

On to *Intensional* Logics

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IFLAI
3/20/2025



In The Logic-and-AI News

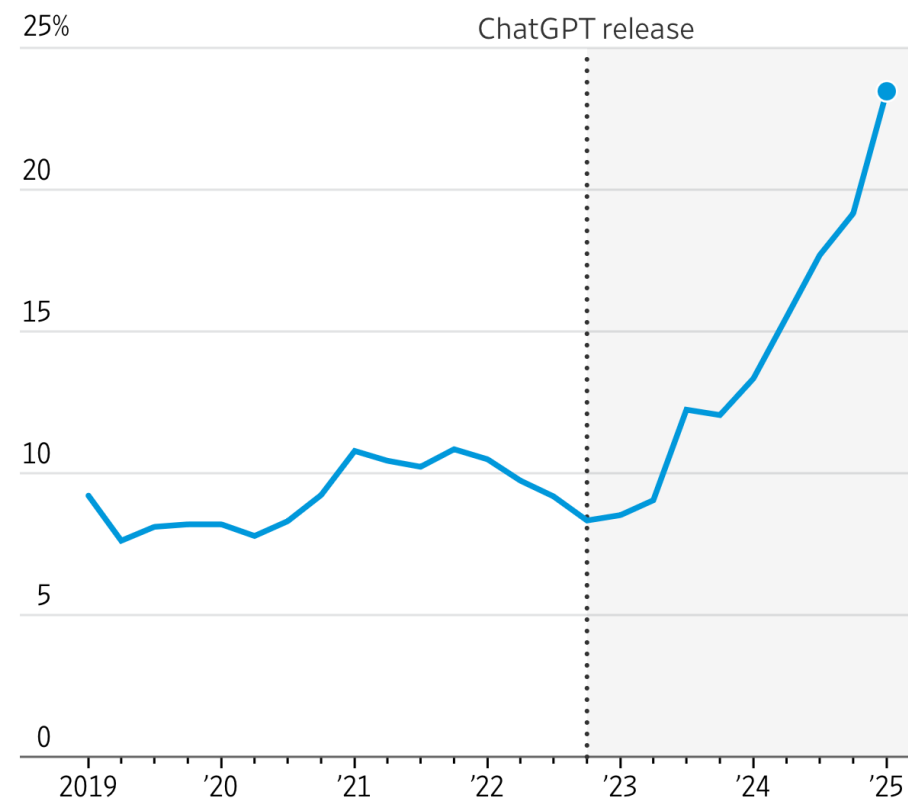
...

TECH

How the AI Talent Race Is Reshaping the Tech Job Market

In industries from finance to retail, companies are seeking artificial-intelligence skills when looking for technology staff

Share of newly listed technology* jobs that are AI-related, quarterly



*Technology jobs are a broad group of computer- and math-related occupations

Note: U.S. only. 1Q 2025 figures are as of January.

Source: UMD-LinkUp AI Maps

By Nate Rattner

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Consider therefore taking ILBAI & I(ML)AI!

*Technology jobs are a broad group of computer- and math-related occupations

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On the esemplastic
extensional-logic ladder ...
questions?

Climbing the k -order Ladder

Climbing the k -order Ladder

a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

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Climbing the k -order Ladder

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

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There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

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Climbing the k -order Ladder

Things x and y , along with the father of x , share a certain property; and, x R^2 s y , where R^2 is a positive property.

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Climbing the k -order Ladder

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FOL $\exists x [Llama(x) \wedge Llama(b) \wedge \textit{Likes}(x, b) \wedge Llama(\textit{fatherOf}(x))]$

\mathcal{L}_1 There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge \textit{Likes}(a, b) \wedge Llama(\textit{fatherOf}(a))$

\mathcal{L}_0 a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

⋮

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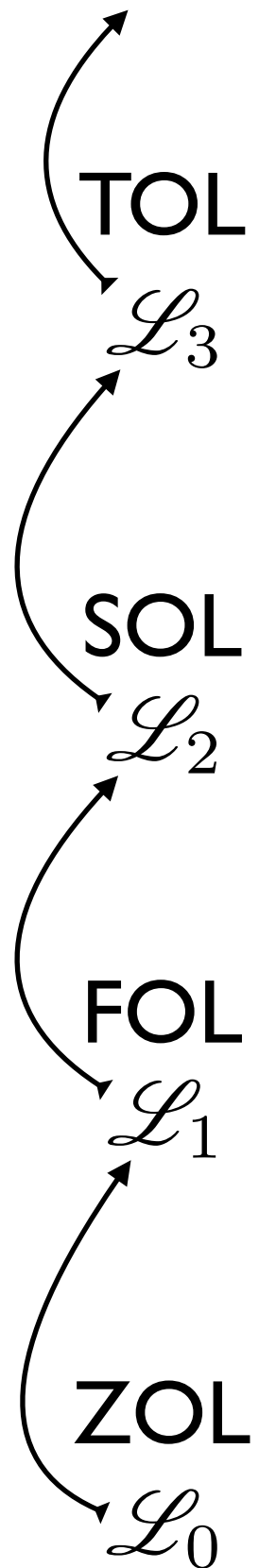
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The Ladder of Expressivity



$\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \text{Positive}(R^2) \wedge R(\text{fatherOf}(x))]$

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$\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \text{Likes}(x, y) \wedge R(\text{fatherOf}(x))]$

Things x and y , along with the father of x , share a certain property (and x likes y).

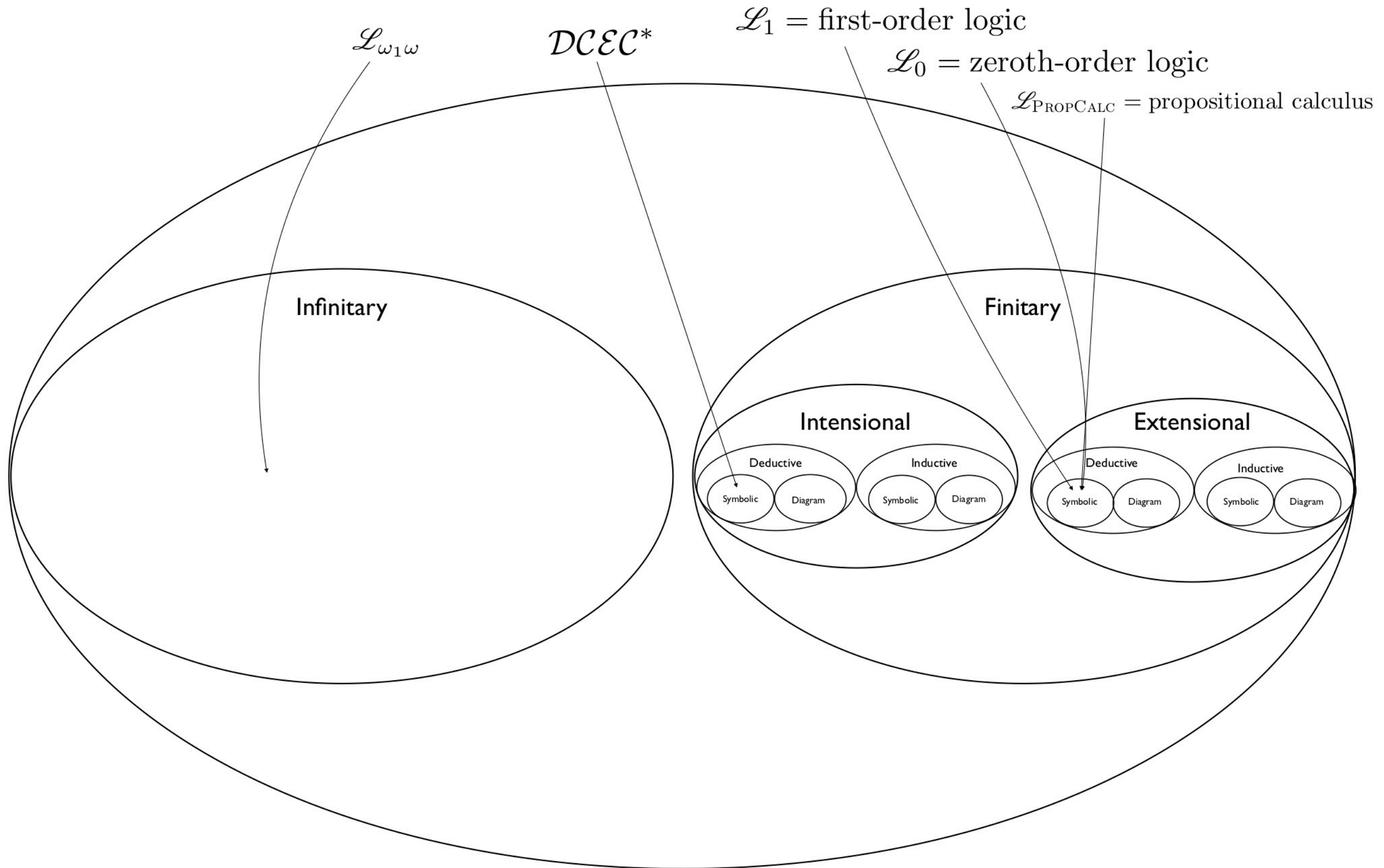
$\exists x [Llama(x) \wedge Llama(b) \wedge \text{Likes}(x, b) \wedge Llama(\text{fatherOf}(x))]$

There's some thing which is a llama and likes *Betty* (which is also a llama), and whose father is a llama too.

$Llama(a) \wedge Llama(b) \wedge \text{Likes}(a, b) \wedge Llama(\text{fatherOf}(a))$

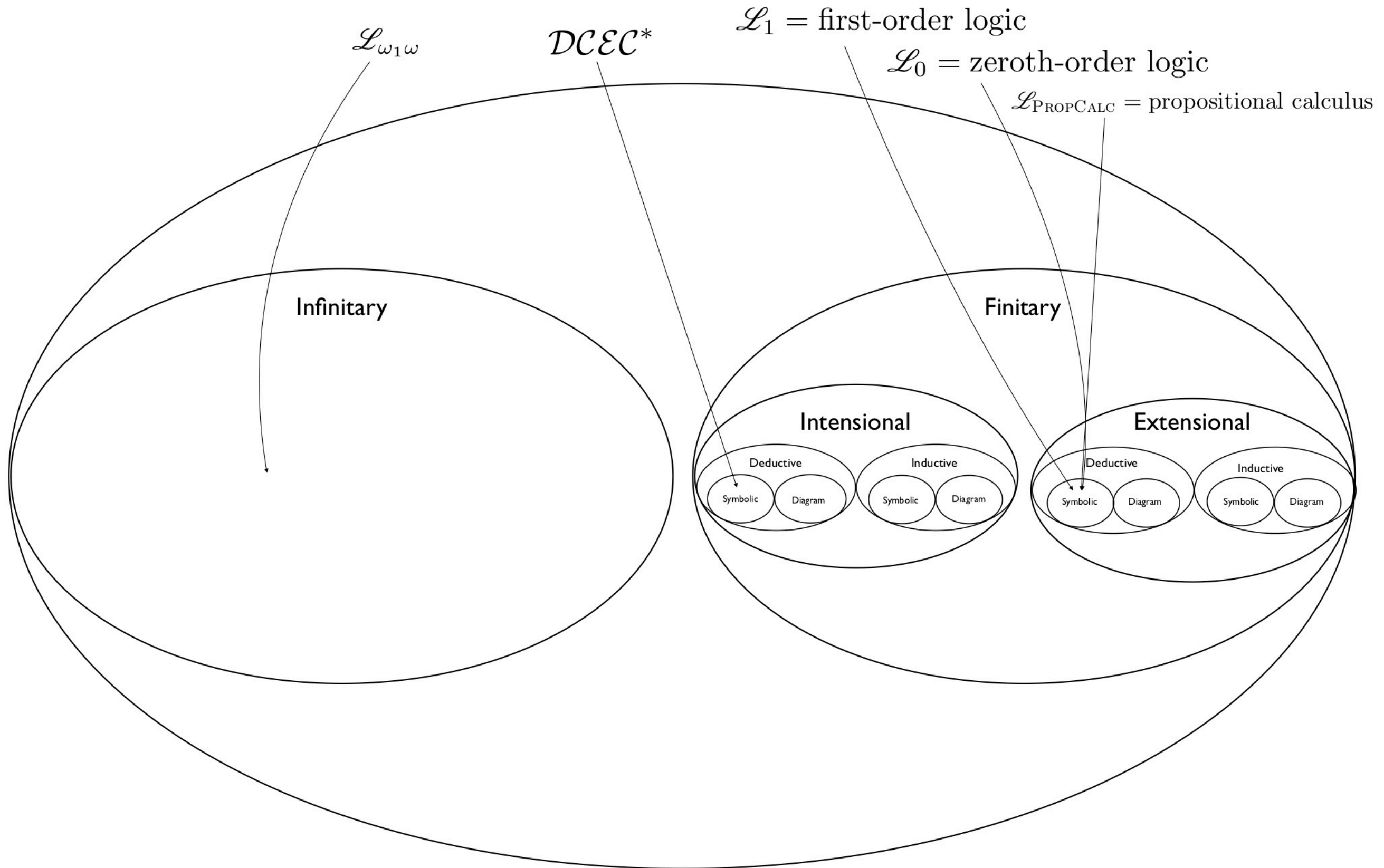
Alvin is a llama, as is *Betty*, *Alvin* likes *Betty*, and the father of *Alvin* is a llama as well.

The Universe of Logics

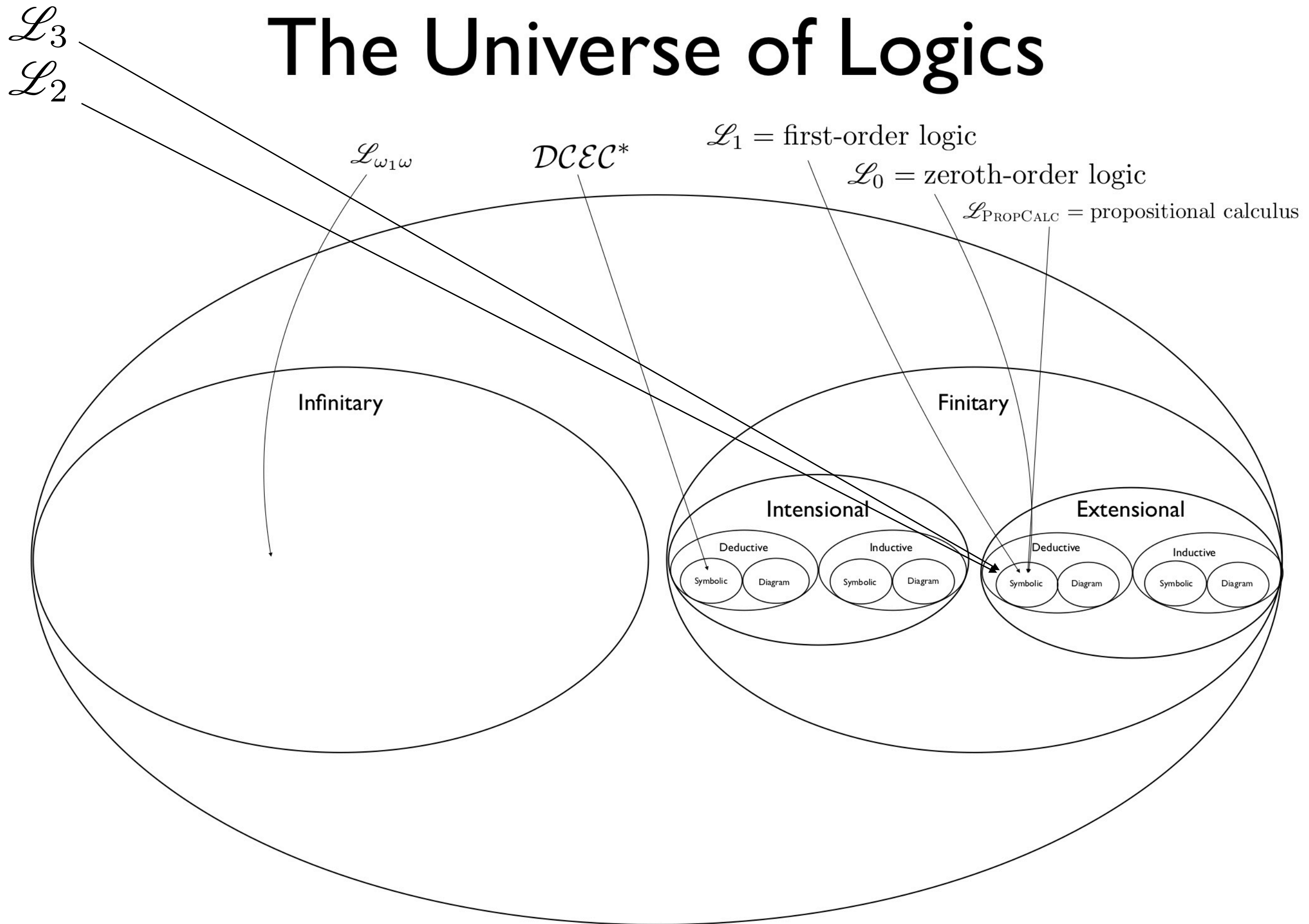


\mathcal{L}_3
 \mathcal{L}_2

The Universe of Logics



The Universe of Logics



Climbing the k -order Ladder

⋮

TOL $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \textit{Positive}(R^2) \wedge R(\textit{fatherOf}(x))]$

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Climbing the k -order Ladder

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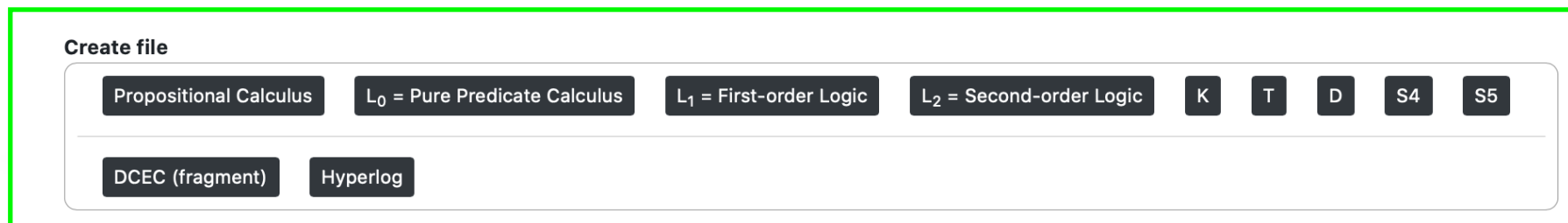
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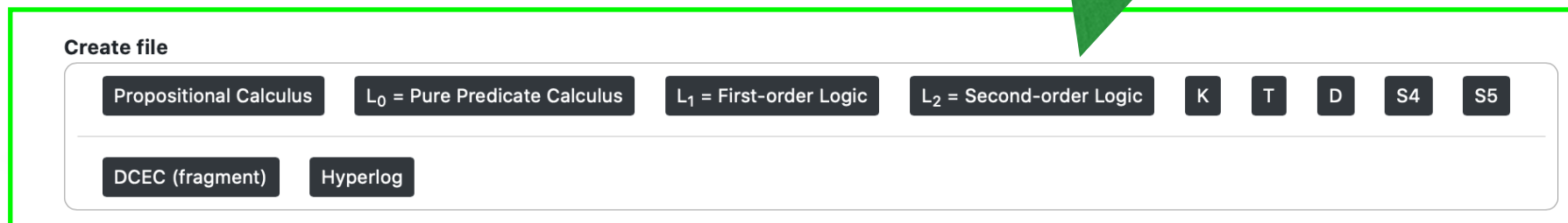
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Climbing the k -order Ladder

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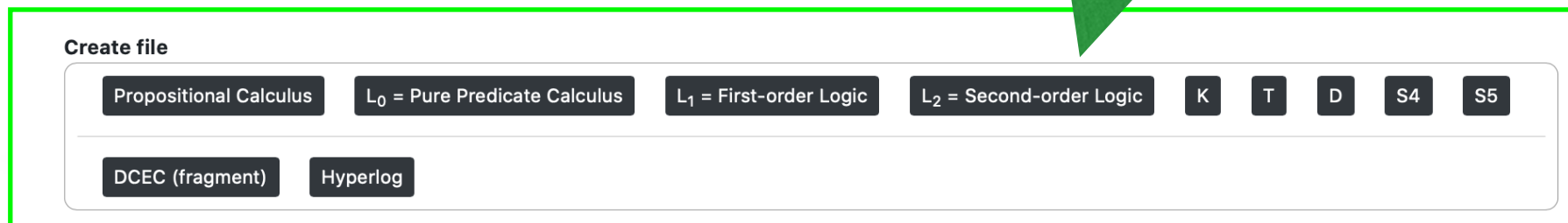
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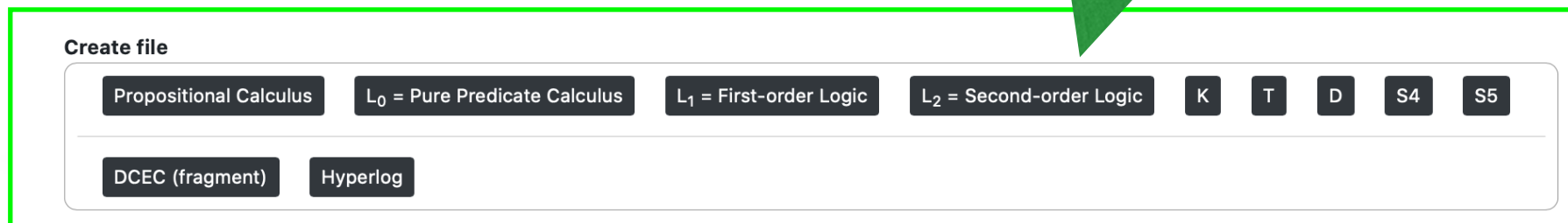
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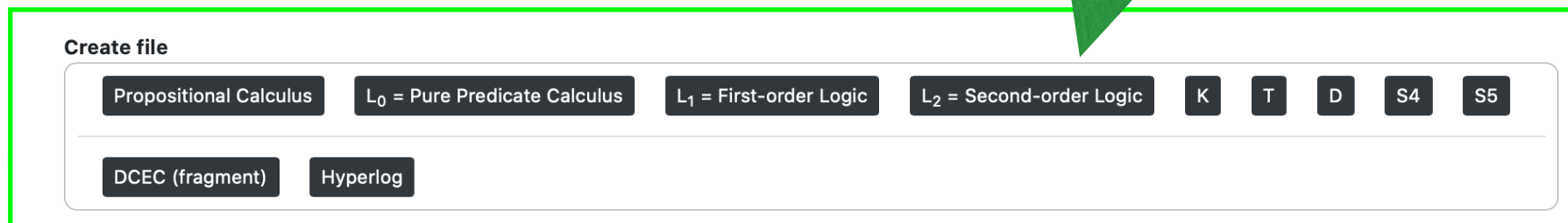
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Climbing the k -order Ladder

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Incomplete!

TOL

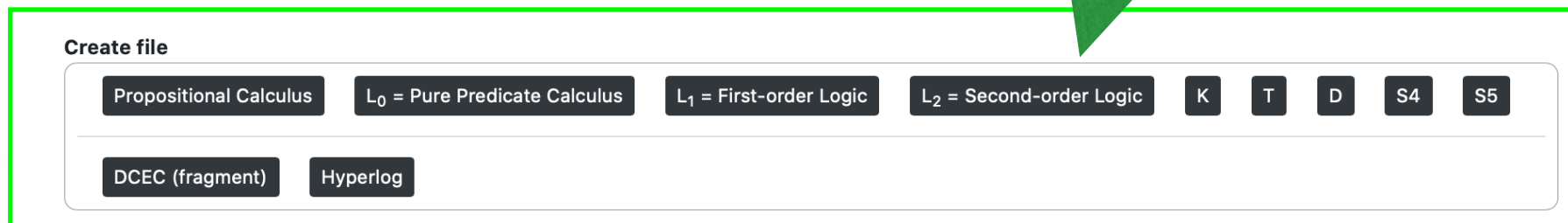
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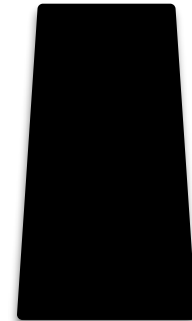
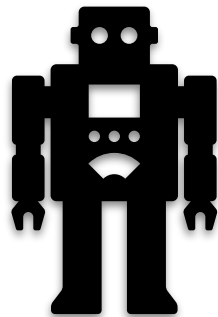
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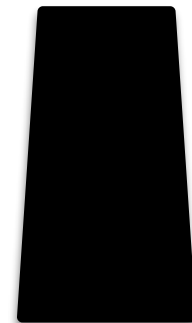
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**Blinky as portal to
intensional logics ...**

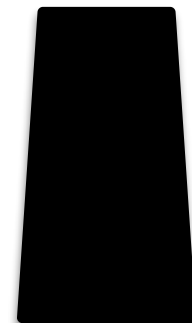
Blinky



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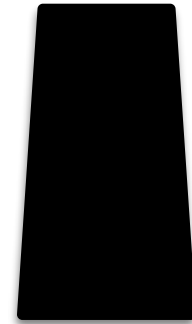
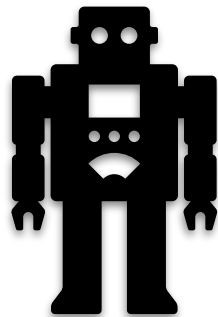


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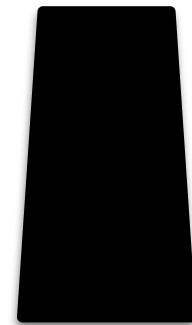


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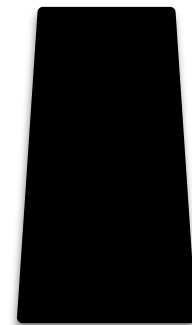
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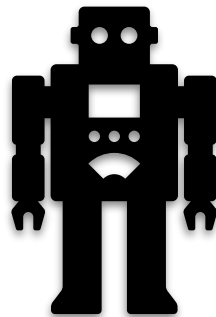
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Blinky



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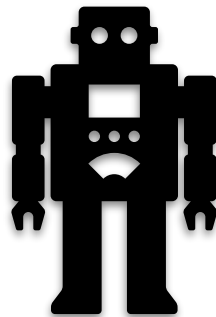


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Blinky



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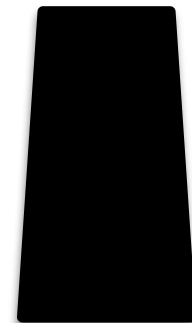
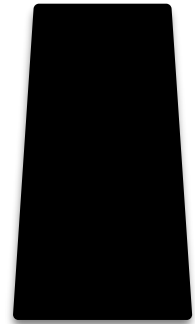
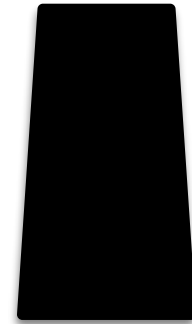
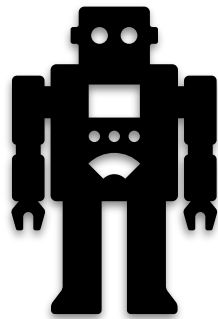


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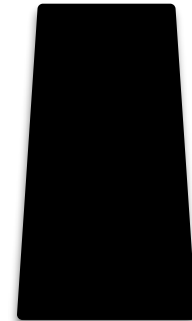
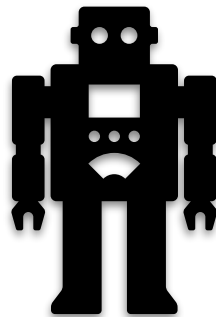
Blinky



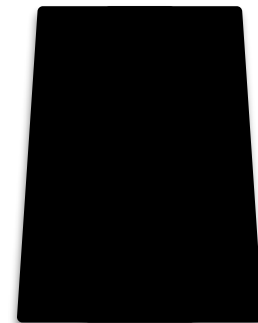
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3

Blinky



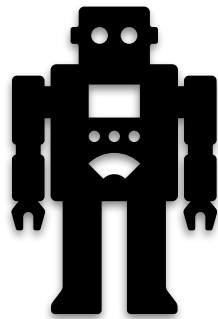
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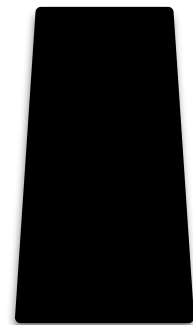
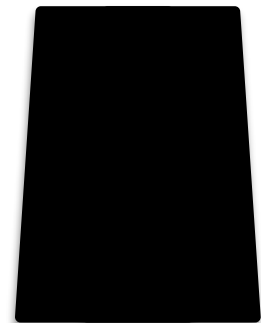
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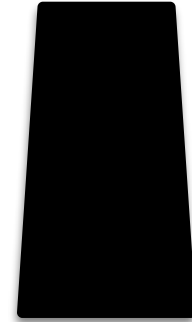
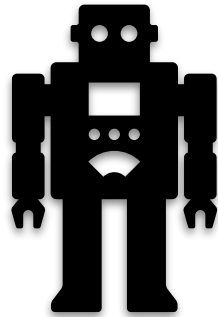
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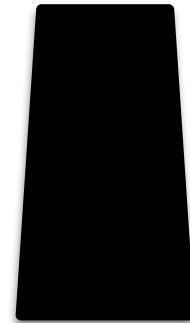
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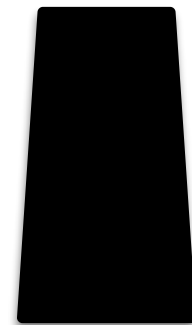
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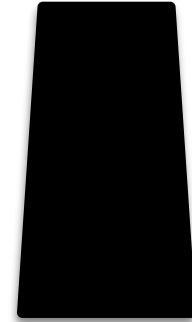
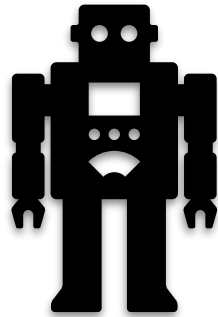


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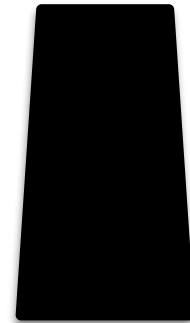


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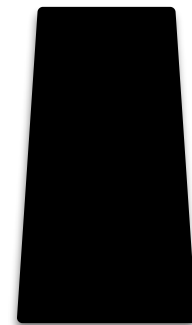
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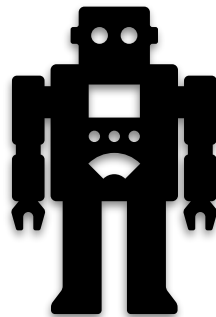


2



3

Blinky



1



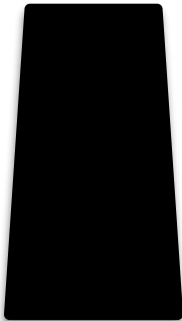
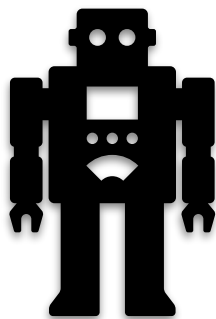
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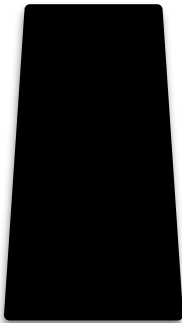
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James believes Blinky believes that the ball is in the cup at location #1.

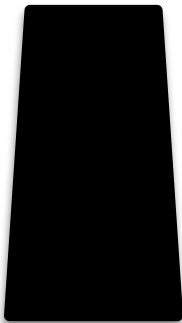
Blinky



1



2

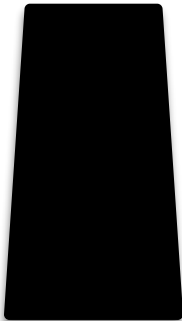
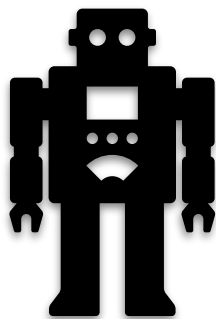


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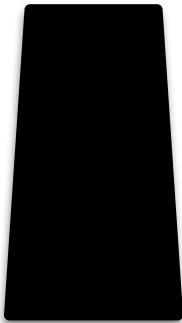
James believes Blinky believes that the ball is in the cup at location #1.

B(blinky, loc-ball-1)

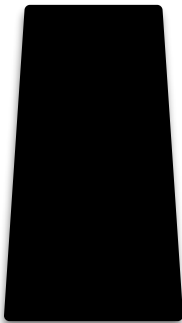
Blinky



1



2



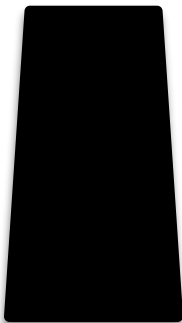
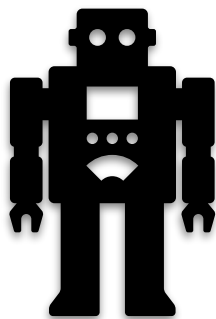
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James believes Blinky believes that the ball is in the cup at location #1.

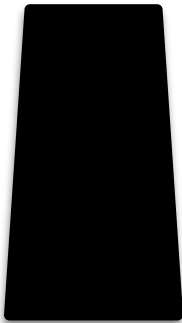
B(blinky, loc-ball-1)

(Believes! blinky loc-ball-1)

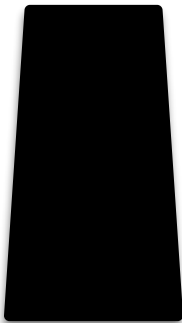
Blinky



1



2



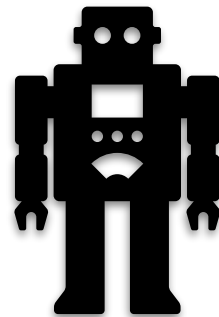
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James believes Blinky believes that the ball is in the cup at location #1.

B(blinky, loc-ball-1)

(Believes! blinky loc-ball-1)

Blinky



1



2



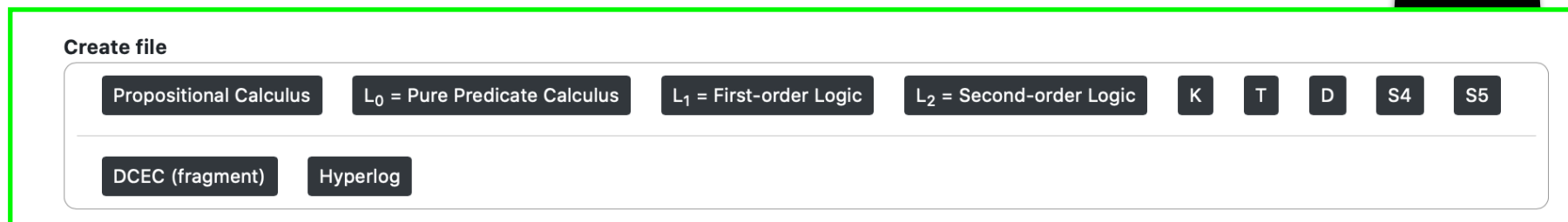
3

In intensional logics, meaning and designation are separated, and compositionality is abandoned.

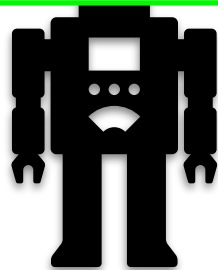
James believes Blinky believes that the ball is in the cup at location #1.

B(blinky, loc-ball-1)

(Believes! blinky loc-ball-1)



Blinky



1

2

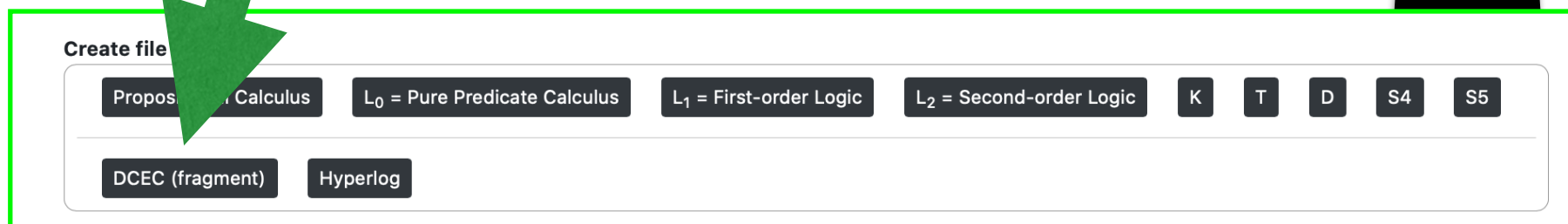
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In intensional logics, meaning and designation are separated, and compositionality is abandoned.

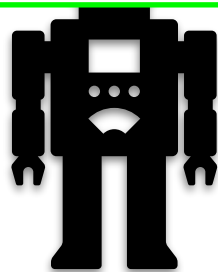
James believes Blinky believes that the ball is in the cup at location #1.

B(blinky, loc-ball-1)

(Believes! blinky loc-ball-1)



Blinky



2

3

In intensional logics, meaning and designation are separated, and compositionality is abandoned.

Blinky believes that the ball is in the cup at location #1.

James believes Blinky believes that the ball is in the cup at location #1.

B(blinky, loc-ball-1)

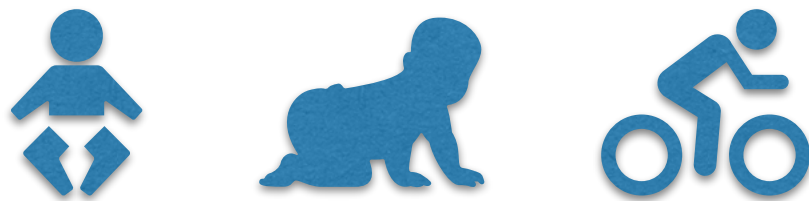
(Believes! blinky loc-ball-1)



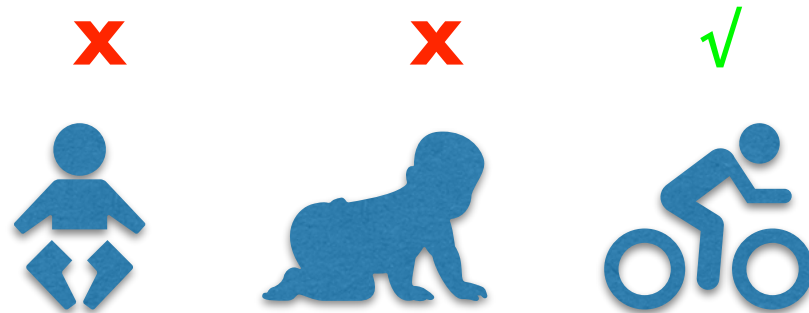
In intensional logics, meaning and designation are separated, and compositionality is abandoned.

False Belief Task Demands Intensional Logic ...

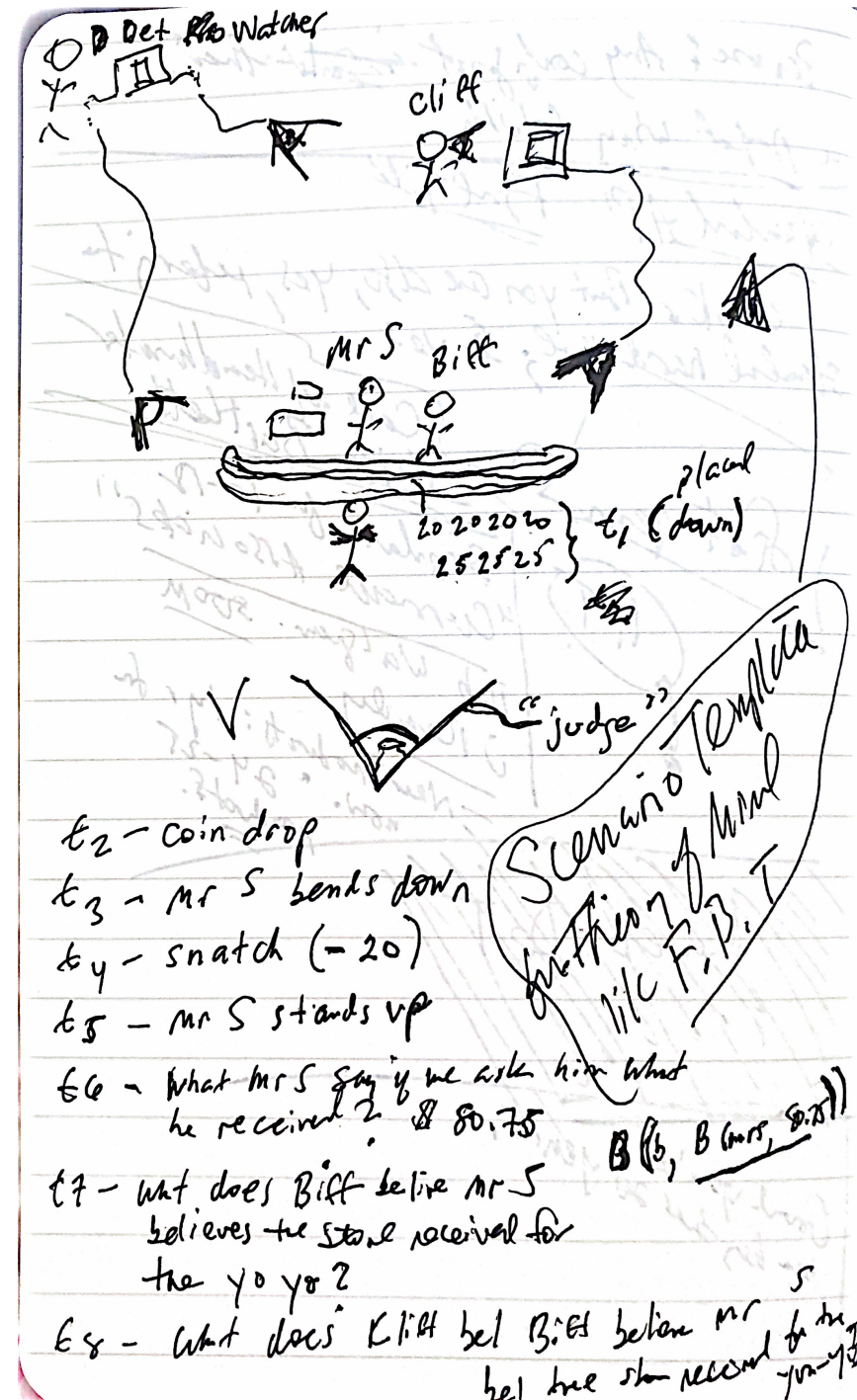
False Belief Task Demands Intensional Logic ...



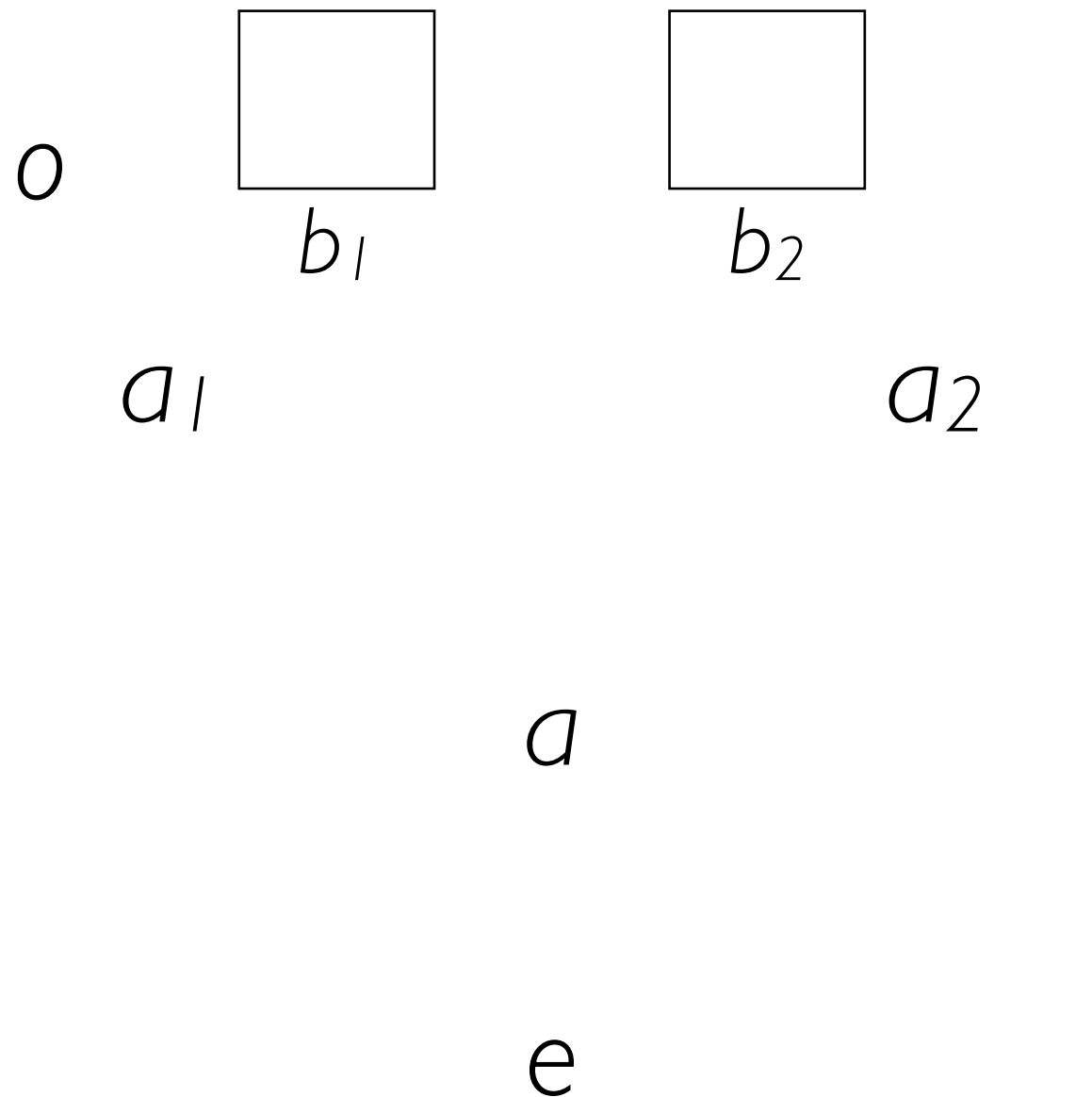
False Belief Task Demands Intensional Logic ...



