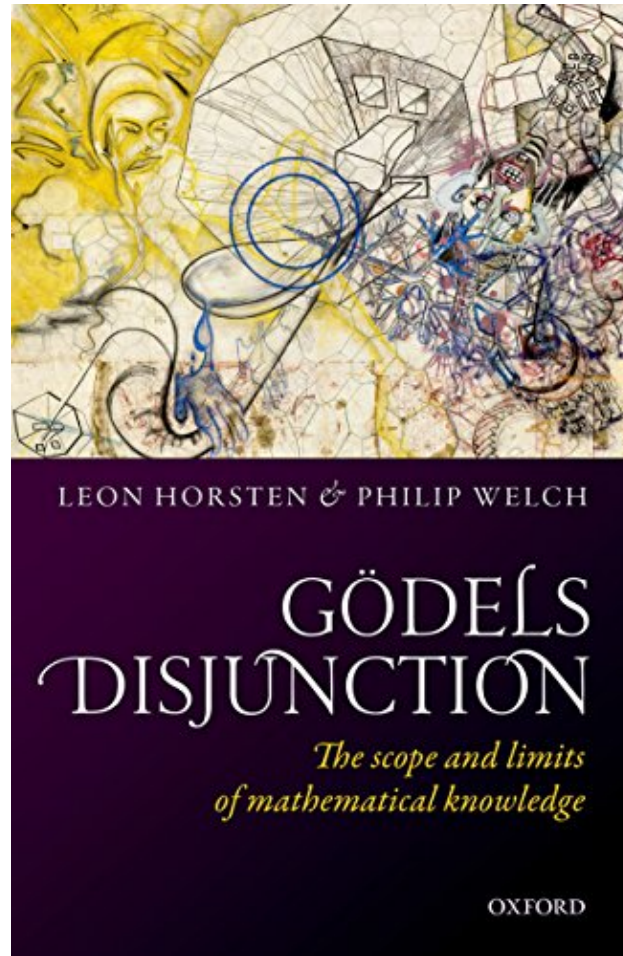
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Entire book on Gödel's Either-Or ...

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Earlier Gödelian Argument for the “No.”



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Outline

Abstract

1. Introduction
2. Clarifying computationalism, the view to be overthro...
3. The essence of hypercomputation: harnessing the in...
4. Gödel on minds exceeding (Turing) machines by “co...
5. Setting the context: the busy beaver problem
6. The new Gödelian argument
7. Objections
8. Conclusion

References

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Tables (1)

Table 1



Applied Mathematics and Computation

Volume 176, Issue 2, 15 May 2006, Pages 516-530



A new Gödelian argument for hypercomputing minds based on the busy beaver problem ☆

Selmer Bringsjord , Owen Kellett, Andrew Shilliday, Joshua Taylor, Bram van Heuveln, Yingrui Yang, Jeffrey Baumes, Kyle Ross

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<https://doi.org/10.1016/j.amc.2005.09.071>

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Abstract

Do human persons hypercompute? Or, as the doctrine of *computationalism* holds, are they information processors at or below the Turing Limit? If the former, given the essence of hypercomputation, persons must in some real way be capable of infinitary information processing. Using as a springboard Gödel's little-known assertion that the human mind has a power “converging to infinity”, and as an anchoring problem Rado's [T. Rado, On non-computable functions, Bell System Technical Journal 41 (1963) 877–884] Turing-uncomputable “busy beaver” (or Σ) function, we present in this short paper a new argument that, in fact, human persons can hypercompute. The argument is intended to be formidable, not conclusive: it brings Gödel's intuition to a greater level of precision, and places it within a sensible case against computationalism.

Finally, finally, ...

Gödel-vs-AI “Scorecard”

The Particular Work	Nutshell Diagnosis	Beyond AI?

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*Philosophical Reasoning	Undeniably beyond foreseeable AI.	Yes

Bringsjord vs. Rapaport ...

Will AI Match (Or Even Exceed) Human Intelligence?



No.



Yes.

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■: “Negative” enumerative induction for $\neg \exists year_k (AI = HI @ year_k)$ from $AI \neq HI @ year_{1958} \wedge \dots \wedge AI \neq HI @ year_{2021}$. Plus the proposition that AI is in fact not improving — relative to the intellectual stuff that matters most.

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4: And finally, the sledgehammer is used: *phenomenal consciousness*.

And now let's wrap up with final logistics:

Required Problems

Test 3 — just 2!!!!!!!

Visit to Final-Grades Algorithm:

E.g., for John Doe:

$$4 \times .10 + 4 \times .15 + 4 \times .25 + 4 \times .10 + 4 \times .40 = 4 = A$$

*Med nok penger, kan
logikk løse alle problemer.*