Gödel's God Theorem, (Selmer's Mental Family?)

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Required; April 17; Help ...

Fun Times @ Penn

"'Proving that God exists is no harder than proving that 2+2=4 from **PA**."



Context ...

Gödel's Great Theorems (OUP)

- Introduction ("The Wager")
- Brief Preliminaries (e.g. the propositional calculus & FOL)
- The Completeness Theorem
- The First Incompleteness Theorem
- The Second Incompleteness Theorem
- The Speedup Theorem
- The Continuum-Hypothesis Theorem
- The Time-Travel Theorem
- Gödel's "God Theorem"
- Could a Finite Machine Match Gödel's Greatness?



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The Ontological/Modal Argument Meets Al

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Theorems ...

Benzmüller-Scott-Gödel

A1	Either a property or its negation is positive, but not both:
	$\forall \phi [P(\neg \phi) \equiv \neg P(\phi)]$
A2	A property necessarily implied by a positive property is posi-
	tive: $\forall \phi \forall \psi [(P(\phi) \land \Box \forall x [\phi(x) \supset \psi(x)]) \supset P(\psi)]$
T 1	Positive properties are possibly exemplified:
	$\forall \phi [P(\phi) \supset \diamondsuit \exists x \phi(x)]$
D1	A God-like being possesses all positive properties:
	$G(x) \equiv \forall \phi[P(\phi) \supset \phi(x)]$
A3	The property of being God-like is positive: $P(G)$
С	Possibly, God exists: $\diamond \exists x G(x)$
A4	Positive properties are necessarily positive:
	$\forall \phi [P(\phi) \supset \Box \ P(\phi)]$
D2	An essence of an individual is a property possessed by it and
	necessarily implying any of its properties:
	$\phi ess. x \equiv \phi(x) \land \forall \psi(\psi(x) \supset \Box \forall y(\phi(y) \supset \psi(y)))$
T2	Being God-like is an essence of any God-like being:
	$\forall x[G(x) \supset G \ ess. \ x]$
D3	Necessary existence of an individ. is the necessary exemplifi-
	cation of all its essences: $NE(x) \equiv \forall \phi [\phi \ ess. \ x \supset \Box \exists y \phi(y)]$
A5	Necessary existence is a positive property: $P(NE)$
T3	Necessarily, God exists: $\Box \exists x G(x)$

Benzmüller-Scott-Gödel

t not both:	1	A1
$(\neg \phi) \equiv \neg P(\phi)$]		
roperty is posi-	2	A2
$\psi(x)]) \supset P(\psi)]$		
	'1	T 1
$(b) \supset \Diamond \exists x \phi(x)]$		
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P (G)	3	A
$\diamond \exists x G(x)$	l ,	С
	4	A۷
$P(\phi) \supset \Box P(\phi)$]		
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$(\phi(y) \supset \psi(y))) \Big $		
being:	2	T2
$(x) \supset G ess. x]$		
sary exemplifi-	3	D3
$x \supset \Box \exists y \phi(y)] \Big $		
P (NE)	5	A5
$\Box \exists x G(x)$	'3	T3

Benzmüller-Scott-Gödel

X

A Victorious Gödelian Variant?

Intelligently extracted from Gödel/Benzmüller's AI; FI a from Oppy.

$$\forall R(Pos(R) \rightarrow \neg Pos(\bar{R}))$$

Gödel/Benzmüller's A2; F2 from Oppy.

$\forall R, R'[Pos(R) \land \Box \forall x(R(x) \rightarrow R'(x))] \rightarrow Pos(R')]$

N2 (Necessary existence is positive):

Pos(NE)

Def of strongly positive:

$$\forall R[Pos!(R) \leftrightarrow Pos(ER)]$$

Theorem 5 — welcome-weak? — from Oppy:

$$\forall R[Pos!(R) \rightarrow \Box \exists x ER(x)]$$

Variant: Positive to Great-making

Intelligently extracted from Gödel/Benzmüller's AI; FI a from Oppy.

$\forall R(GM(R) \to \neg GM(\bar{R}))$

Gödel/Benzmüller's A2; F2 from Oppy.

$\forall R, R'[GM(R) \land \Box \forall x(R(x) \to R'(x))] \to GM(R')]$

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Are Formal Inductive Arguments Even Better?



"The Other Way"

A New Family of Mental Arguments^{*}

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version 1204231154NY

A New Family of Mental Arguments*

⁴I refer, note, to the *original* series; but I do so without loss of generality, since nothing fundamentally changes in subsequent spinoffs.

⁵For a full definition of personhood, see (Bringsjord 1997, Bringsjord, Noel & Caporale 2000) (or any other credible account; e.g., see Dennett 1978, Chisholm 1978). Here, without the surrounding discussion from that book, is the definition, amended slightly for the present paper: x is a person if and only if x has the *capacity*

- 1. to "will," to make choices and decisions, set plans and projects autonomously;
- 2. for consciousness,⁶ for experiencing pain and sorrow and happiness, and a thousand other emotions love, passion, gratitude, and so on;
- 3. for *self*-consciousness, for being aware of his/her states of mind, inclinations, preferences, etc., and for grasping the concept of him/herself;
- 4. to communicate through a language;
 - Note: The language here should at minimum be at the level of one determined by a mildly Type-0 grammar. For now (I return below to the issue), I leave this formal constraint aside, and mention only that one of the extraordinary things about human persons is that the natural languages over which they have command are at least at this level, when viewed through the lens of formal logic. From the point of view of the present paper, the greatness of us, on the linguistic side, can be viewed as at least partially revealed in the rather famous (Chomsky 1956). However, many philosophers and logicians will know that so-called "Type 0" grammars in Chomsky's hierarchy were being specified, probed, and understood by Post (himself, of course, a human person) in the 1920's. Post didn't publish these grammars till much later, in (Post 1943).
- 5. to know and believe propositions of great complexity,
 - Note: I leave at this sport the concept of complexity informal. It would be easy enough to pin things down via both extensional (e.g. quantificational complexity regimented by the standard Δ_i, Σ_j, Π_k categorization) and intensional (e.g. layers of epistemic and other modal operators) complexity measures for formulae that capture propositions. I return to this below.

and to believe things about what others believe (second-order beliefs), and to believe things about what others believe about one's beliefs (third-order beliefs), and so on;

- 6. to desire not only particular objects and events, but also changes in his or her character, and in the character of others;
- 7. to reason (for example, in the fashion exhibited in the writing and reading/studying of this very paper).

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