Better, But Embryonic: The ToM Pawn Shop

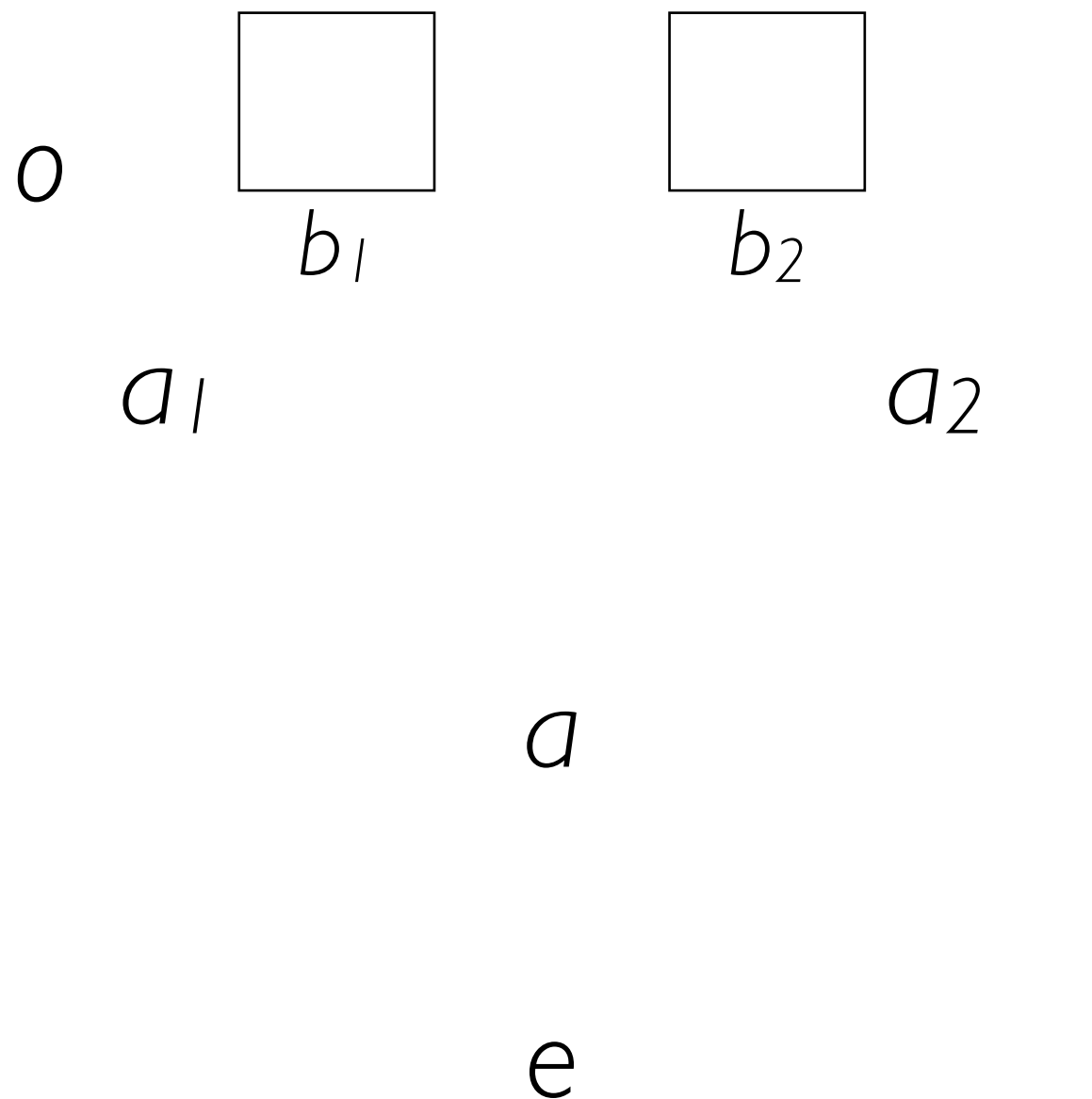


Framework for FBT^0_1



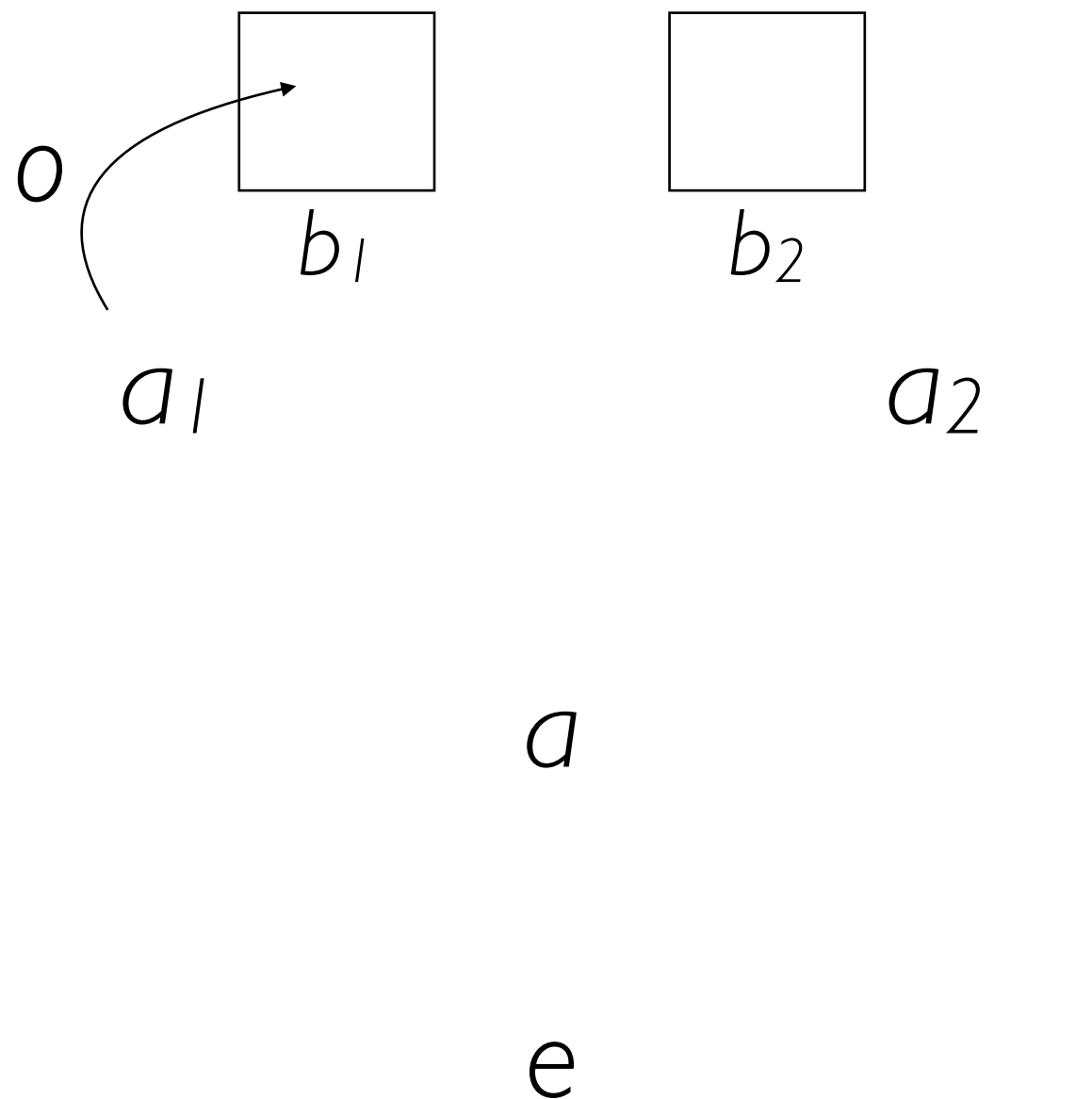
Framework for FBT^0_1

(five timepoints)



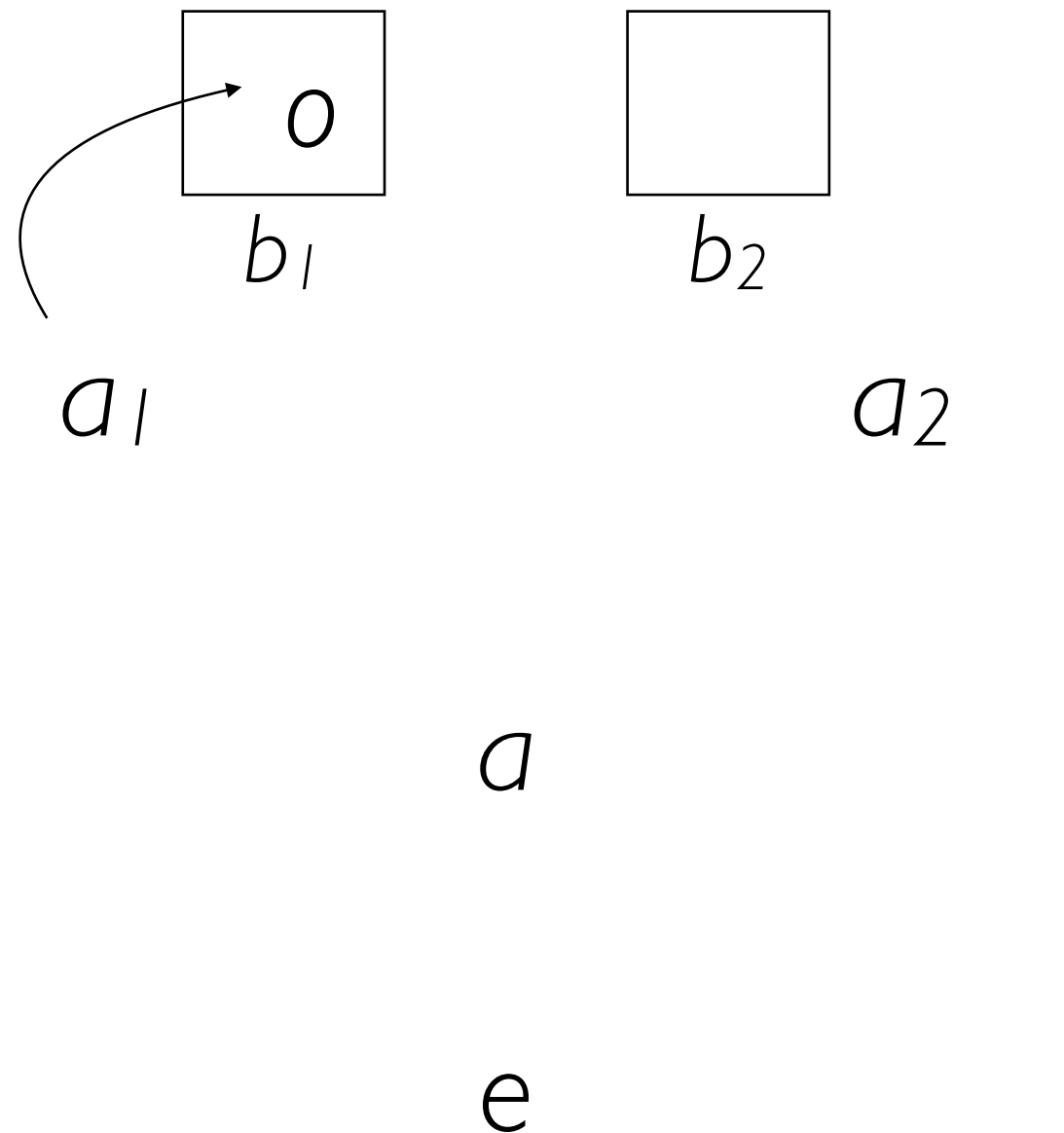
Framework for FBT^0_1

(five timepoints)



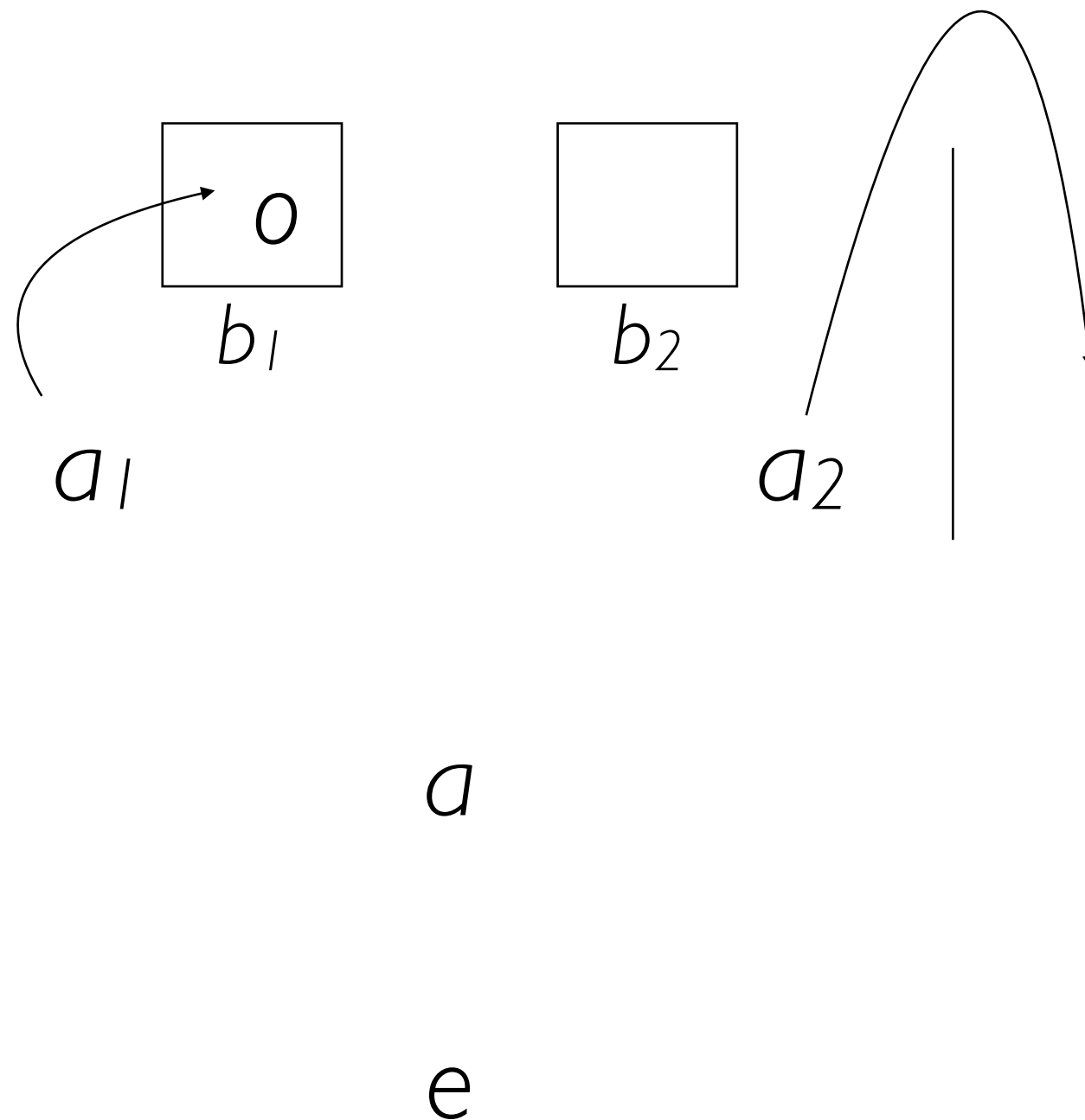
Framework for FBT^0_1

(five timepoints)



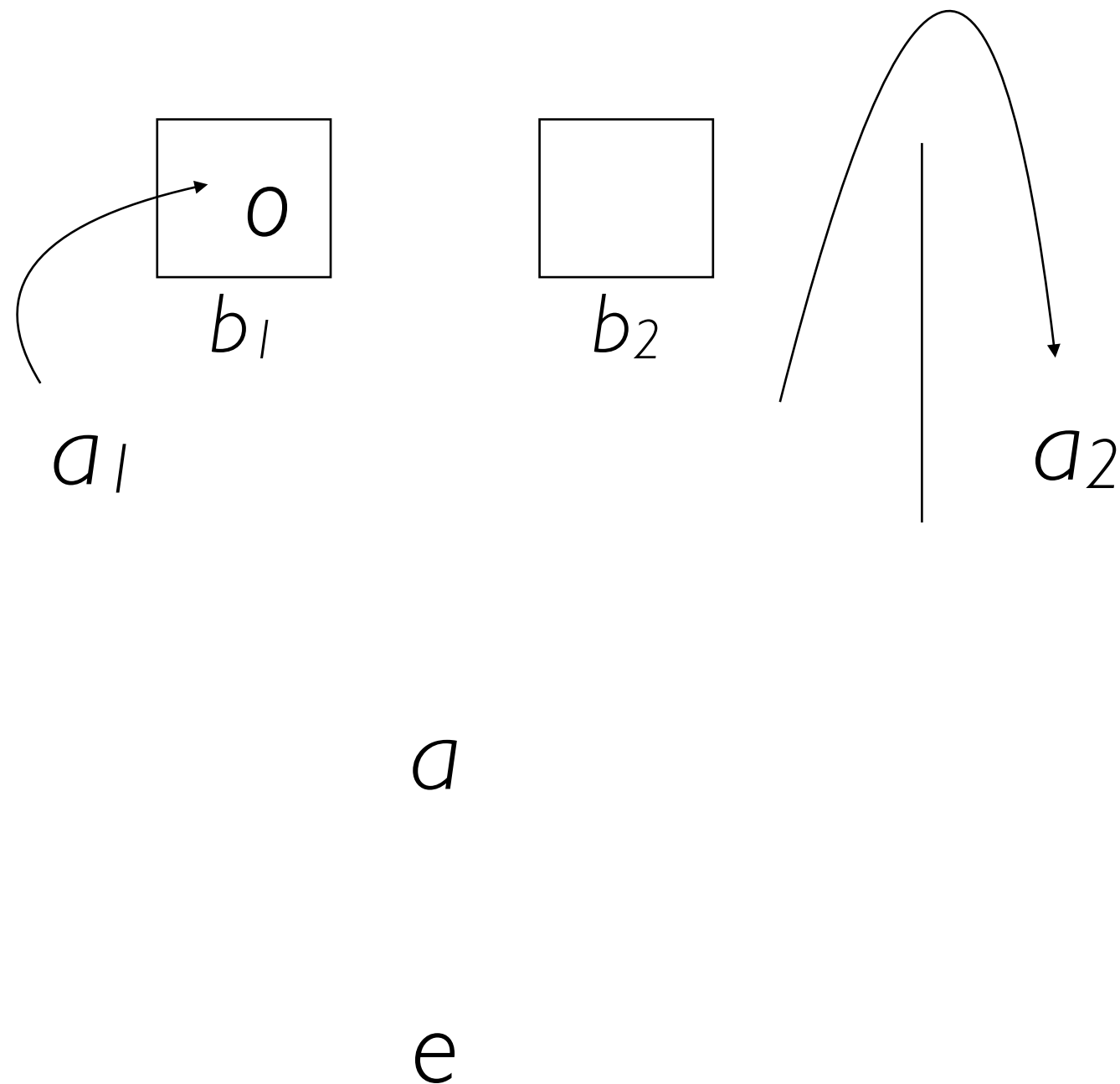
Framework for FBT^0_1

(five timepoints)



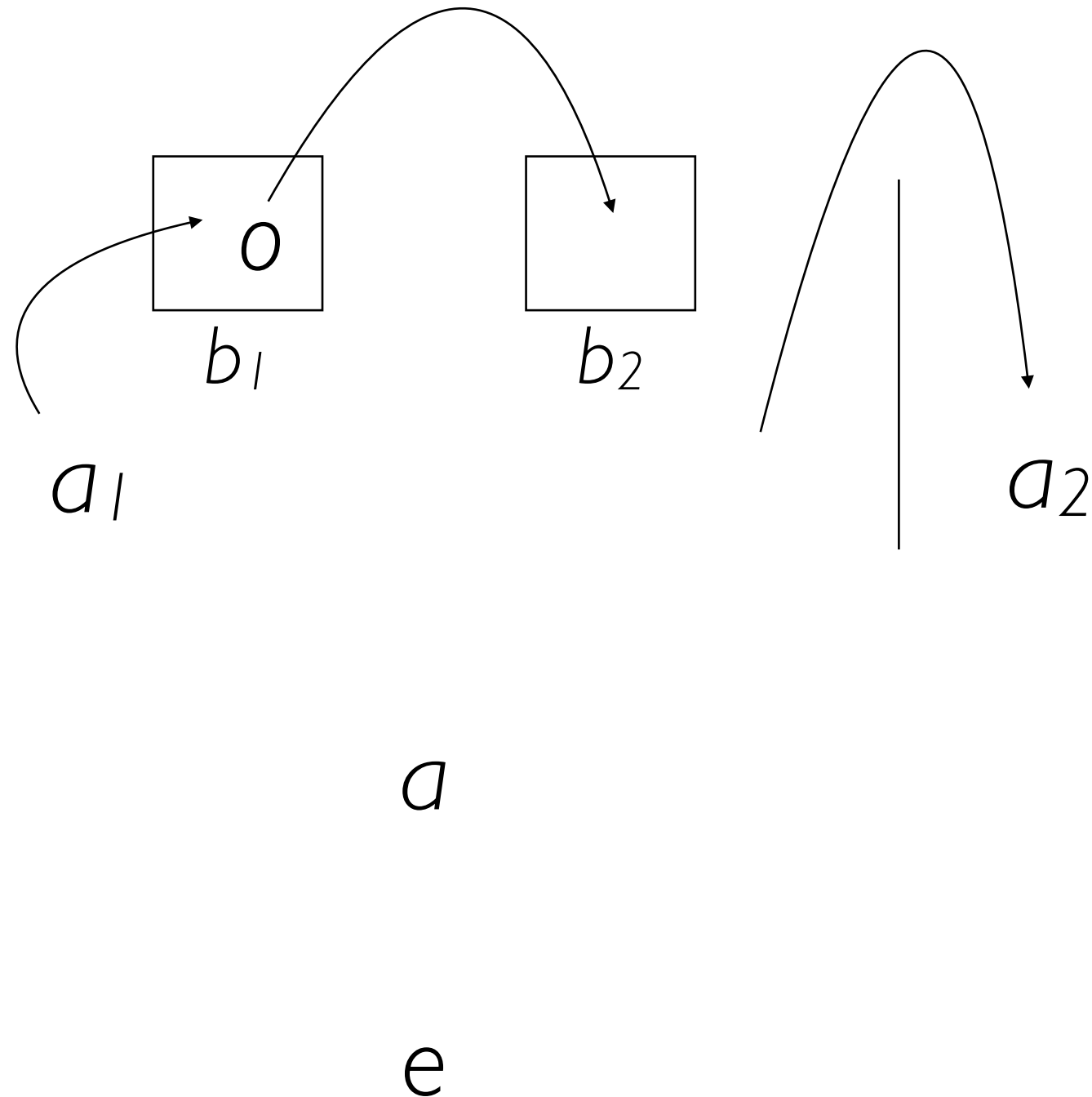
Framework for FBT^0_1

(five timepoints)



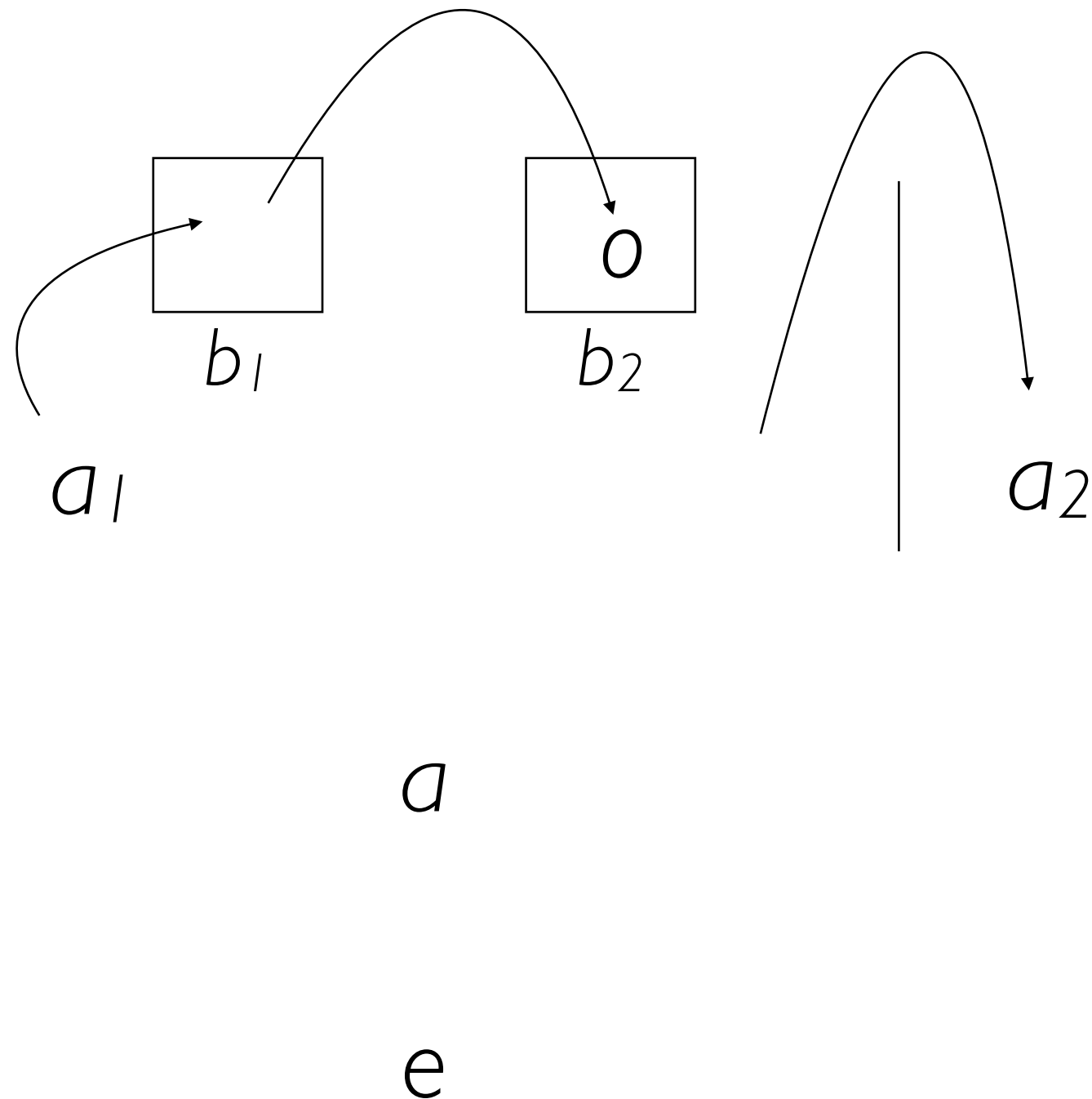
Framework for FBT^0_1

(five timepoints)



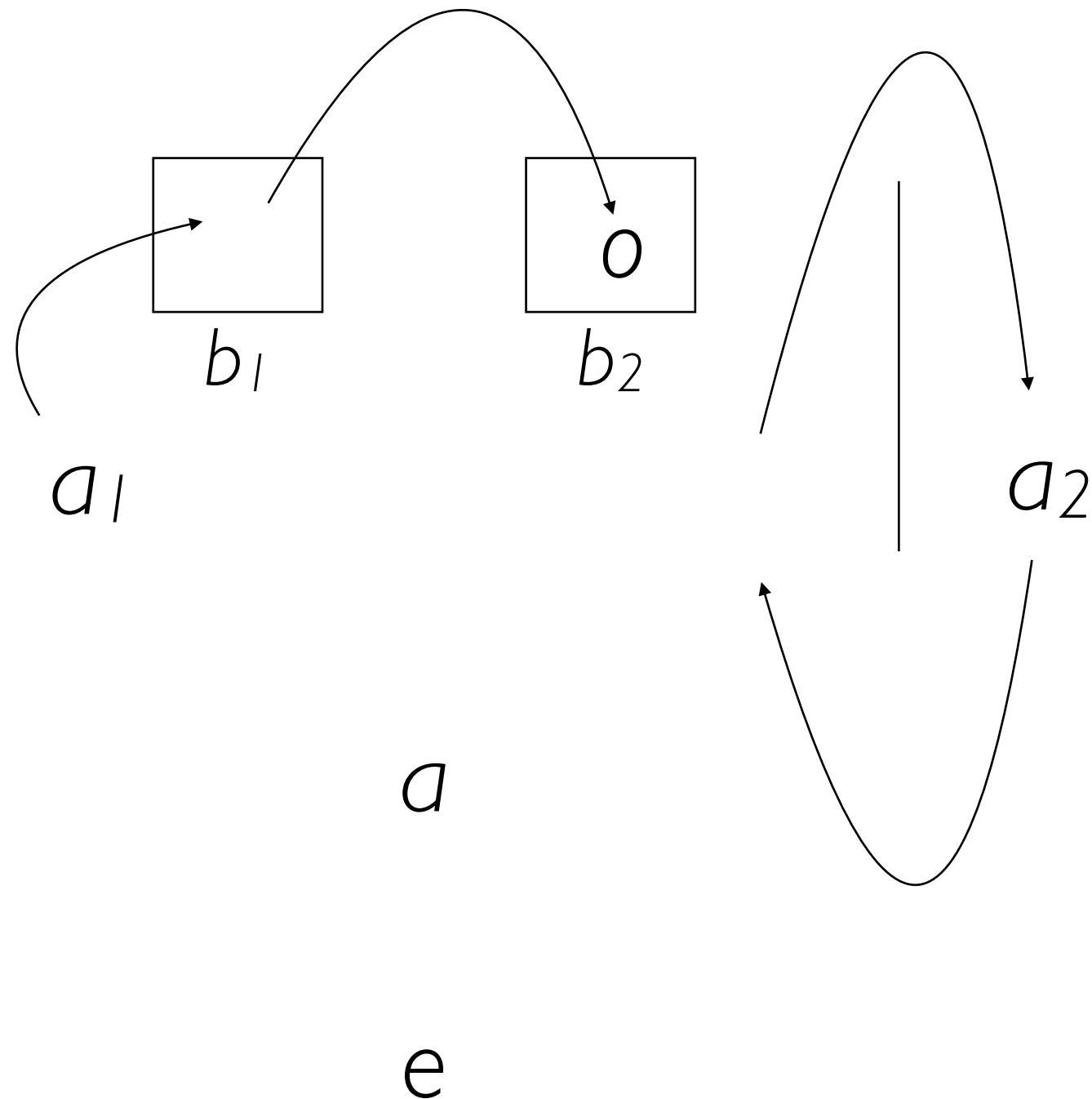
Framework for FBT^0_1

(five timepoints)



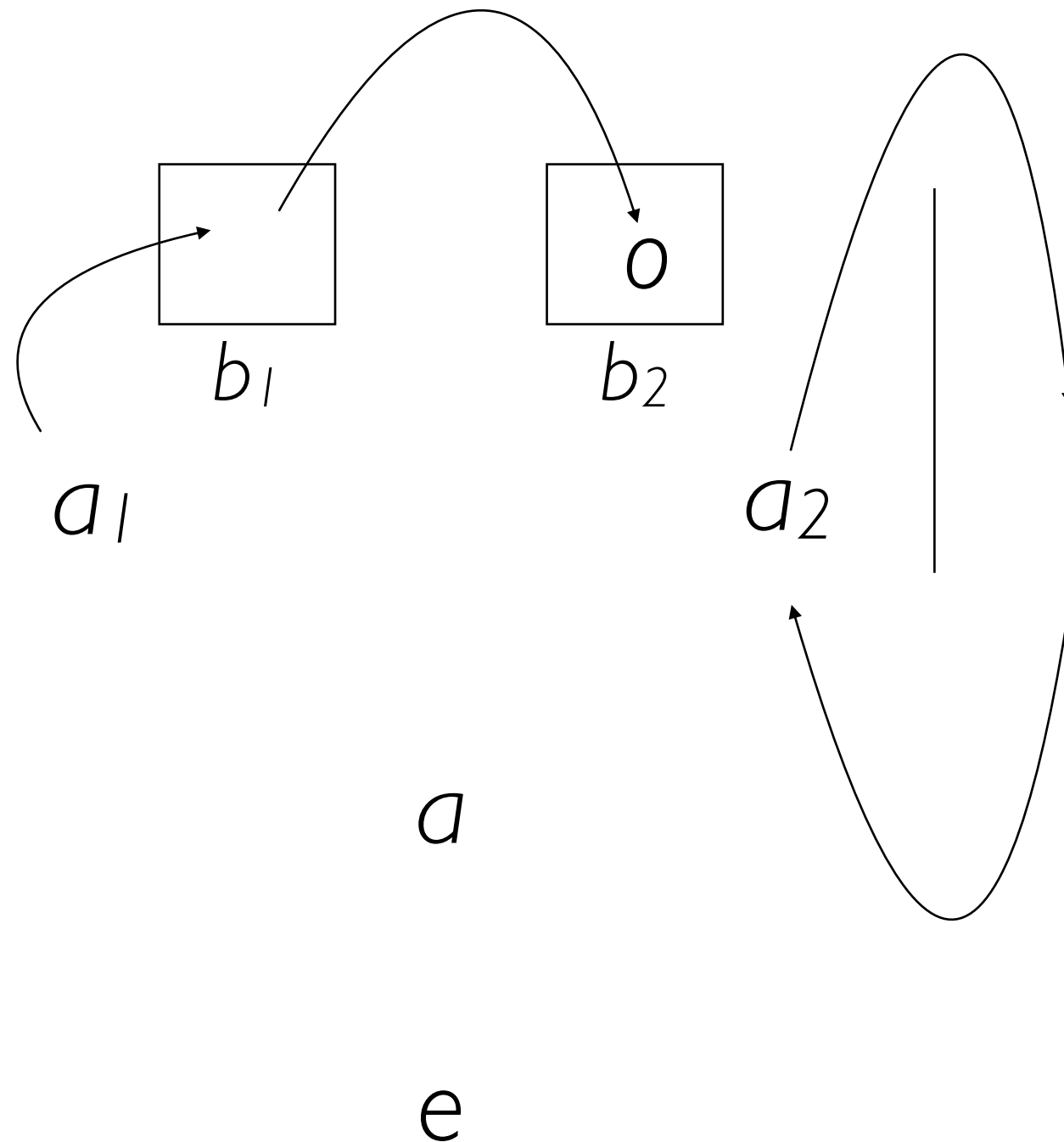
Framework for FBT^0_1

(five timepoints)



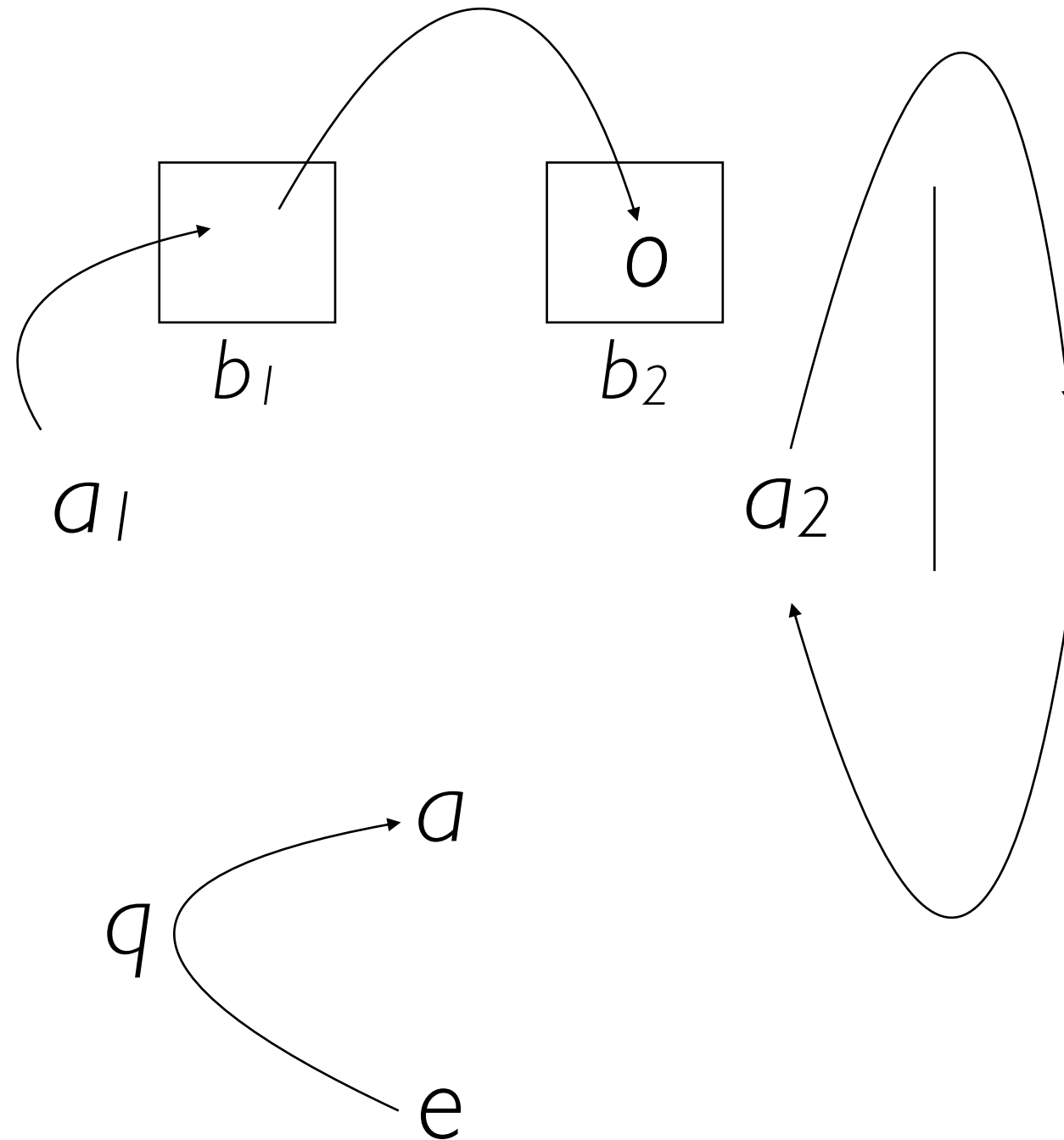
Framework for FBT^0_1

(five timepoints)



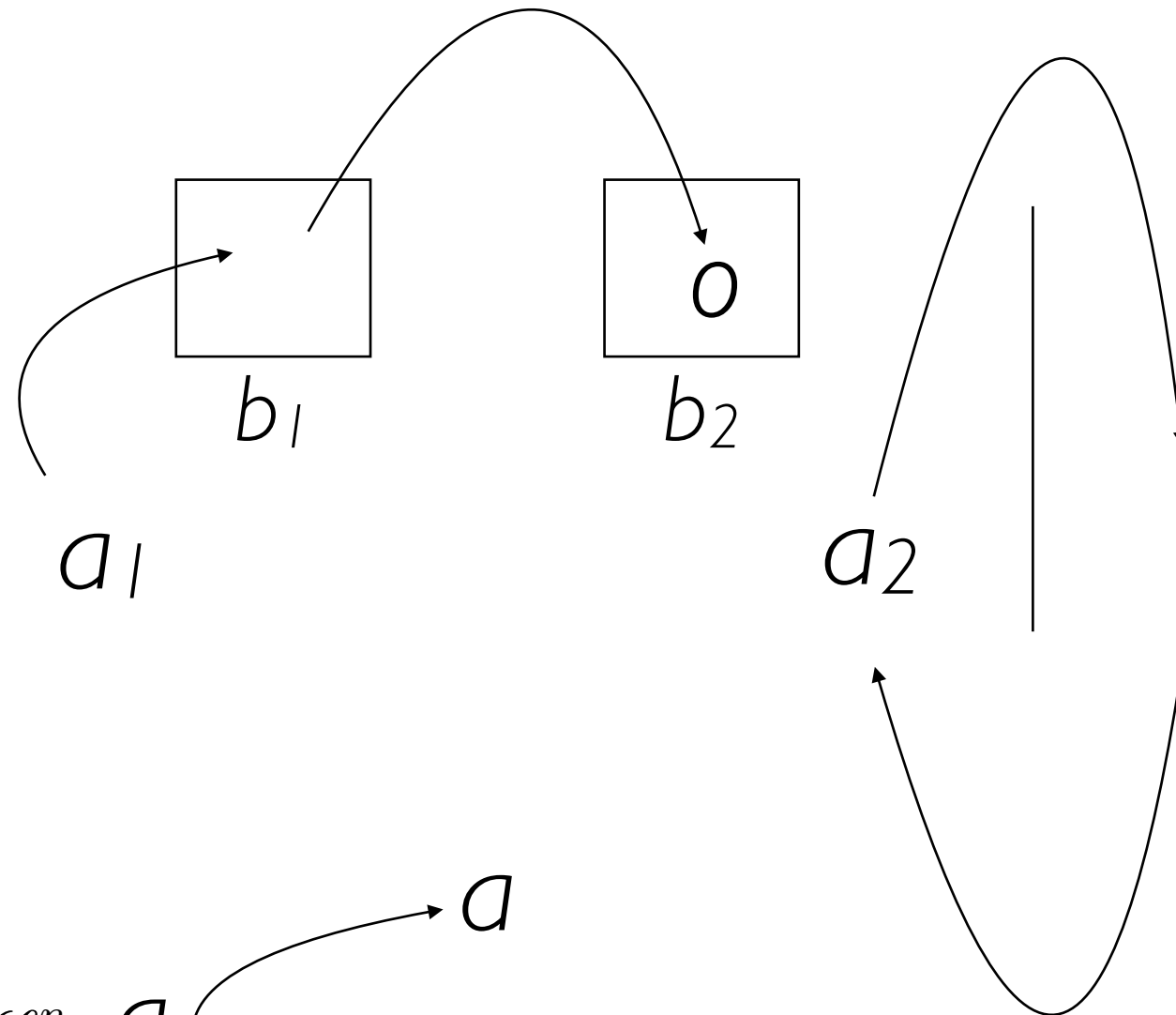
Framework for FBT^0_1

(five timepoints)

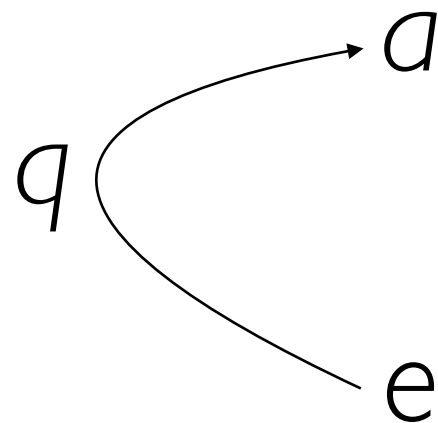


Framework for FBT^0_1

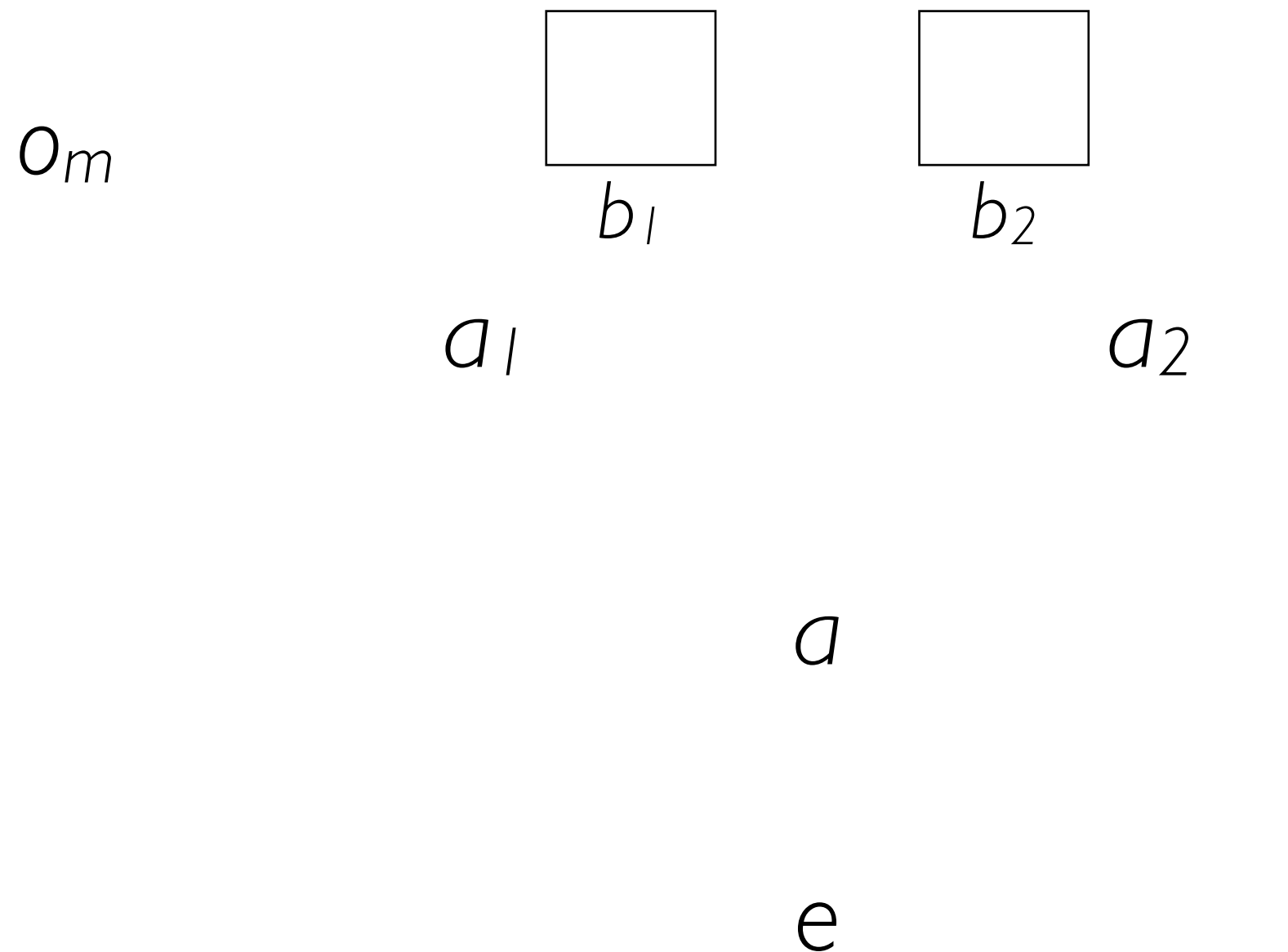
(five timepoints)



q a formula in modal \mathcal{L}^n

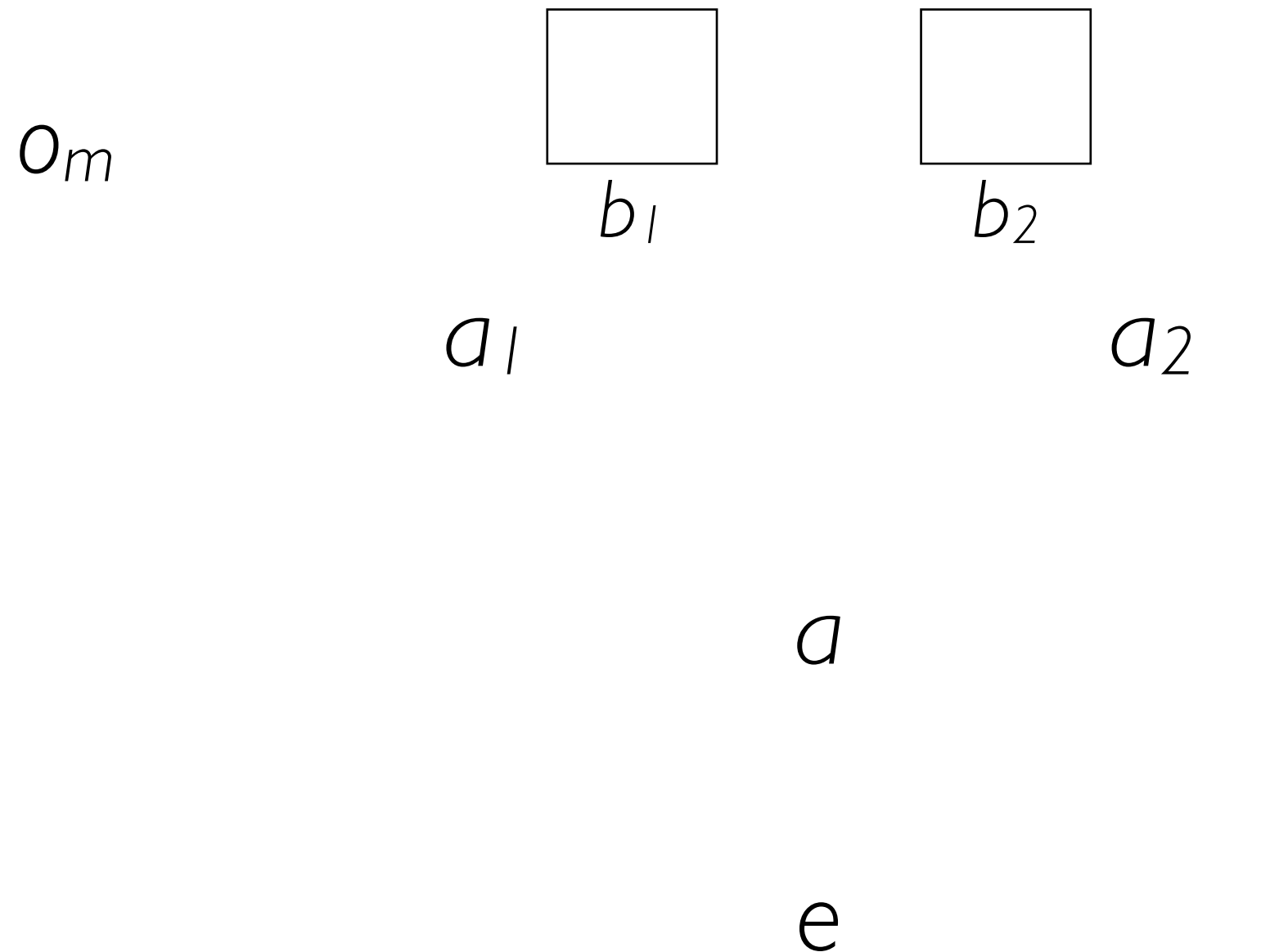


Framework for FBT₁



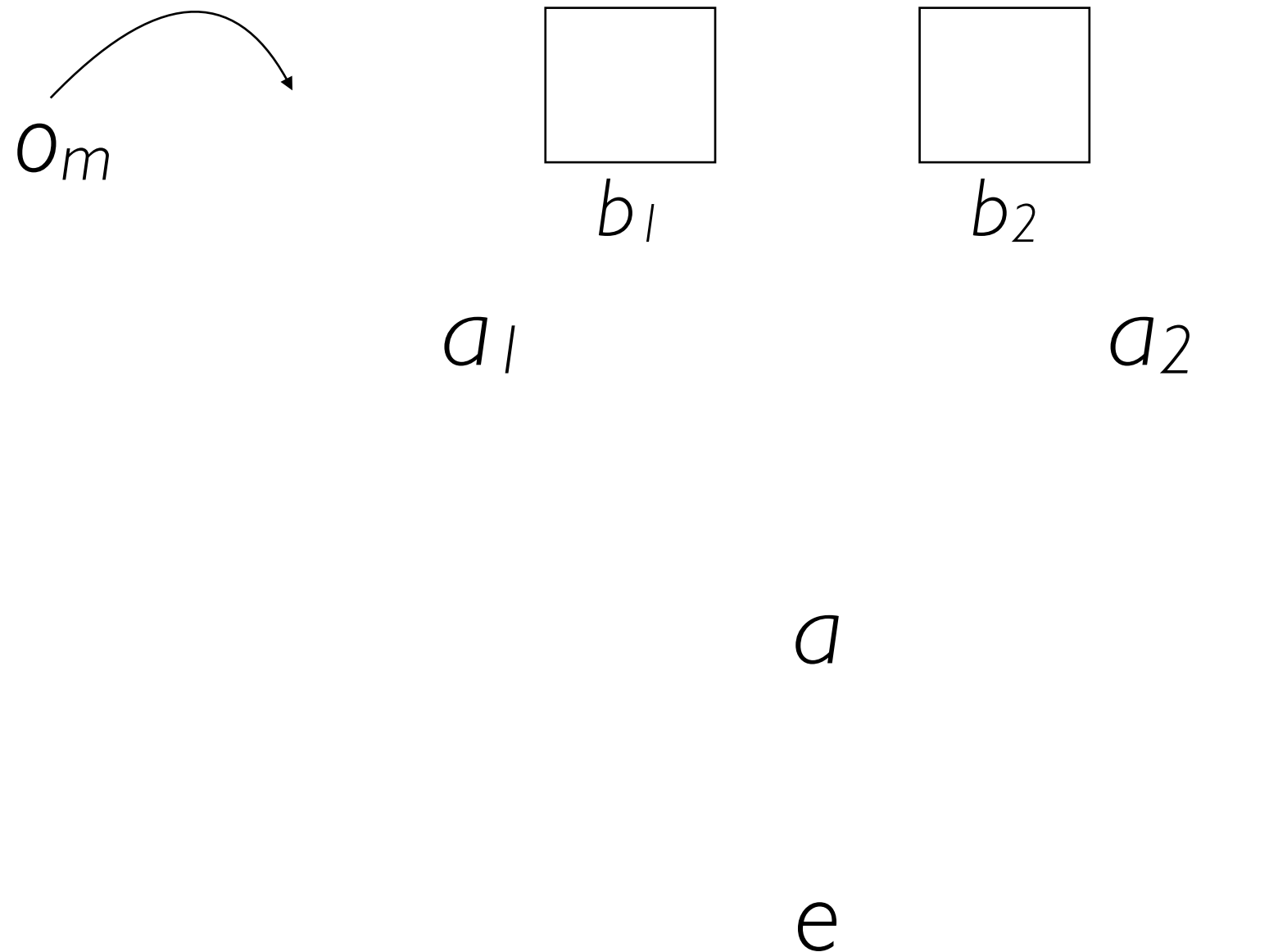
Framework for FBT₁

(six timepoints)



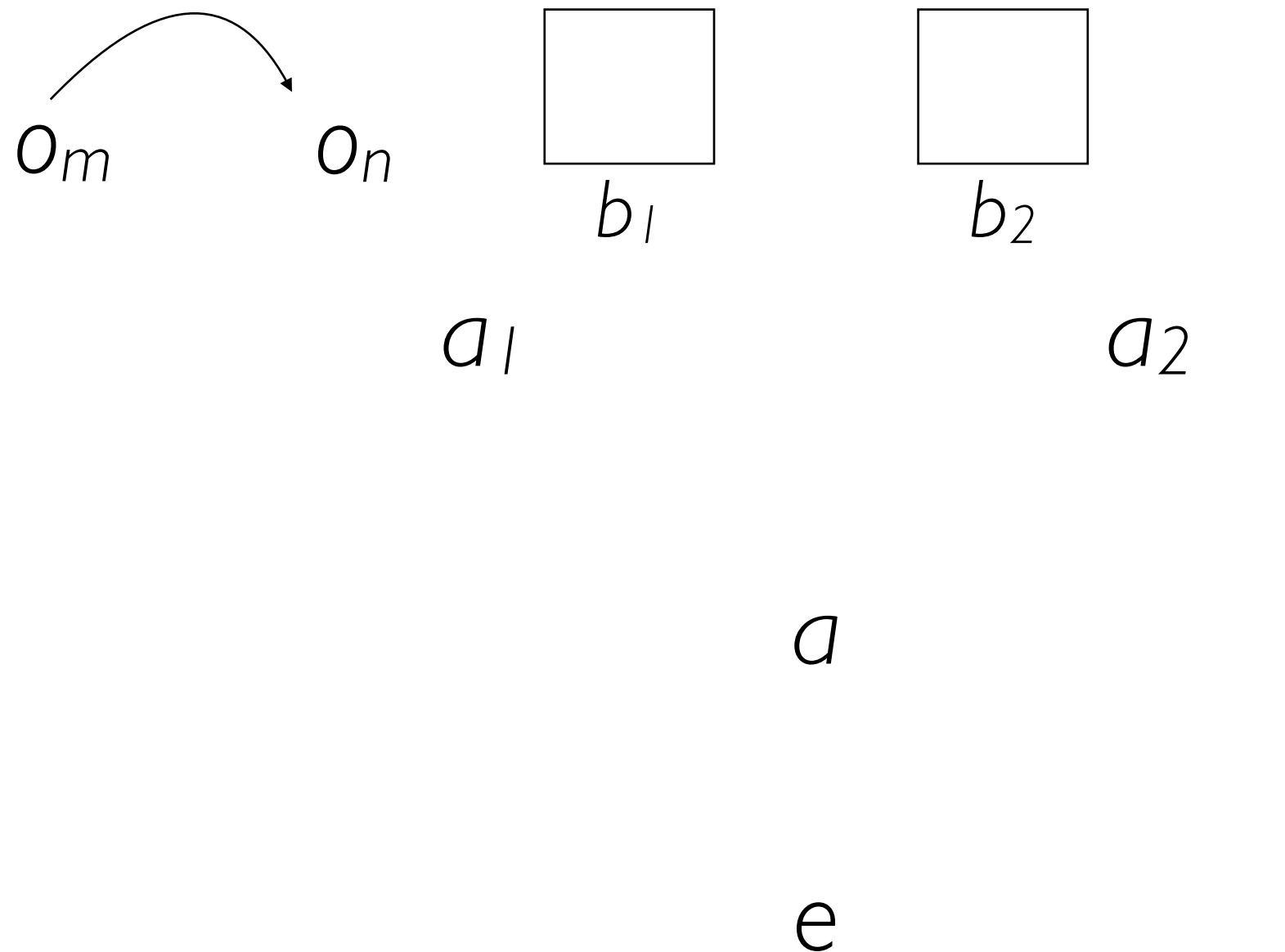
Framework for FBT₁

(six timepoints)



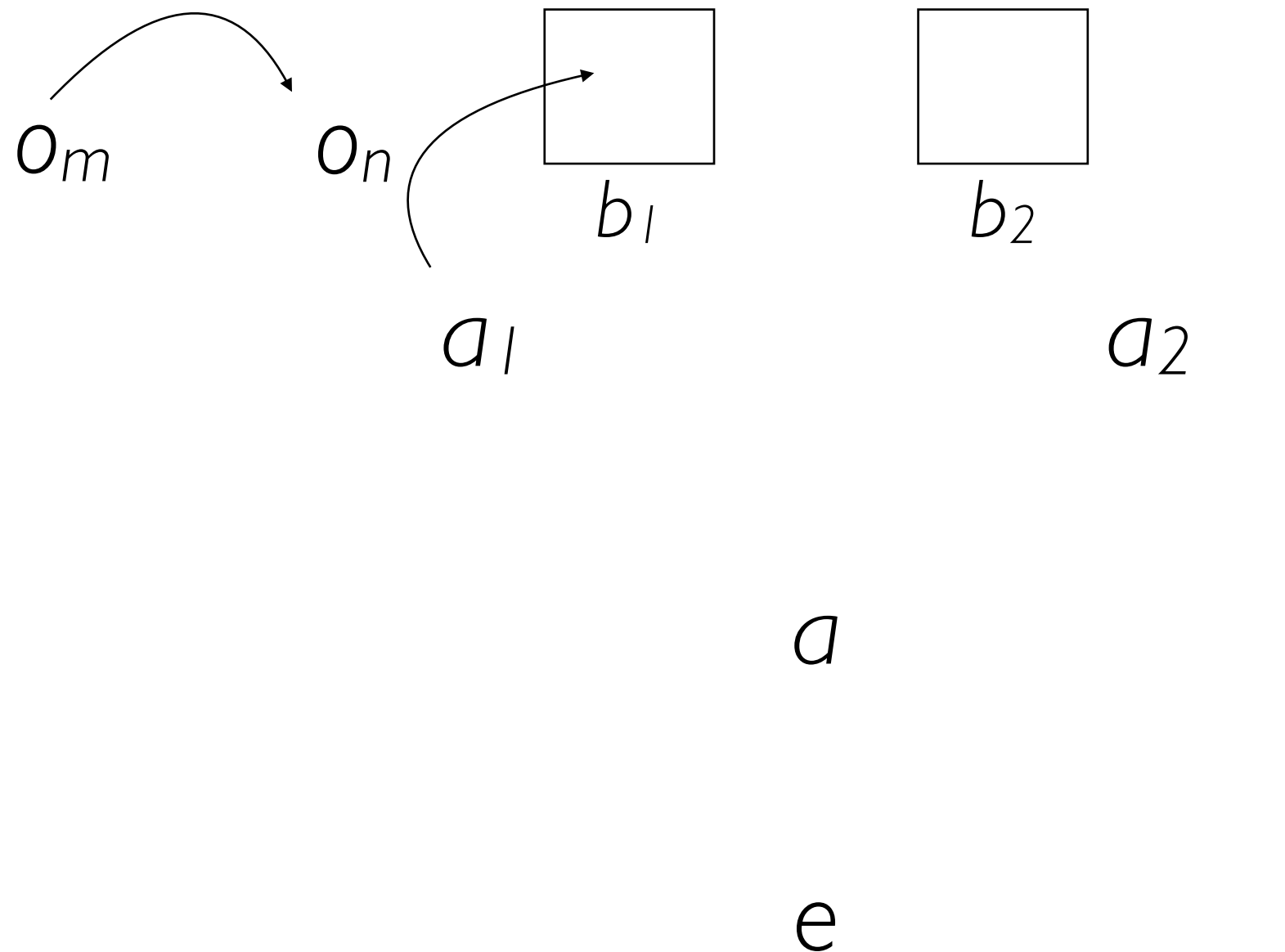
Framework for FBT₁

(six timepoints)



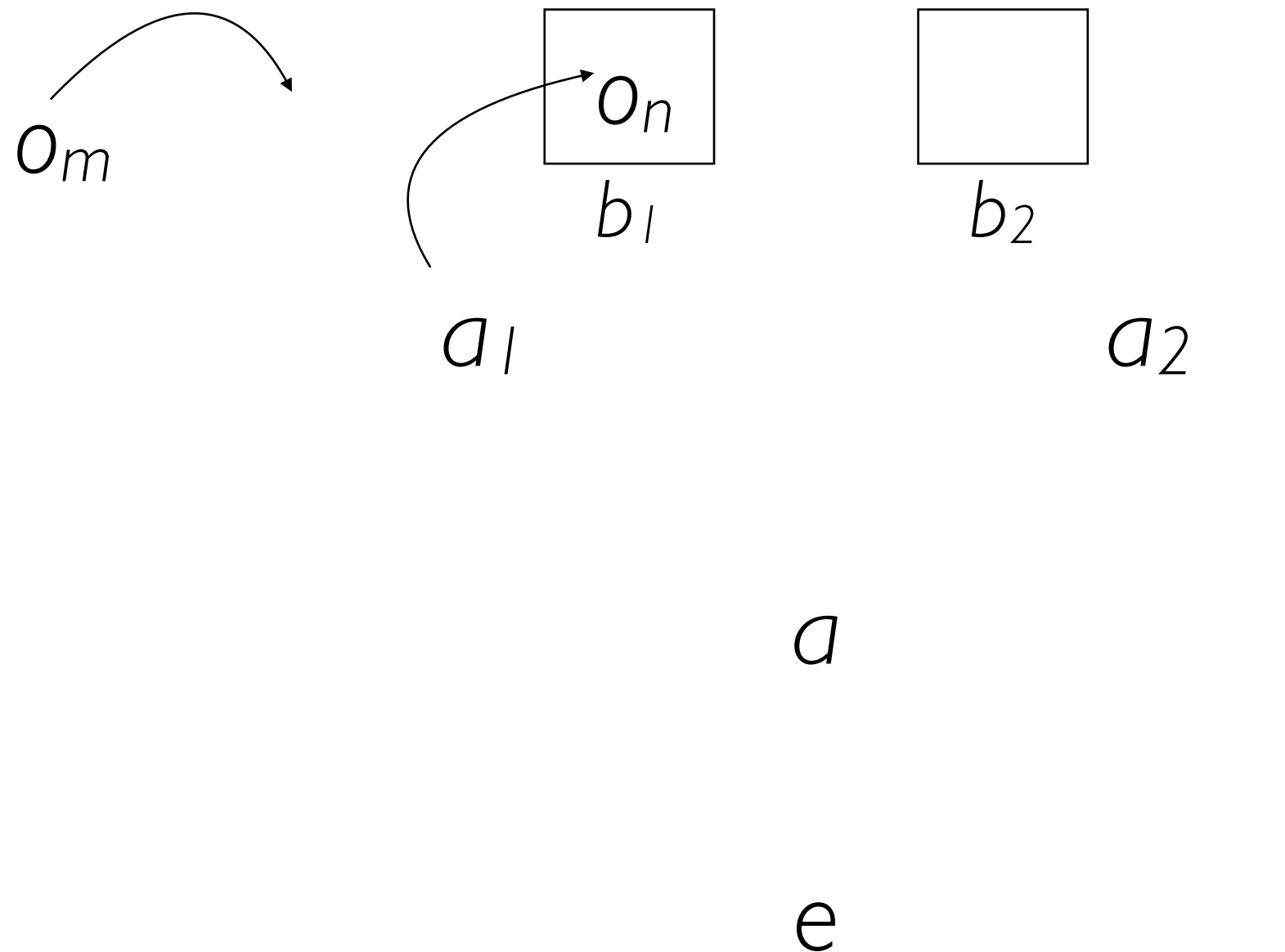
Framework for FBT₁

(six timepoints)



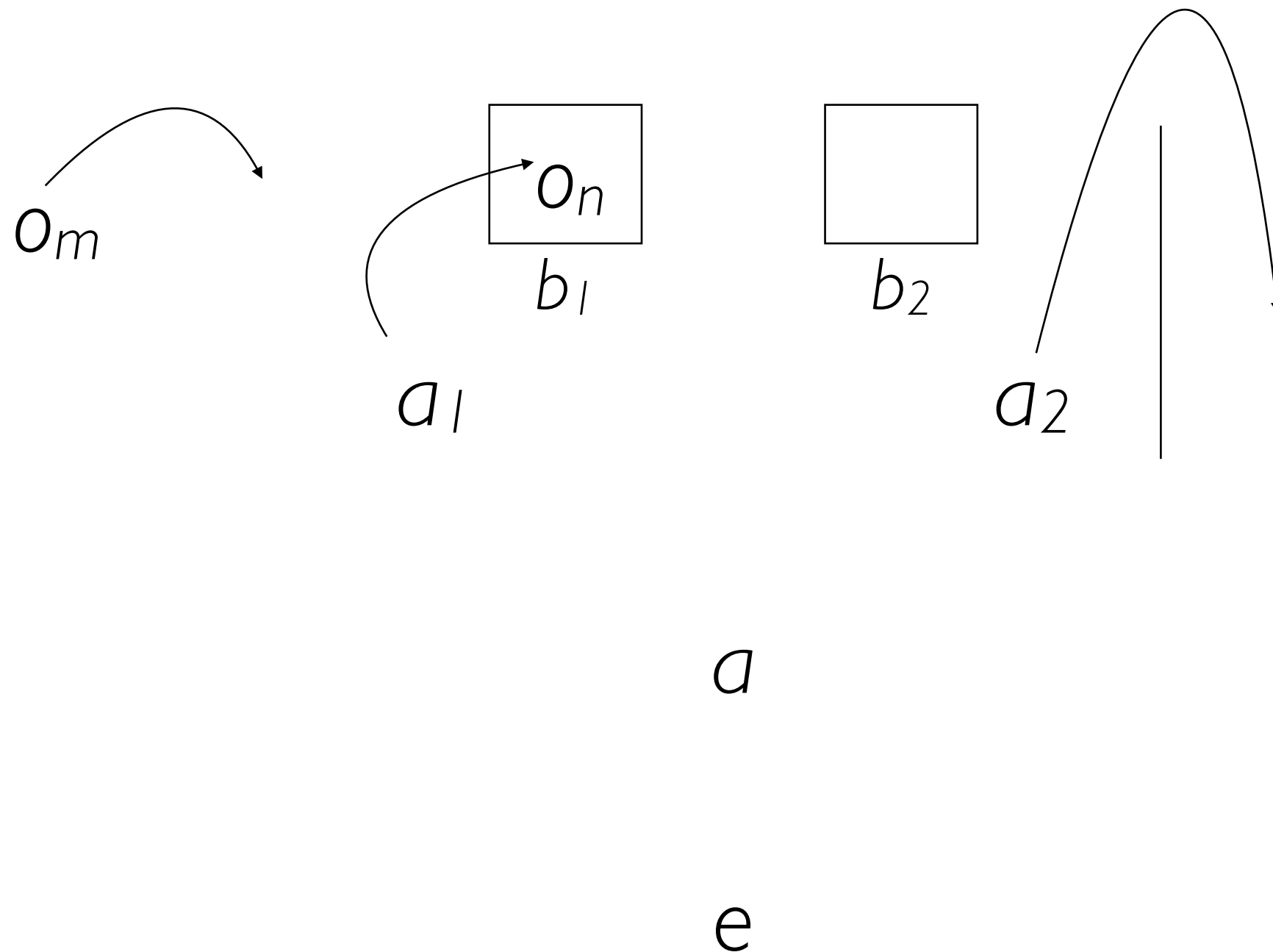
Framework for FBT₁

(six timepoints)



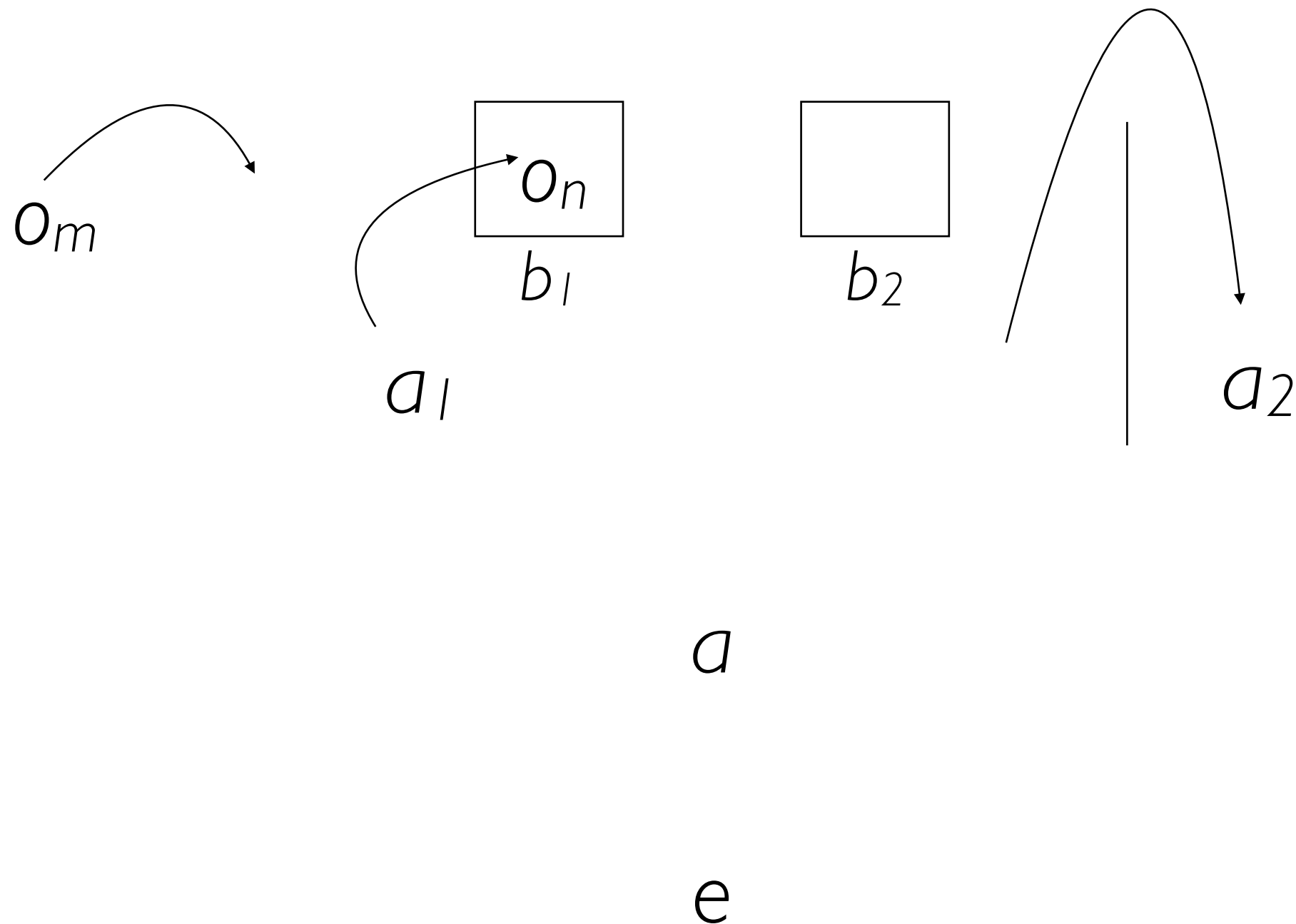
Framework for FBT₁

(six timepoints)



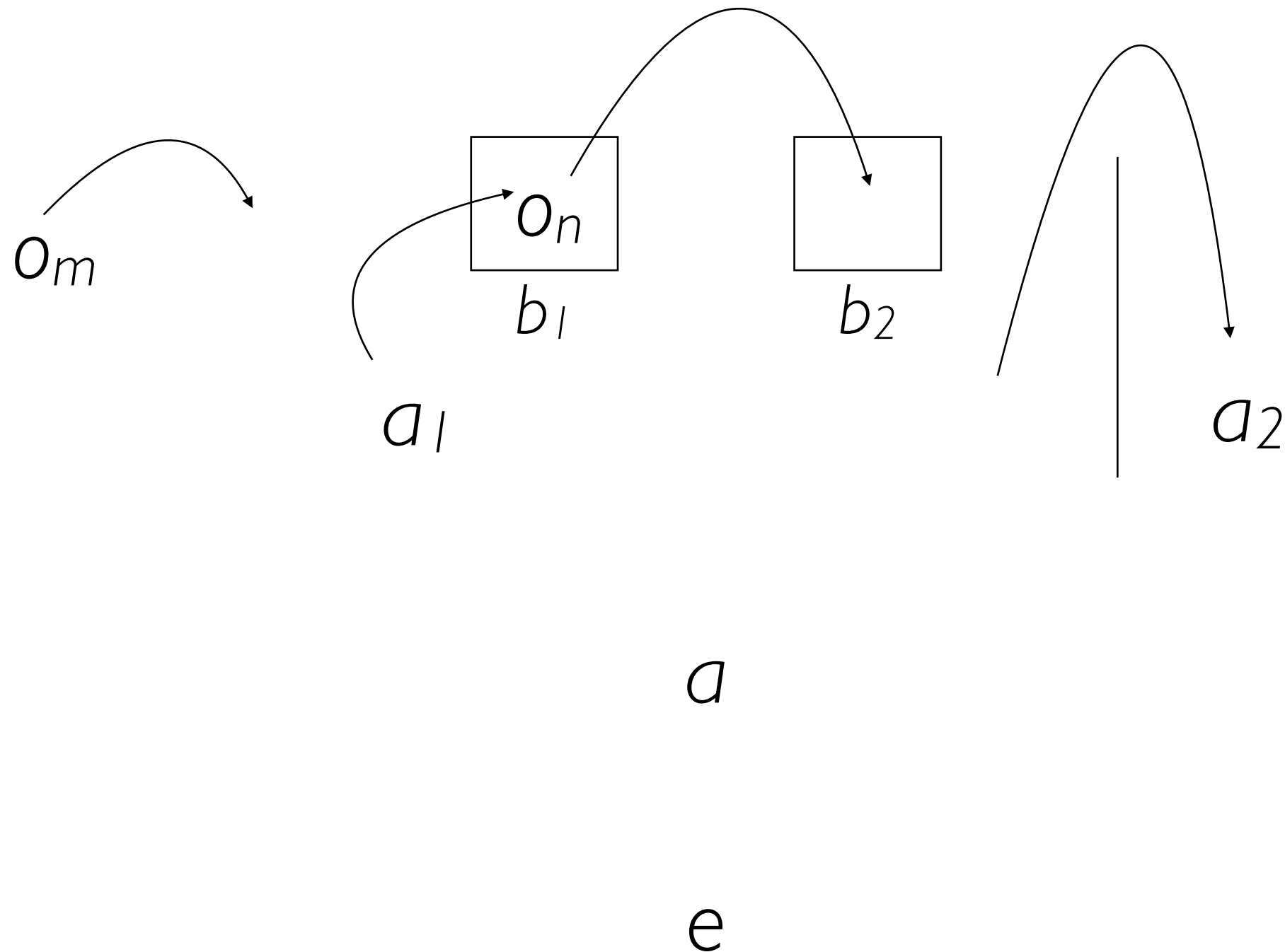
Framework for FBT₁

(six timepoints)



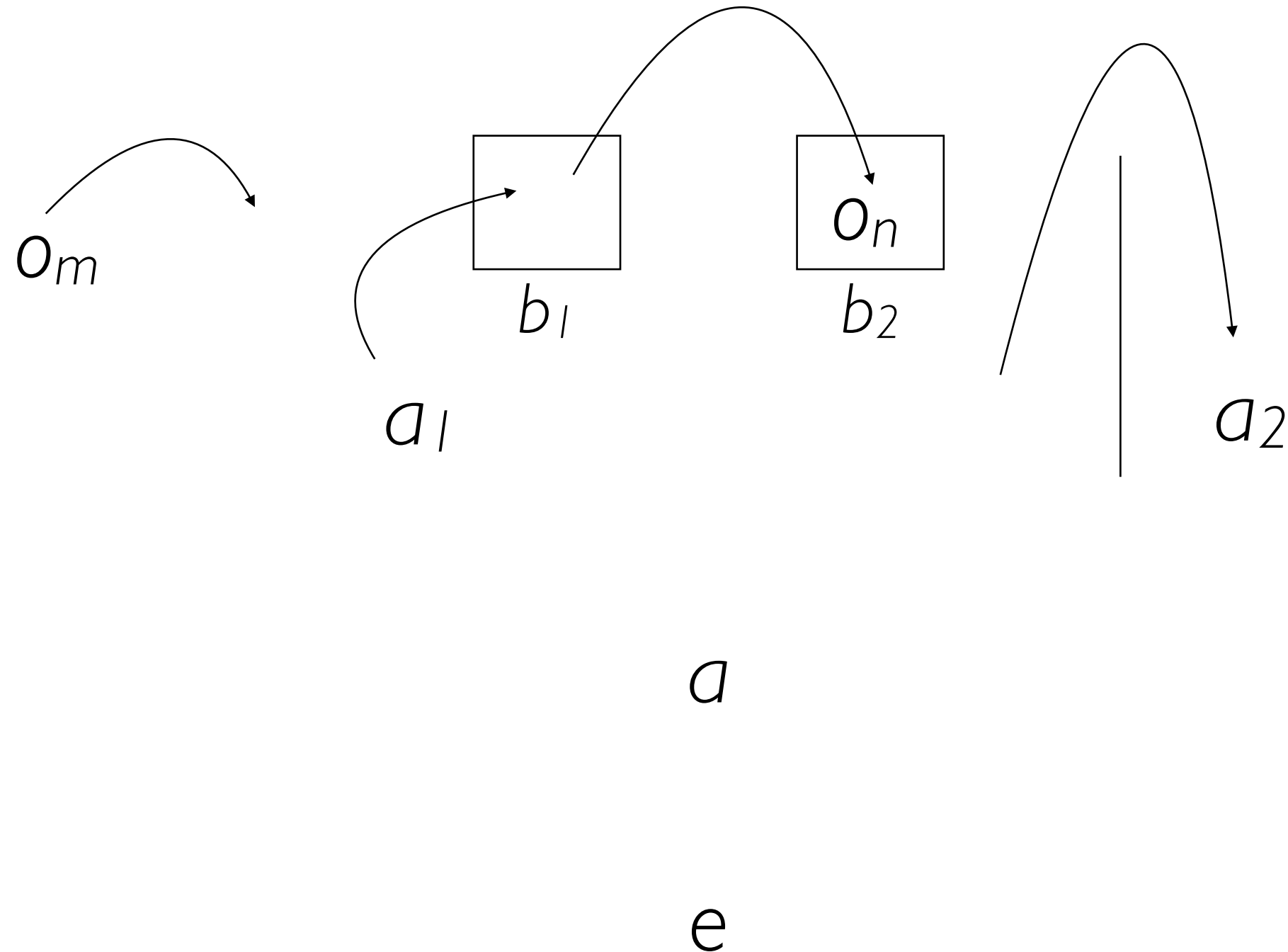
Framework for FBT₁

(six timepoints)



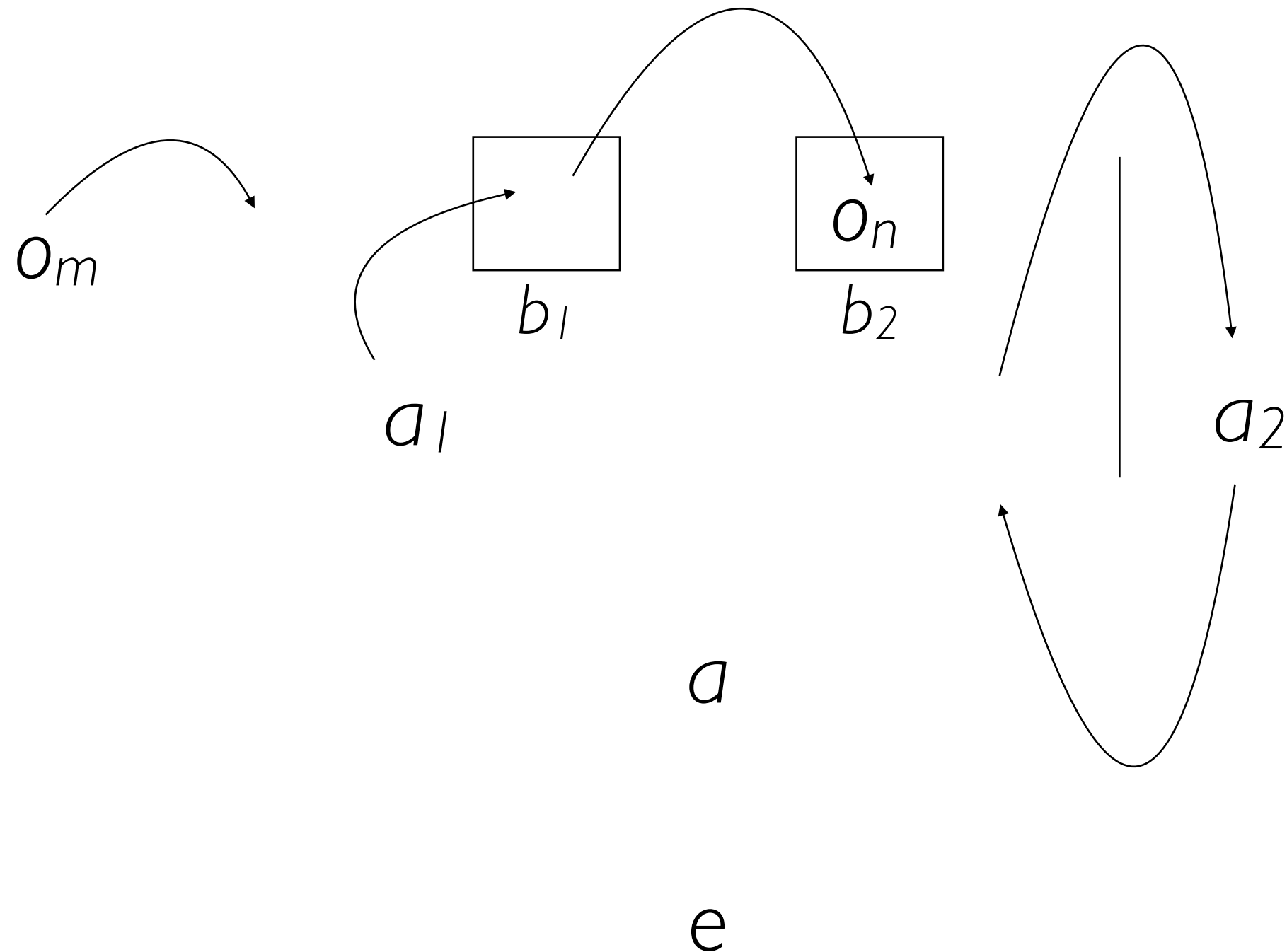
Framework for FBT₁

(six timepoints)



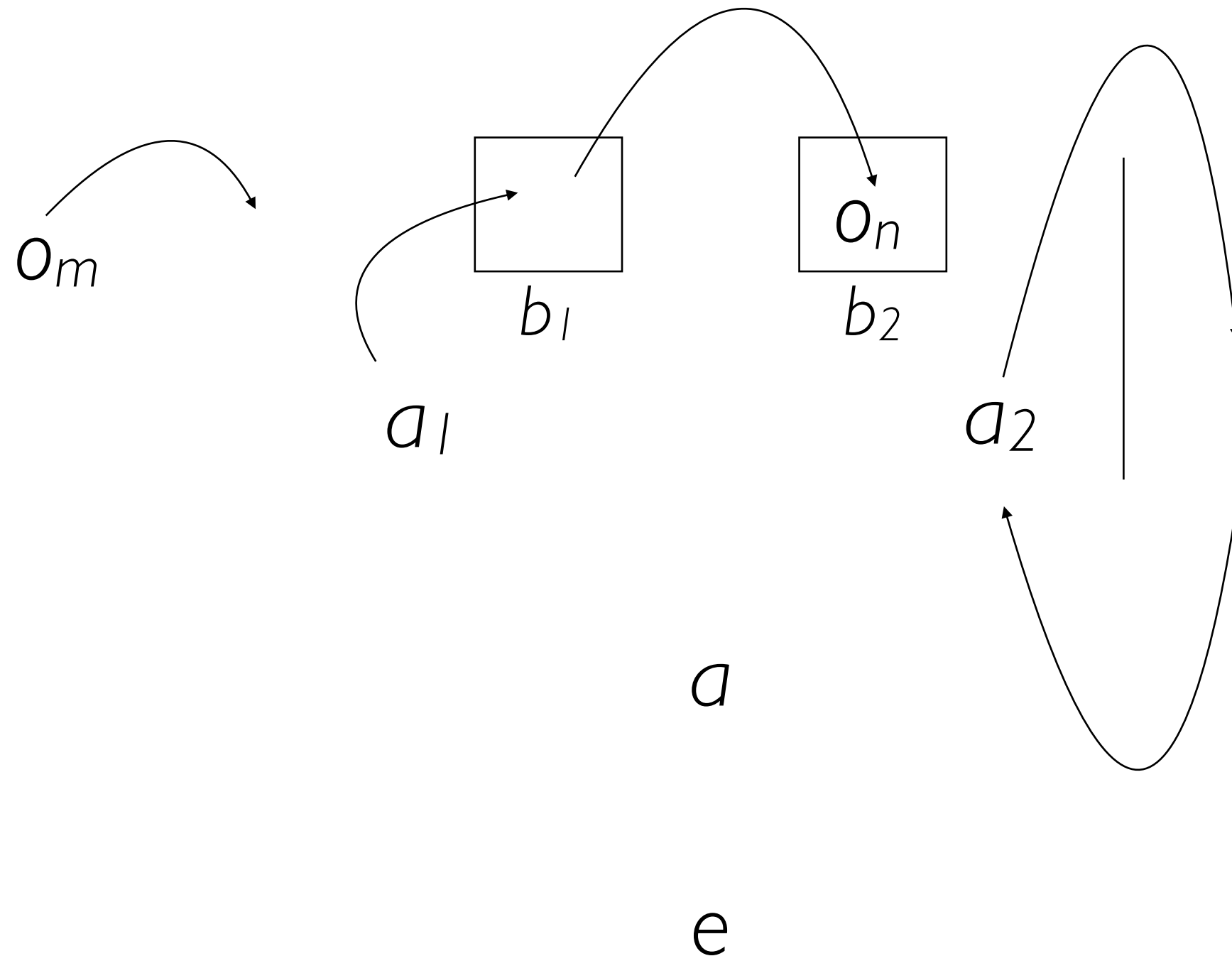
Framework for FBT₁

(six timepoints)



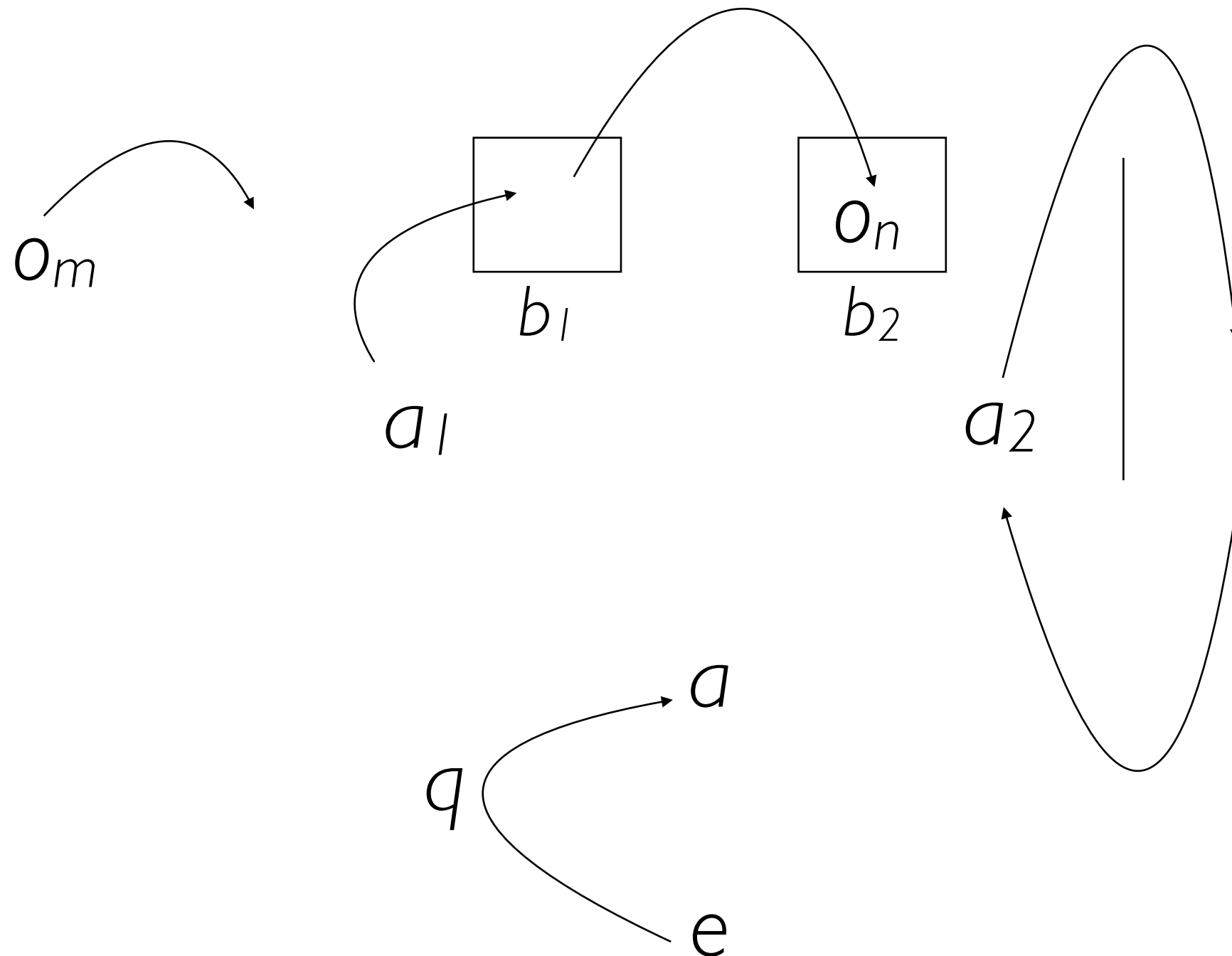
Framework for FBT₁

(six timepoints)



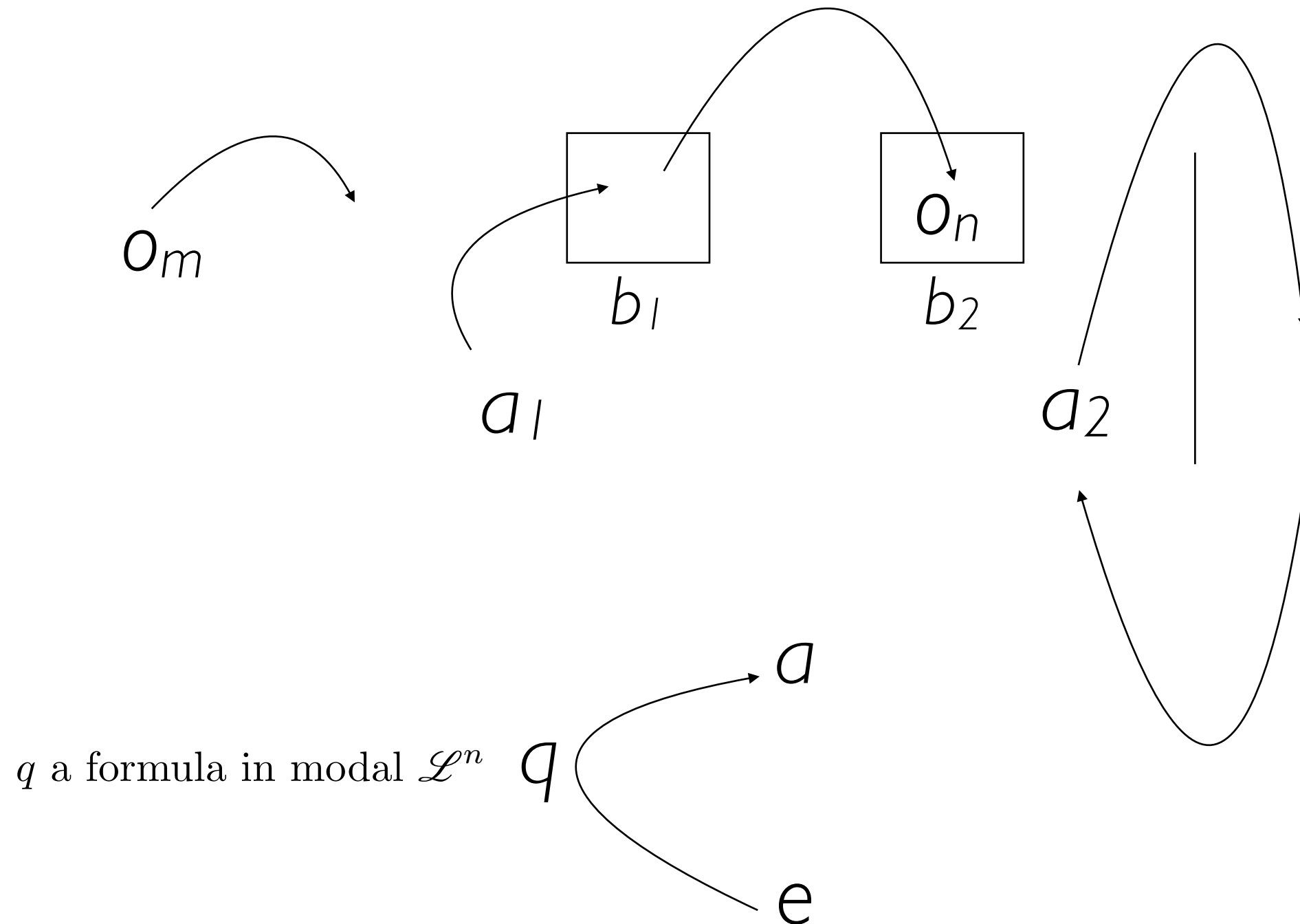
Framework for FBT₁

(six timepoints)



Framework for FBT^1_1

(six timepoints)



Done, a Decade Ago, Formally & Implementation/Simulation

Arkoudas, K. & Bringsjord, S.
(2009) “Propositional
Attitudes and Causation”
*International Journal of Software
and Informatics* **3.1**: 47–65.

http://kryten.mm.rpi.edu/PRICAI_w_sequentcalc_041709.pdf

Propositional attitudes and causation

Konstantine Arkoudas and Selmer Bringsjord

Cognitive Science and Computer Science Departments, RPI
arkouk@rpi.edu, brings@rpi.edu

Abstract. Predicting and explaining the behavior of others in terms of mental states is indispensable for everyday life. It will be equally important for artificial agents. We present an inference system for representing and reasoning about mental states, and use it to provide a formal analysis of the false-belief task. The system allows for the representation of information about events, causation, and perceptual, doxastic, and epistemic states (vision, belief, and knowledge), incorporating ideas from the event calculus and multi-agent epistemic logic. Unlike previous AI formalisms, our focus here is on mechanized proofs and proof programmability, not on metamathematical results. Reasoning is performed via relatively cognitively plausible inference rules, and a degree of automation is achieved by general-purpose inference methods and by a syntactic embedding of the system in first-order logic.

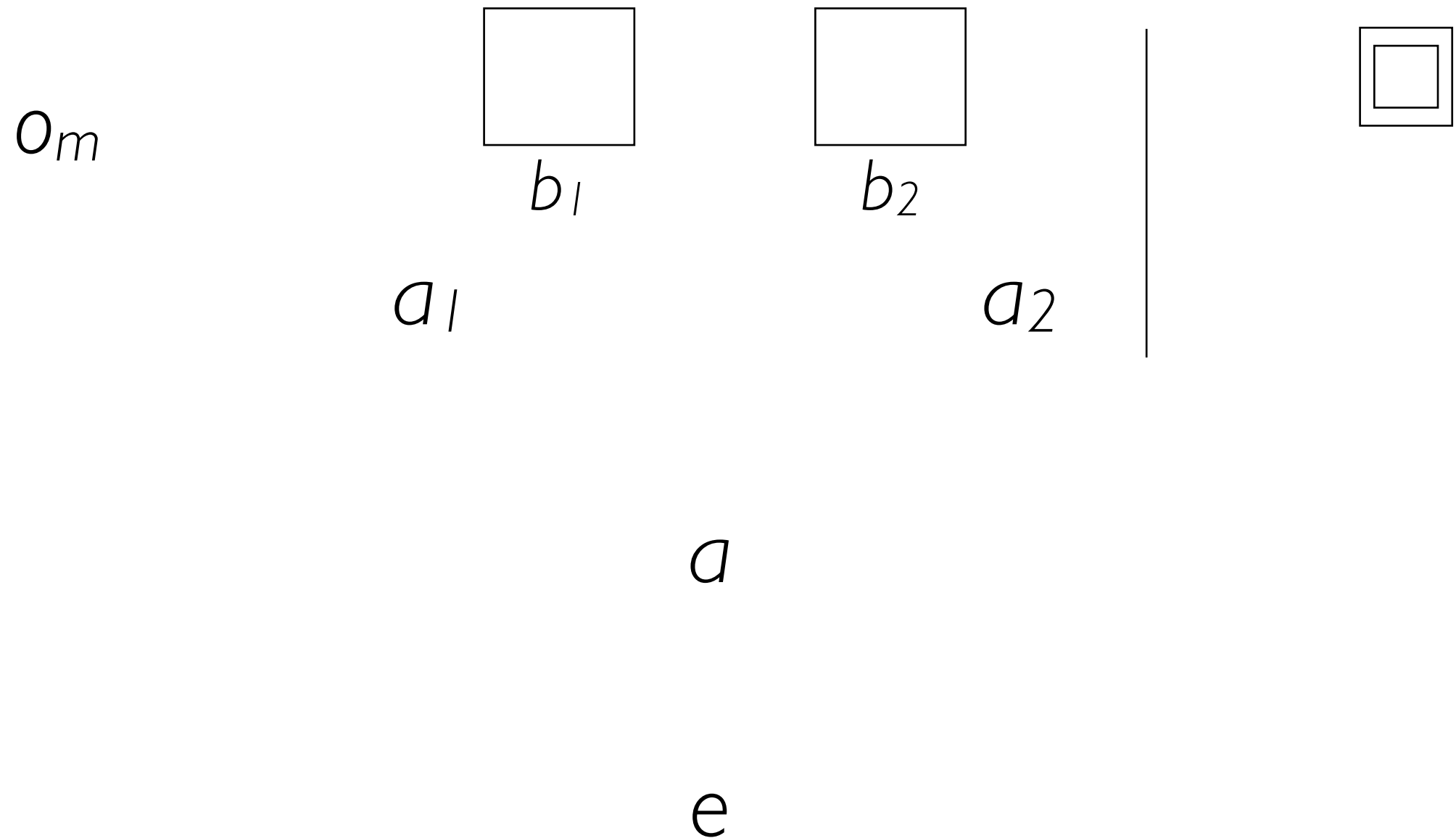
1 Introduction

Interpreting the behavior of other people is indispensable for everyday life. It is something that we do constantly, on a daily basis, and it helps us not only to make sense of human behavior, but also to predict it and—to a certain extent—to control it. How exactly do we manage that? That is not currently known, but many have argued that the ability to ascribe mental states to others and to reason about such mental states is a key component of our capacity to understand human behavior. In particular, all social transactions, from engaging in commerce and negotiating to making jokes and empathizing with other people's pain or joy, appear to require at least a rudimentary grasp of common-sense psychology (CSP), i.e., a large body of truisms such as the following: When an agent a (1) wants to achieve a certain state of affairs p , and (2) believes that some action c can bring about p , and (3) a knows how to carry out c ; then, *ceteris paribus*,¹ a will carry out c ; when a sees that p , a knows that p ; when a fears that p and a discovers that p is the case, a is disappointed; and so on.

Artificial agents without a mastery of CSP would be severely handicapped in their interactions with humans. This could present problems not only for artificial agents trying to interpret human behavior, but also for artificial agents trying to interpret the behavior of one another. When a system exhibits a complex but rational behavior, and detailed knowledge of its internal structure is not

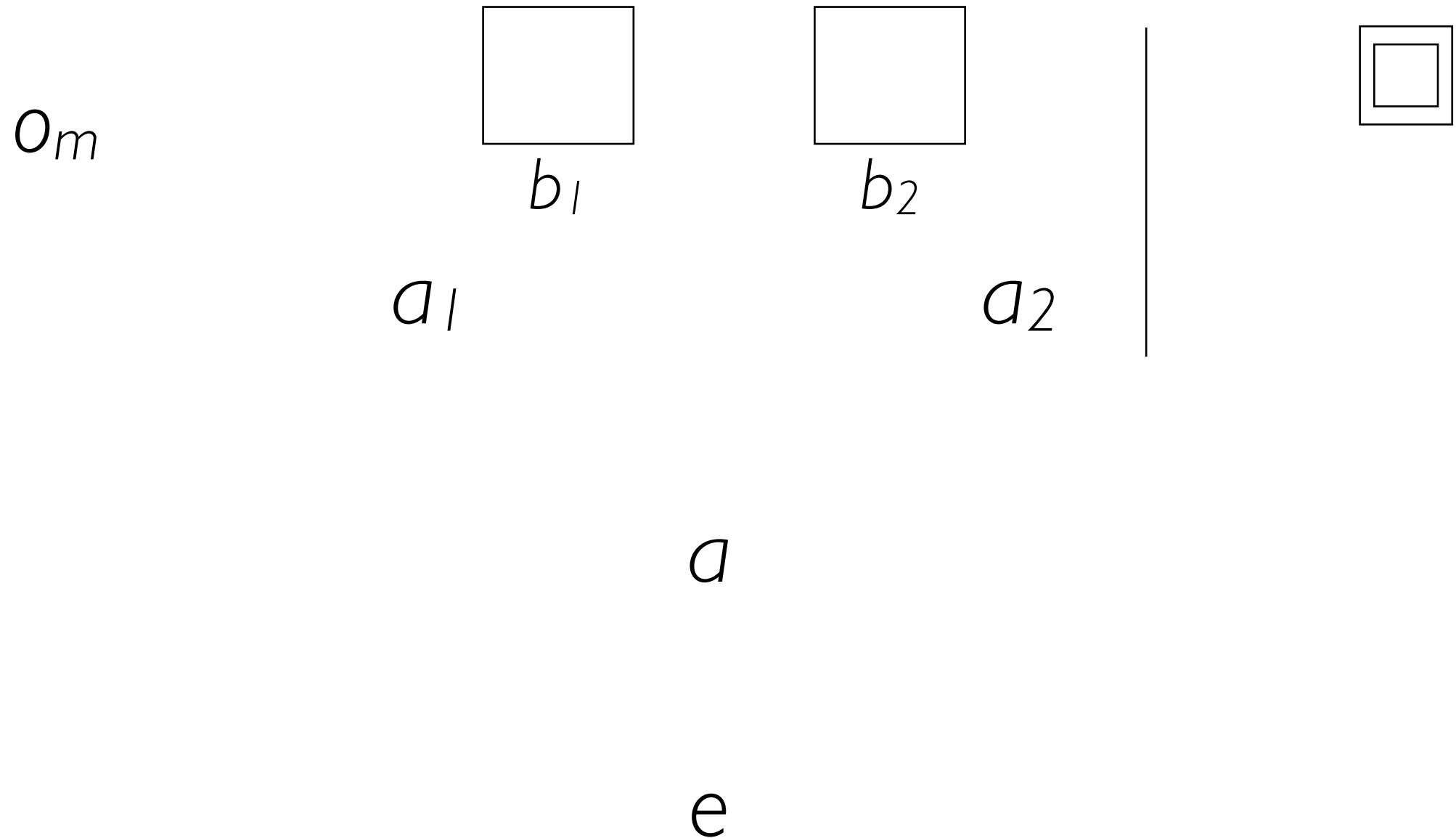
¹ Assuming that a is able to carry out c , that a has no conflicting desires that override his goal that p ; and so on.

Framework for FBT₂



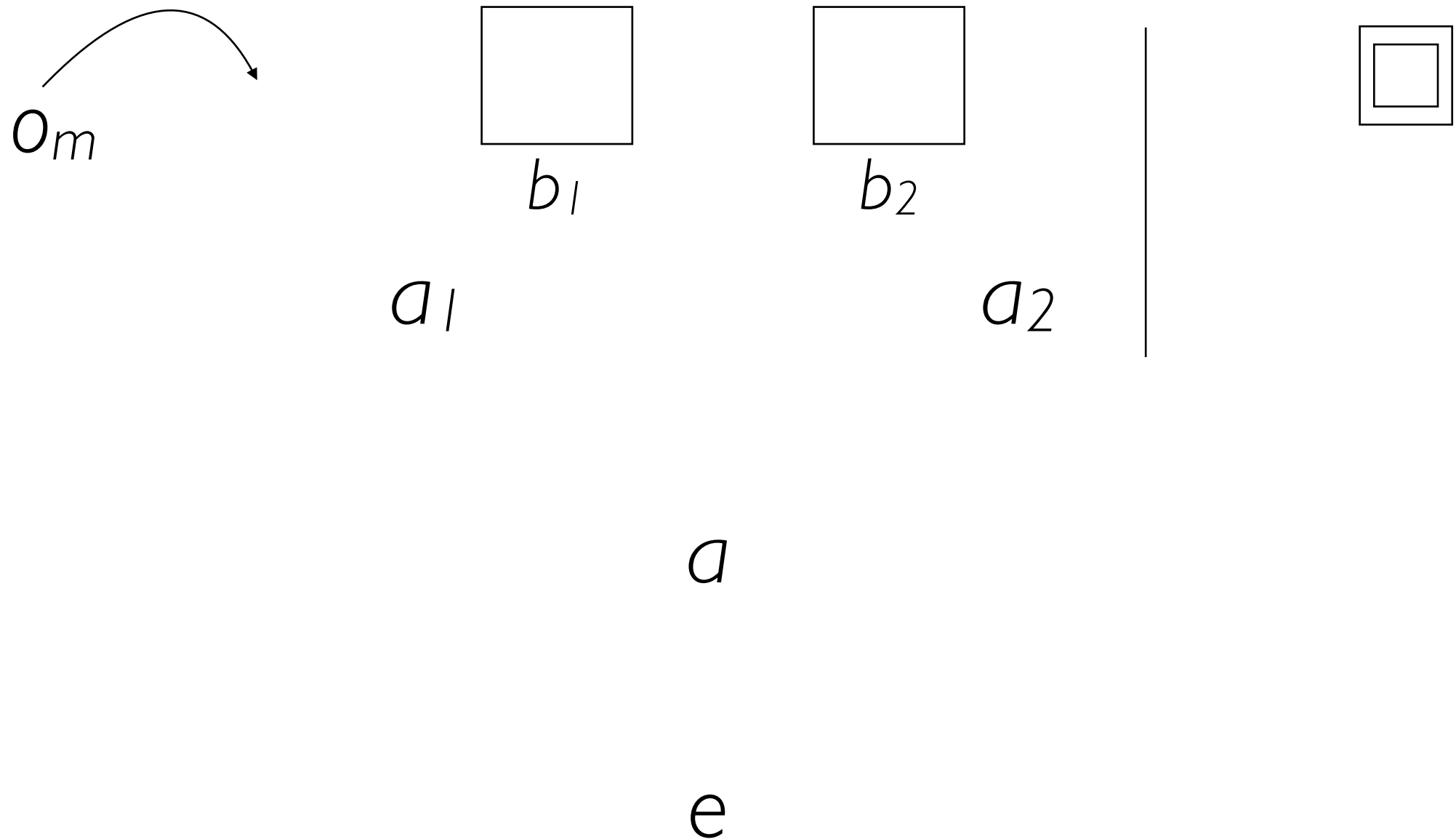
Framework for FBT₂

(seven timepoints)

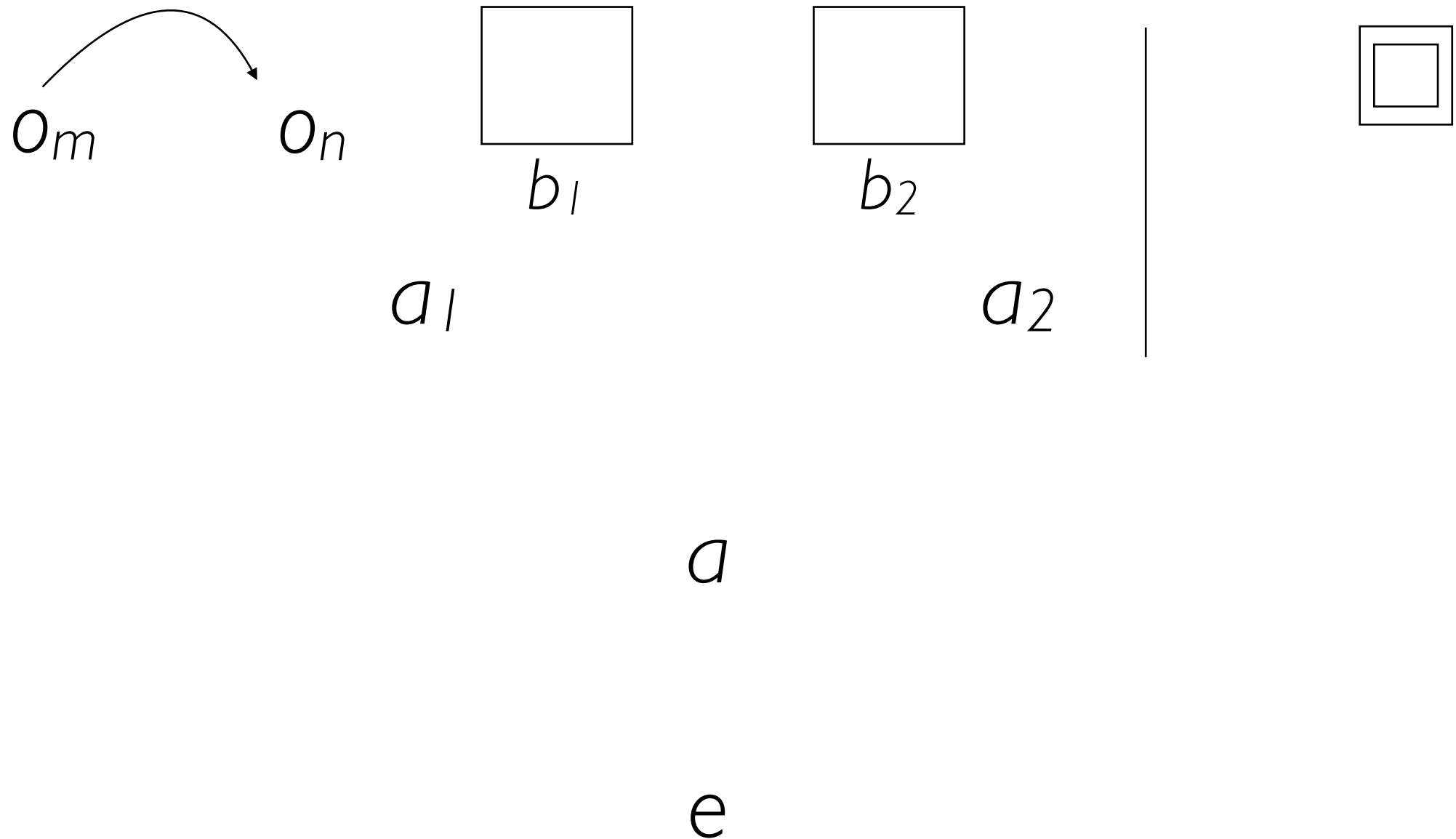


Framework for FBT^1_2

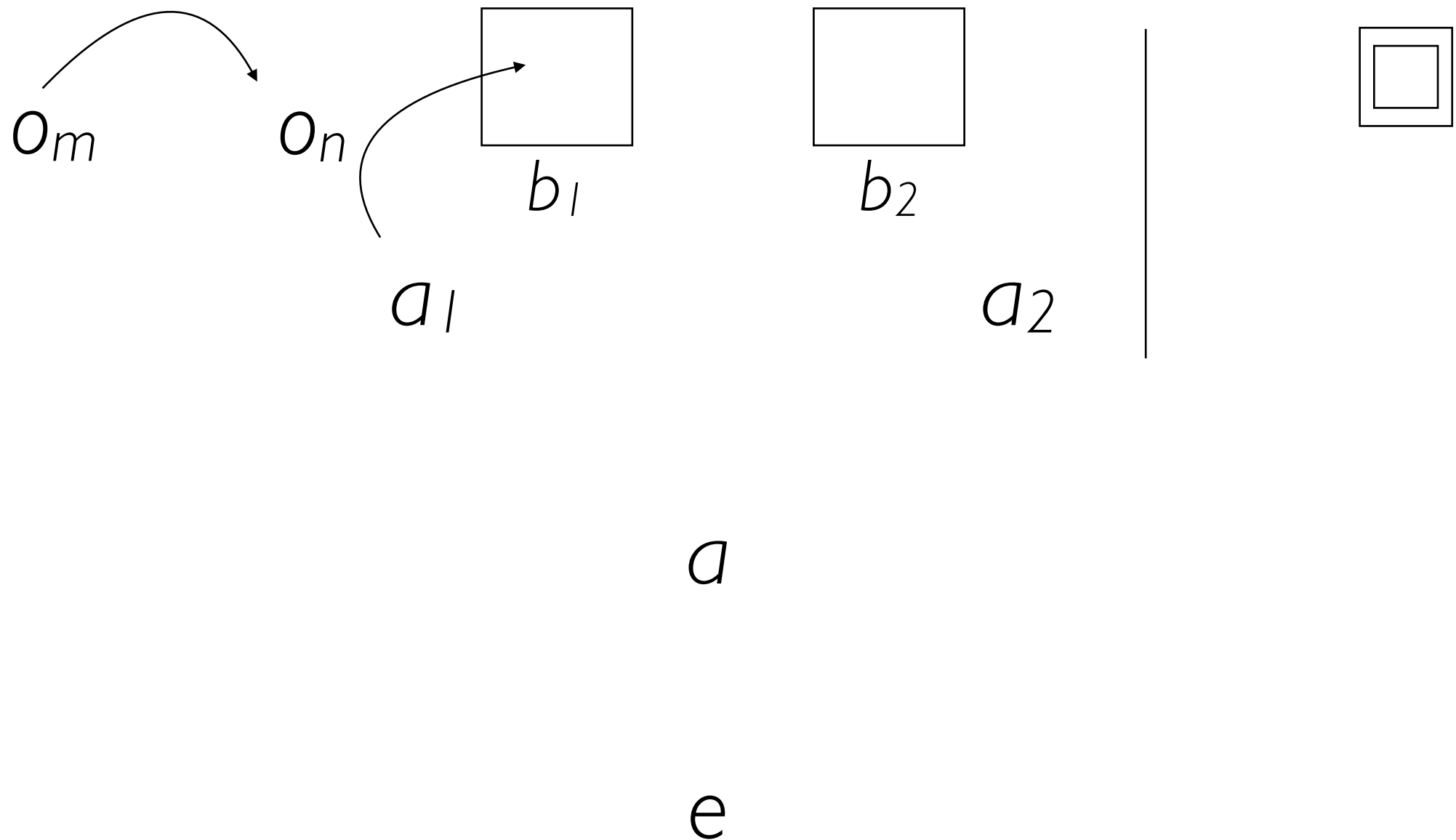
(seven timepoints)



Framework for FBT₂ (seven timepoints)

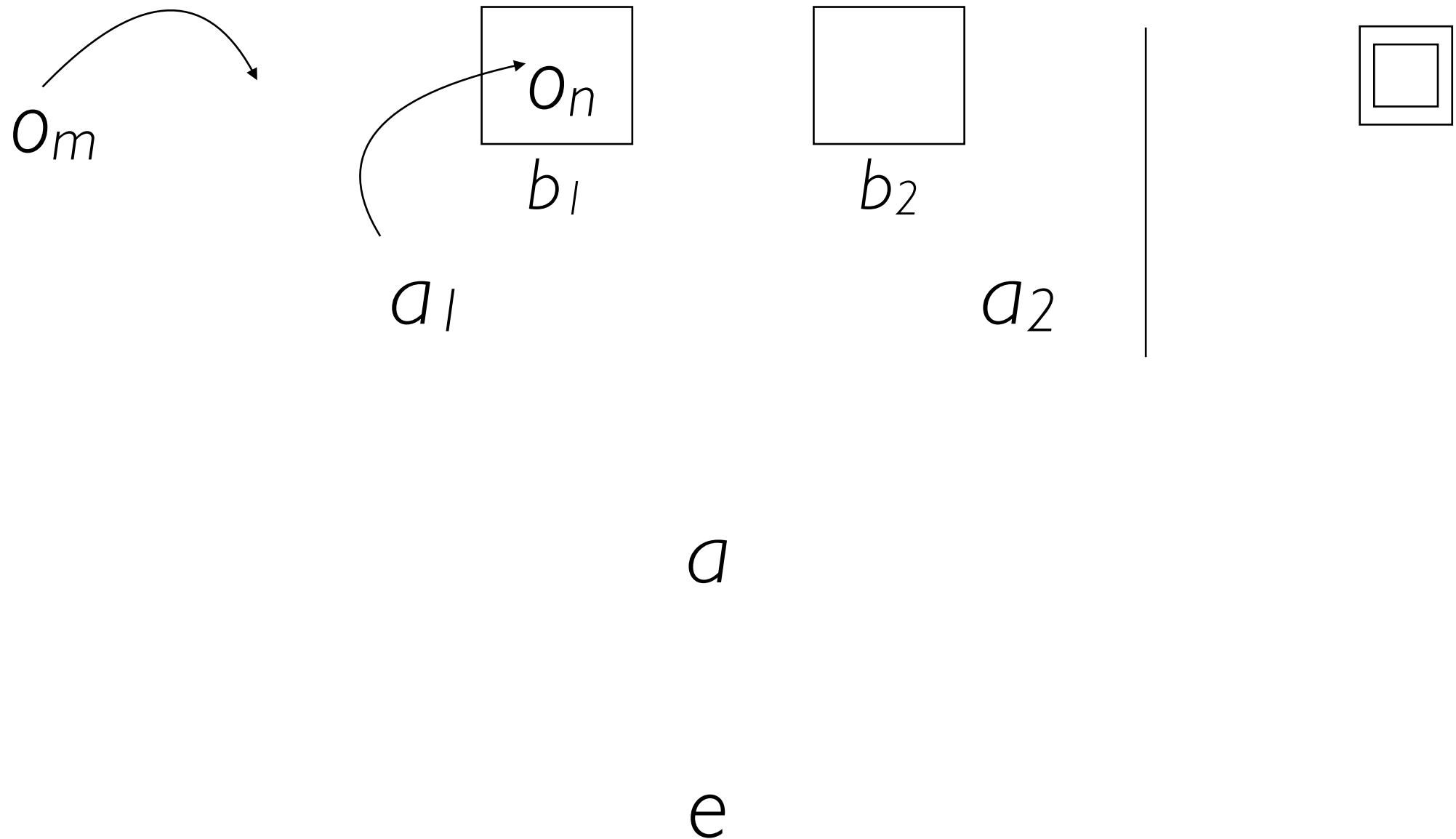


Framework for FBT₂ (seven timepoints)



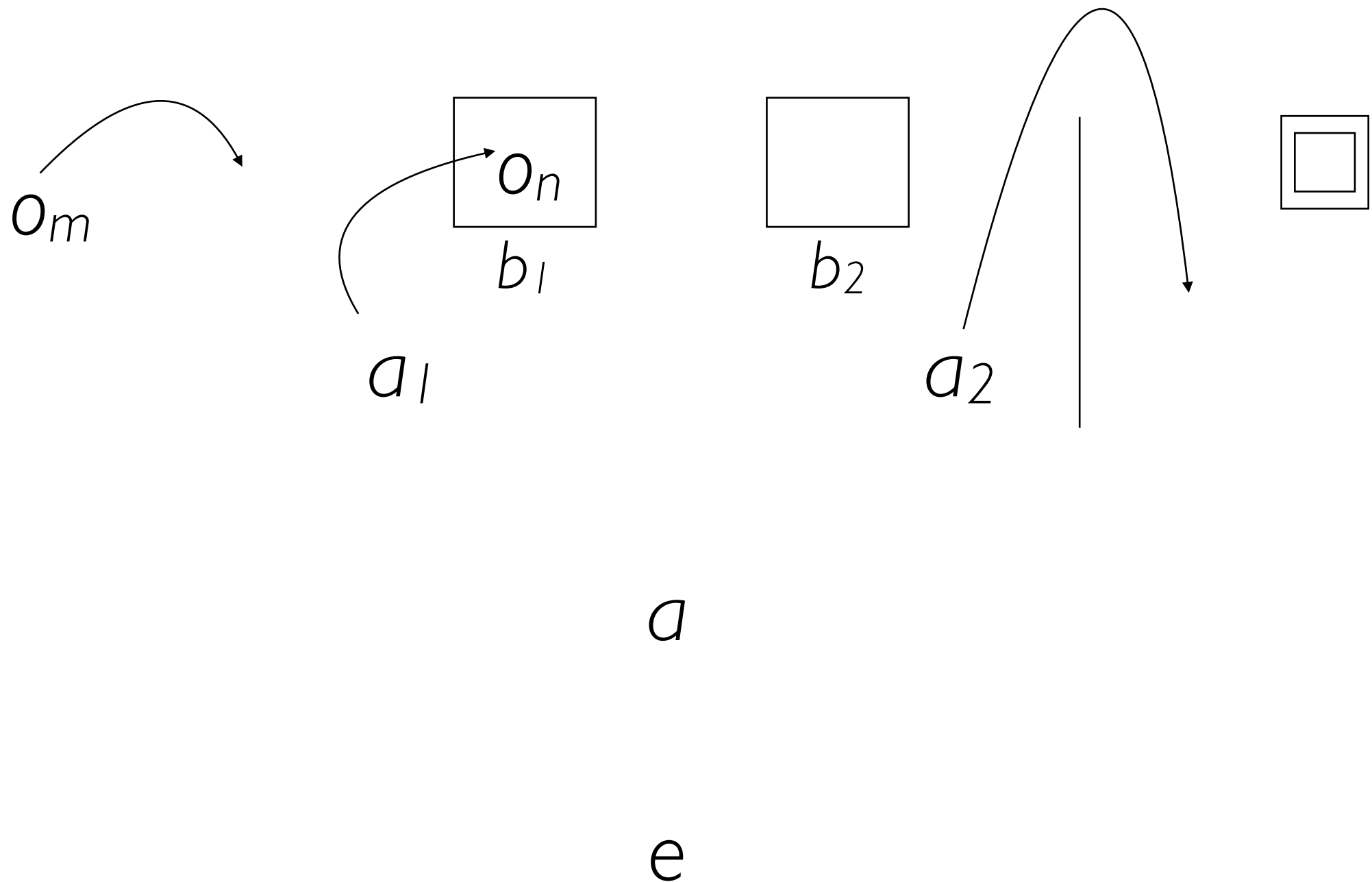
Framework for FBT^1_2

(seven timepoints)



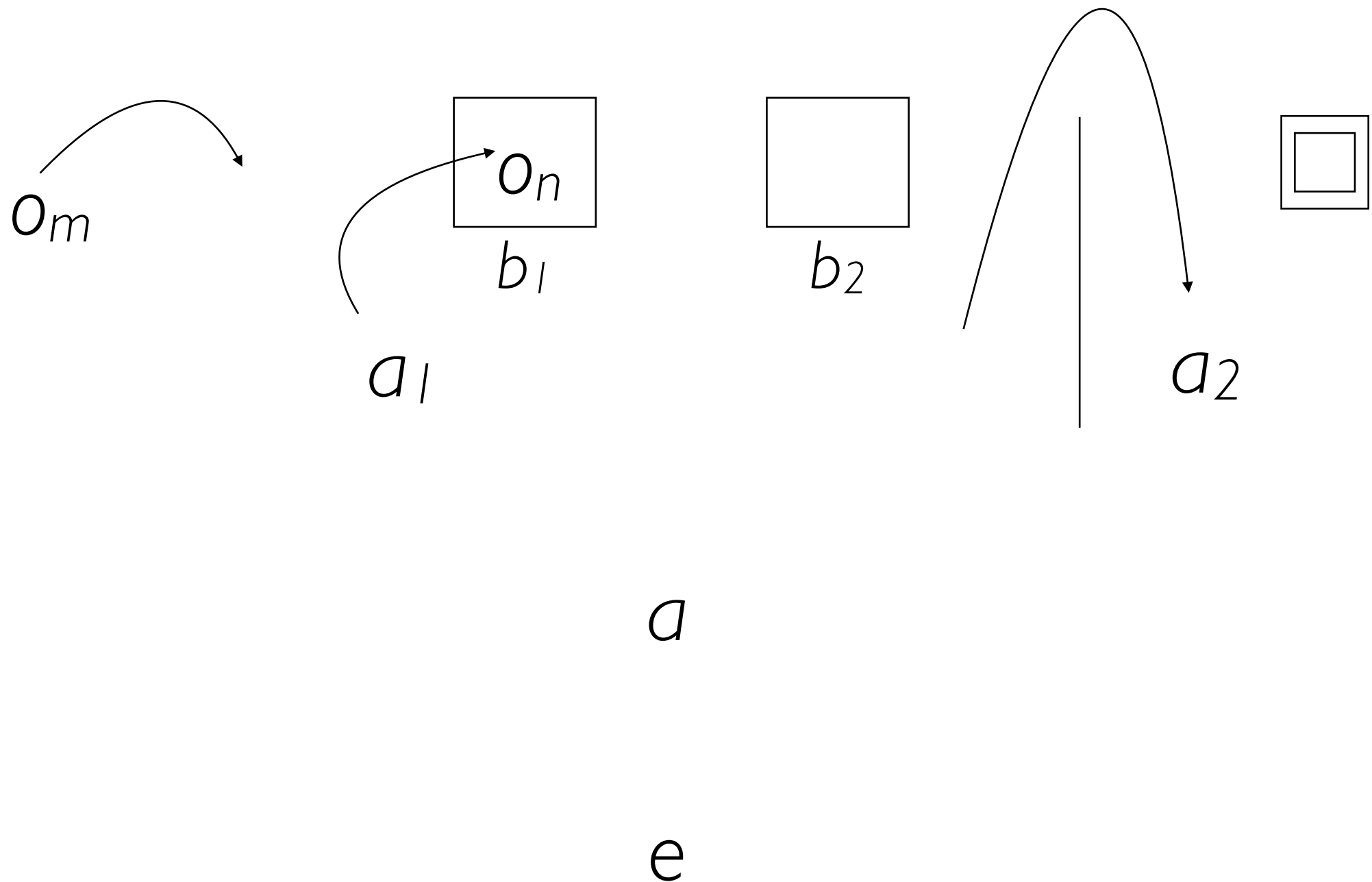
Framework for FBT^I_2

(seven timepoints)



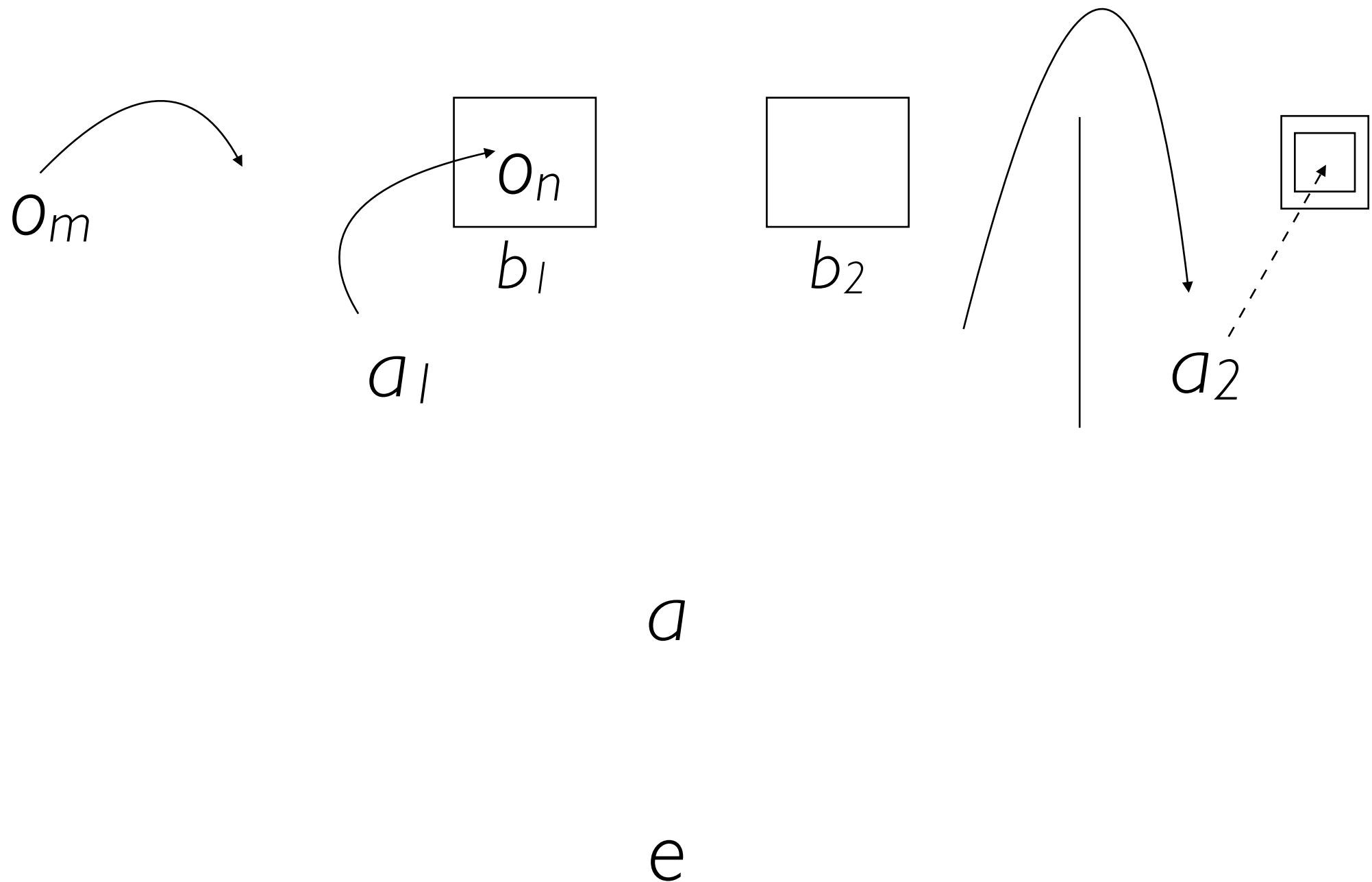
Framework for FBT^1_2

(seven timepoints)



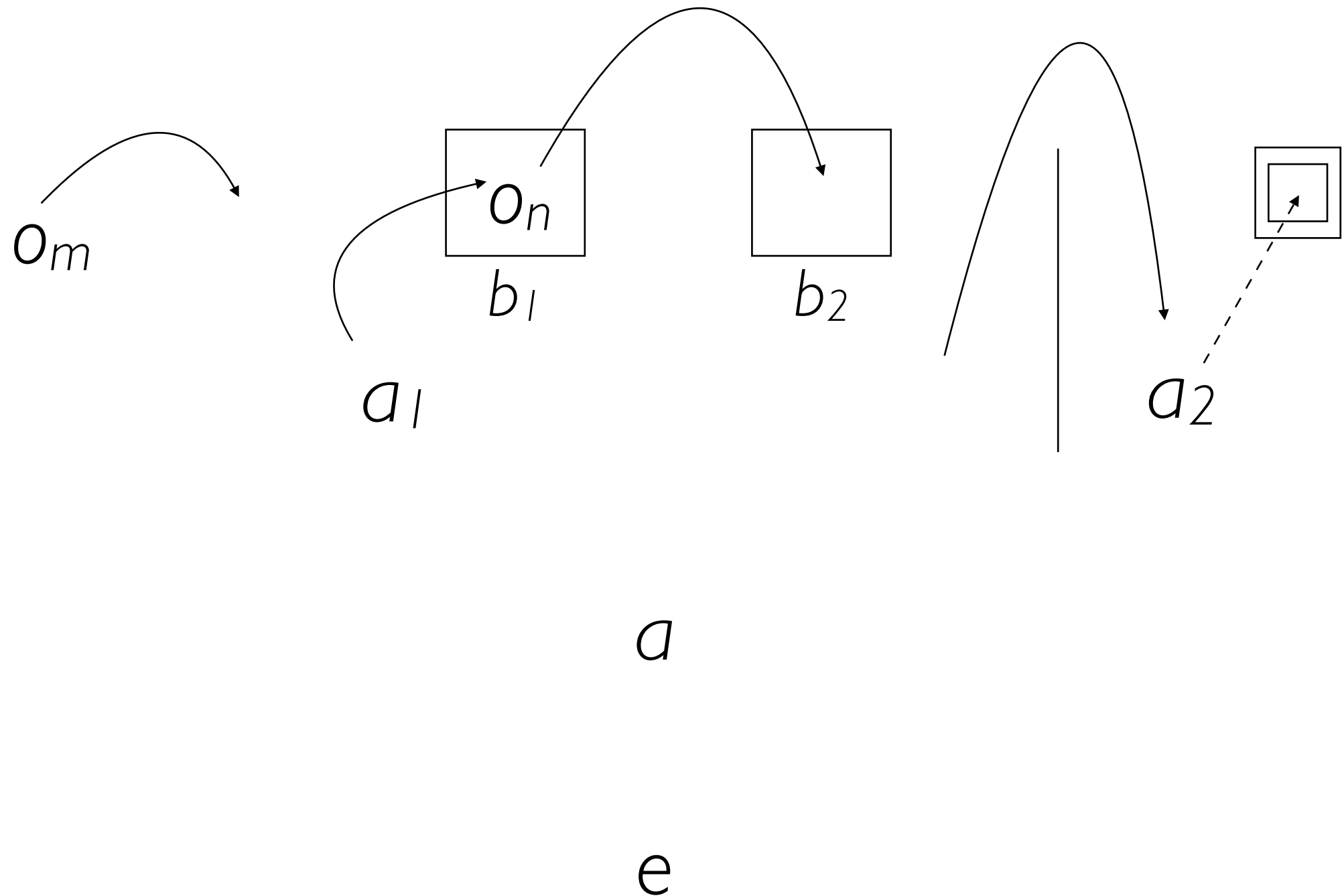
Framework for FBT^1_2

(seven timepoints)



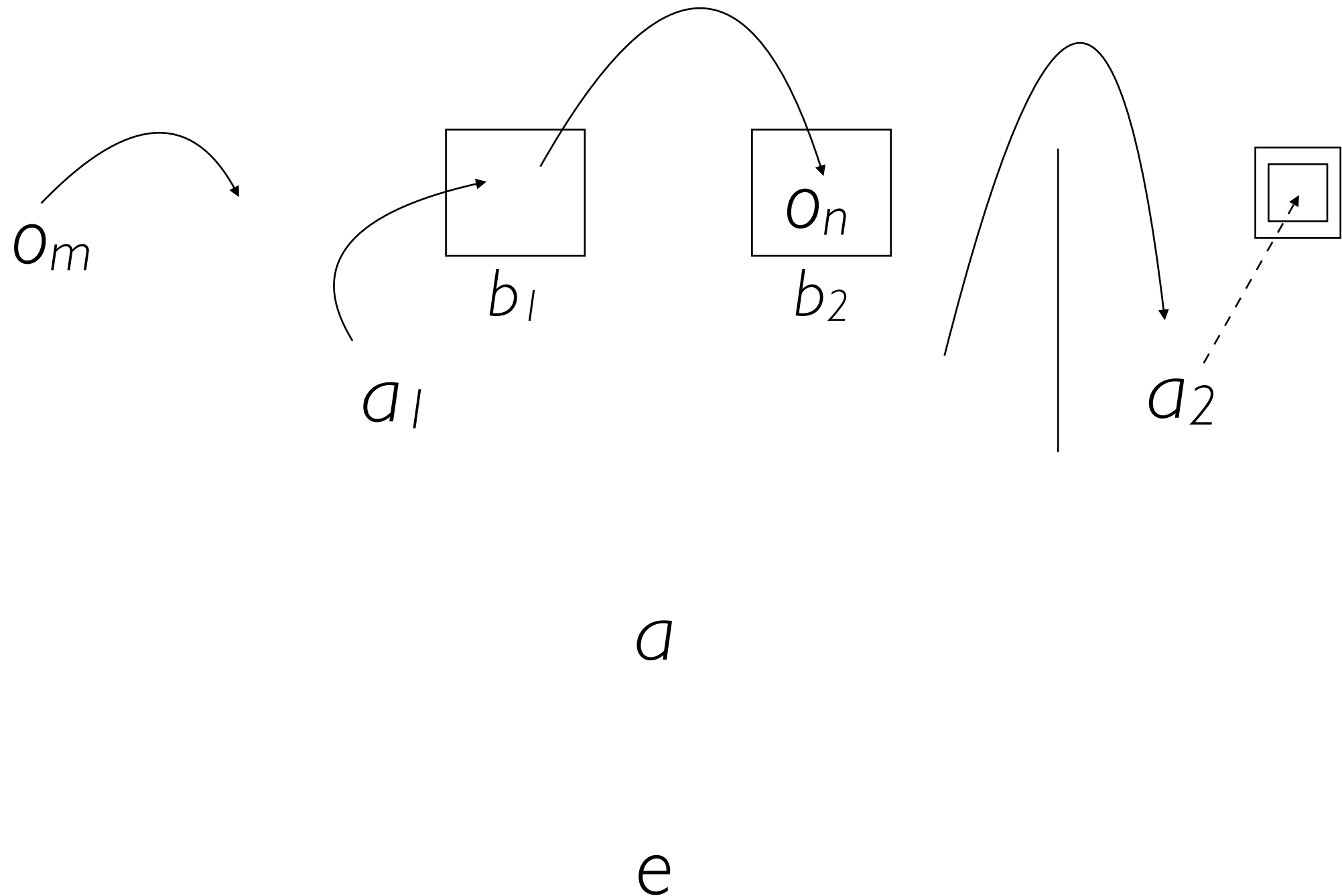
Framework for FBT^1_2

(seven timepoints)



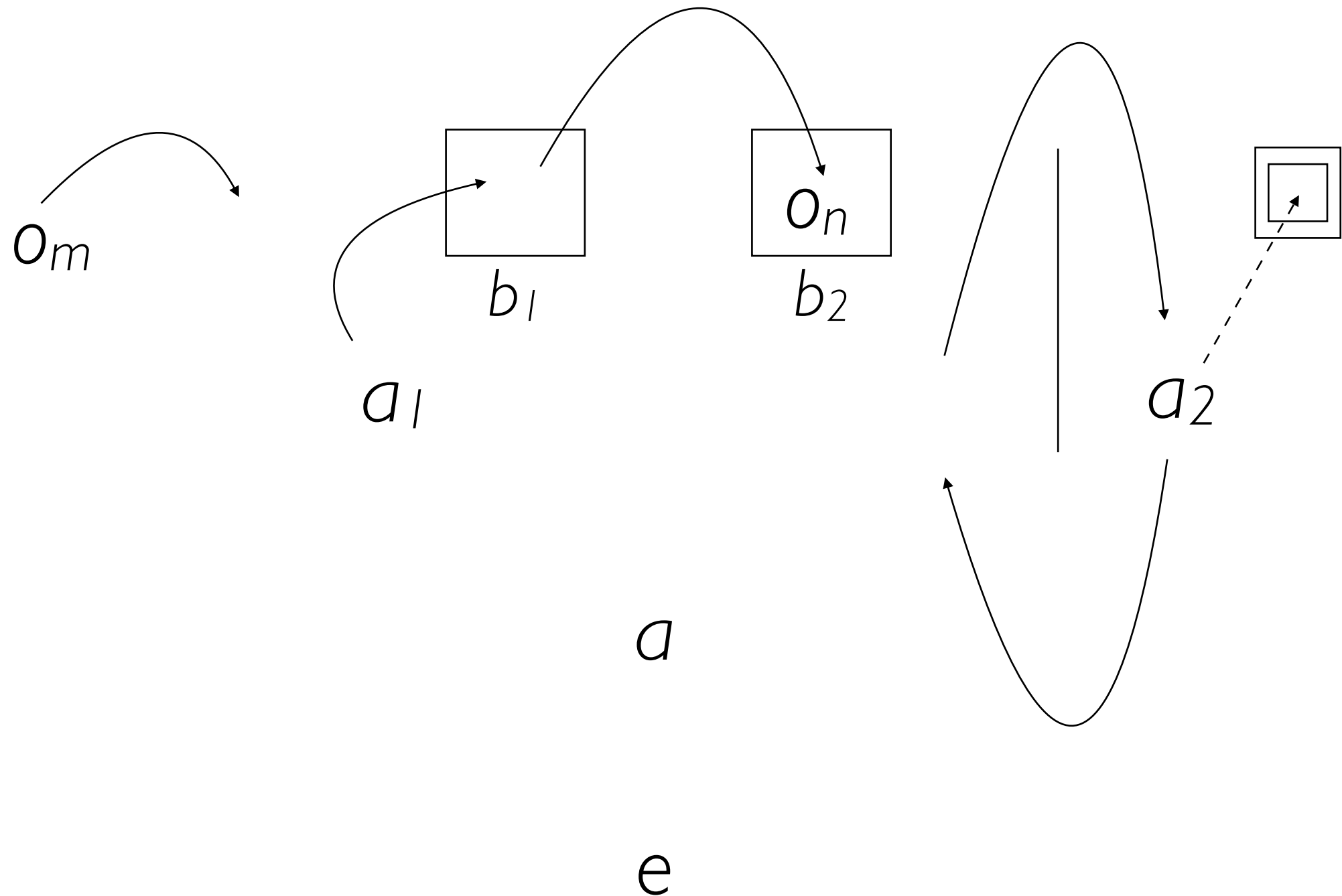
Framework for FBT^1_2

(seven timepoints)



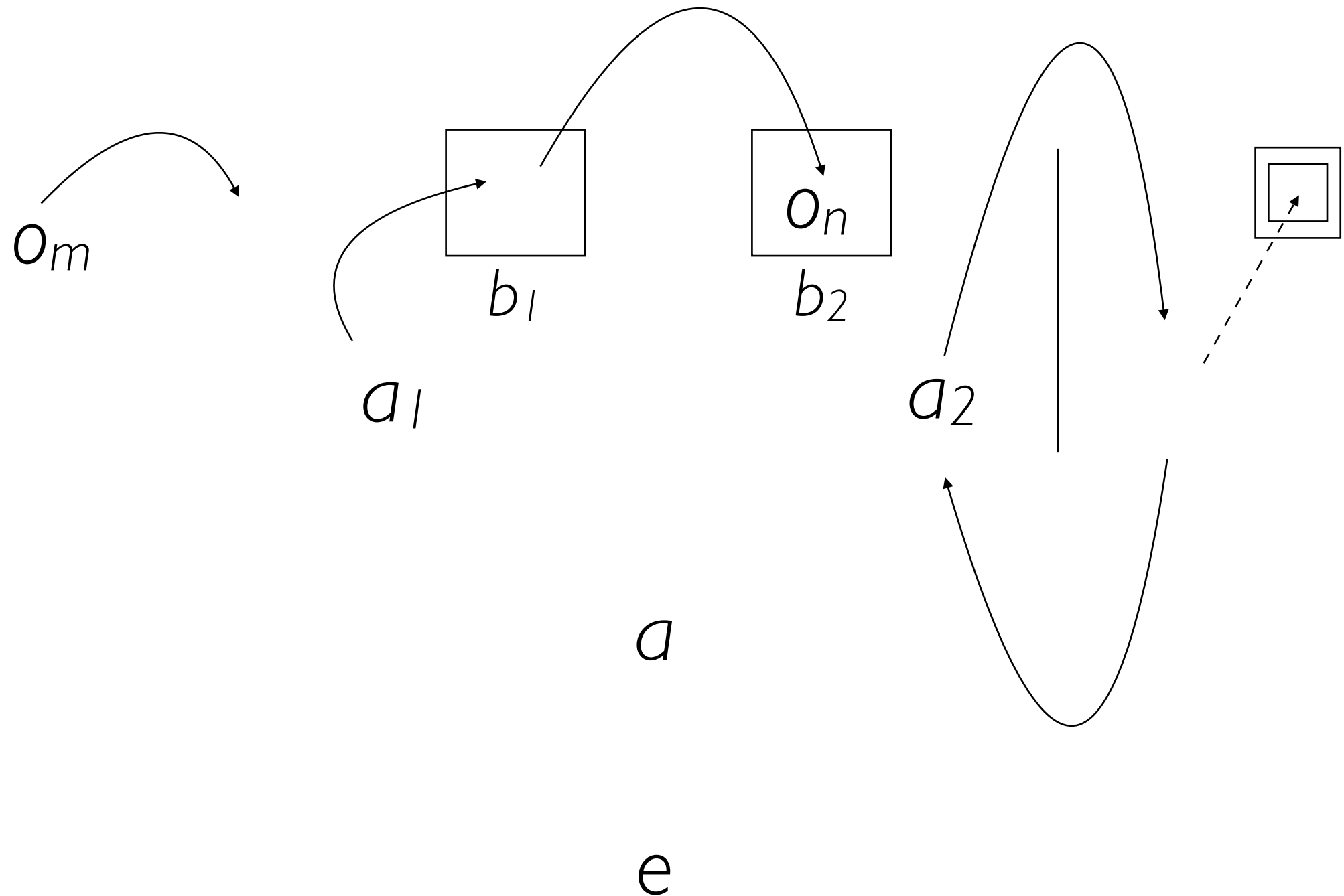
Framework for FBT^1_2

(seven timepoints)



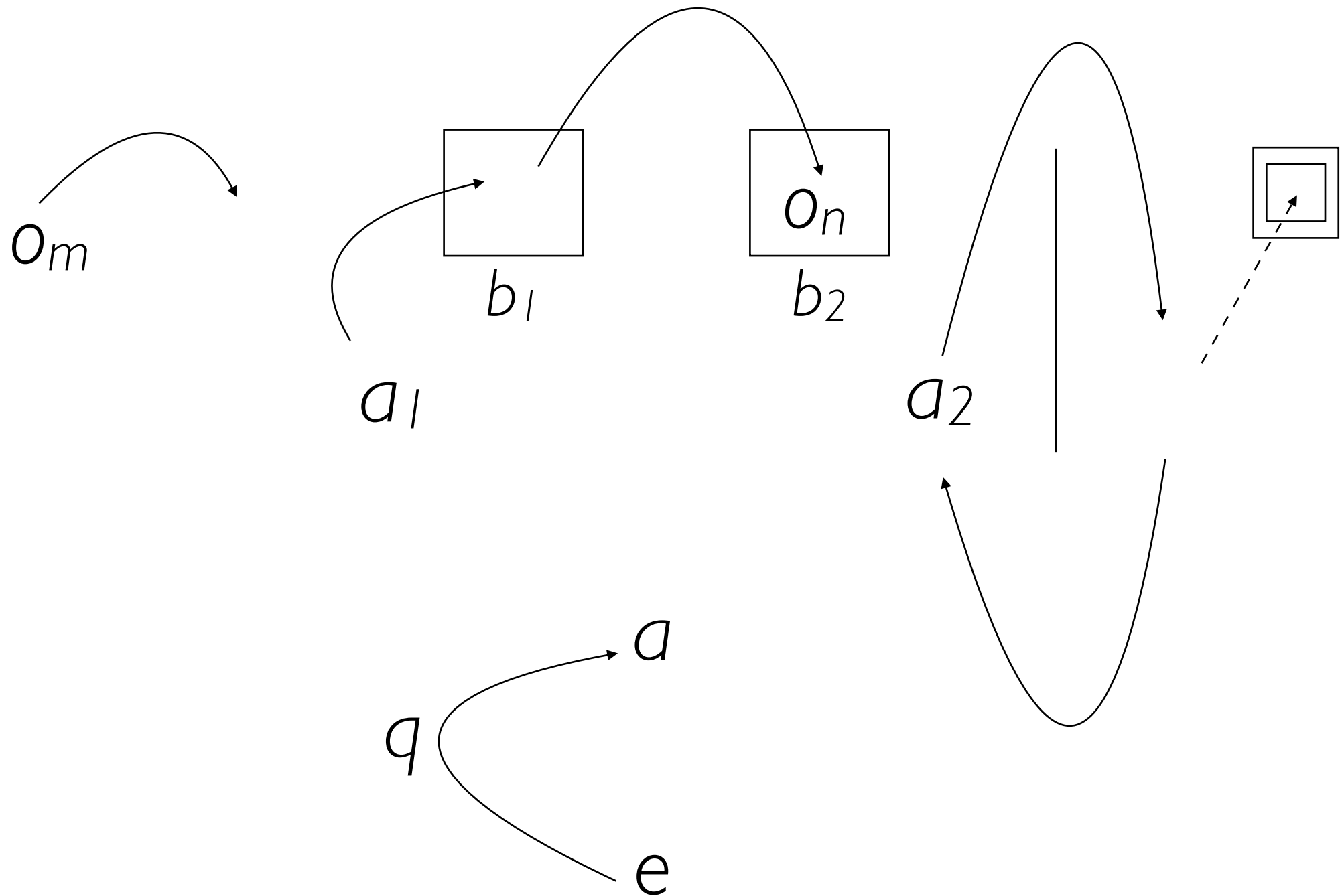
Framework for FBT^1_2

(seven timepoints)



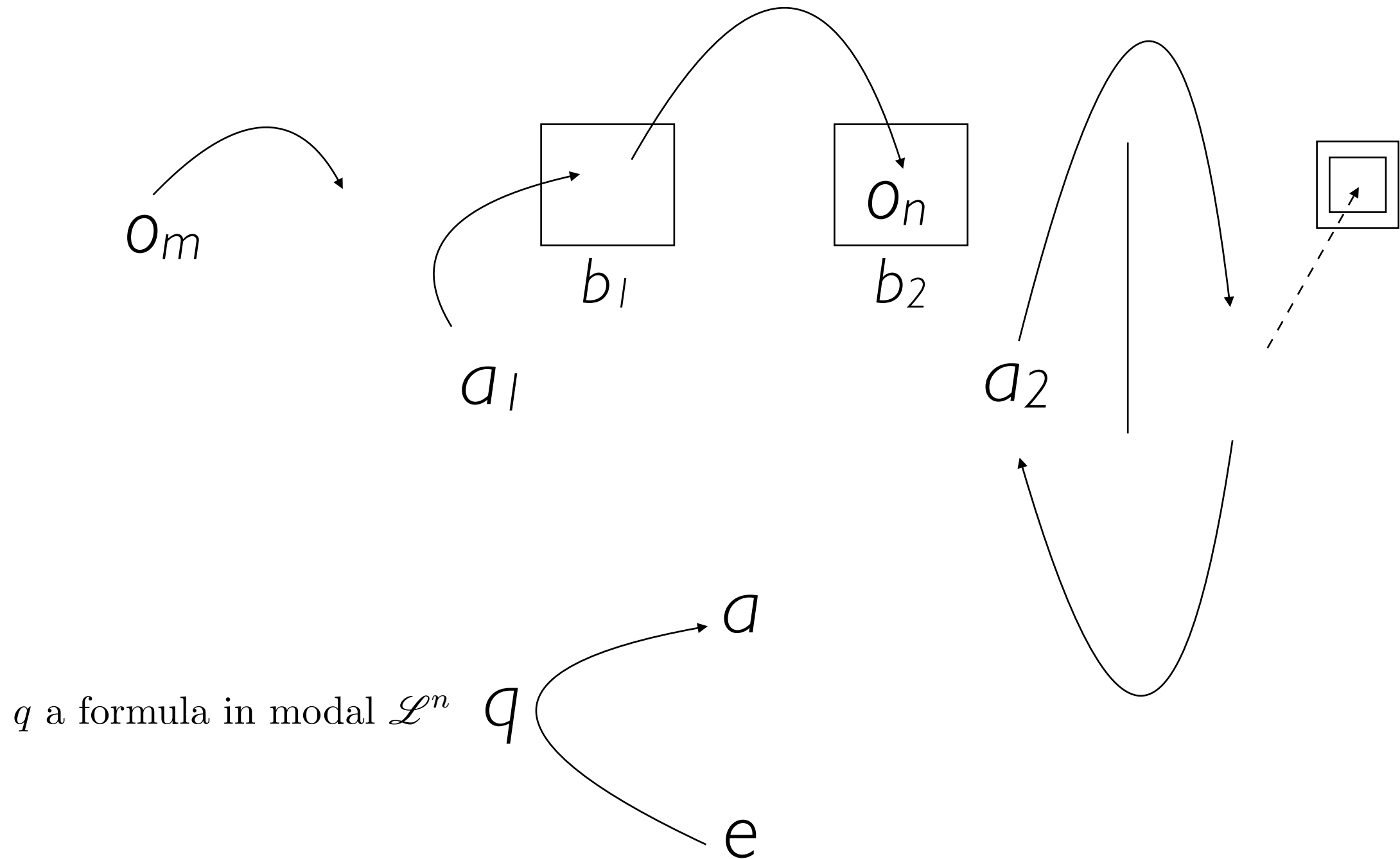
Framework for FBT^1_2

(seven timepoints)

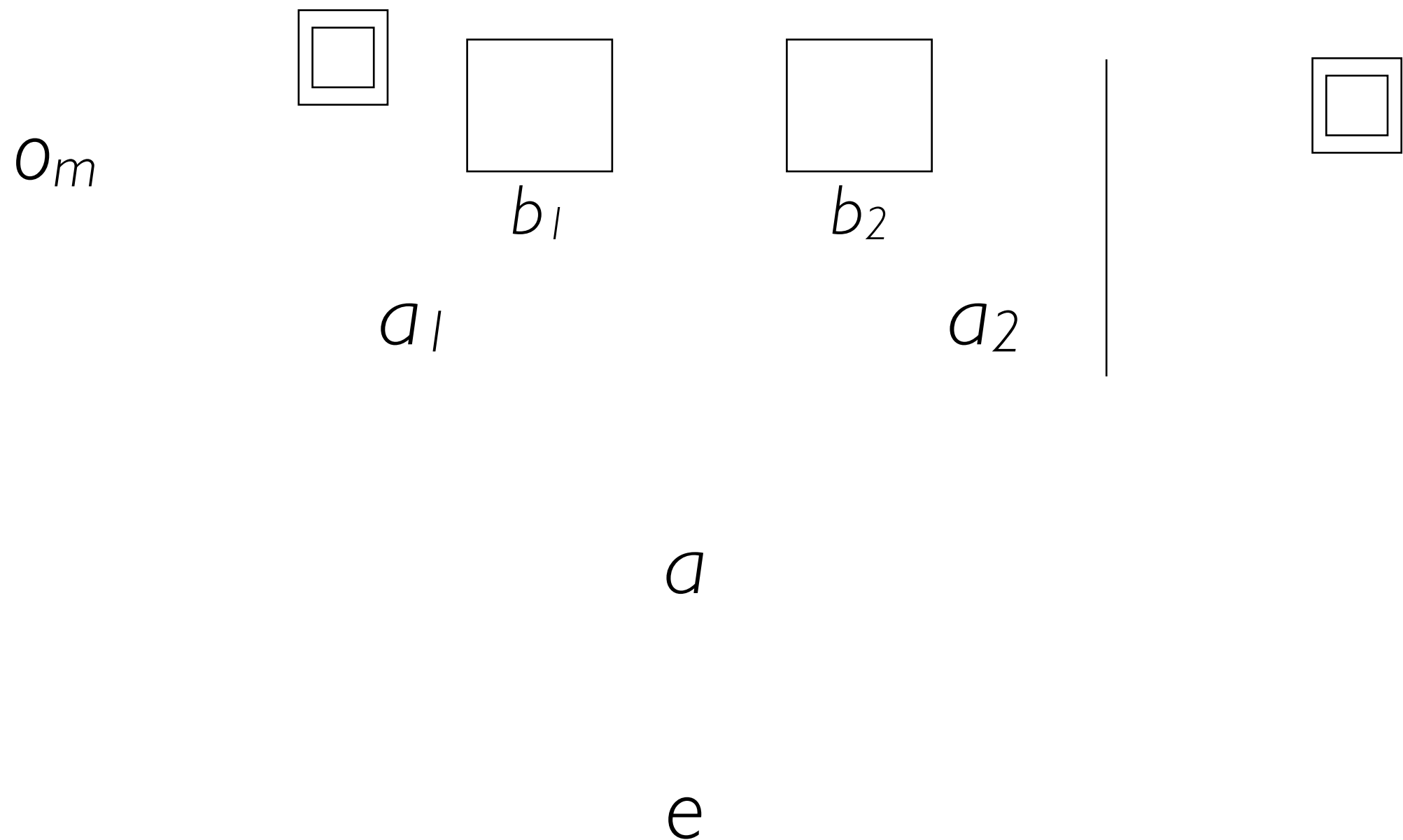


Framework for FBT^1_2

(seven timepoints)

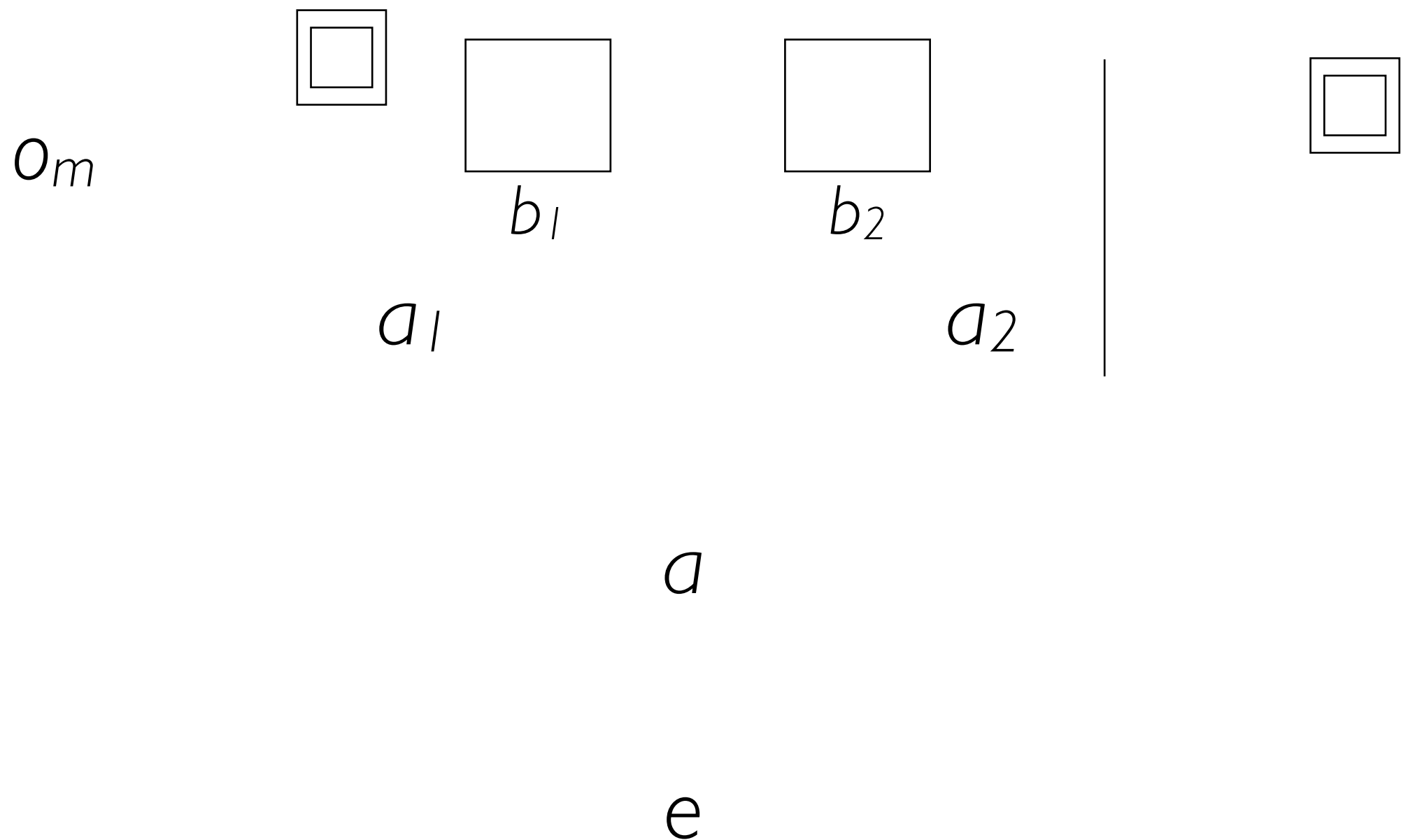


Framework for FBT^I_3



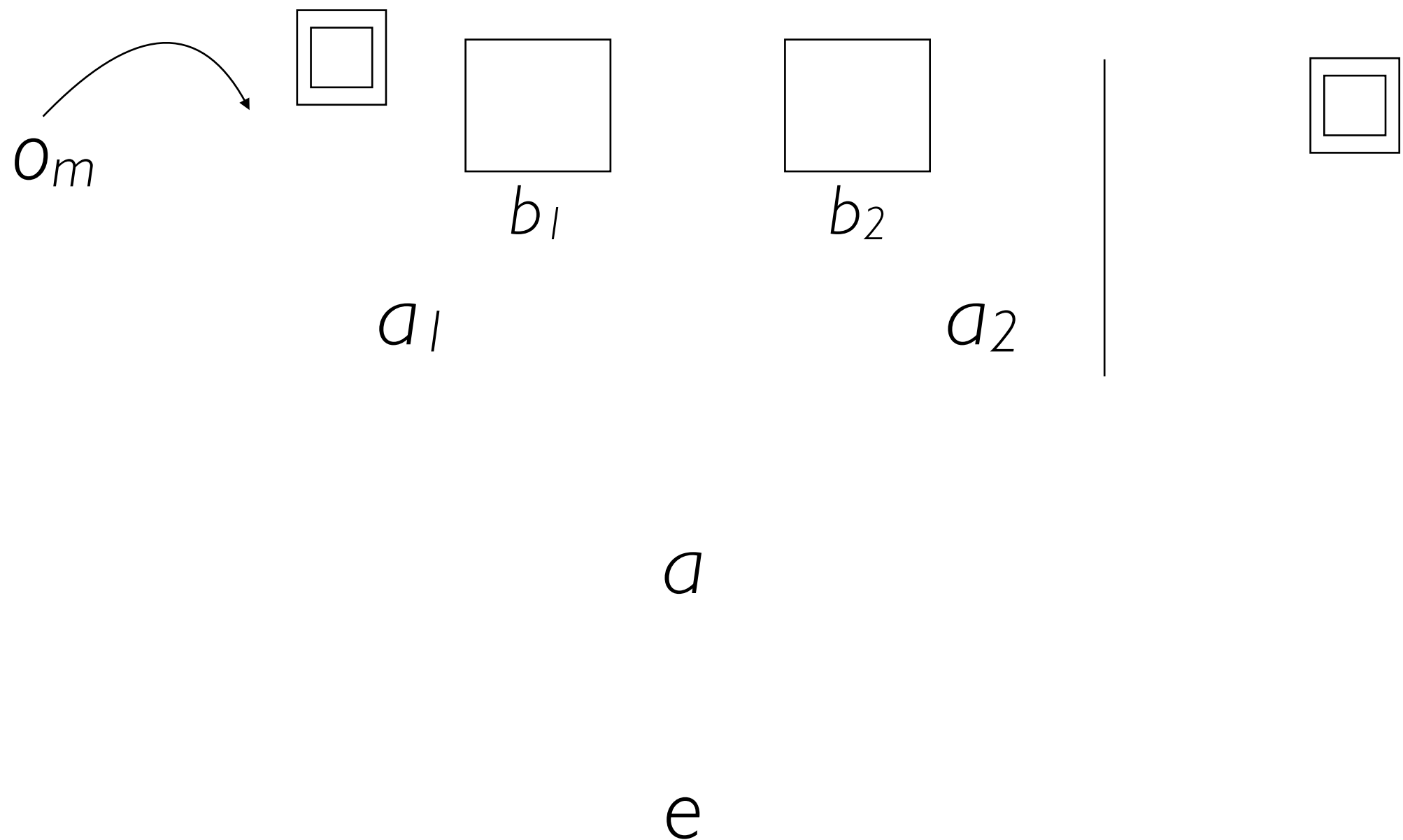
Framework for FBT^I_3

(eight timepoints)



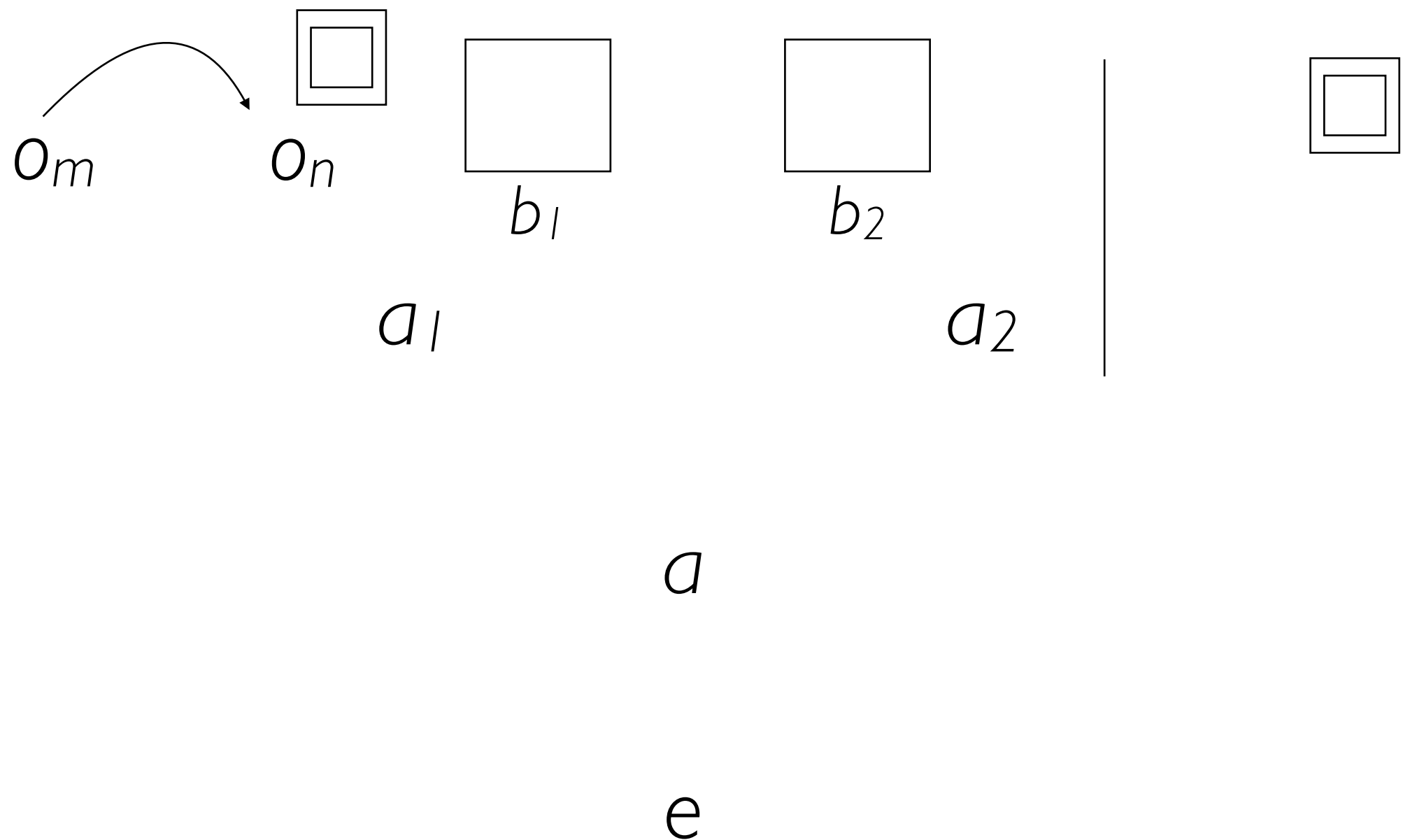
Framework for FBT^I_3

(eight timepoints)



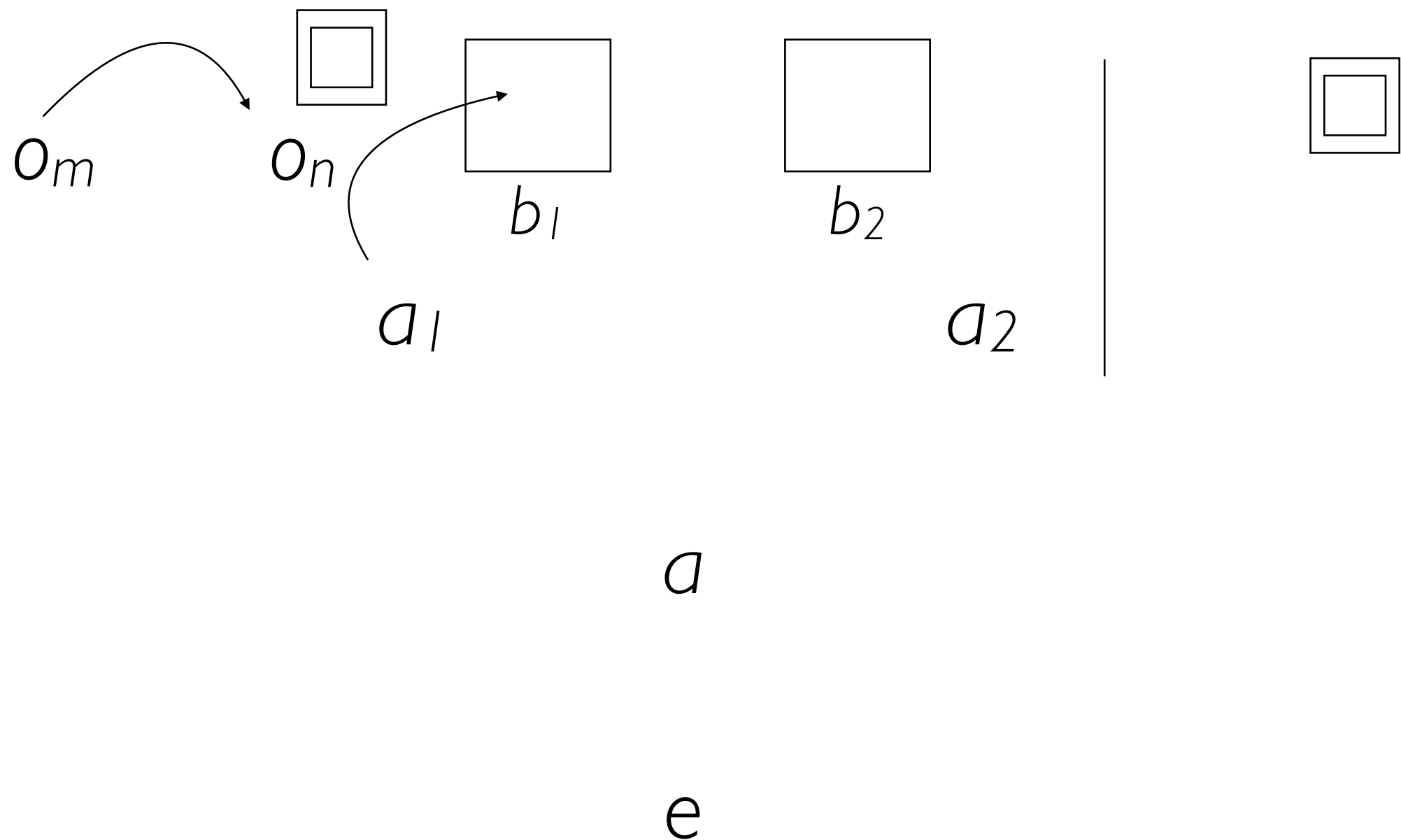
Framework for FBT^I_3

(eight timepoints)



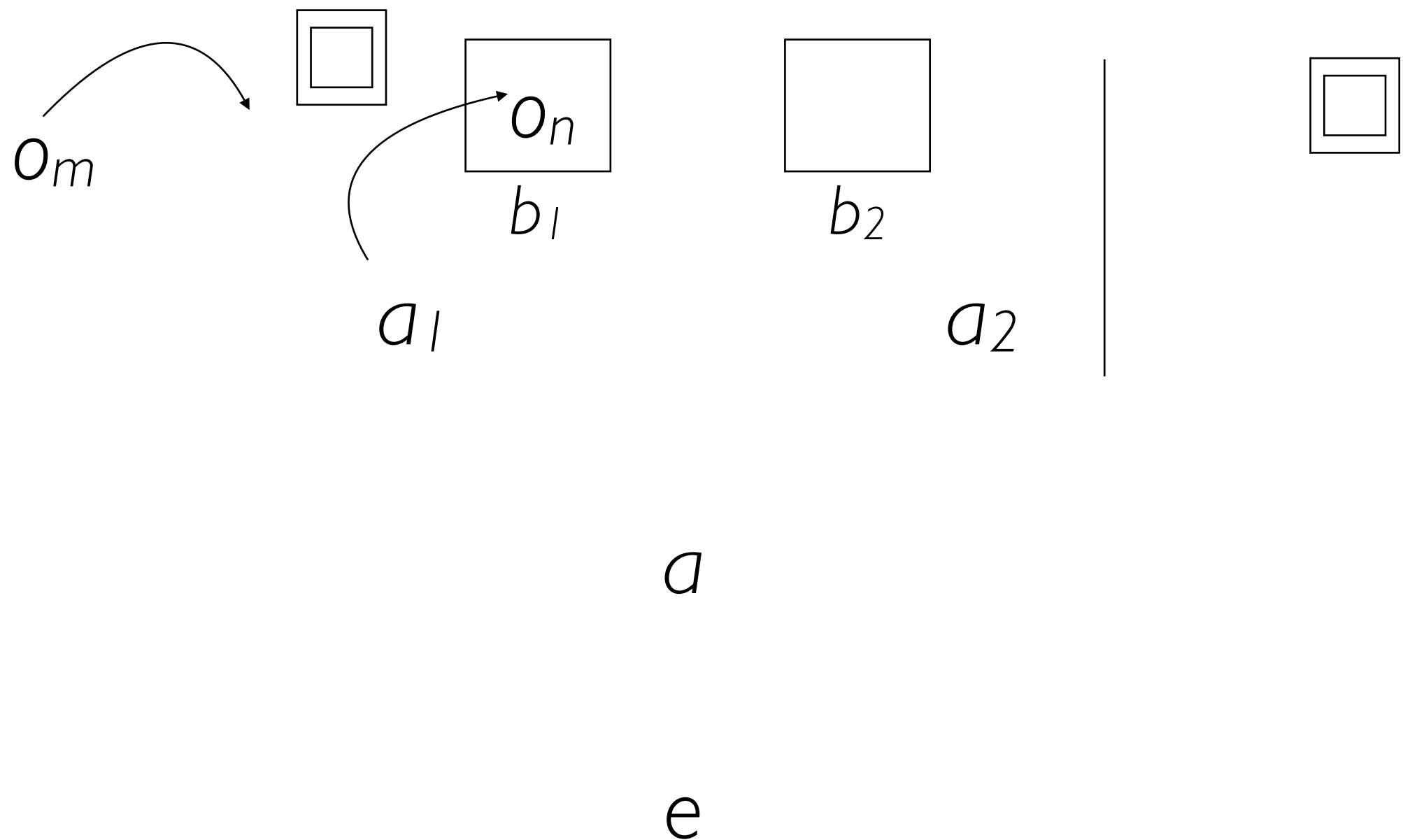
Framework for FBT^I_3

(eight timepoints)



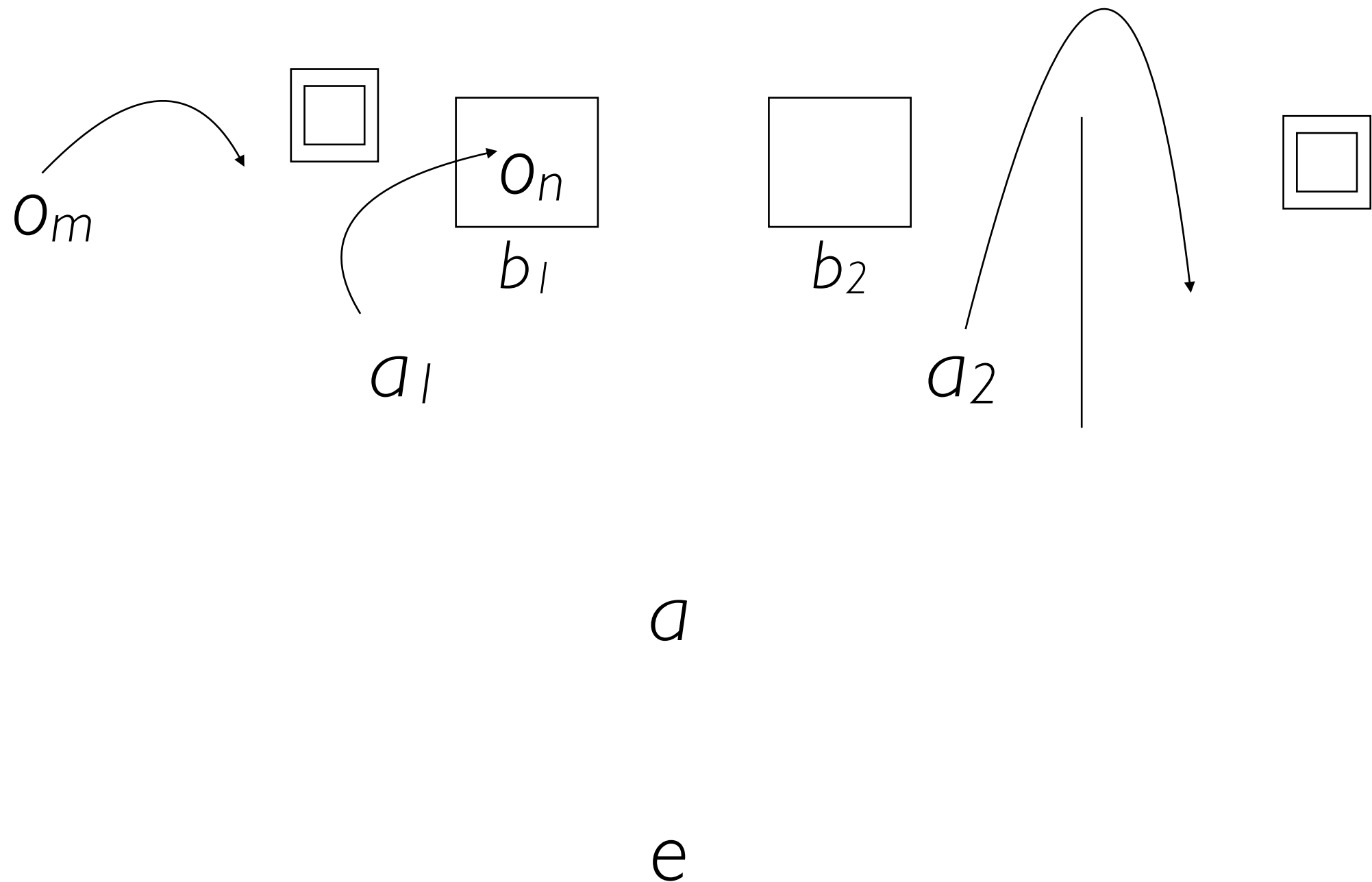
Framework for FBT^I_3

(eight timepoints)



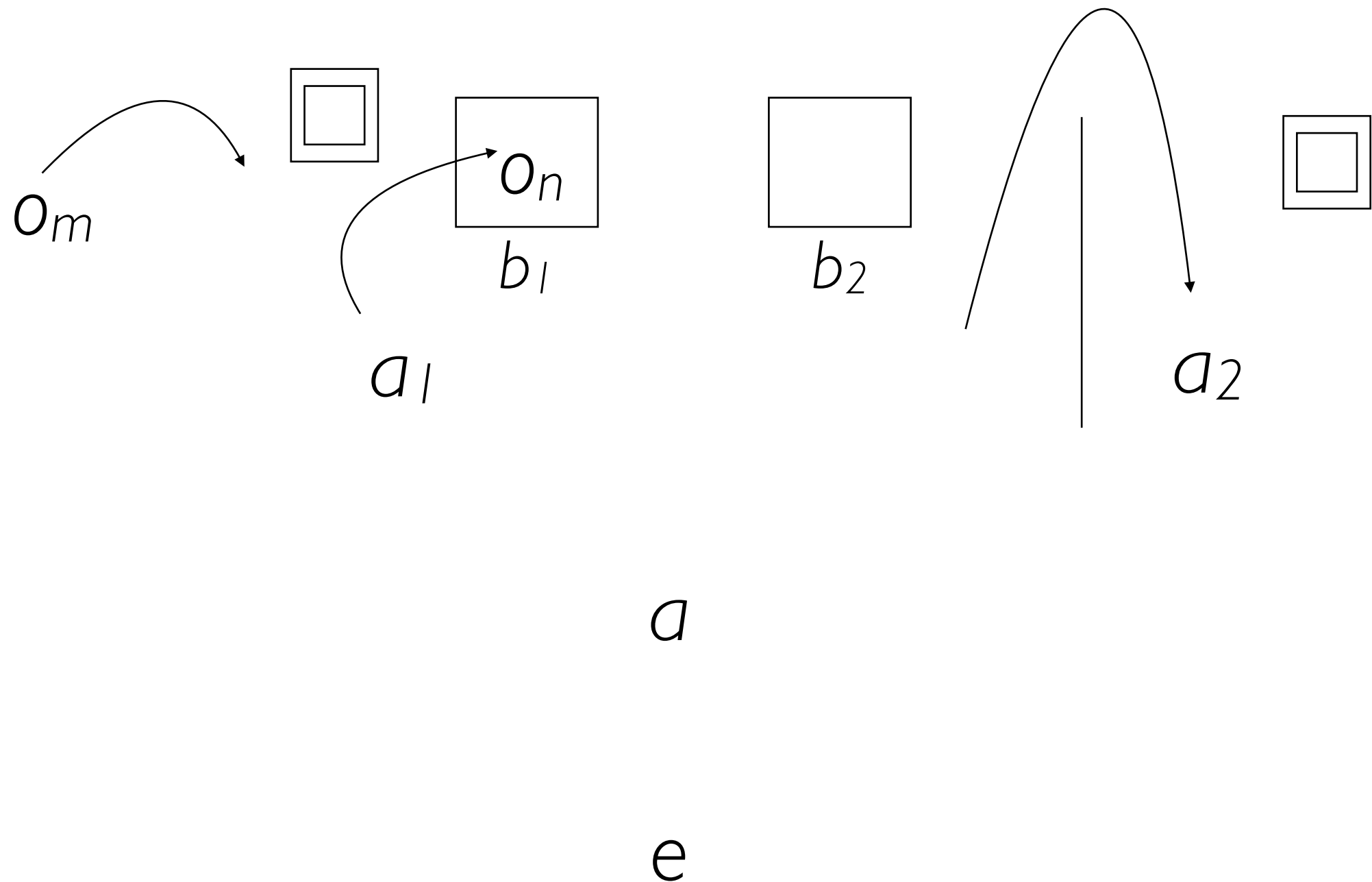
Framework for FBT^I_3

(eight timepoints)



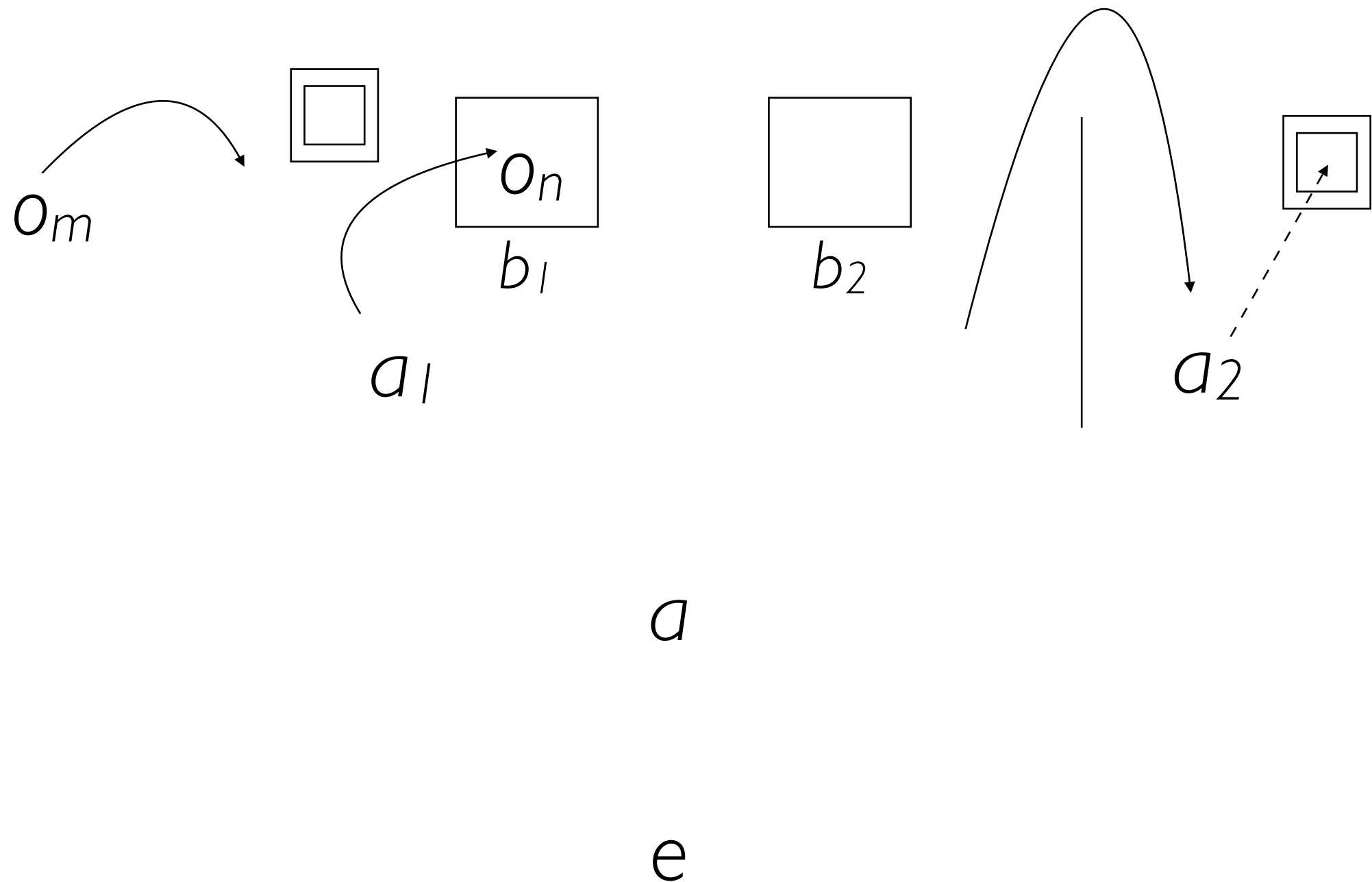
Framework for FBT^I_3

(eight timepoints)



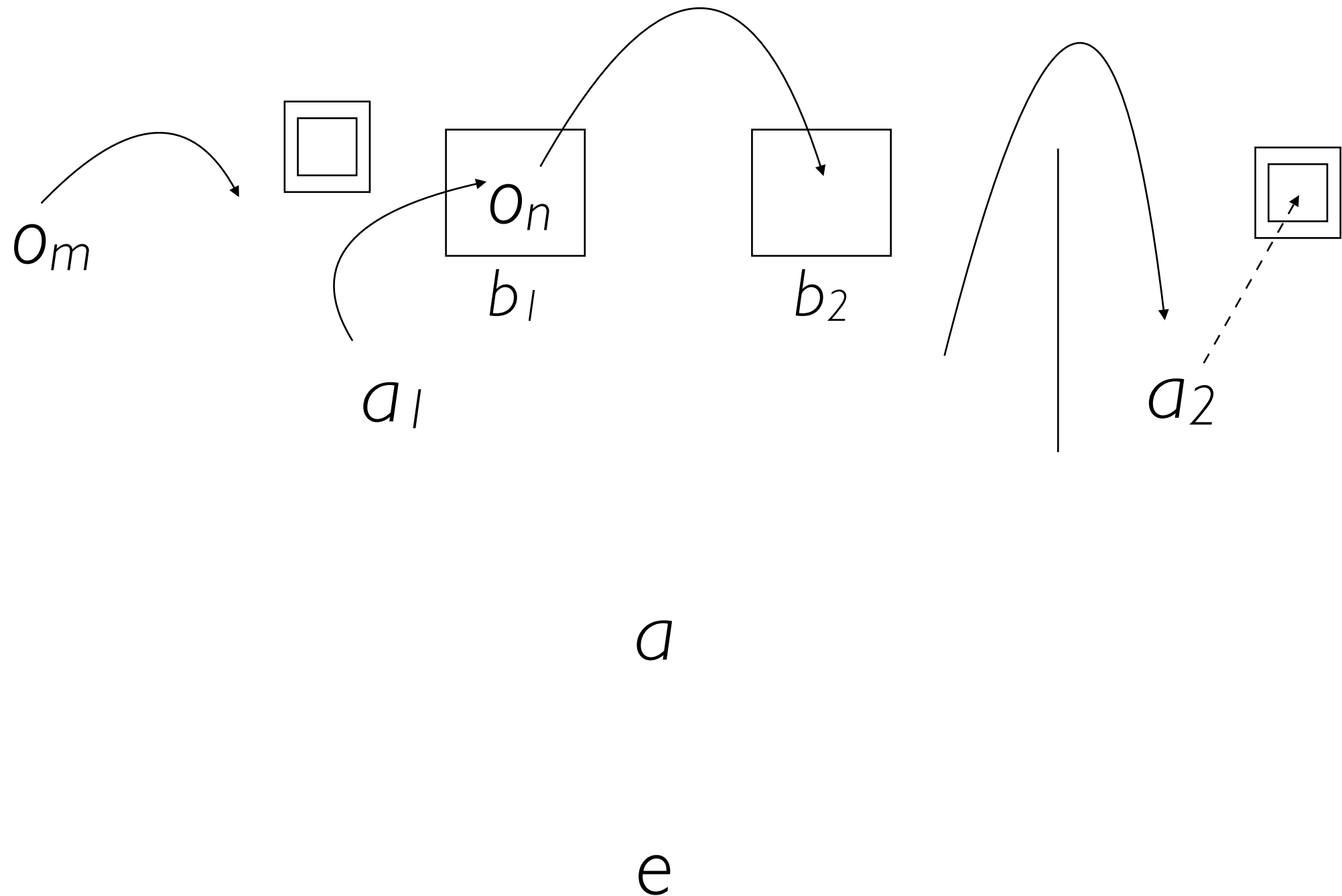
Framework for FBT^I_3

(eight timepoints)



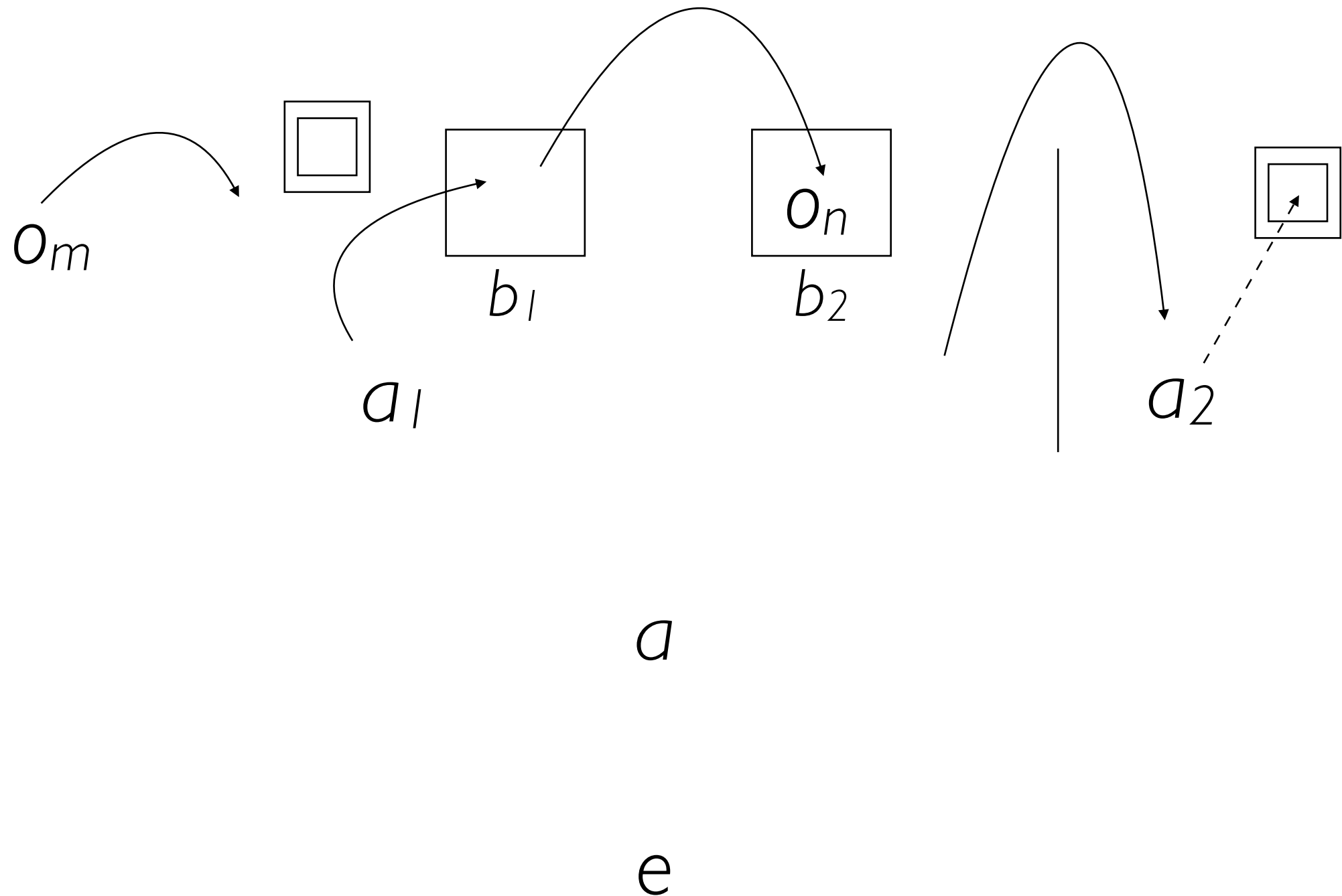
Framework for FBT^I_3

(eight timepoints)



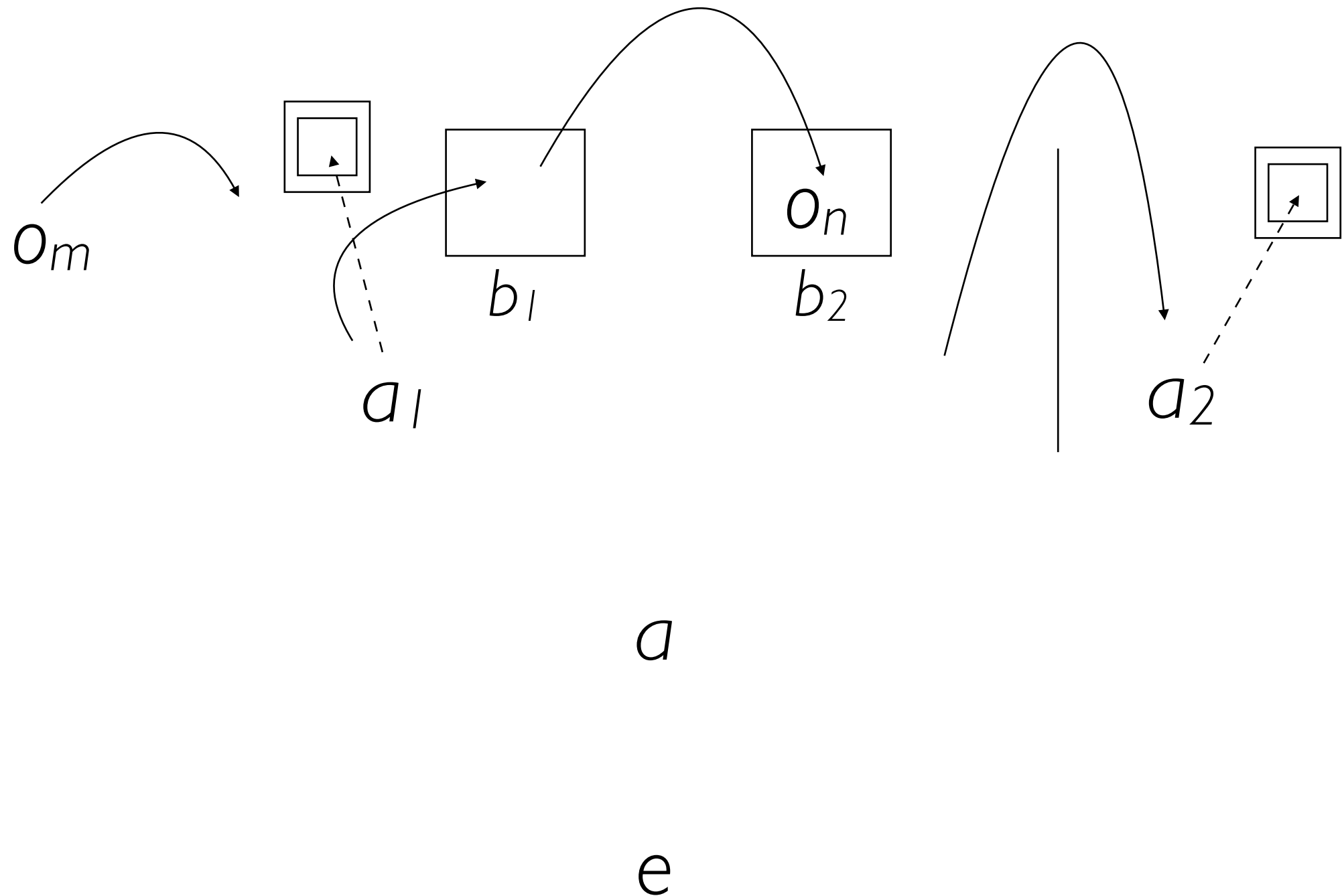
Framework for FBT^I_3

(eight timepoints)



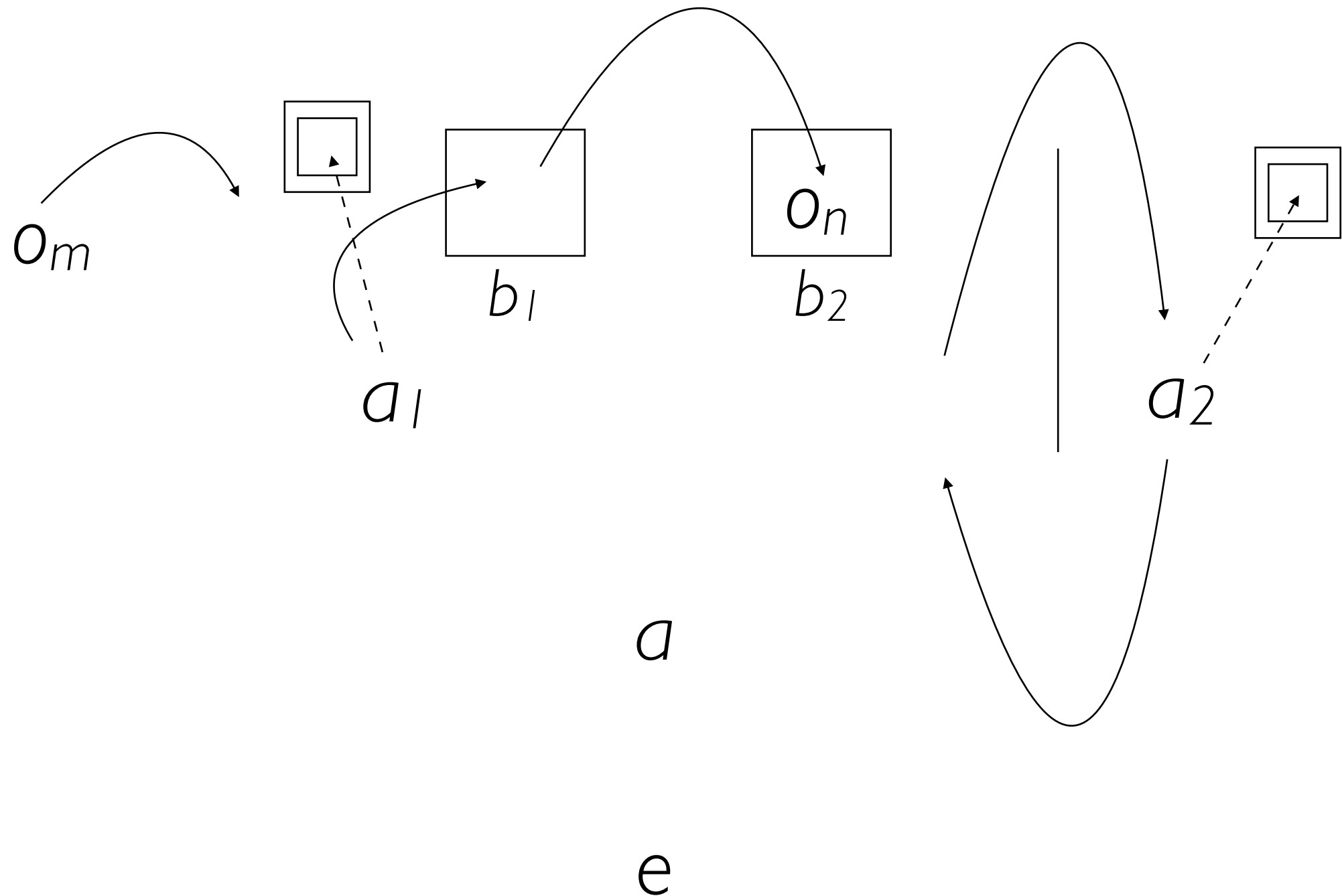
Framework for FBT^I_3

(eight timepoints)



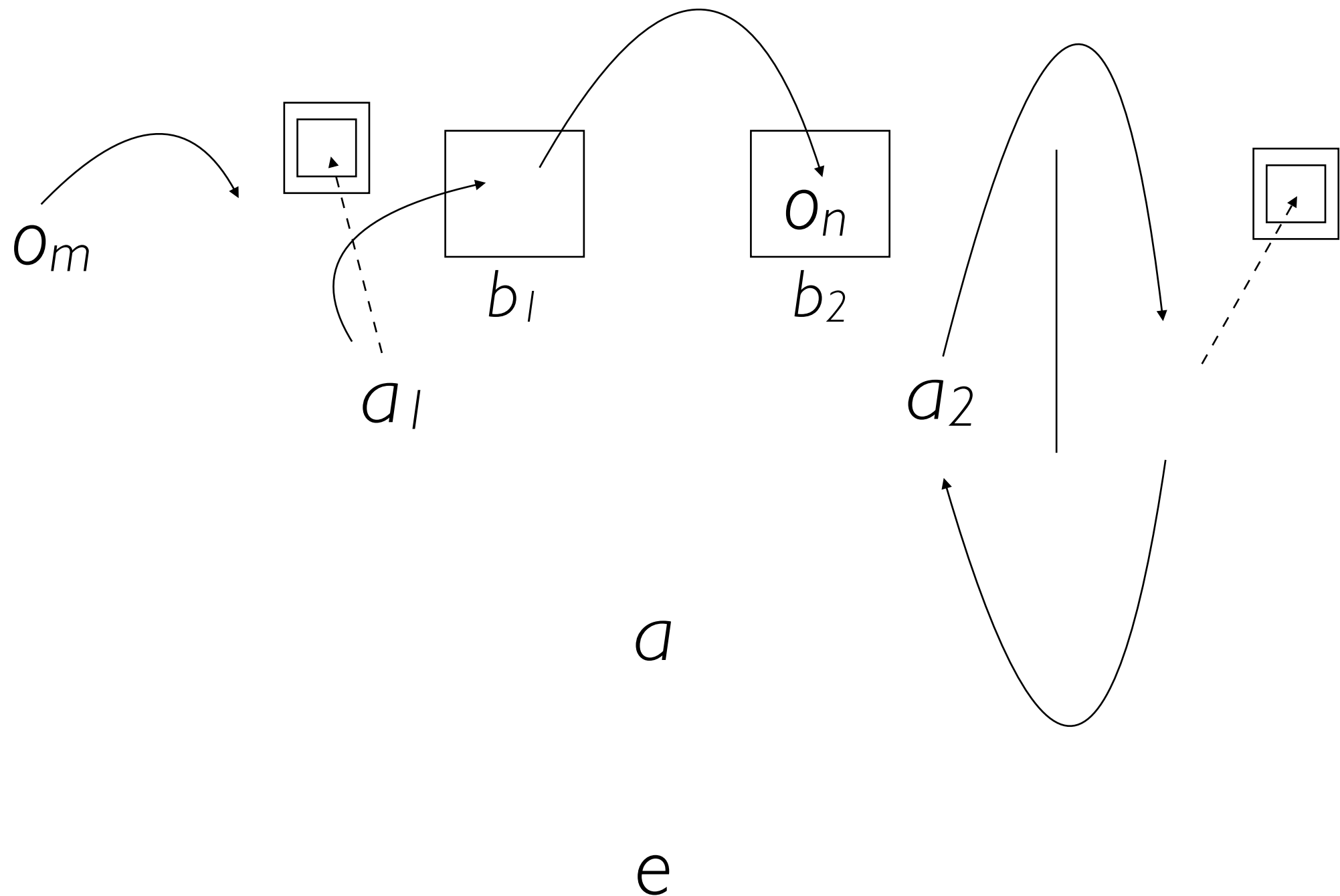
Framework for FBT^I_3

(eight timepoints)



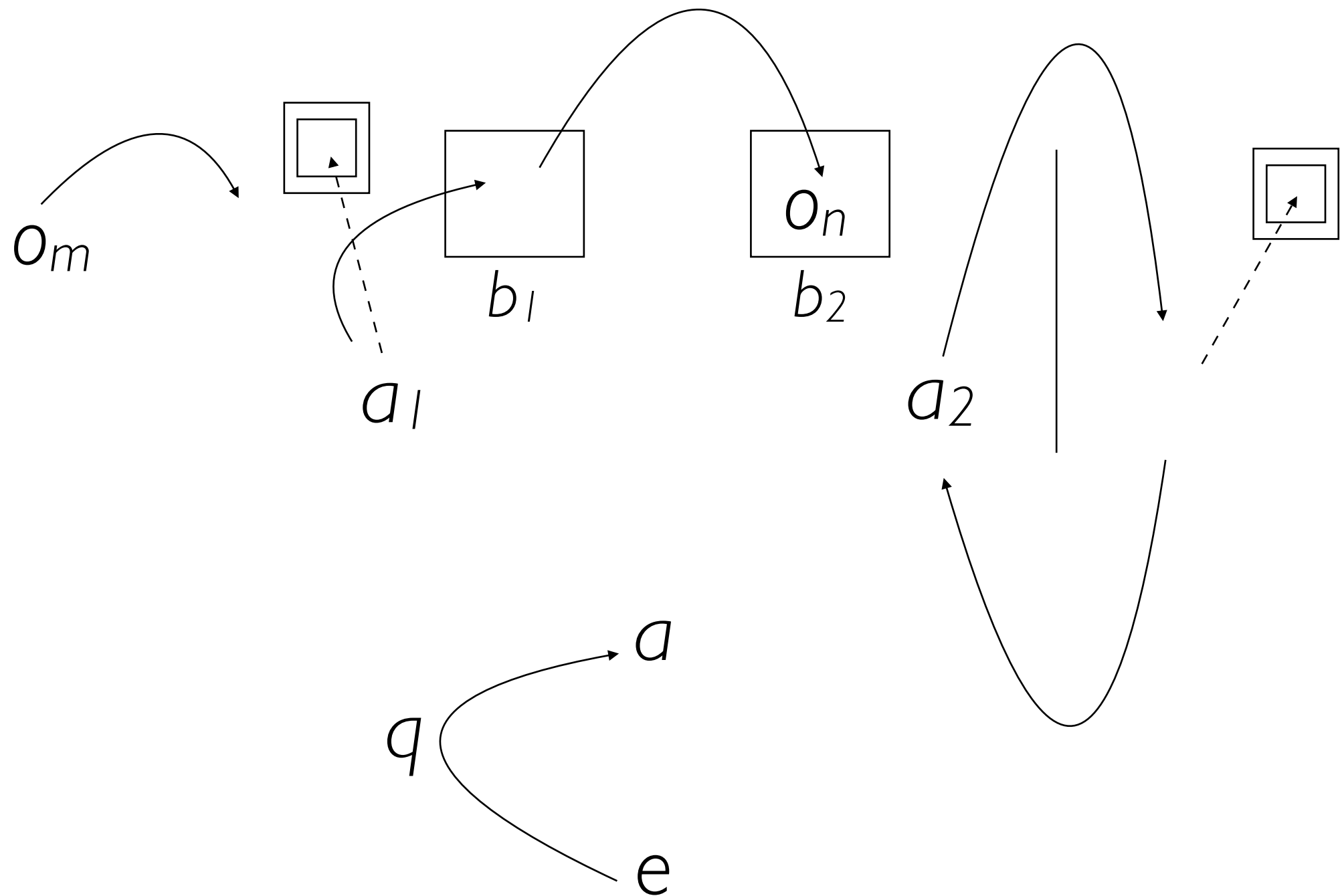
Framework for FBT^I_3

(eight timepoints)



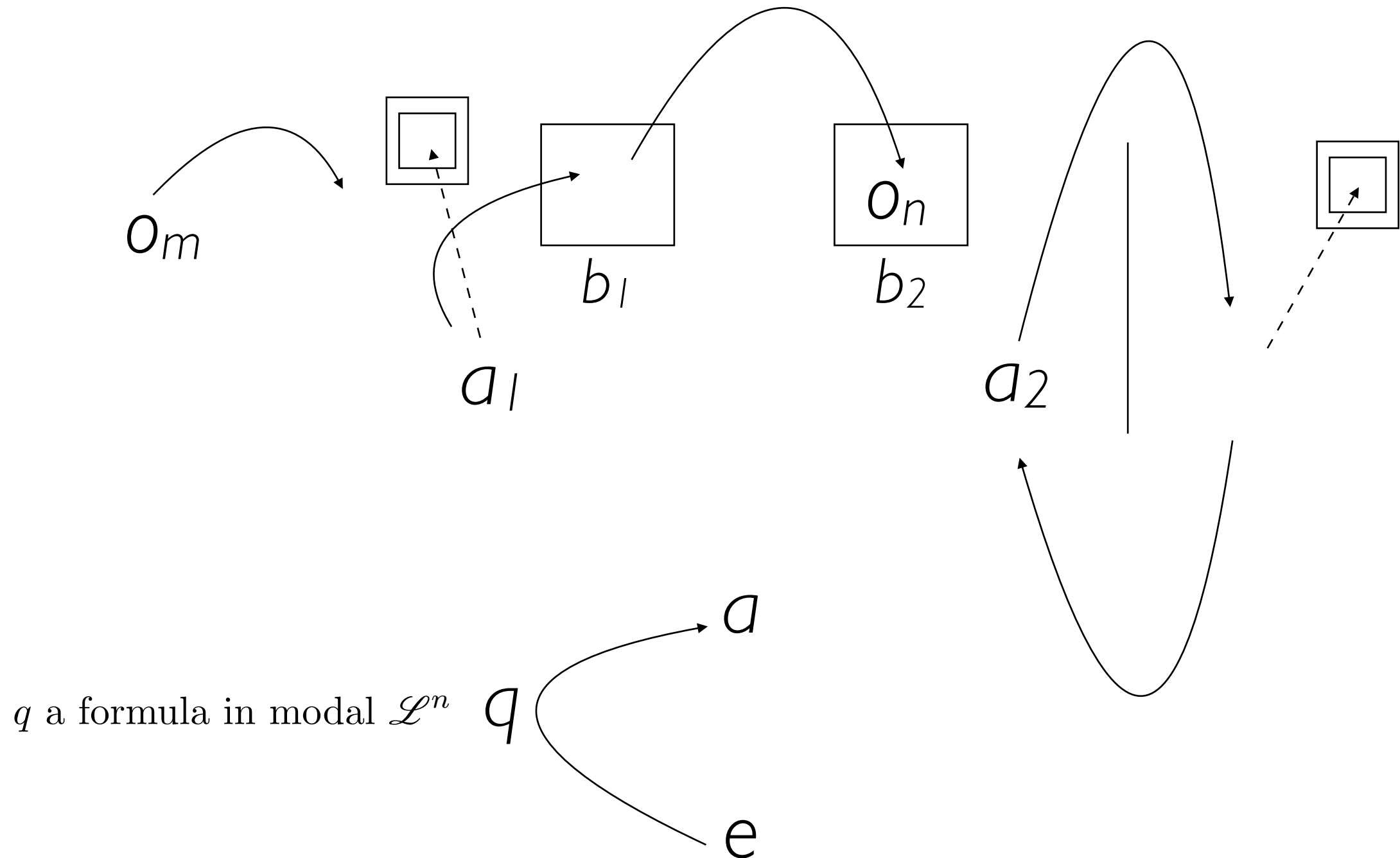
Framework for FBT^I_3

(eight timepoints)

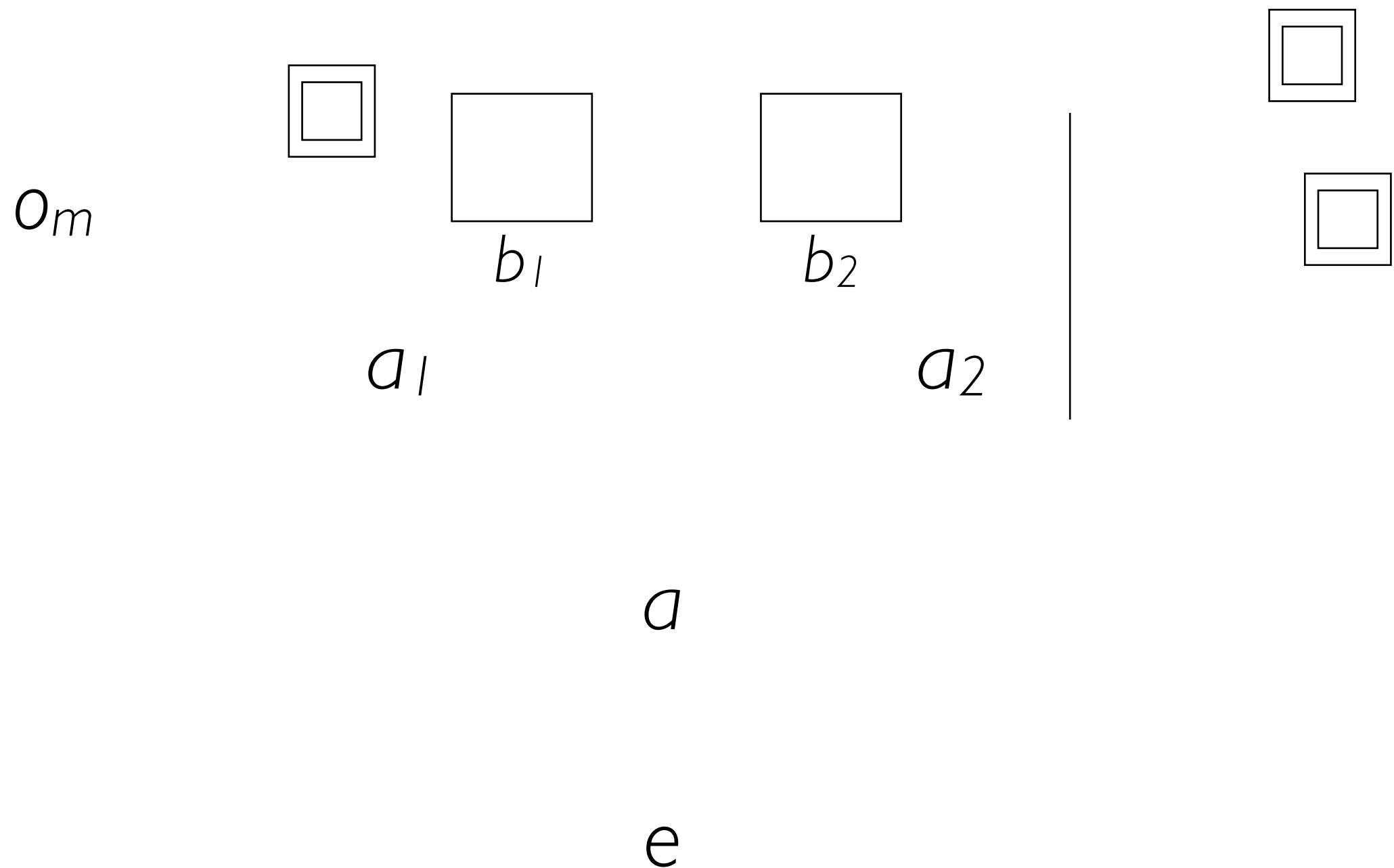


Framework for FBT^I_3

(eight timepoints)

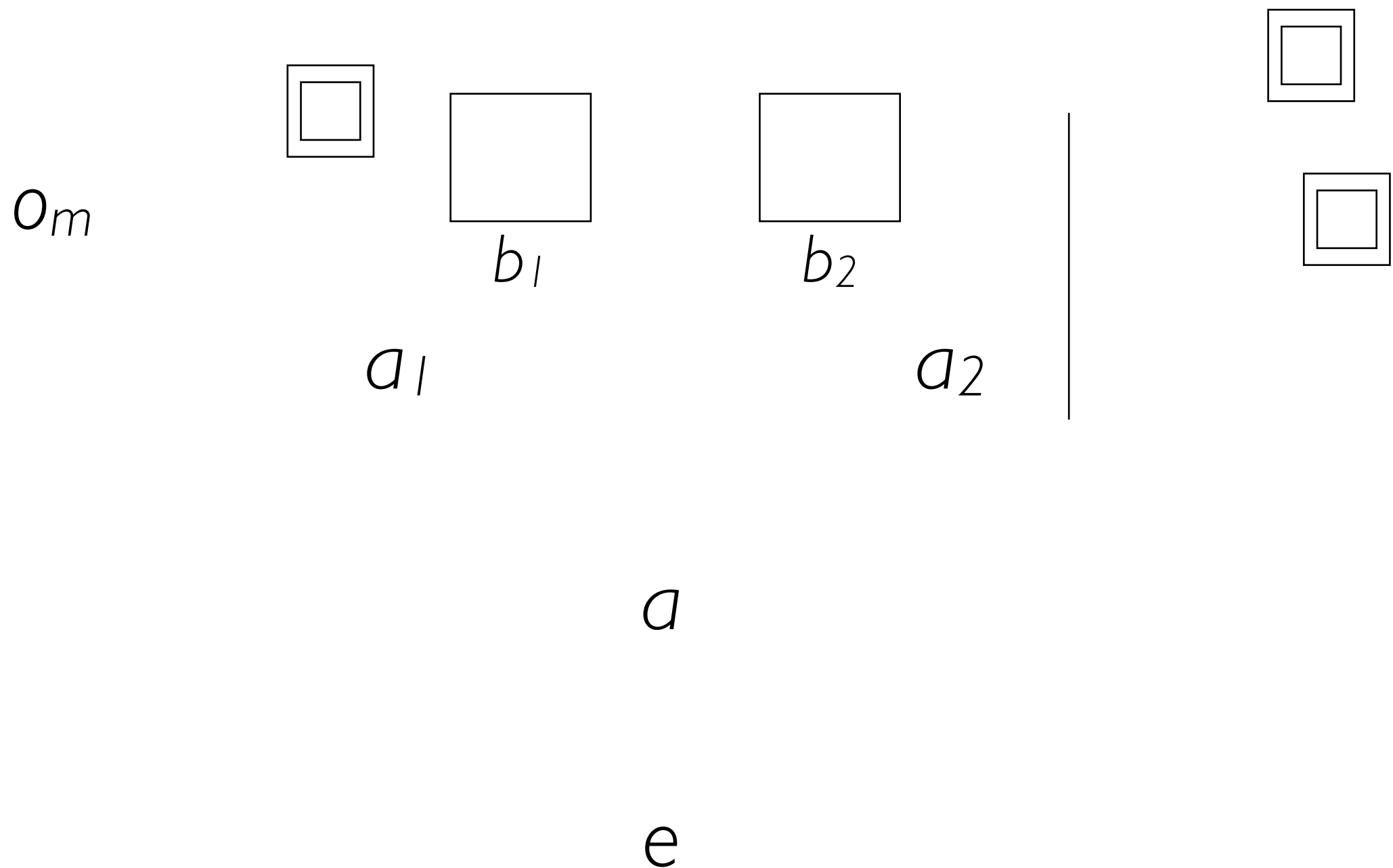


Framework for FBT^I_4



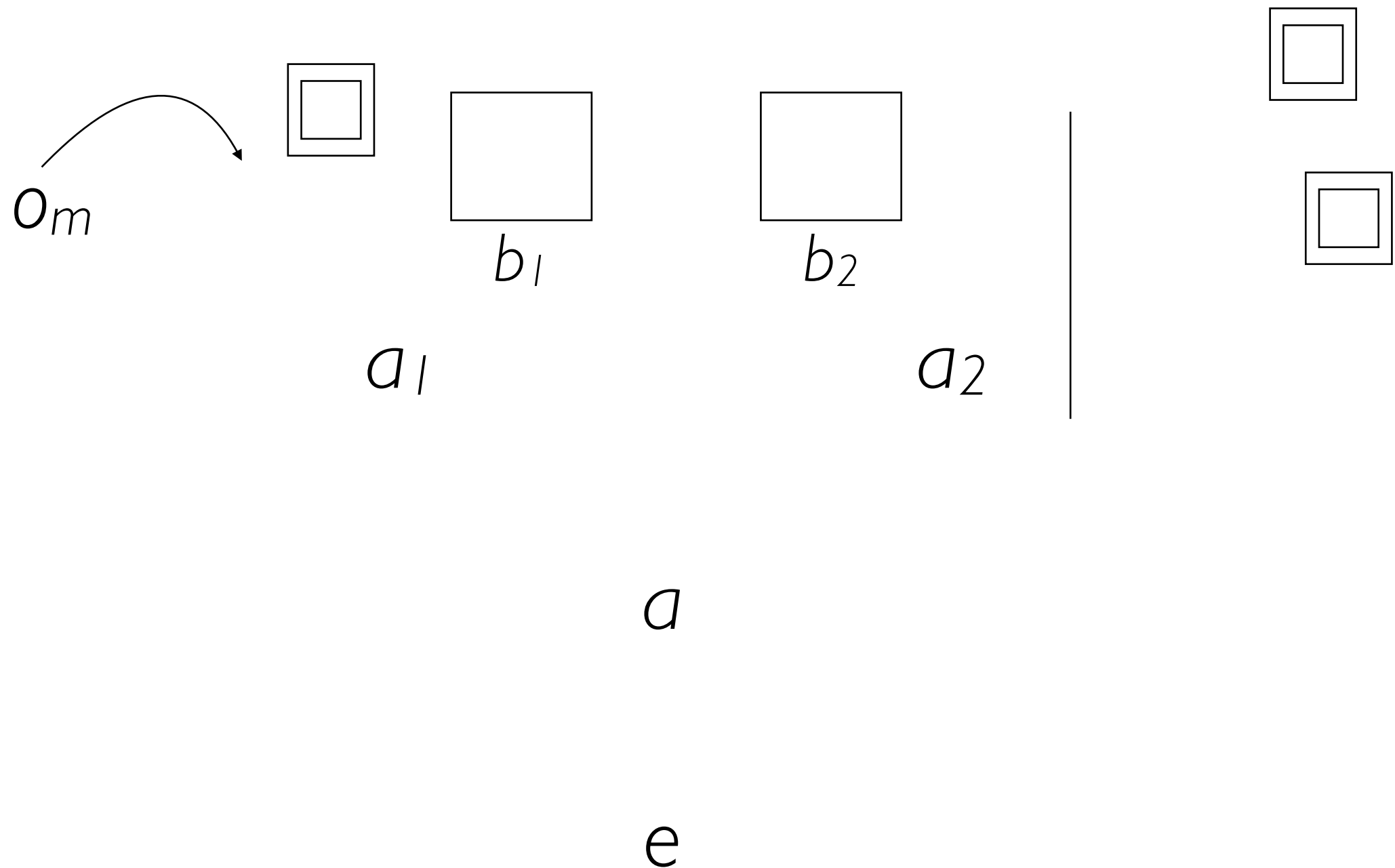
Framework for FBT^I_4

(nine timepoints)



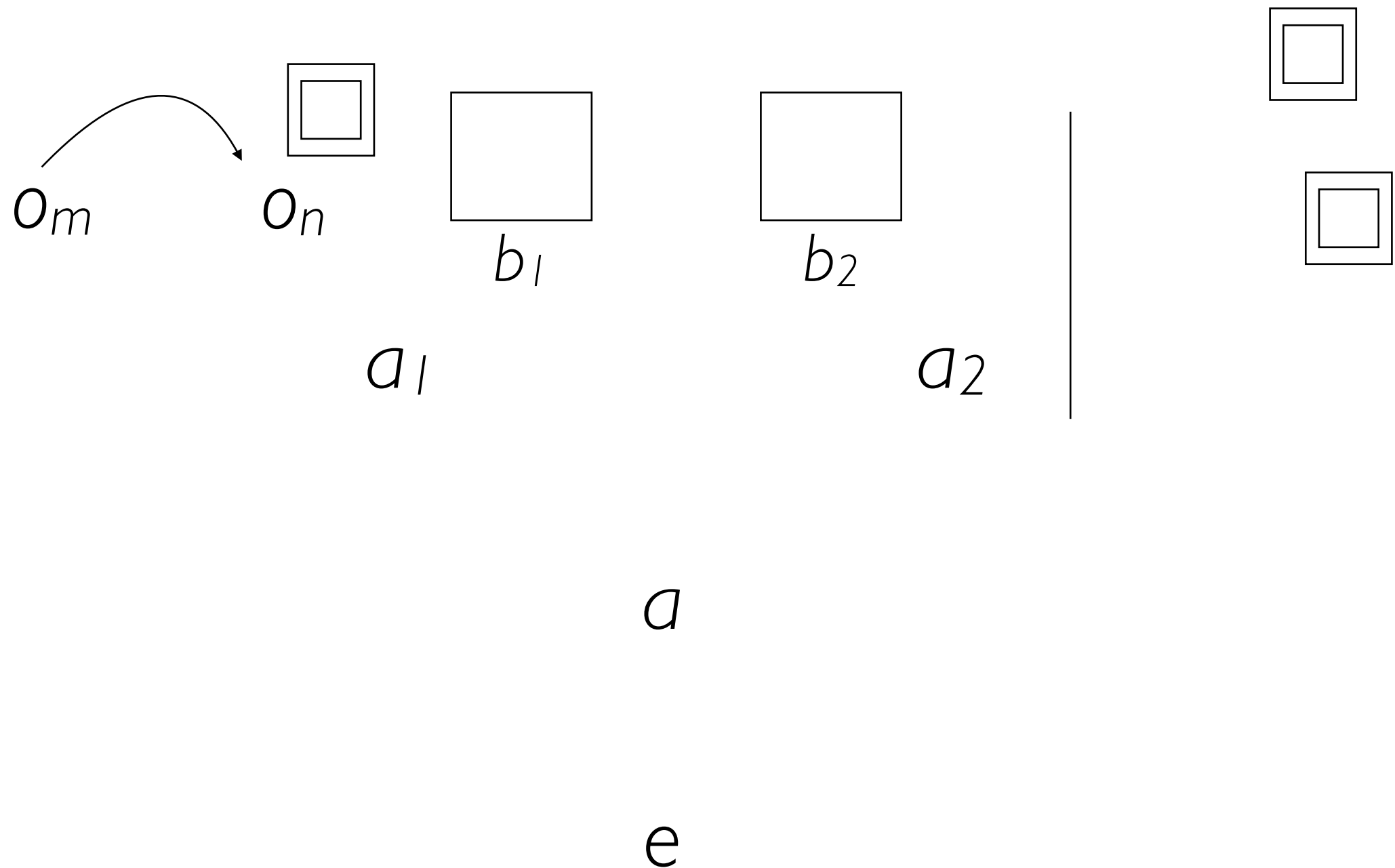
Framework for FBT^I_4

(nine timepoints)



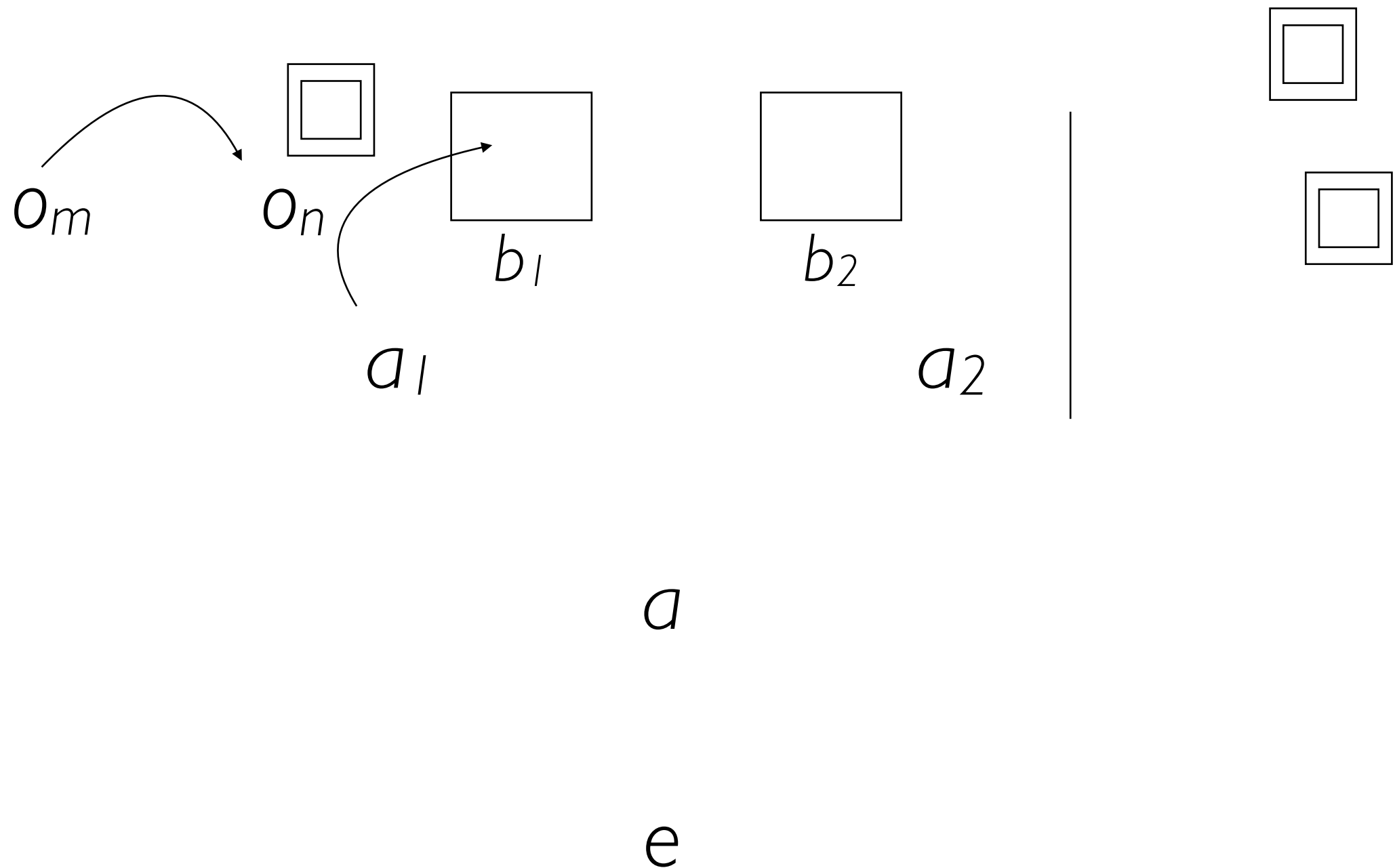
Framework for FBT^I_4

(nine timepoints)



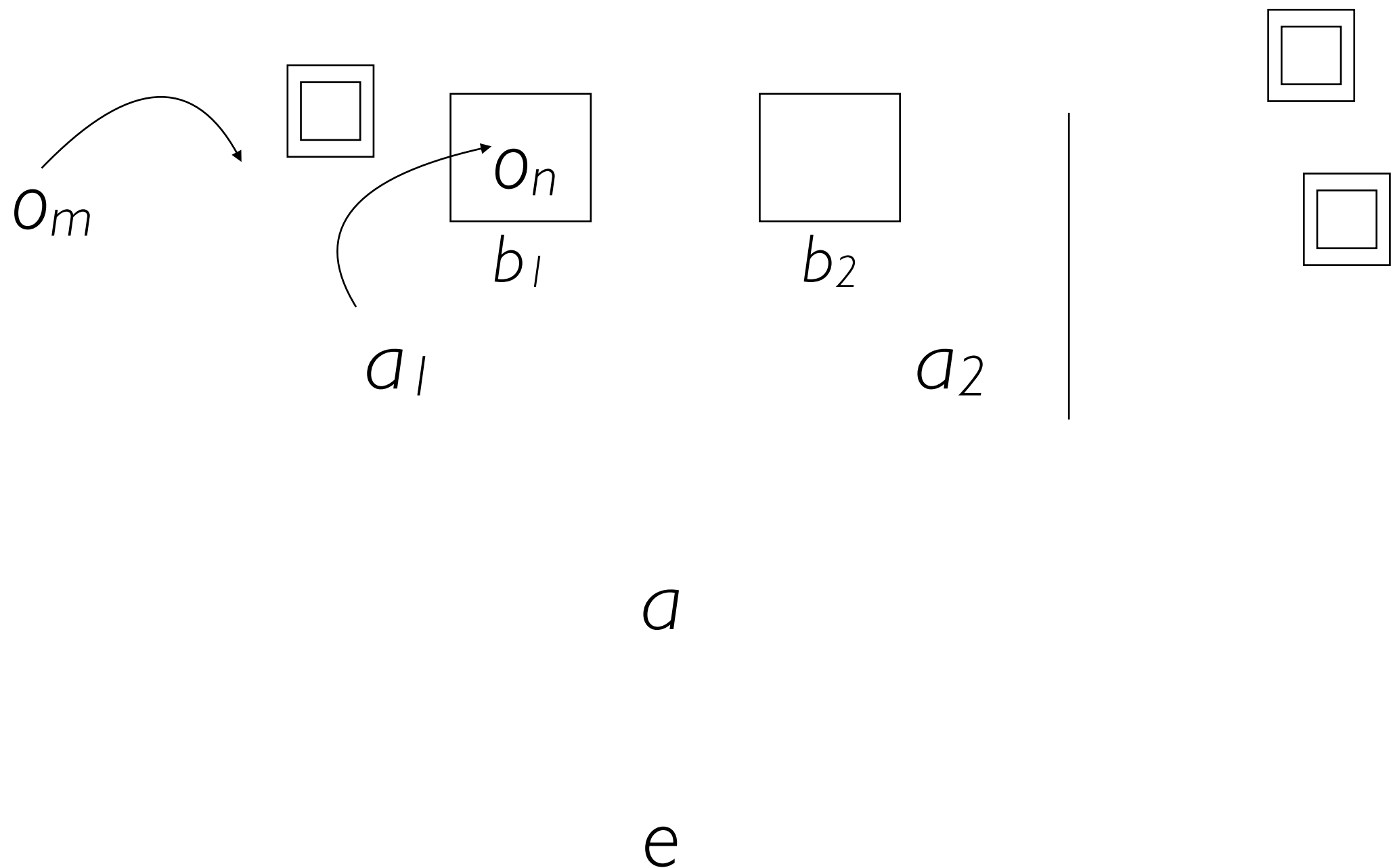
Framework for FBT^I_4

(nine timepoints)



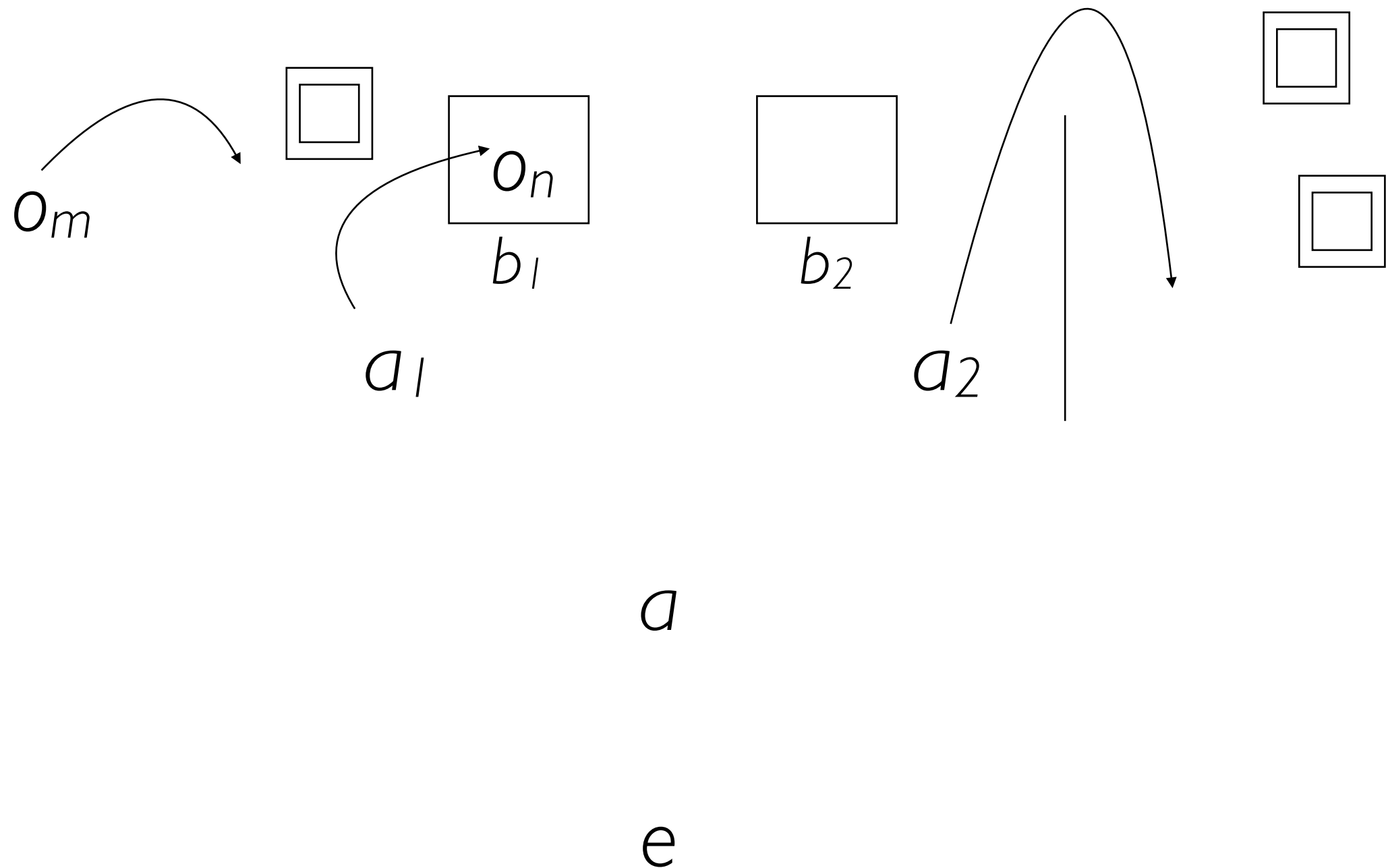
Framework for FBT^I_4

(nine timepoints)



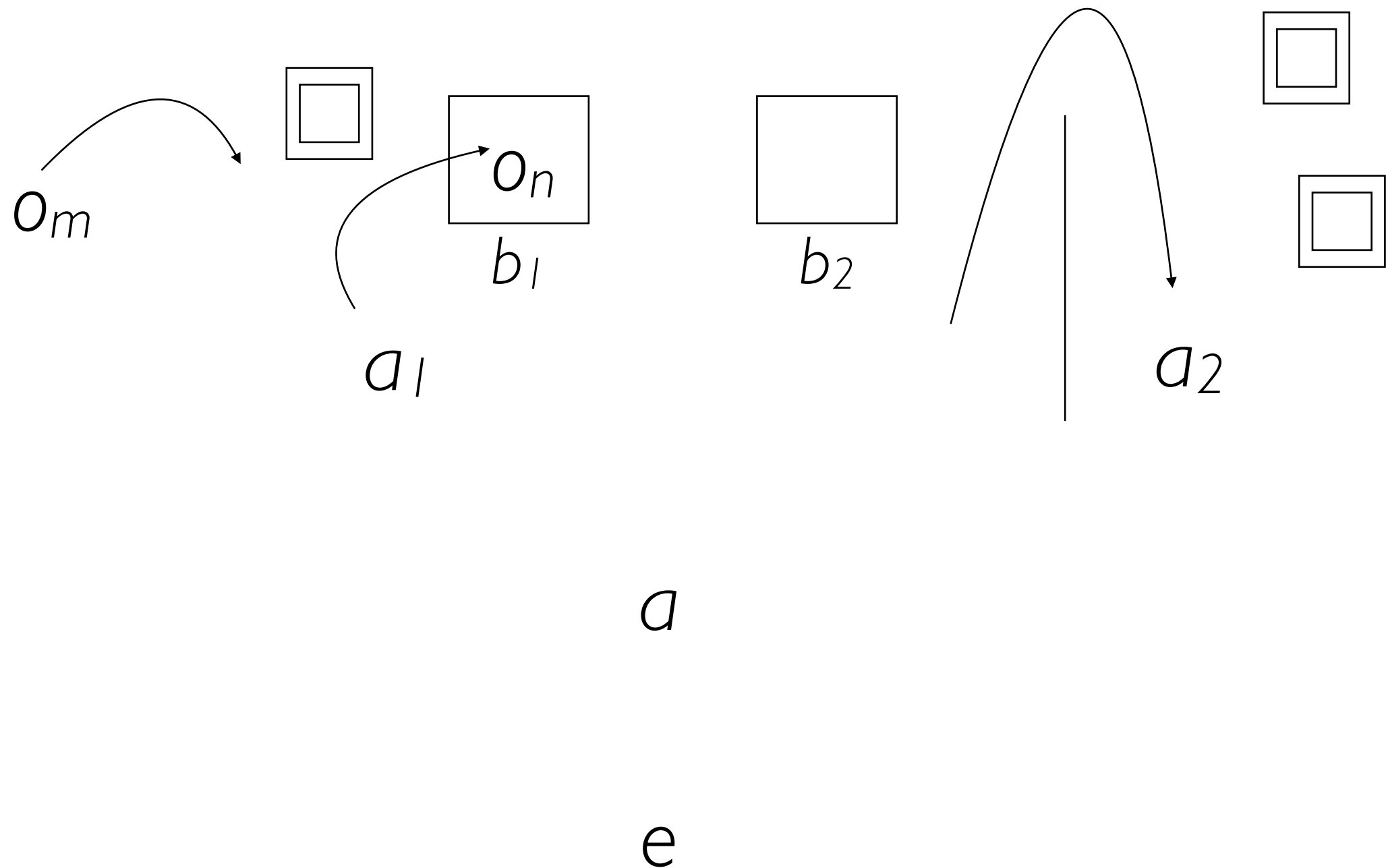
Framework for FBT^I_4

(nine timepoints)



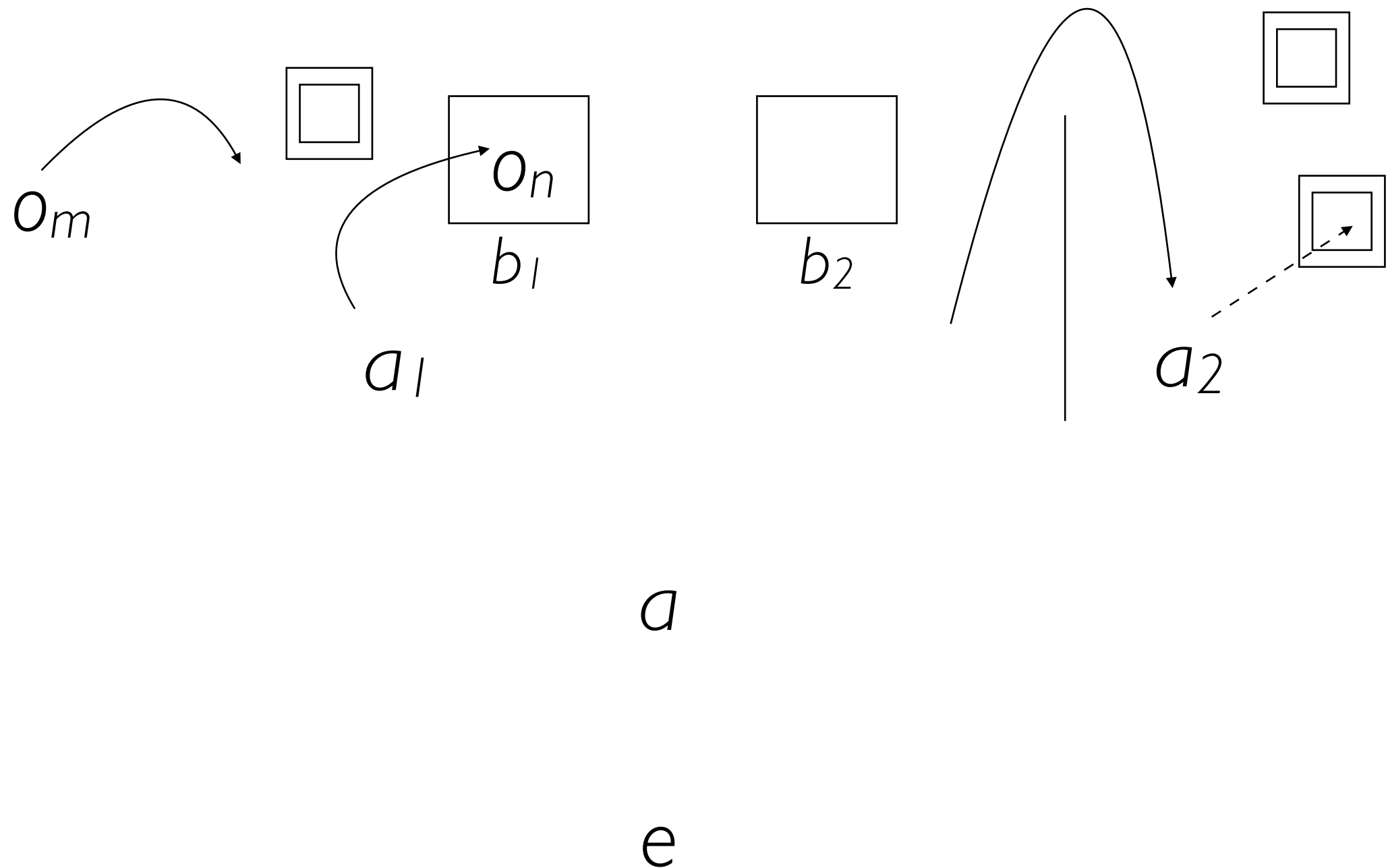
Framework for FBT^I_4

(nine timepoints)



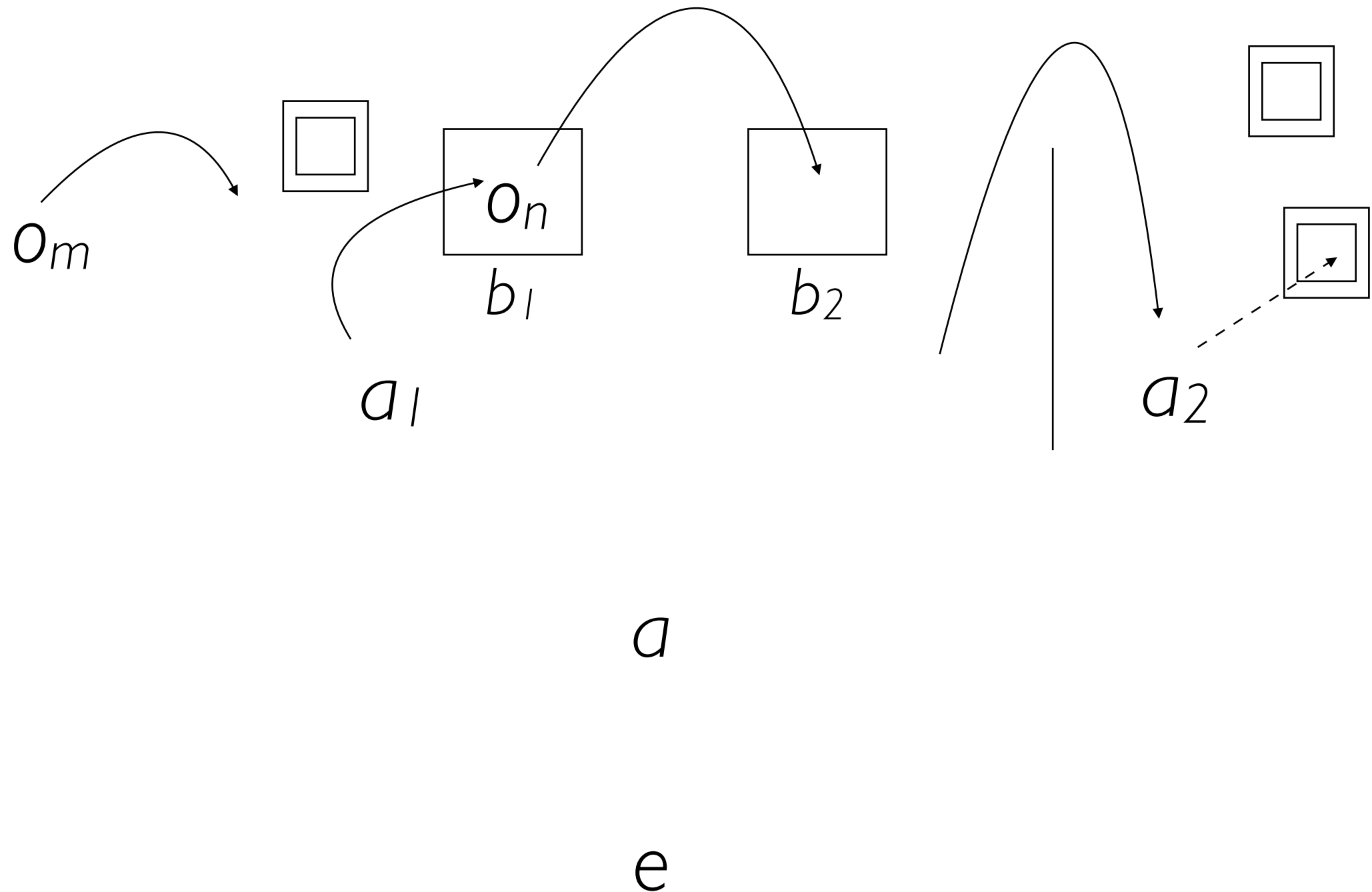
Framework for FBT^I_4

(nine timepoints)



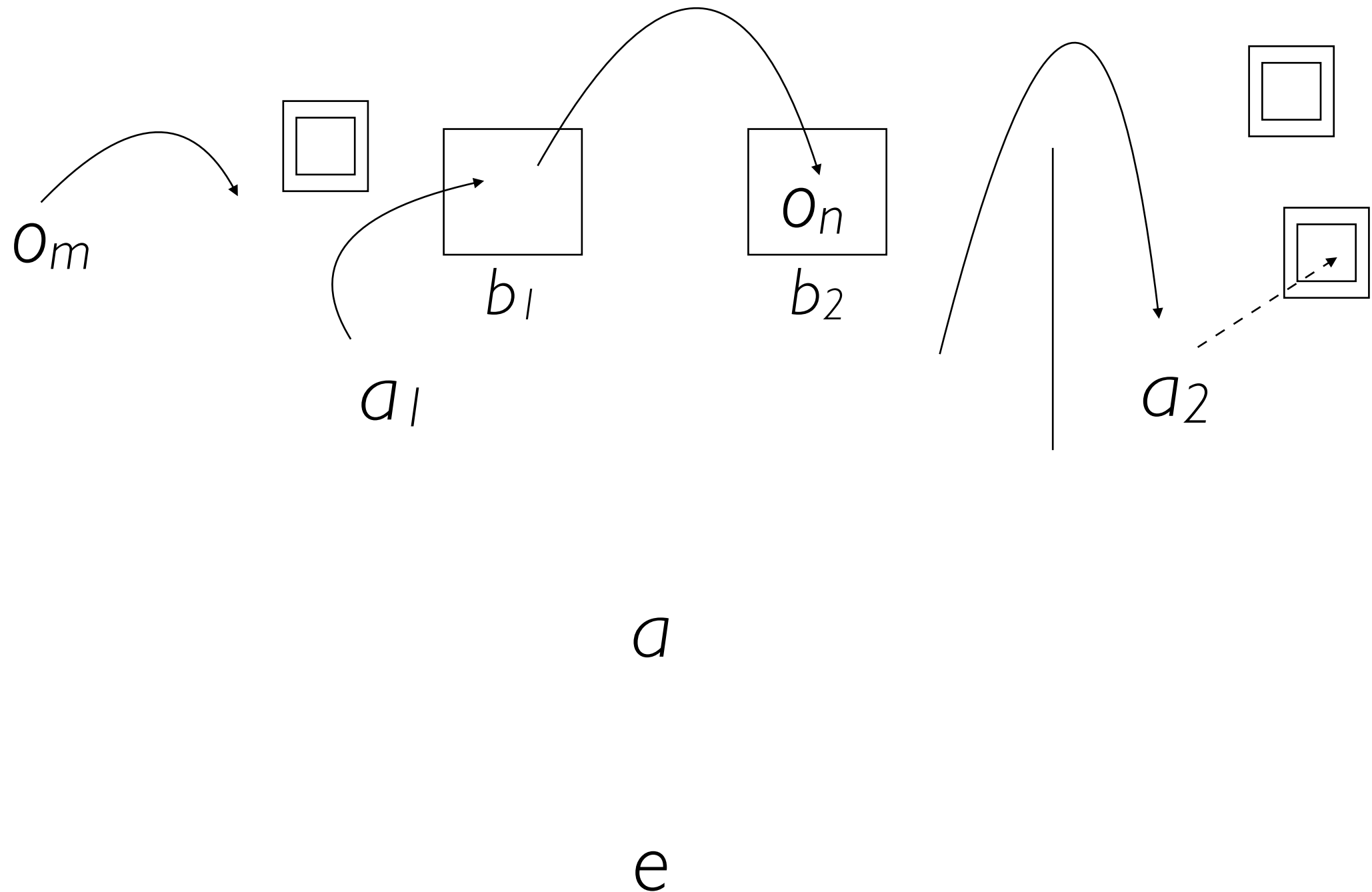
Framework for FBT^I_4

(nine timepoints)



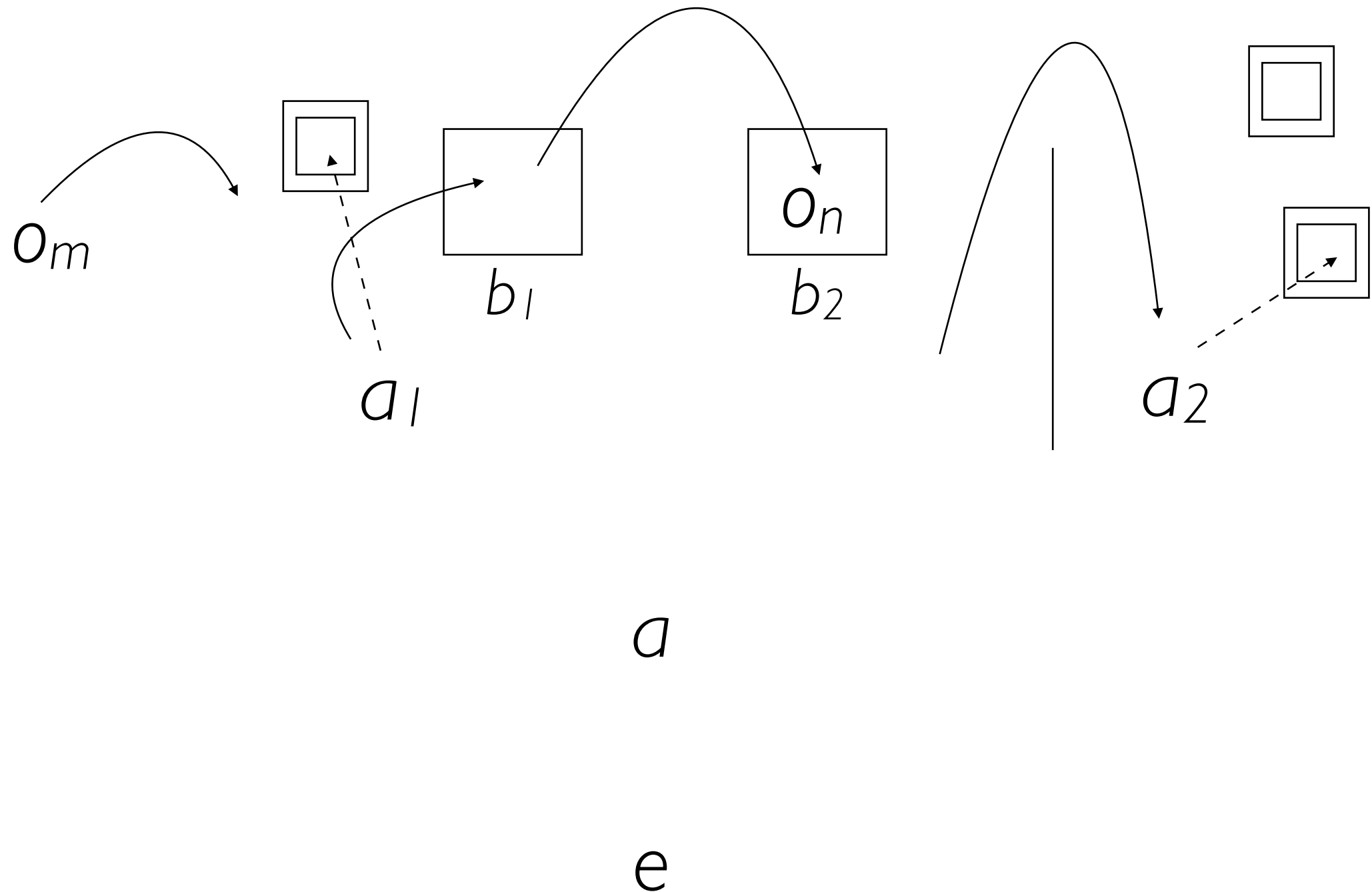
Framework for FBT^I_4

(nine timepoints)



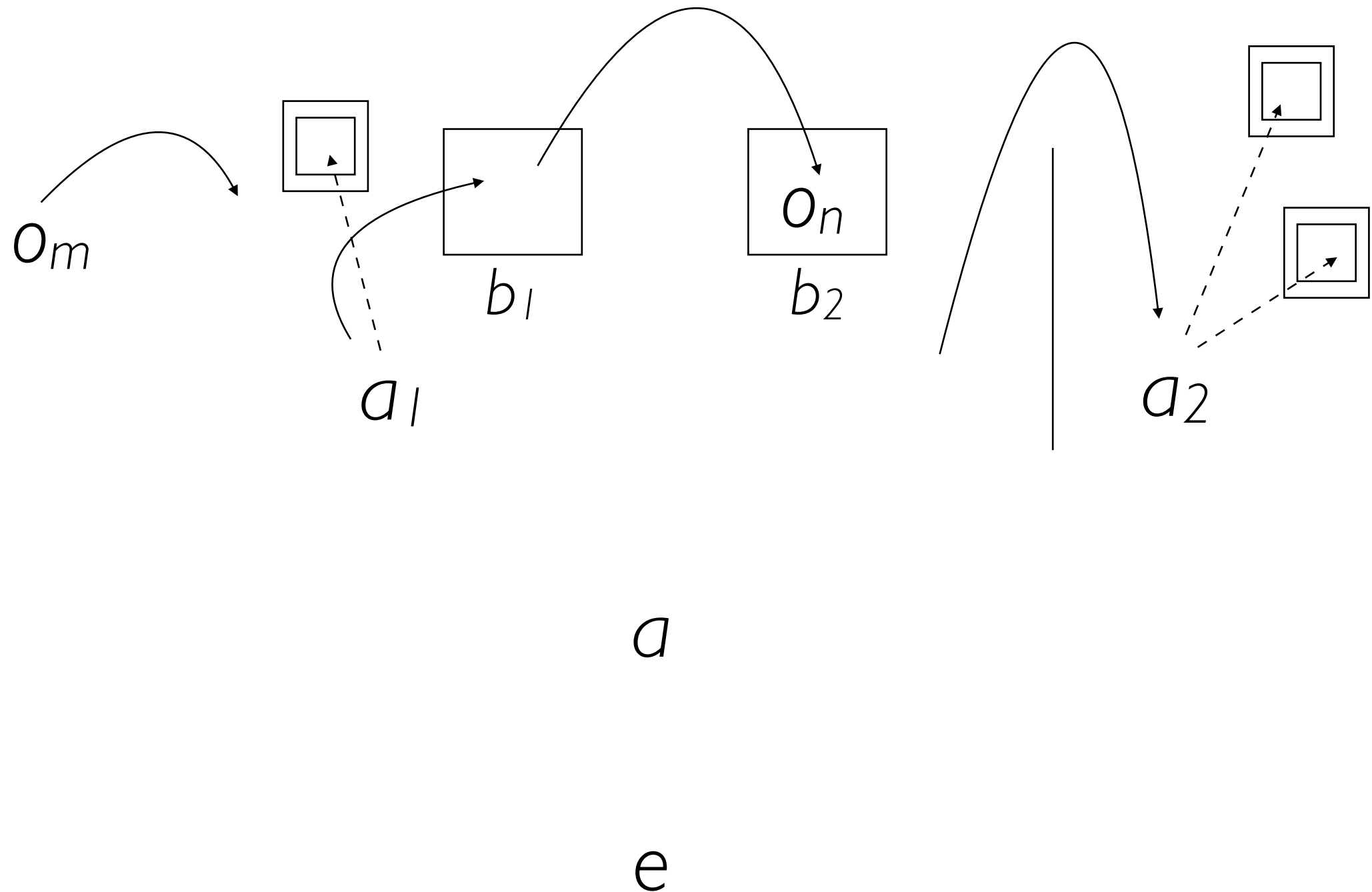
Framework for FBT^I_4

(nine timepoints)



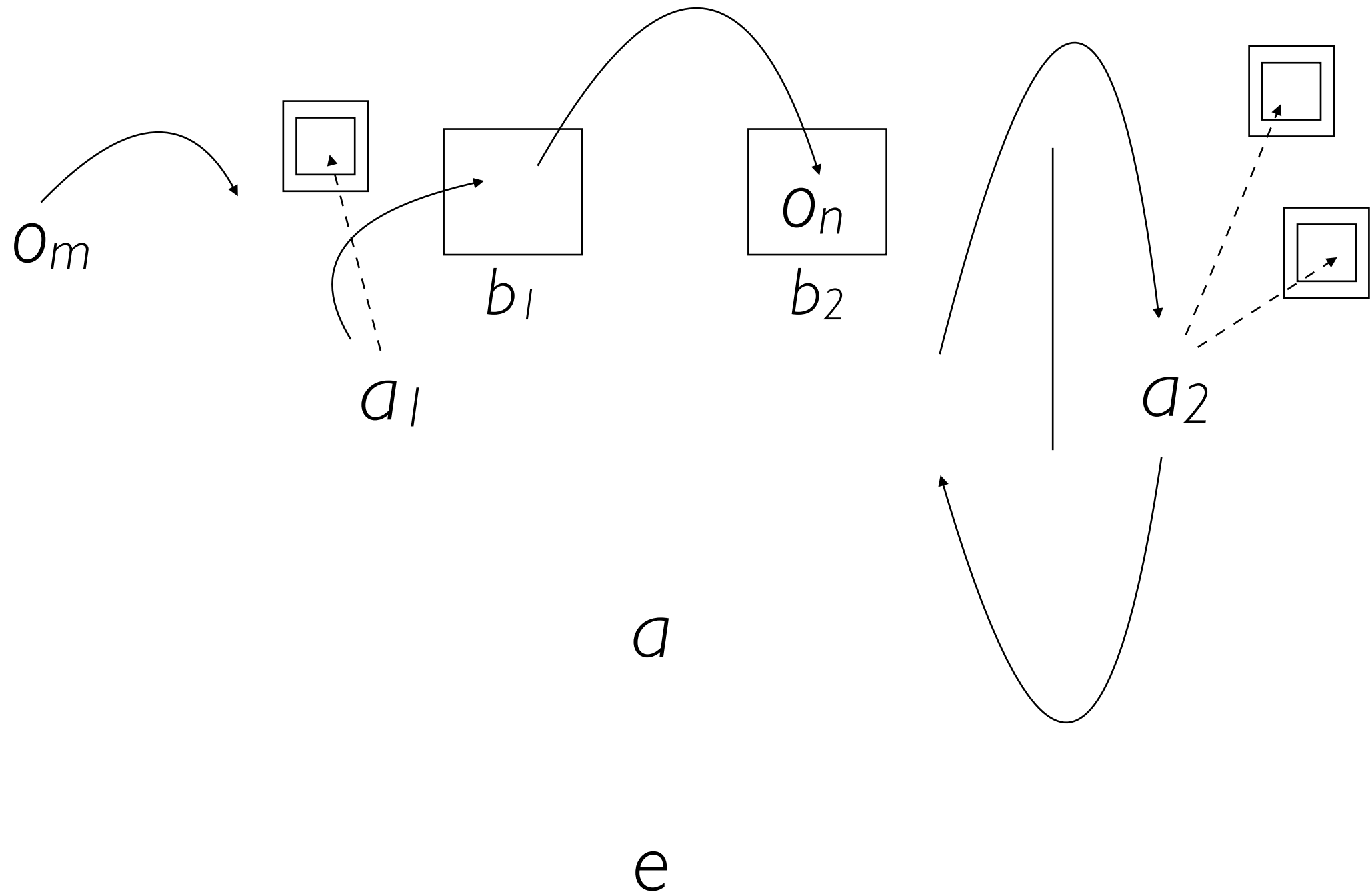
Framework for FBT^I_4

(nine timepoints)



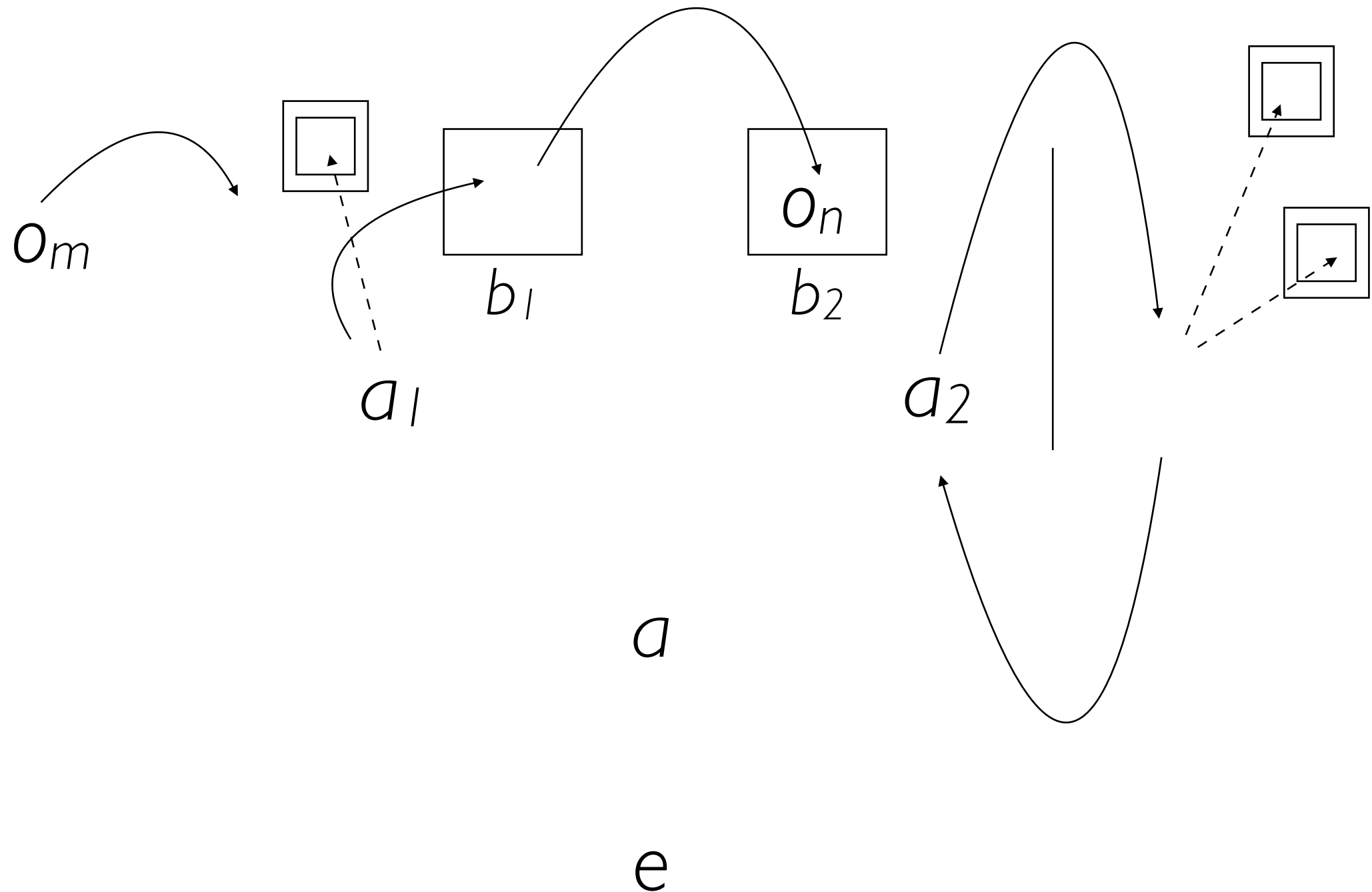
Framework for FBT^I_4

(nine timepoints)



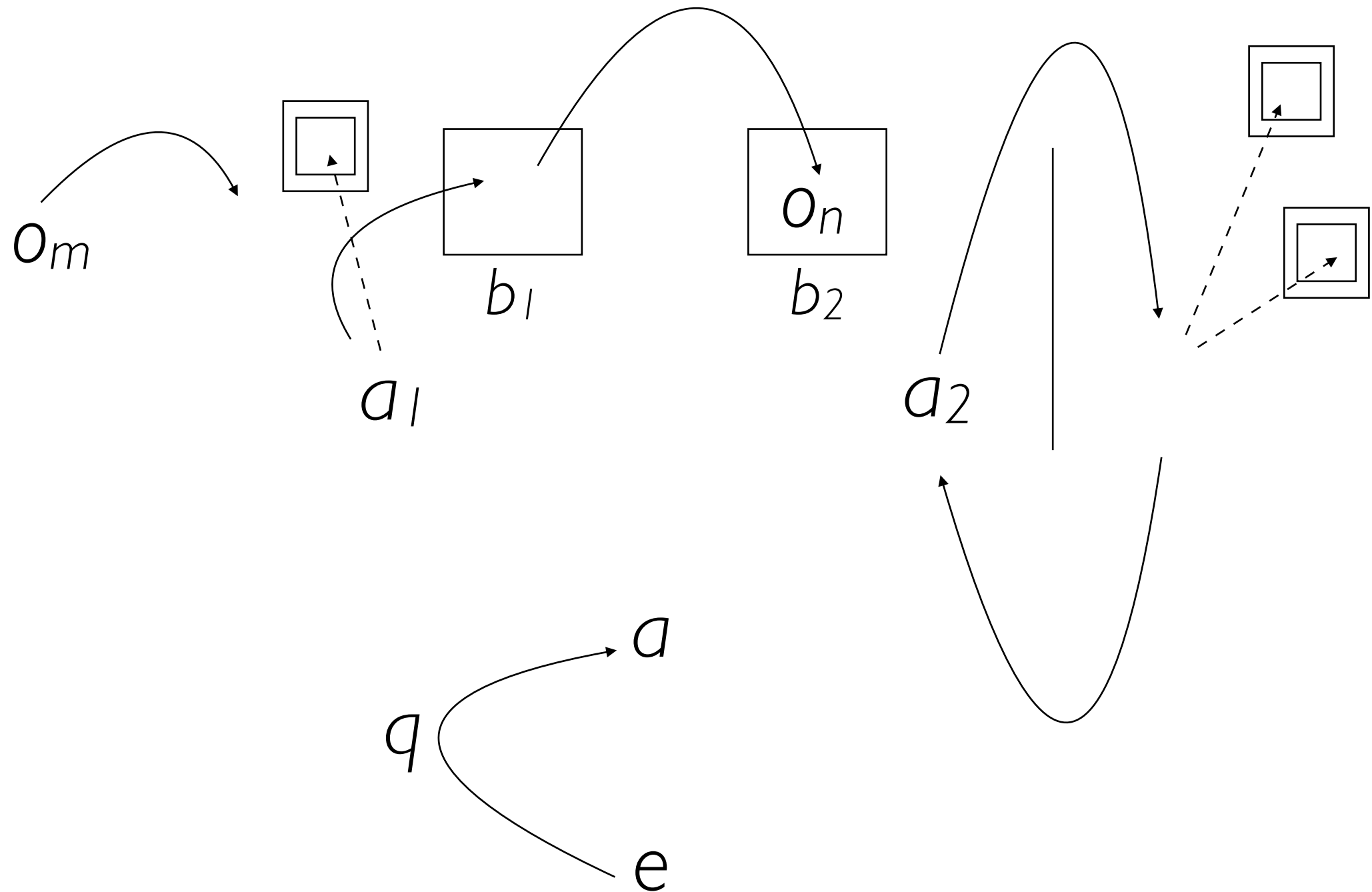
Framework for FBT^I_4

(nine timepoints)



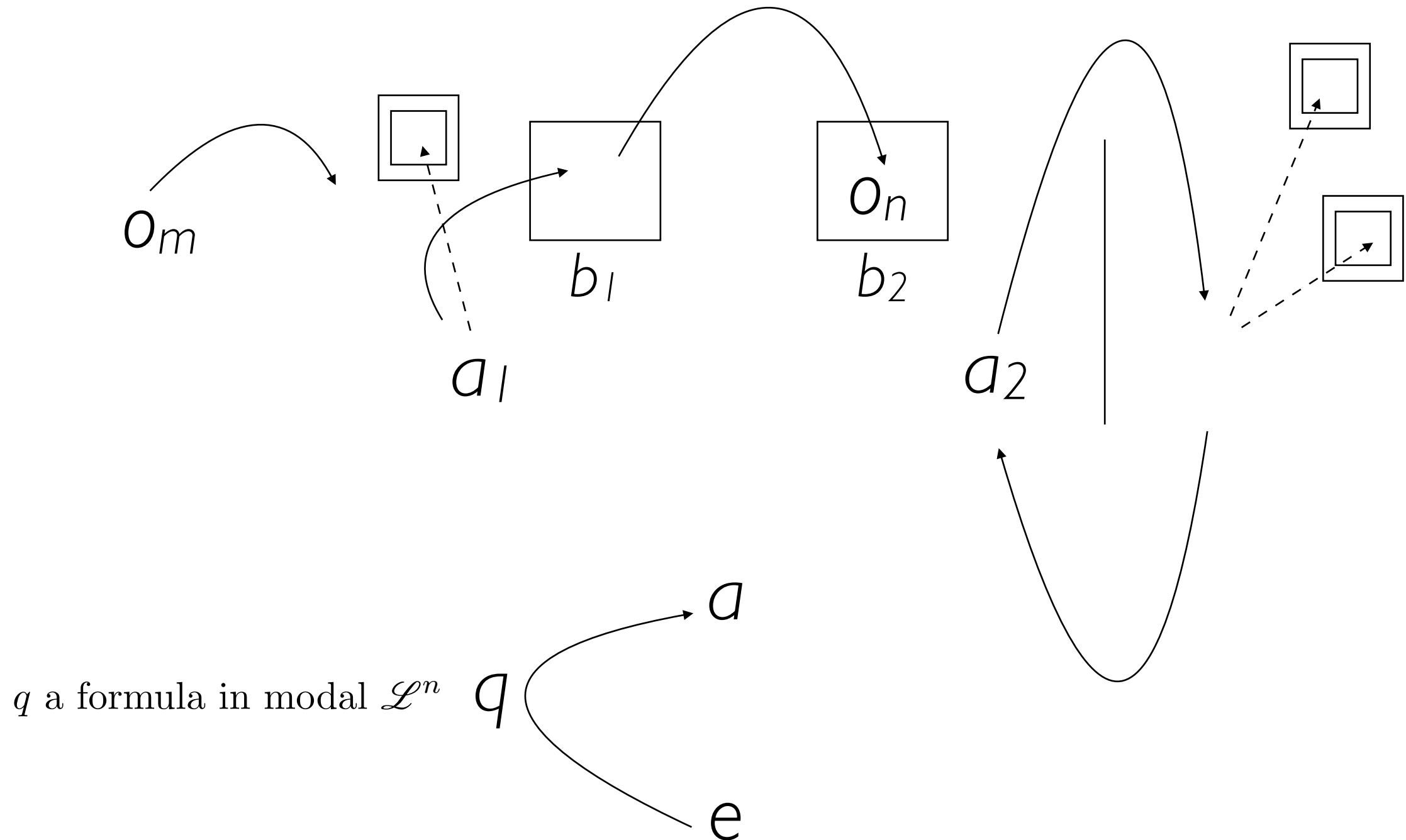
Framework for FBT^I_4

(nine timepoints)

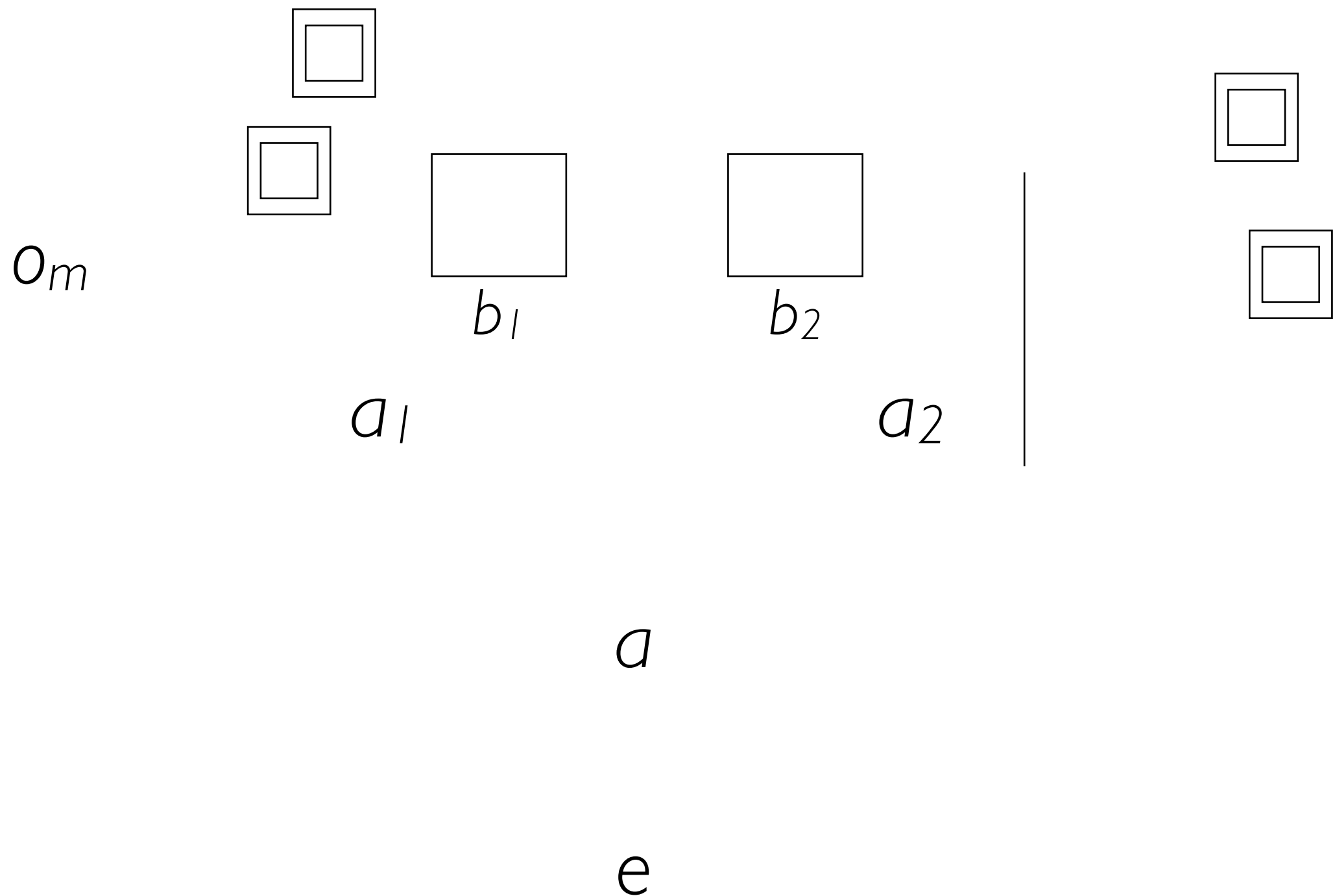


Framework for FBT^I_4

(nine timepoints)

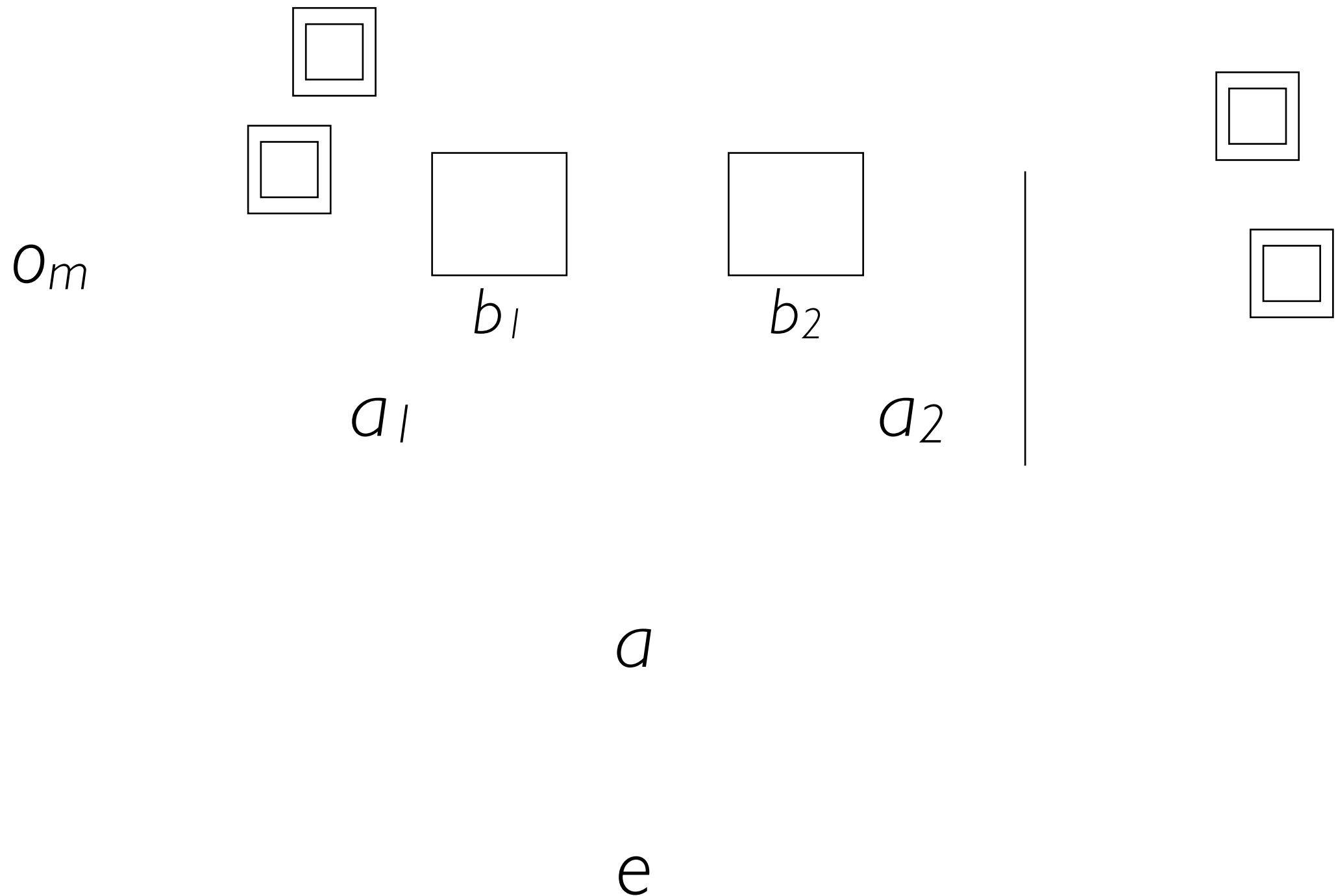


Framework for FBT^I_5



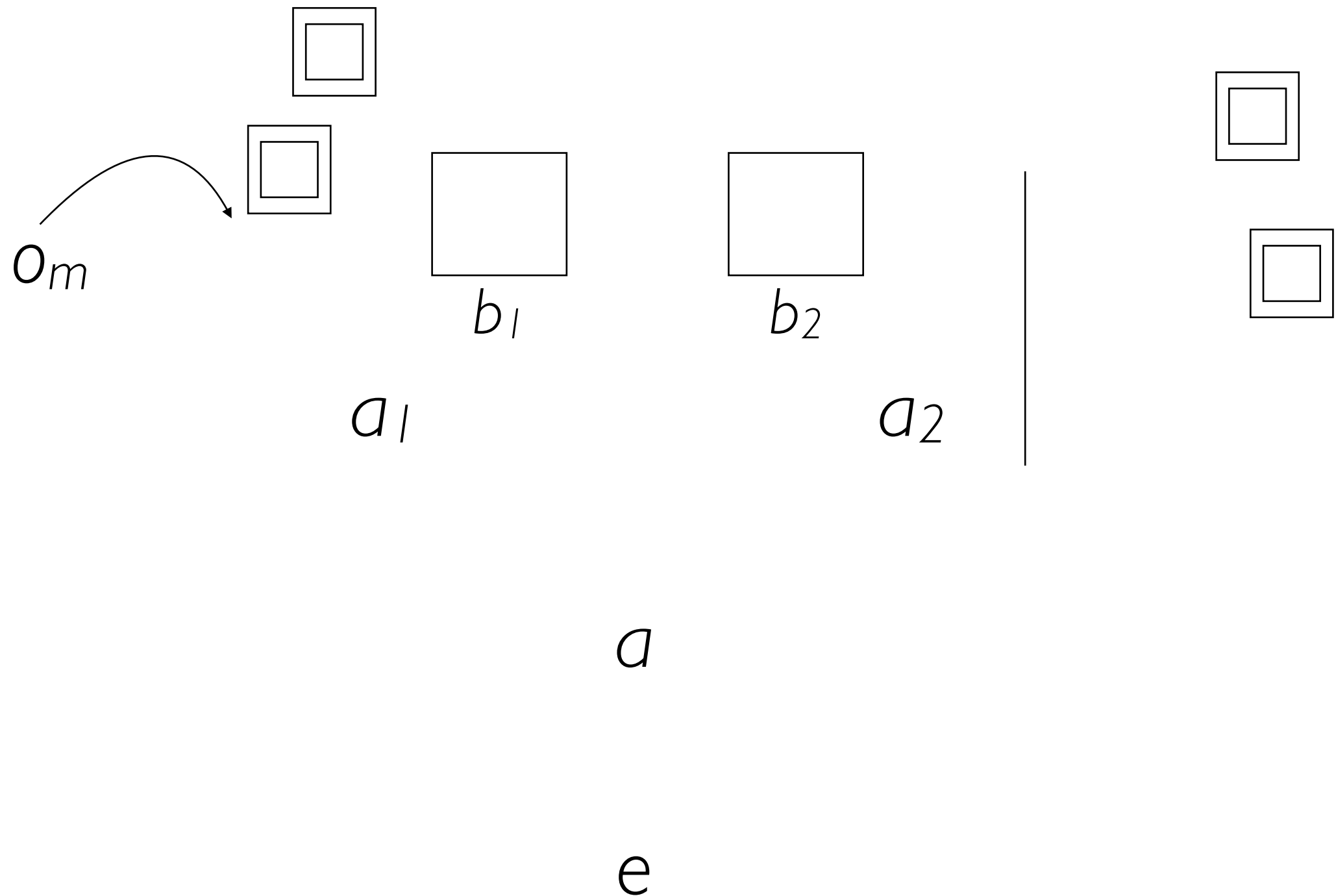
Framework for FBT^I_5

(ten timepoints)



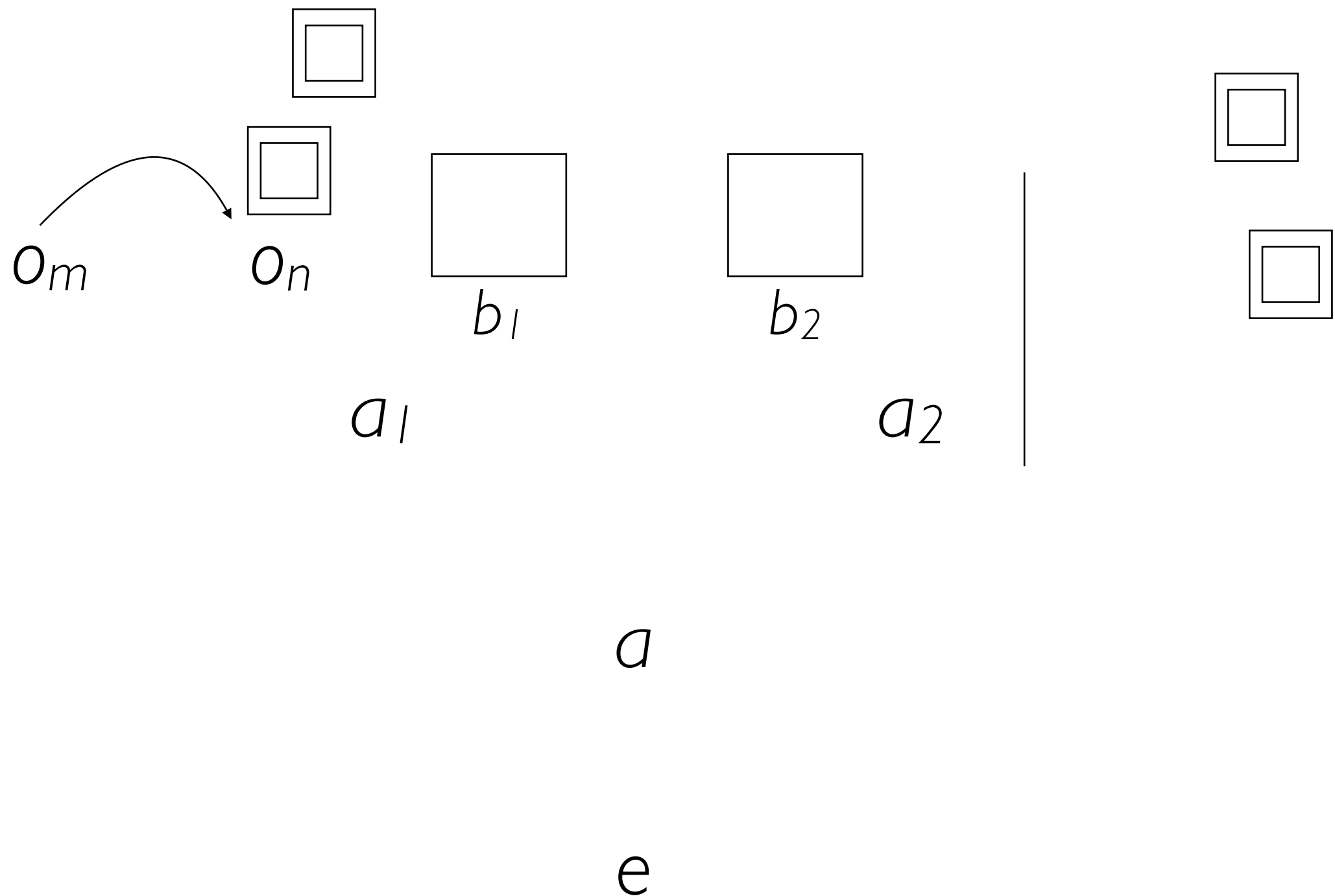
Framework for FBT^I_5

(ten timepoints)



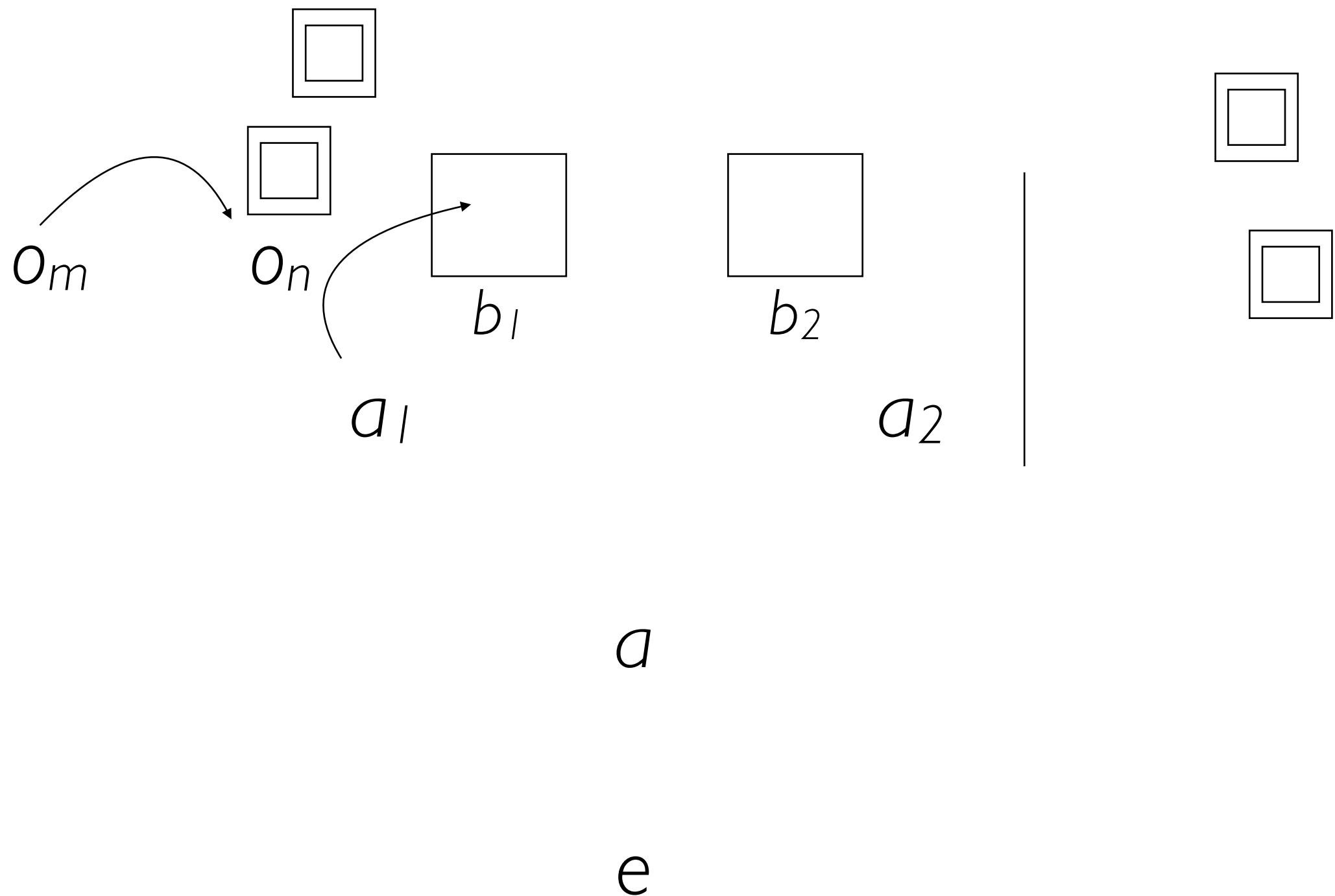
Framework for FBT^I_5

(ten timepoints)



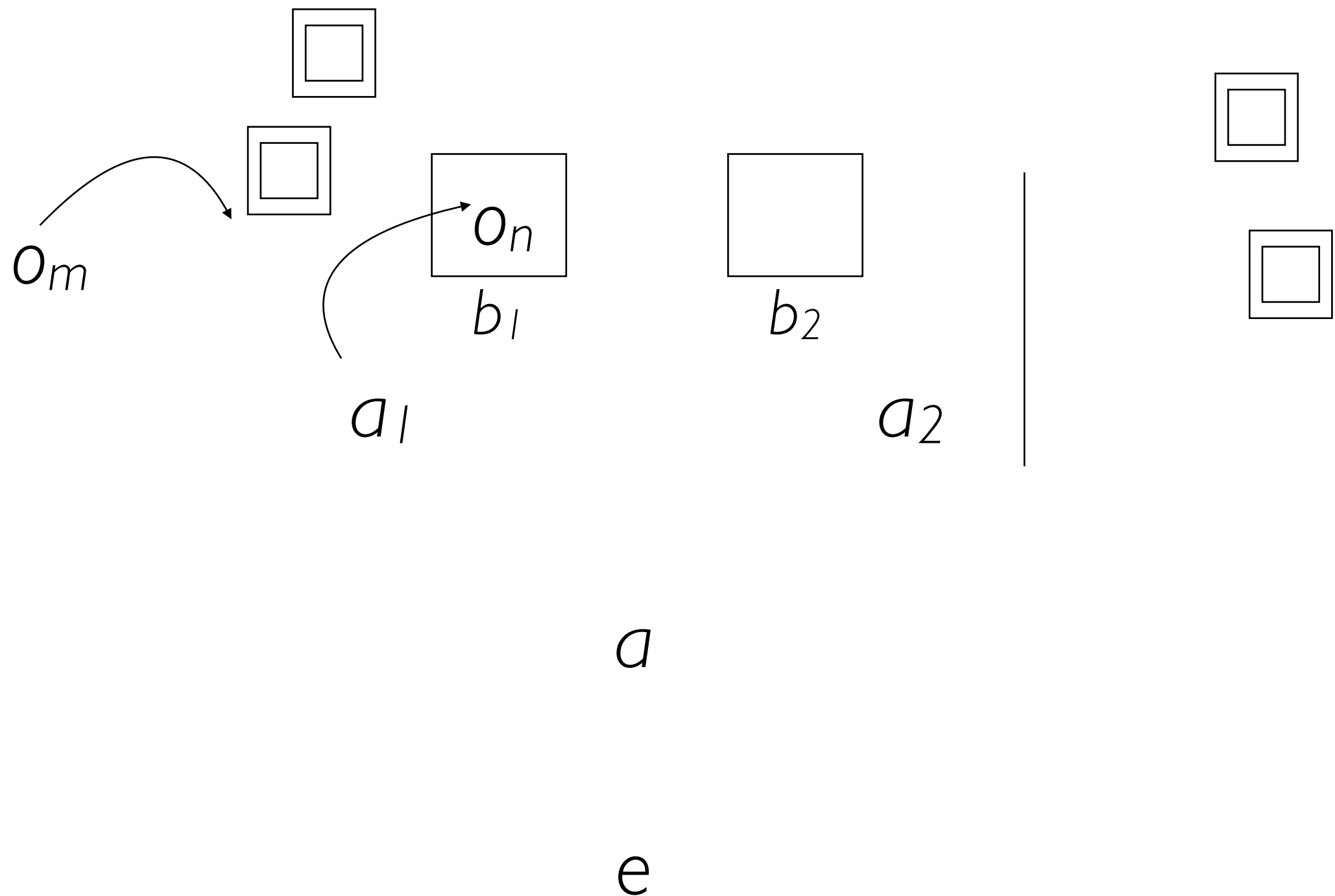
Framework for FBT^I_5

(ten timepoints)



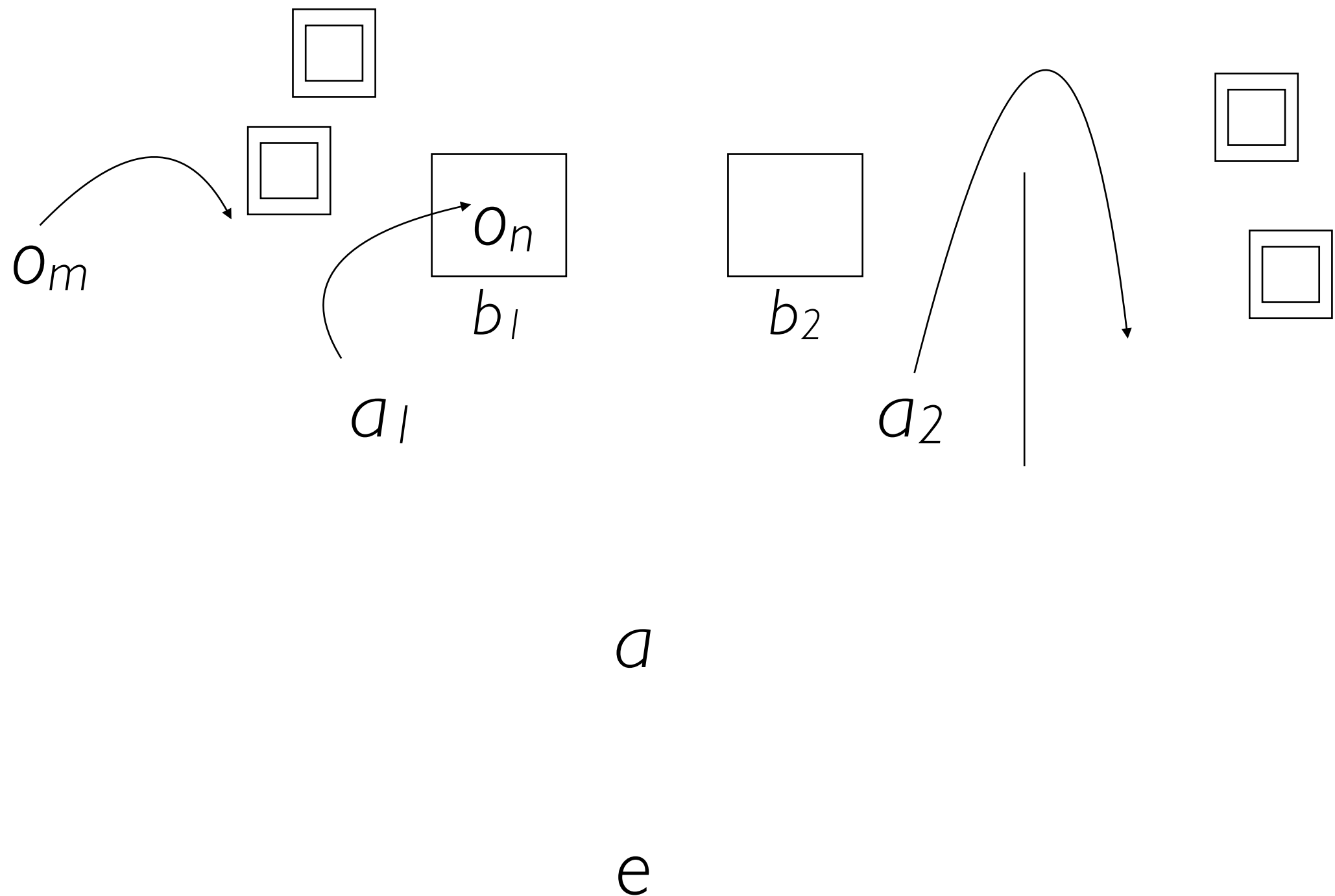
Framework for FBT^I_5

(ten timepoints)



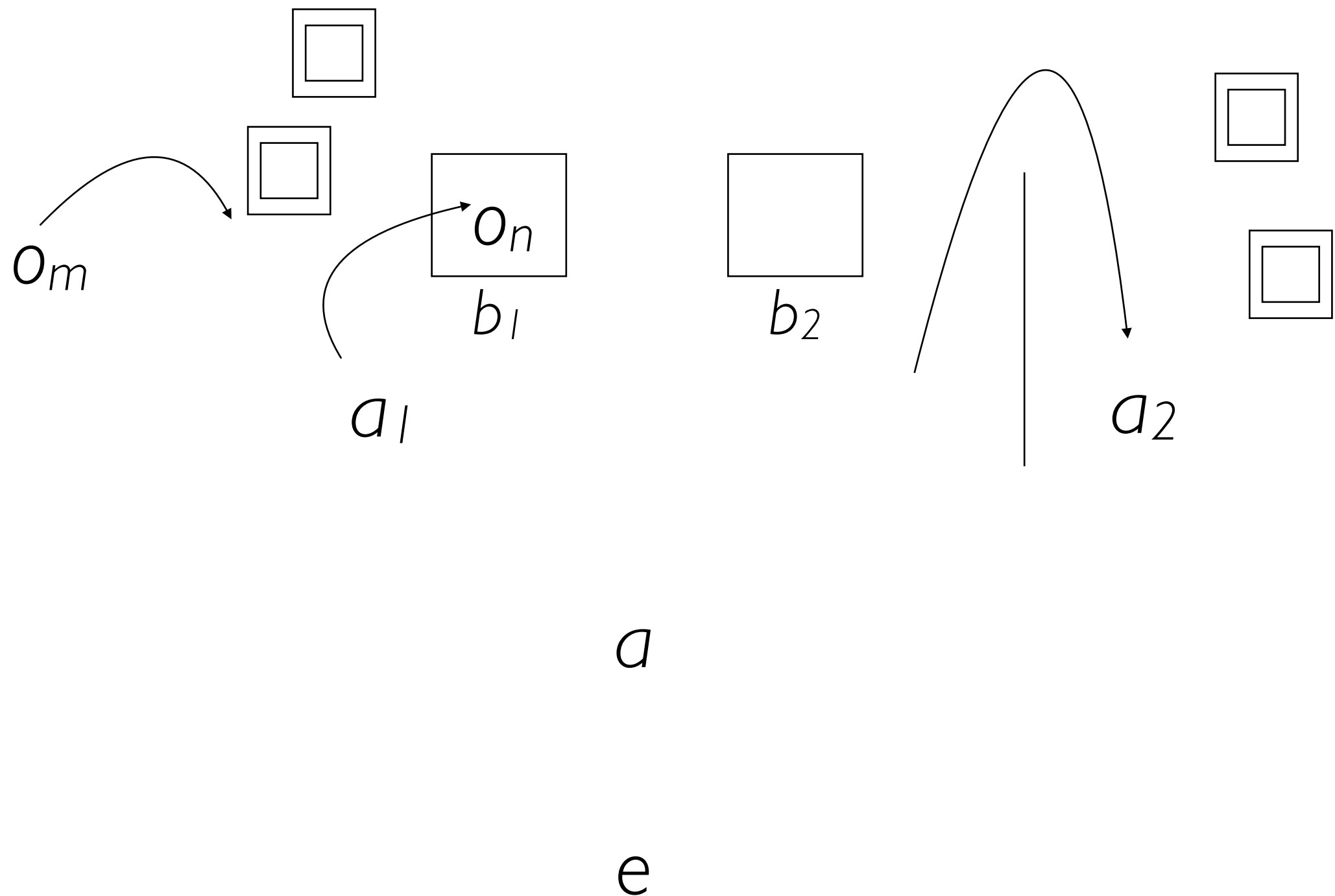
Framework for FBT^I_5

(ten timepoints)



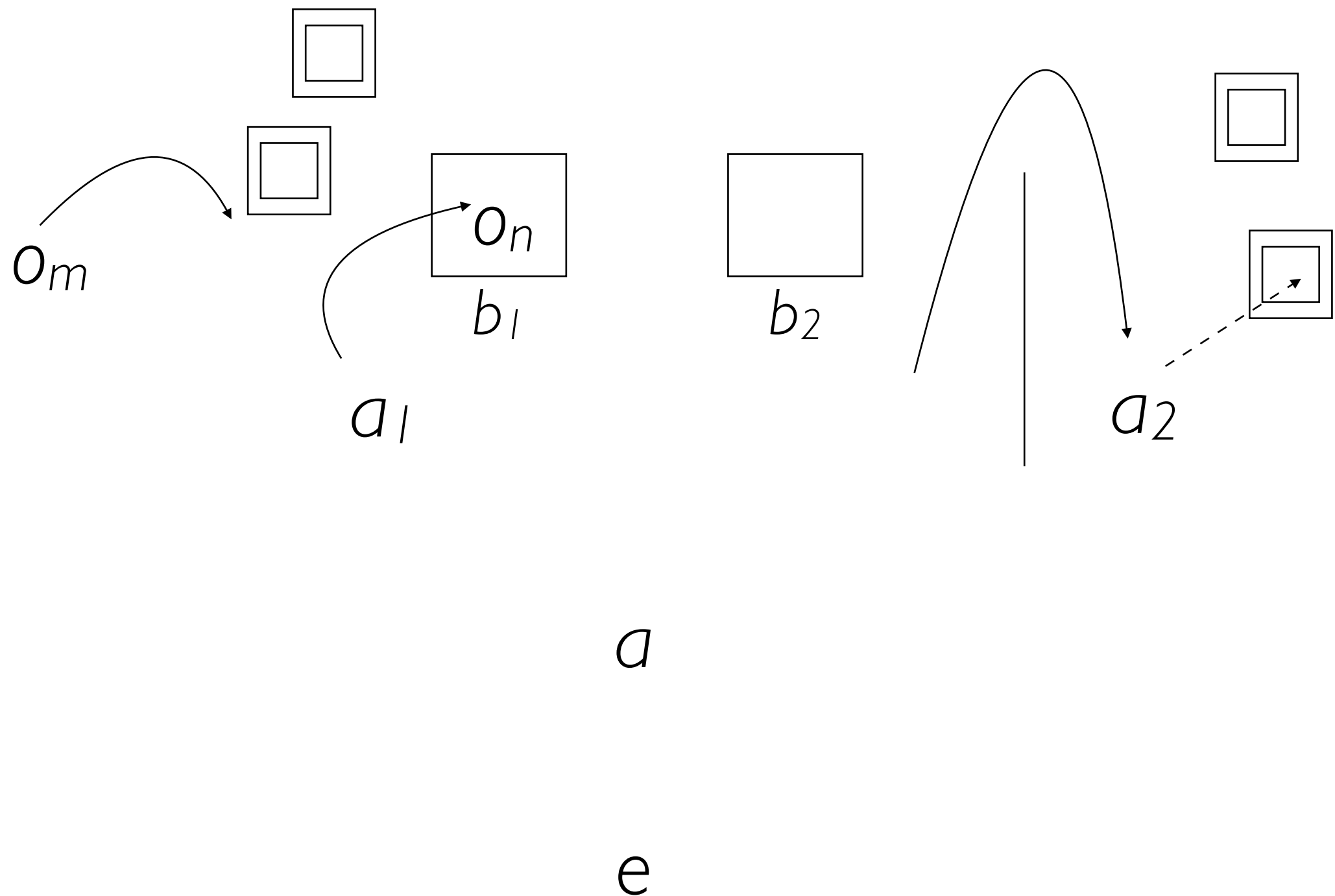
Framework for FBT^I_5

(ten timepoints)



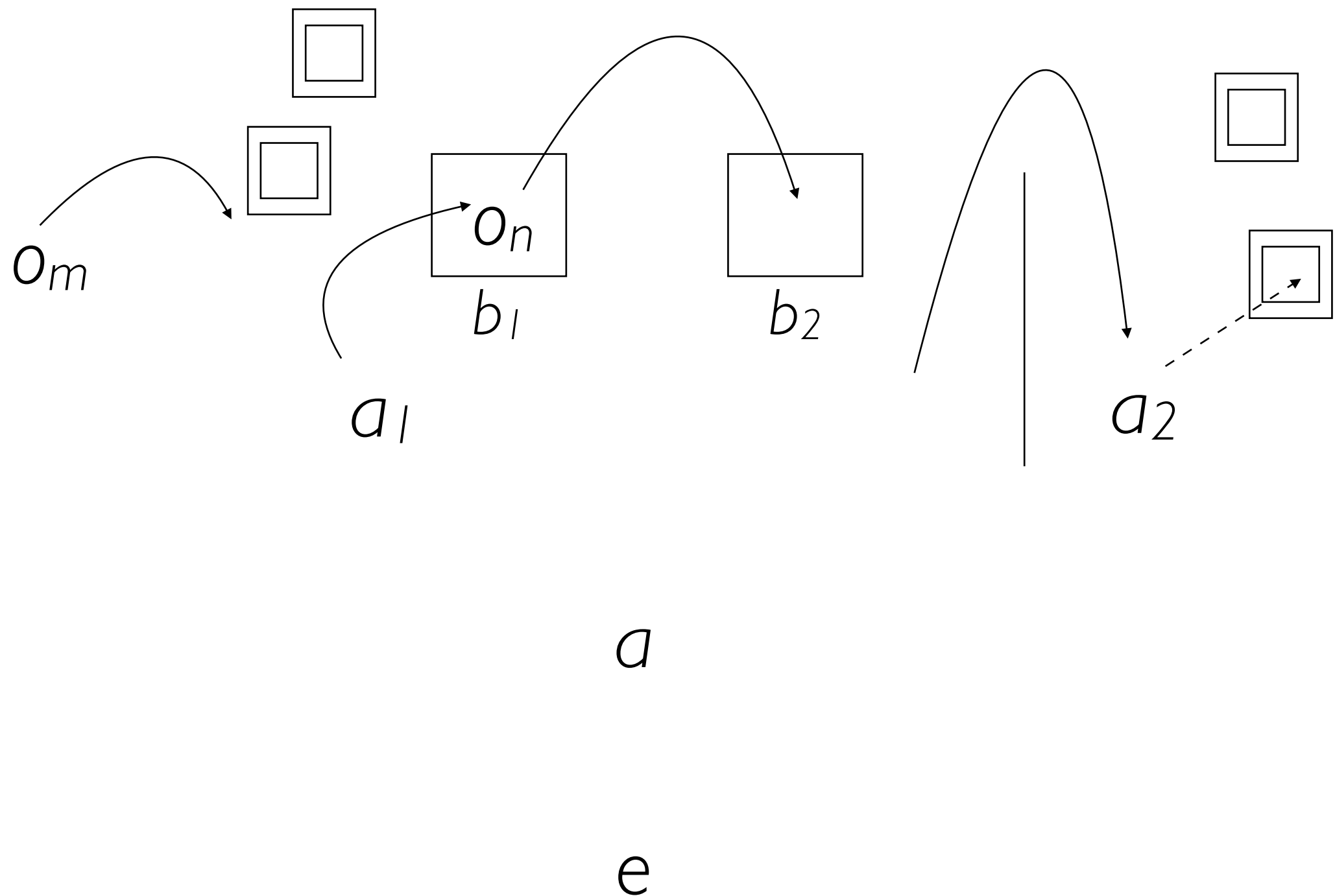
Framework for FBT^I_5

(ten timepoints)



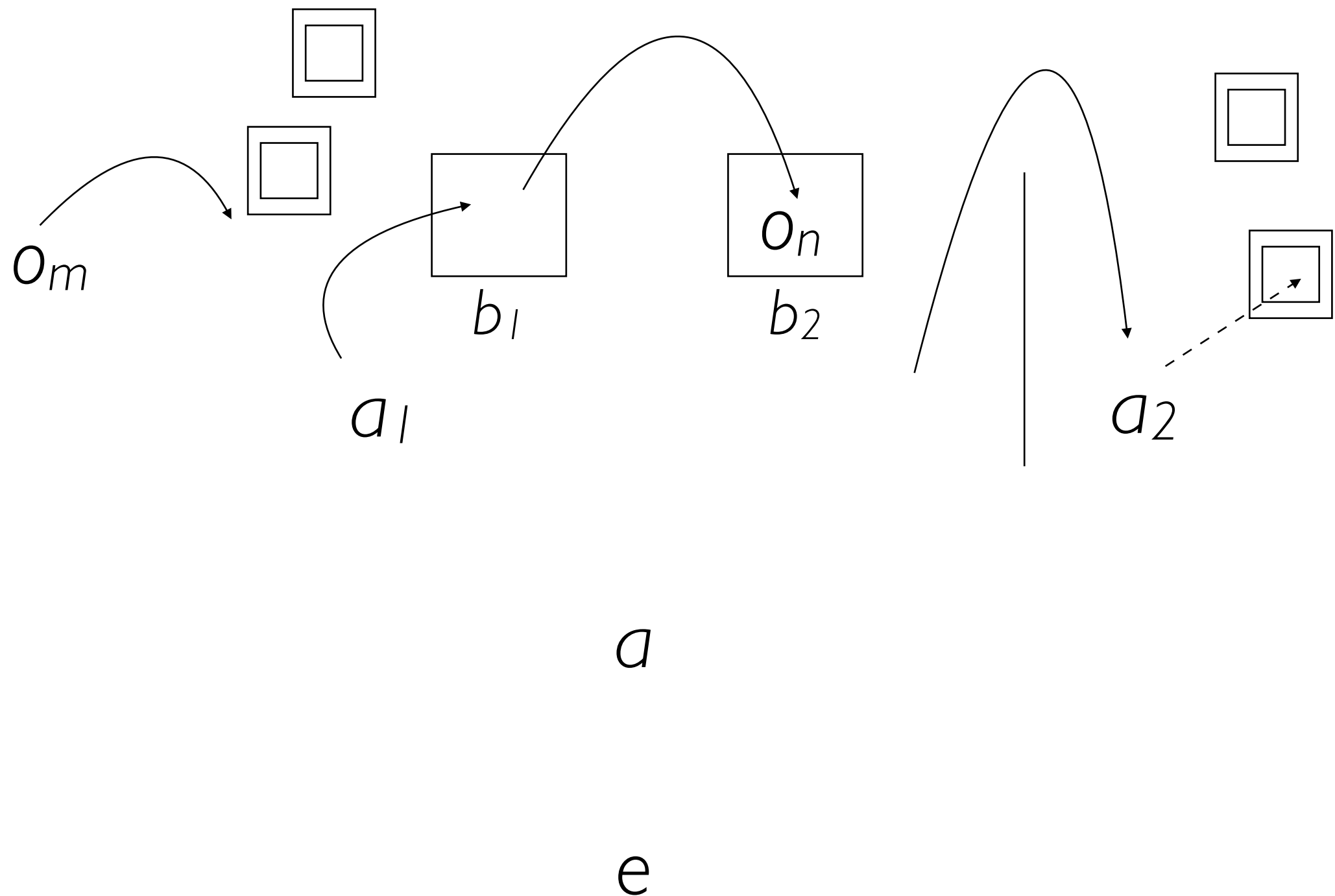
Framework for FBT^I_5

(ten timepoints)



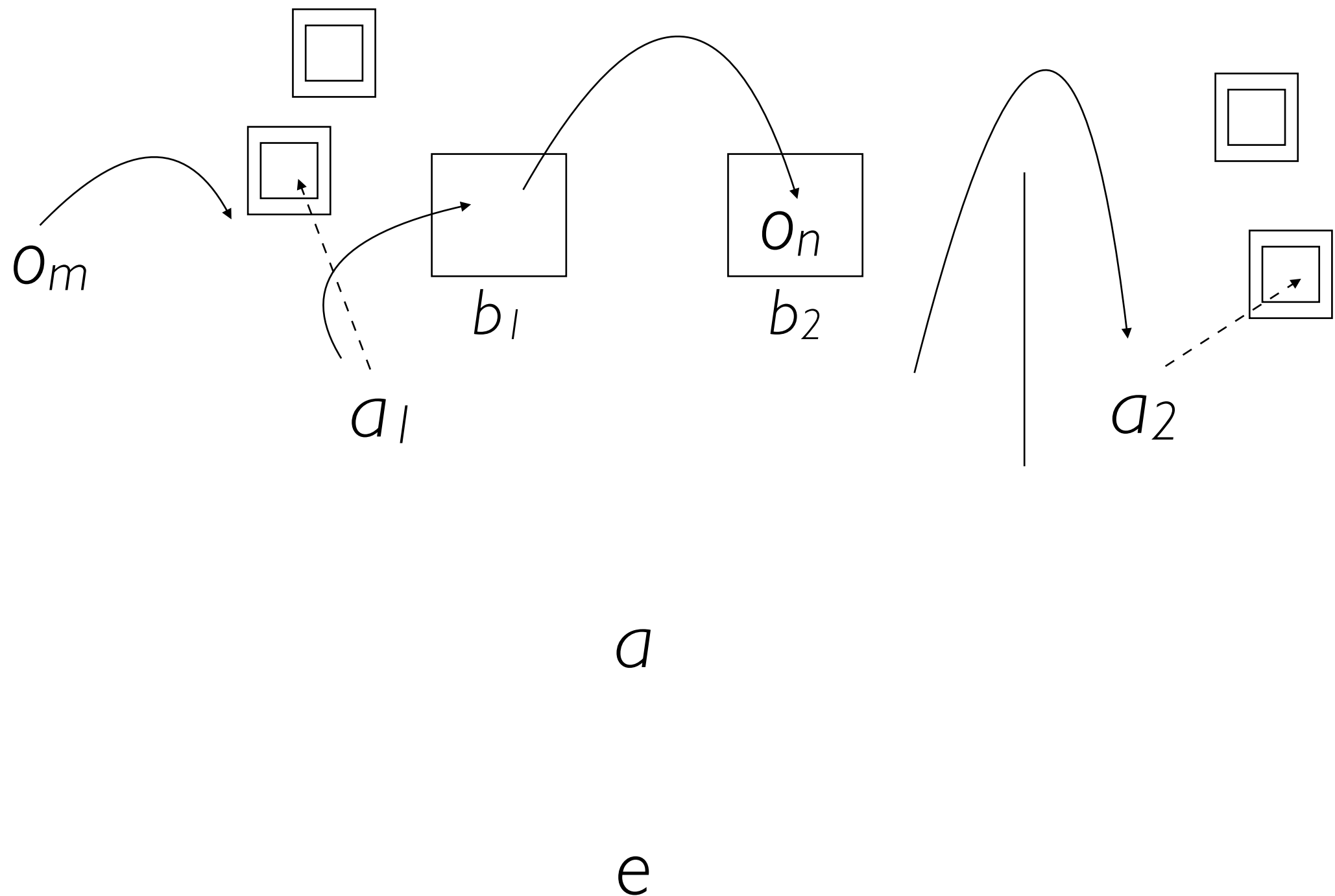
Framework for FBT^I_5

(ten timepoints)



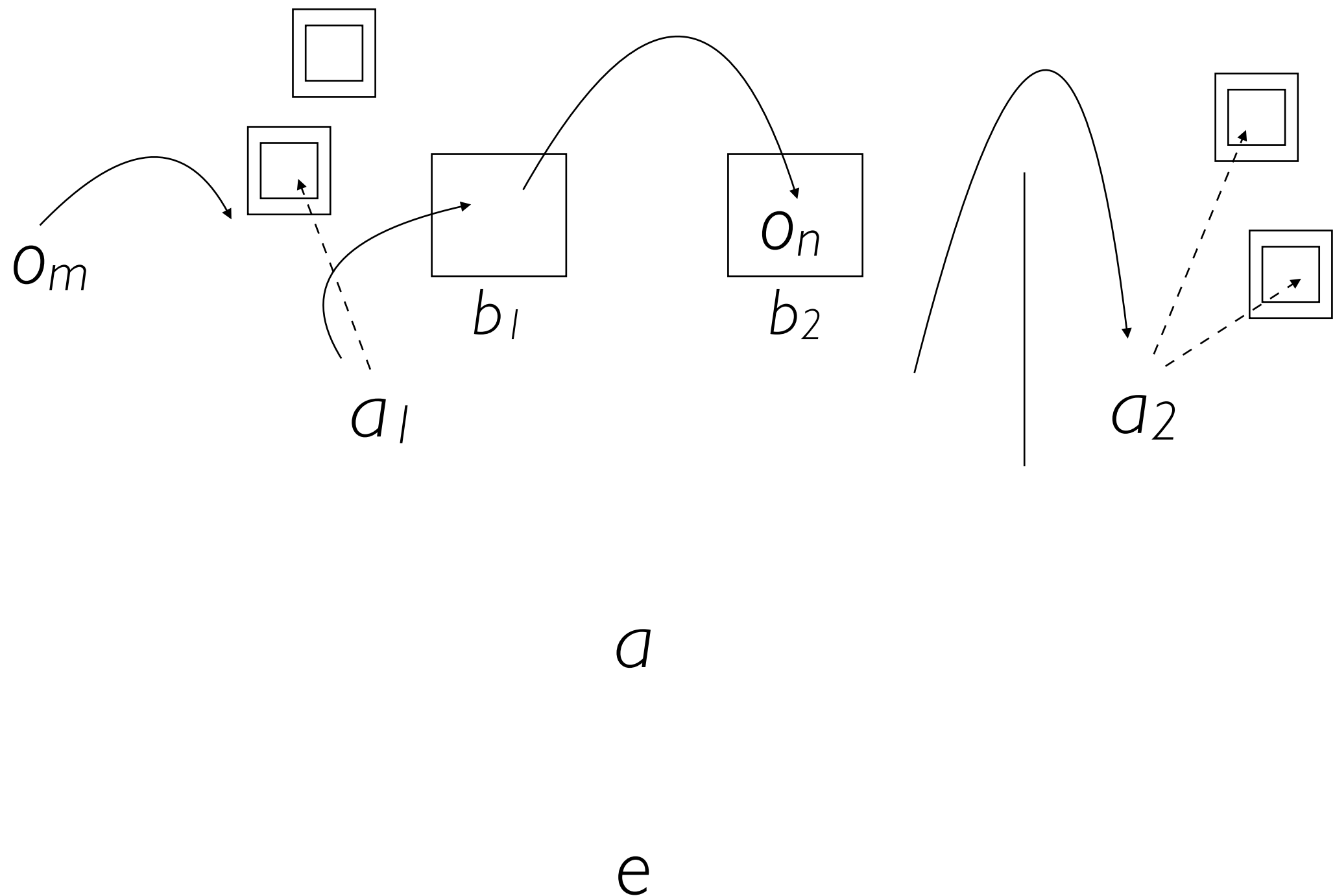
Framework for FBT^I_5

(ten timepoints)



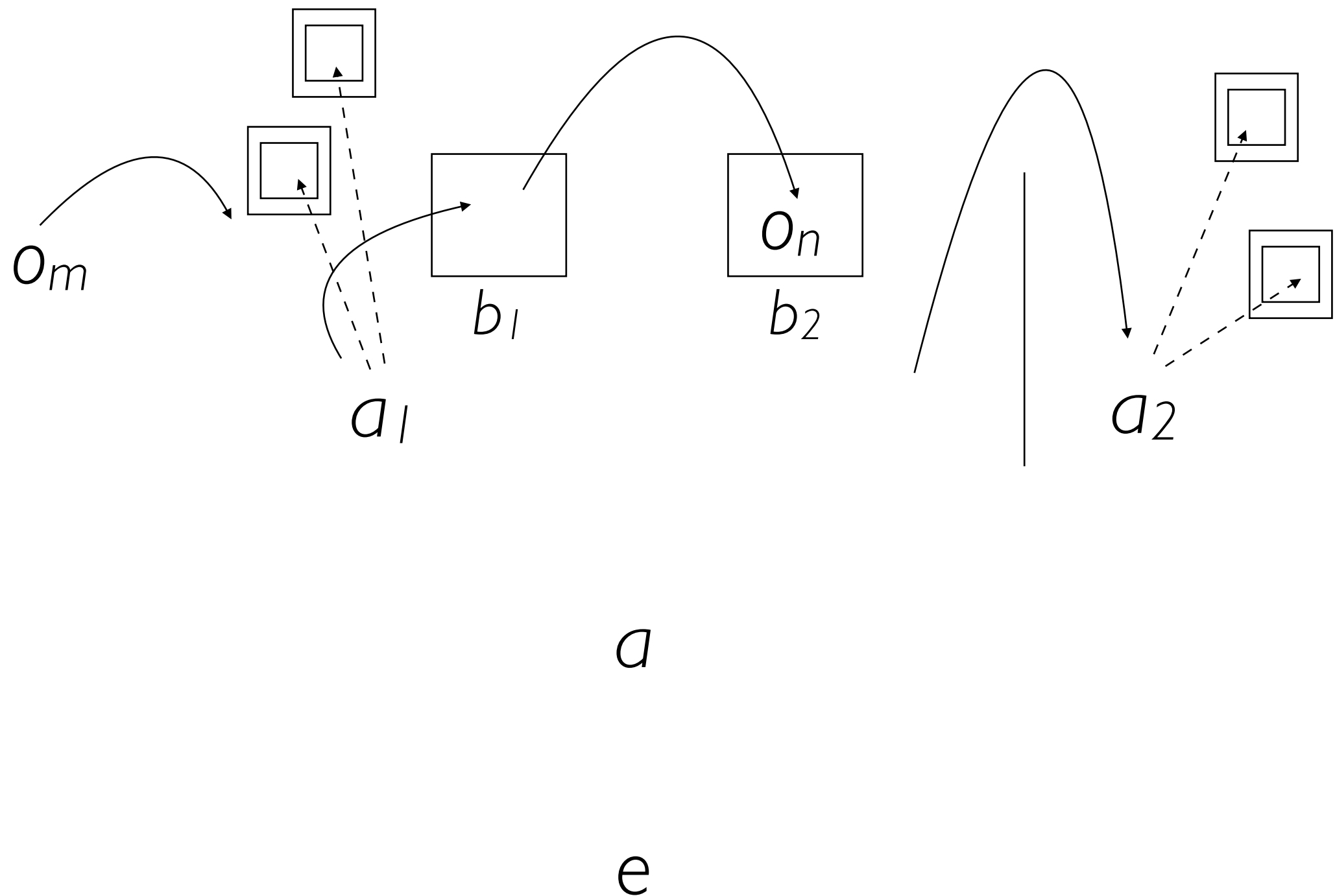
Framework for FBT^I_5

(ten timepoints)



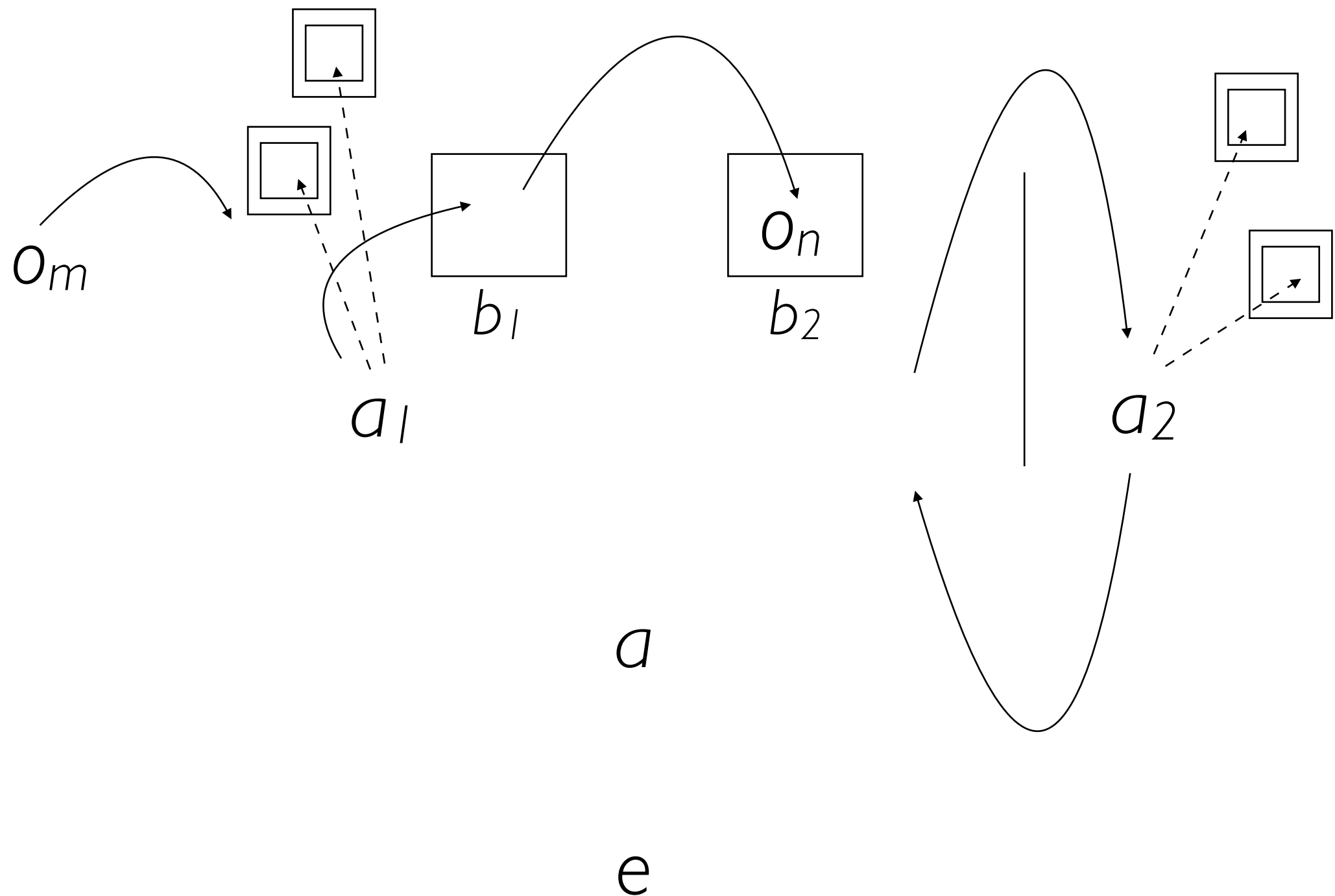
Framework for FBT^I_5

(ten timepoints)



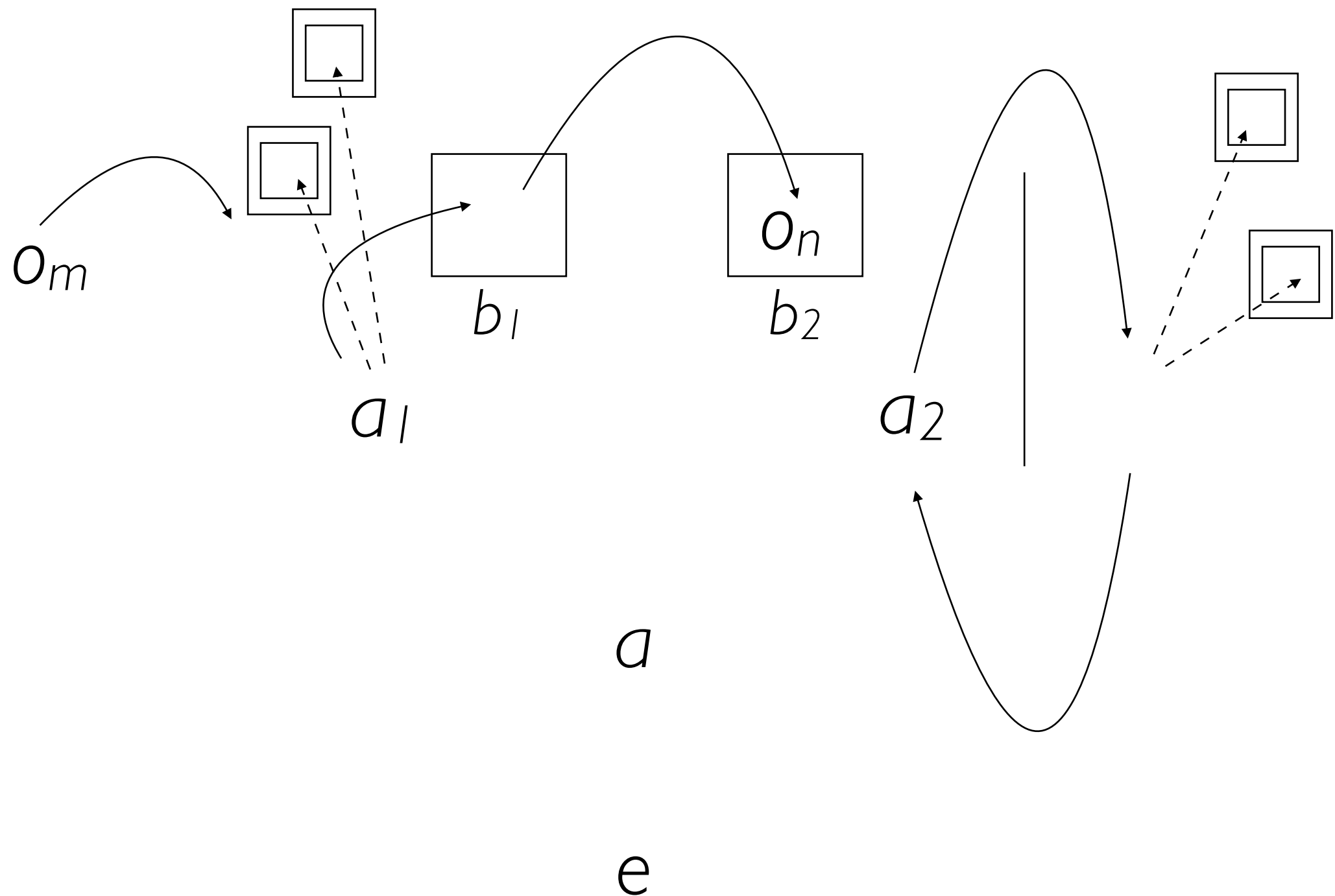
Framework for FBT^I_5

(ten timepoints)



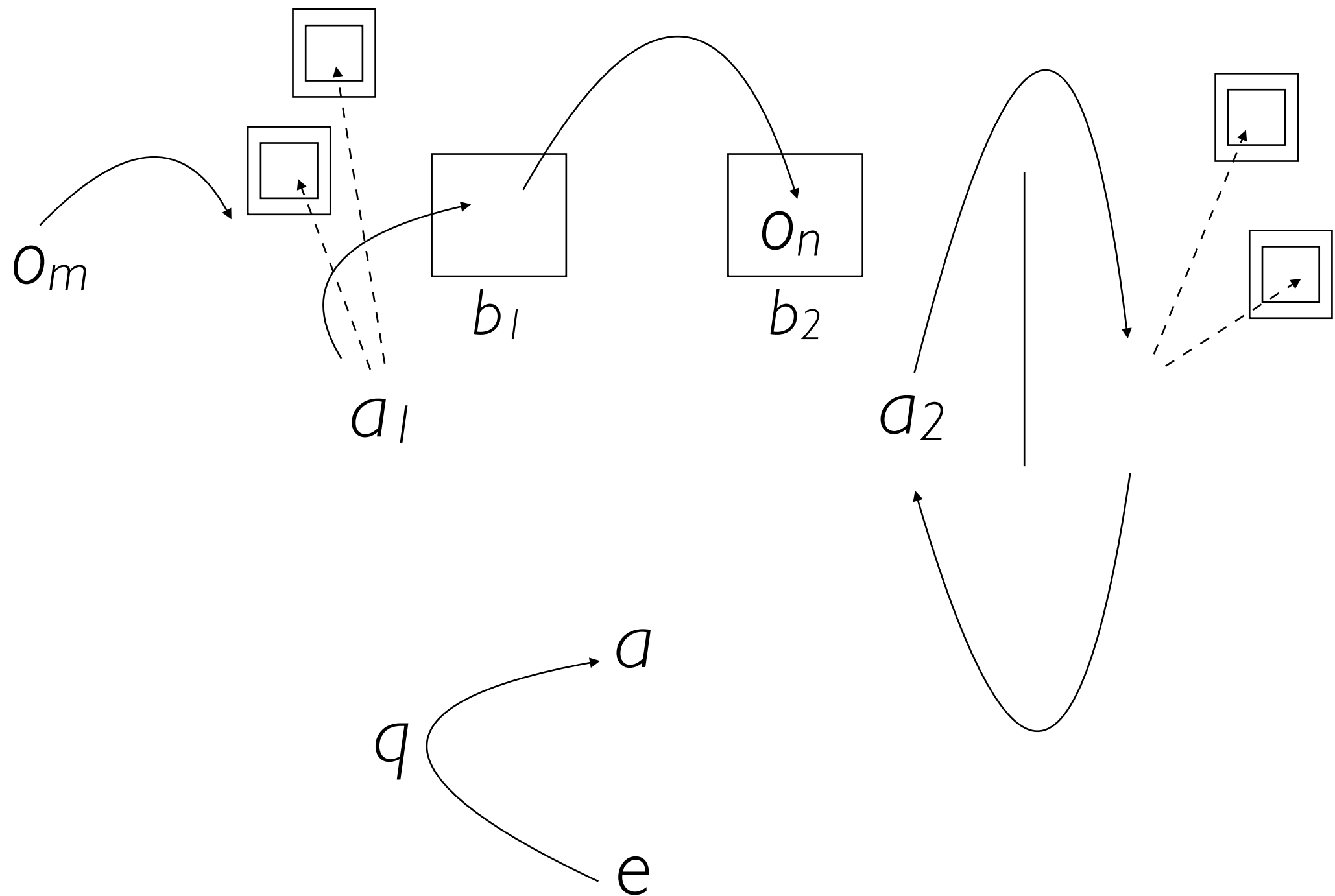
Framework for FBT^I_5

(ten timepoints)



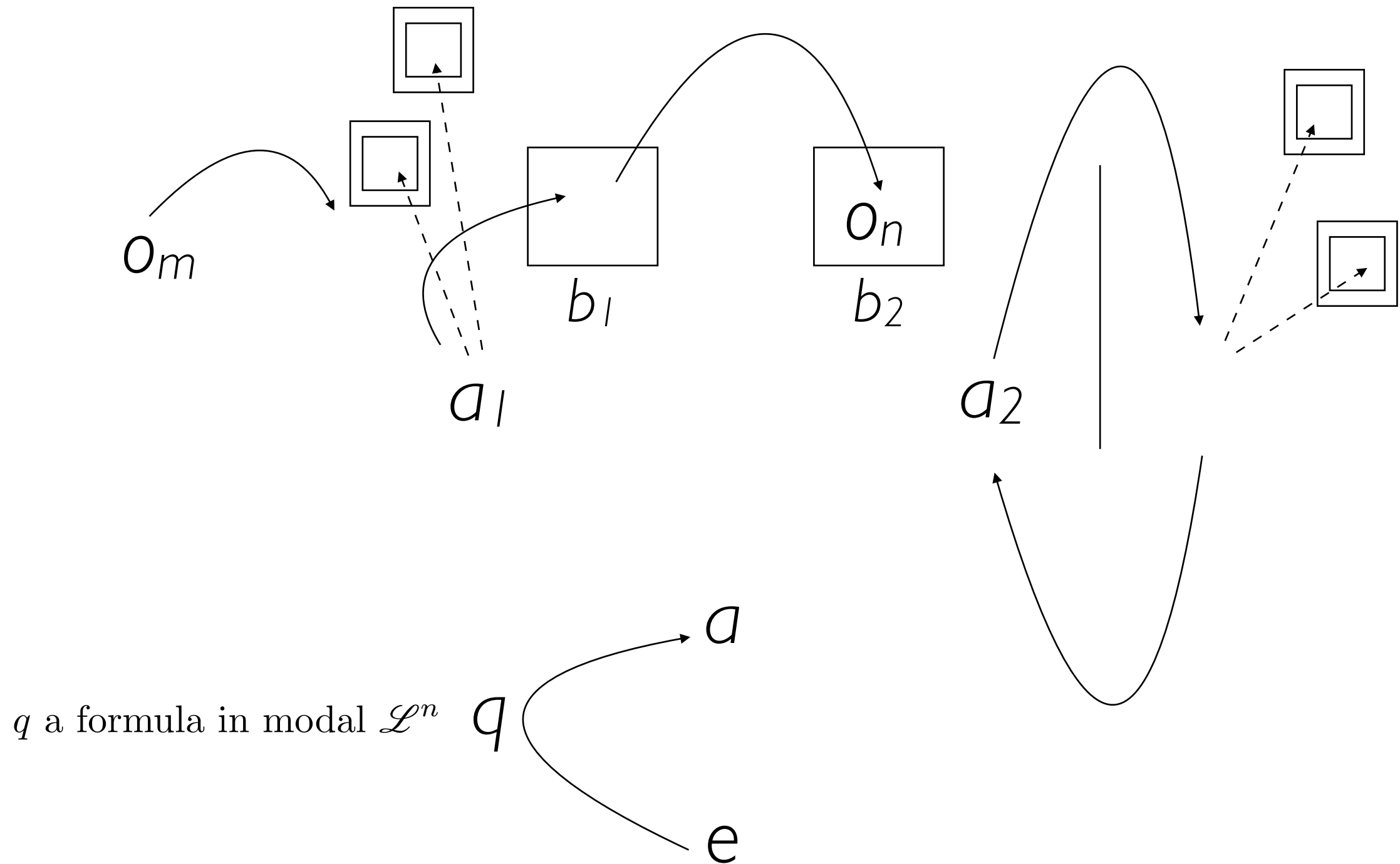
Framework for FBT^I_5

(ten timepoints)



Framework for FBT^I_5

(ten timepoints)



Humans Can Succeed

Neurobiologically normal, nurtured, educated, and sufficiently motivated humans can correctly answer any relevant query q for the infinite progression, and prove that their answer is correct. For the obvious subclass of queries (the form of which appear in the box below), they can prove and exploit the following lemma.

Lemma: Suppose $\text{FBT}_k, k \in \mathbb{Z}^+$, holds; (i.e. that level k of FBT holds). Then, if k is even, $\mathbf{B}_2\mathbf{B}_1 \dots \mathbf{B}_2 \vdash$, where there are $k + 1$ iterated \mathbf{B}_i operators; otherwise $\mathbf{B}_1\mathbf{B}_2 \dots \mathbf{B}_1\mathbf{B}_2 \vdash$, where there again there are $k + 1$ iterated \mathbf{B}_i operators.

Passing to Probing Mastery of the Specific Subclass

Experimenter to a : “At level k ,
from which box will a_2 attempt to
retrieve the objects o_n ? Prove it!”

Theoretical Machine Success on Infinite FBT!

Theorem: $\forall q \in \mathcal{CC}, \mathfrak{M}$ can correctly answer and justify q .
I.e., \mathfrak{M} can pass FBT_ω .

Ok, so this logic machine exists in the *mathematical* universe; but does there exist an *implemented* machine with this power?

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Ok, so this logic machine exists in the *mathematical* universe; but does there exist an *implemented* machine with this power?

Simulation Courtesy of ...

ShadowProver!



Level I

```
:name      "Level 1: False Belief Task "

:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level1 Belief: a1 believes a2 believes o is in b1.
"

:date      "Monday July 22, 2019"

:assumptions {
    :P1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1)))

    :P2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))

    :P3 (holds (In o b1) t1)

    :C1 (Common! t0 (forall [?f ?t2 ?t2]
        (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
            (holds ?f ?t2))))

    :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))
}

:goal      (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))}
```

Level 2

```
{:name      "Level 2: False Belief Task "

:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level2 Belief: a2 believes a1 believes a2 believes o is in b1.
"

:date      "Monday July 22, 2019"

:assumptions {

    :P1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))

    :P2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1)))))))

    :P3 (holds (In o b1) t1)

    :C1 (Common! t0
        (forall [?f ?t2 ?t2]
            (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
                (holds ?f ?t2))))

    :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))

:goal      (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))}
```

Level 3

```
{:name "Level 3: False Belief Task "

:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level3 Belief: a2 believes a1 believes a2 believes o is in b1.
"

:date "Monday July 22, 2019"

:assumptions {

    :P1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))))
    :P2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))))))
    :P3 (holds (In o b1) t1)

    :C1 (Common! t0
        (forall [?f ?t2 ?t2]
            (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
                (holds ?f ?t2))))

    :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}

:goal (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))}
```

Level 4

```
{:name      "Level 4: False Belief Task "

:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level4 Belief: a2 believes a1 believes a2 believes a1 believes a2 believes o is in b1.
"

:date      "Monday July 22, 2019"

:assumptions {

    :P1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))))
    :P2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))))))
    :P3 (holds (In o b1) t1)

    :C1 (Common! t0
        (forall [?f ?t2 ?t2]
            (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
                (holds ?f ?t2))))

    :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}

:goal      (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))}
```

Level 5

```
{:name "Level 5: False Belief Task "

:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level5 Belief: a1 believes a2 believes a1 believes a2 believes a1 believes a2 believes o is in b1.
"

:date "Monday July 22, 2019"

:assumptions {

  :P1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1)))))))
  :P2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))))))
  :P3 (holds (In o b1) t1)

  :C1 (Common! t0
    (forall [?f ?t2 ?t2]
      (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
        (holds ?f ?t2))))

  :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}

:goal (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3)))))))))}
```



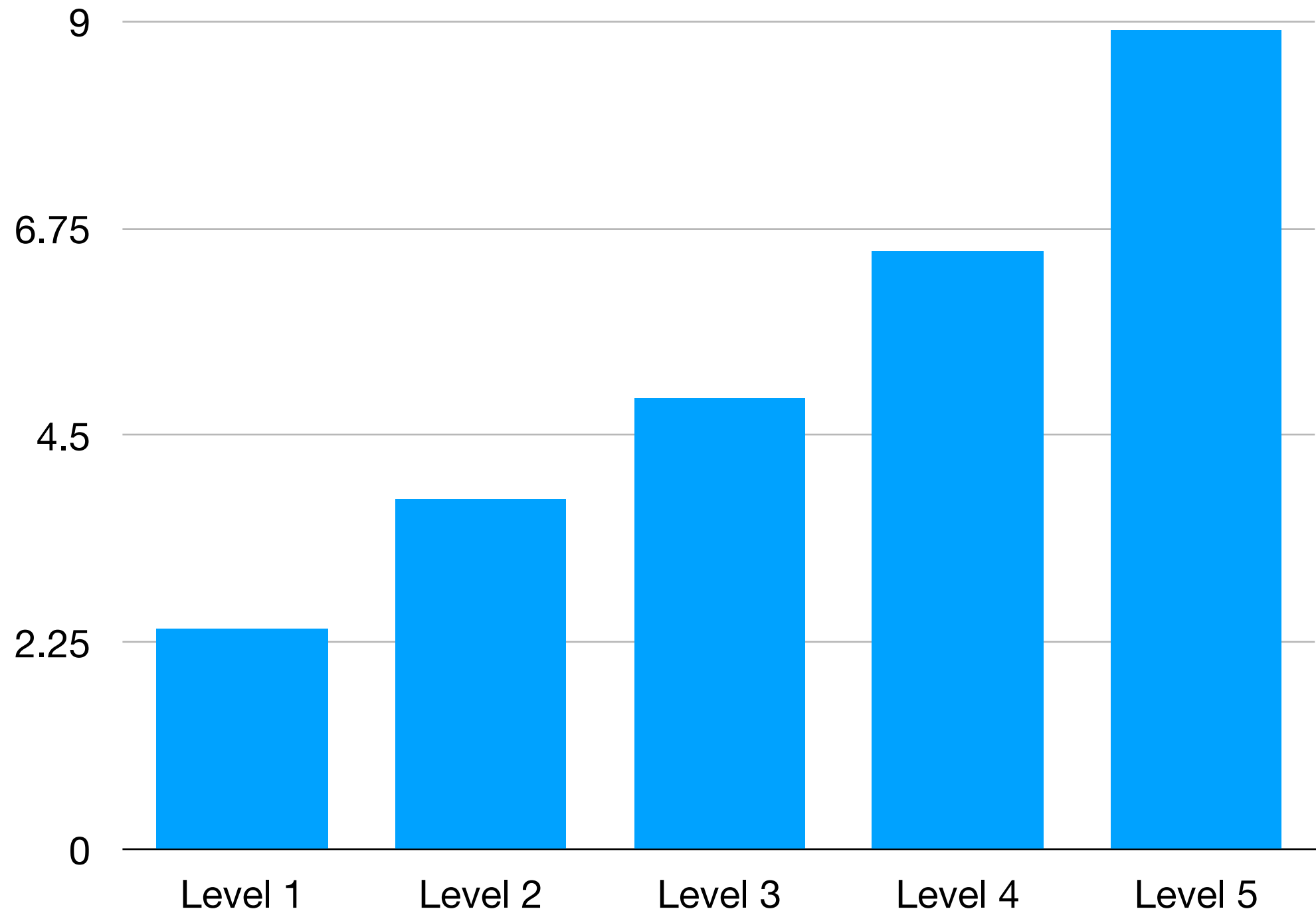
```
(Common! t0  
    (forall [?f ?t2 ?t2]  
        (if (and (not (exists [?e] (terminates ?e ?f)))  
            (holds ?f ?t1) (< ?t1 ?t2))
```

```

    :goal
      (Common! t0
        (forall [?f ?t2 ?t2]
          (if (and (not (exists [?e] (terminates ?e ?f)))
            (holds ?f ?t1) (< ?t1 ?t2))
            (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))}
      )
    )
  )

```

Time (in seconds) to Prove



Simulation of Level 5 in Real Time

```
/Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java ...  
objc[16653]: Class JavaLaunchHelper is implemented in both /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java (0x102a2d4c0) and /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/jre/lib/libinstrument.dylib (0x102ab94e0)  
----- Level 5 -----
```

Simulation of Level 5 in Real Time

```
/Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java ...  
objc[16653]: Class JavaLaunchHelper is implemented in both /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java (0x102a2d4c0) and /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/jre/lib/libinstrument.dylib (0x102ab94e0)  
----- Level 5 -----
```

Encapsulation

Slate - K.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
---	---	---	---

Encapsulation

The image shows two overlapping Slate editor windows. The top window is titled 'Slate - K.slt' and contains four boxes, each with a modal logic formula and its status in the K system. The bottom window is titled 'Slate - T.slt' and contains the same four boxes, but with their status in the T system.

Formula	K System Status	T System Status
K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	$K \vdash \checkmark \infty \Box$	$M \vdash \checkmark \infty \Box$
T. $\Box\varphi \rightarrow \varphi$	$K \vdash \times \infty \Box$	$M \vdash \checkmark \infty \Box$
4. $\Box\varphi \rightarrow \Box\Box\varphi$	$K \vdash \times \infty \Box$	$M \vdash \times \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	$K \vdash \times \infty \Box$	$M \vdash \times \infty \Box$

Encapsulation

The image displays three overlapping windows, each representing a different modal logic system. Each window contains four boxes, each with a formula and its status in that system (derivable or not, with a checkmark or cross and a symbol).

Slate - K.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
 $K \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
 $K \vdash \times \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
 $K \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
 $K \vdash \times \infty \Box$

Slate - T.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
 $M \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
 $M \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
 $M \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
 $M \vdash \times \infty \Box$

Slate - D.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
 $D \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
 $D \vdash \times \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$
 $D \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
 $D \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
 $D \vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$
 $D \vdash \checkmark \infty \Box$

Encapsulation

Slate - K.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
---	---	---	---

Slate - T.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
---	---	---	---

Slate - D.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$	D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$		INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$	

Slate - S4.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$	D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$		INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ {INTER} Assume \checkmark	

Encapsulation

K

T

D

4 = S4

5 = S5

The image shows five overlapping Slate windows, each displaying a set of modal logic axioms and their derivability status for a specific system. The windows are titled 'Slate - K.slt', 'Slate - T.slt', 'Slate - D.slt', 'Slate - S4.slt', and 'Slate - S5.slt'.

- Slate - K.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (K $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (K $\vdash \times \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (K $\vdash \times \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (K $\vdash \times \infty \Box$)
- Slate - T.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (M $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (M $\vdash \checkmark \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (M $\vdash \times \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (M $\vdash \times \infty \Box$)
- Slate - D.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (D $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (D $\vdash \times \infty \Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ (D $\vdash \checkmark \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (D $\vdash \times \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (D $\vdash \times \infty \Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ (D $\vdash \checkmark \infty \Box$)
- Slate - S4.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S4 $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S4 $\vdash \times \infty \Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume \checkmark)
- Slate - S5.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S5 $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (S5 $\vdash \checkmark \infty \Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ ({D} Assume \checkmark)
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ ({4} Assume \checkmark)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S5 $\vdash \checkmark \infty \Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume \checkmark)

Encapsulation

K

T

D

4 = S4

5 = S5

The image shows a stack of five Slate IDE windows, each containing a set of modal logic formulas and their derivability status in a specific system. The windows are titled 'Slate - K.slt', 'Slate - T.slt', 'Slate - D.slt', 'Slate - S4.slt', and 'Slate - S5.slt'. The 'Slate - D.slt' window is highlighted with a red border.

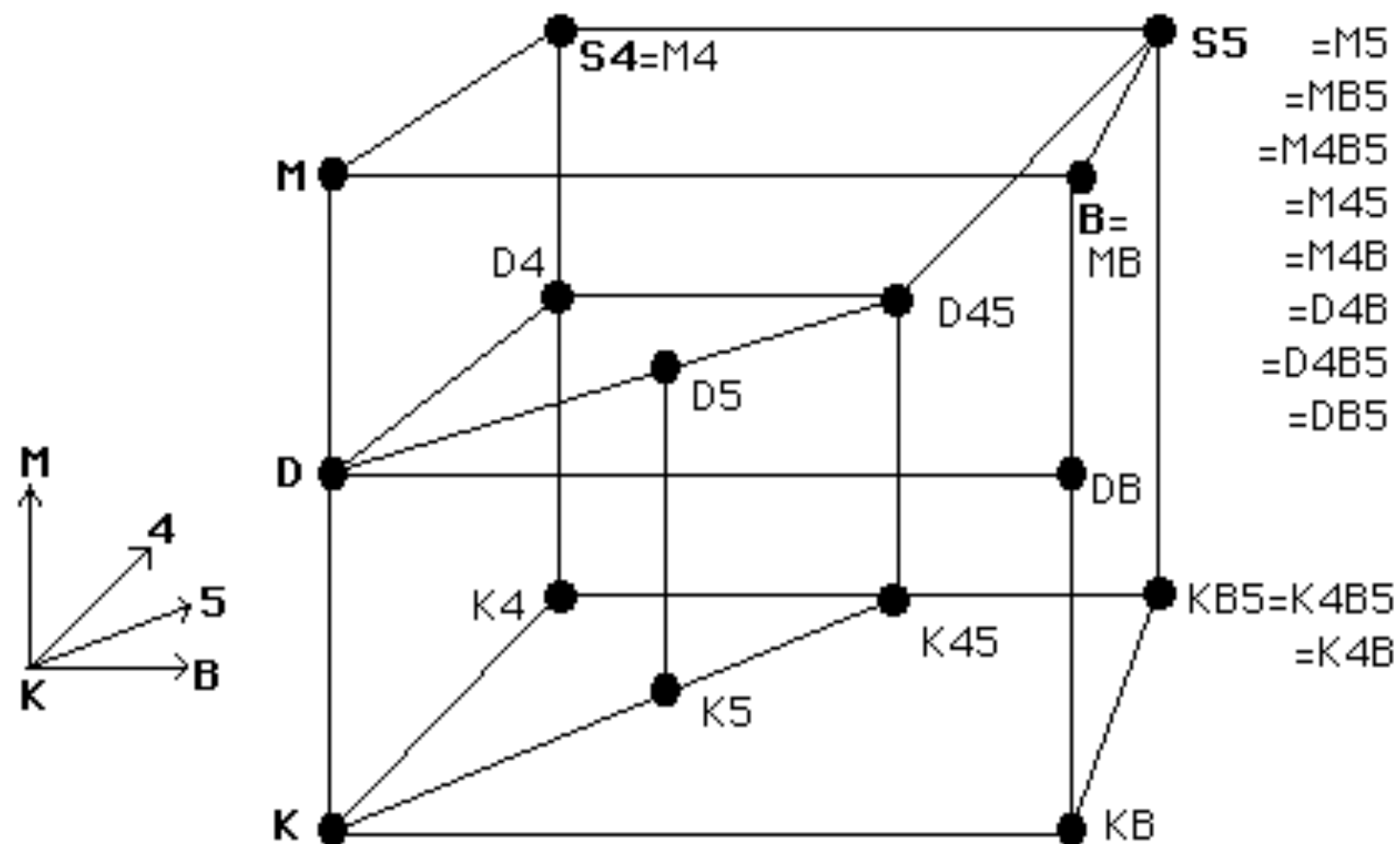
Window Title	Formula	System	Status
Slate - K.slt	K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	K	$\vdash \checkmark \infty \Box$
	T. $\Box\varphi \rightarrow \varphi$	K	$\vdash \times \infty \Box$
	4. $\Box\varphi \rightarrow \Box\Box\varphi$	K	$\vdash \times \infty \Box$
	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	K	$\vdash \times \infty \Box$
Slate - T.slt	K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	M	$\vdash \checkmark \infty \Box$
	T. $\Box\varphi \rightarrow \varphi$	M	$\vdash \checkmark \infty \Box$
	4. $\Box\varphi \rightarrow \Box\Box\varphi$	M	$\vdash \times \infty \Box$
	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	M	$\vdash \times \infty \Box$
Slate - D.slt	K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	D	$\vdash \checkmark \infty \Box$
	T. $\Box\varphi \rightarrow \varphi$	D	$\vdash \times \infty \Box$
	D. $\Box\varphi \rightarrow \Diamond\varphi$	D	$\vdash \checkmark \infty \Box$
	4. $\Box\varphi \rightarrow \Box\Box\varphi$	D	$\vdash \times \infty \Box$
	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	D	$\vdash \times \infty \Box$
INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$	D	$\vdash \checkmark \infty \Box$	
Slate - S4.slt	K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	S4	$\vdash \checkmark \infty \Box$
	T. $\Box\varphi \rightarrow \varphi$	S4	$\vdash \checkmark \infty \Box$
	D. $\Box\varphi \rightarrow \Diamond\varphi$	S4	$\vdash \checkmark \infty \Box$
	4. $\Box\varphi \rightarrow \Box\Box\varphi$	S4	$\vdash \checkmark \infty \Box$
	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	S4	$\vdash \times \infty \Box$
INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$	{INTER}	Assume \checkmark	
Slate - S5.slt	K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	S5	$\vdash \checkmark \infty \Box$
	T. $\Box\varphi \rightarrow \varphi$	S5	$\vdash \checkmark \infty \Box$
	D. $\Box\varphi \rightarrow \Diamond\varphi$	{D}	Assume \checkmark
	4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	{4}	Assume \checkmark
	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	S5	$\vdash \checkmark \infty \Box$
INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$	{INTER}	Assume \checkmark	

Encapsulation

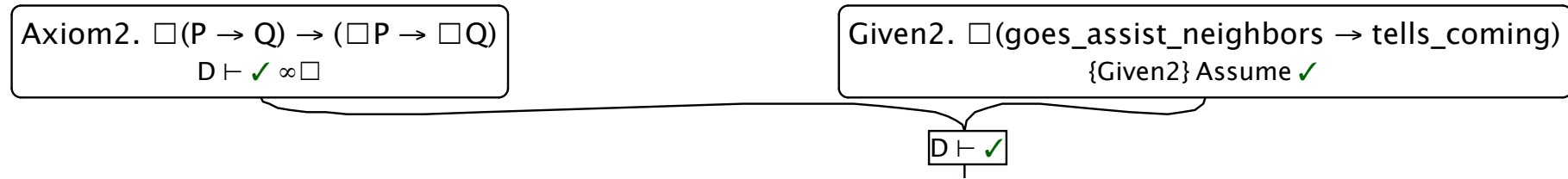
K
T
D
4 = S4
5 = S5

The screenshot displays five windows of the HyperSlate interface, each representing a different modal logic system. Each window contains a grid of logical formulas and their derivability status (indicated by a green checkmark for true and a red X for false).

- Slate - K.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
- Slate - T.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
- Slate - D.slt** (highlighted with a red border)
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$
- Slate - S4.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$
- Slate - S5.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $\{D\} \text{ Assume } \checkmark$
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $\{4\} \text{ Assume } \checkmark$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$



Chisholm's Paradox

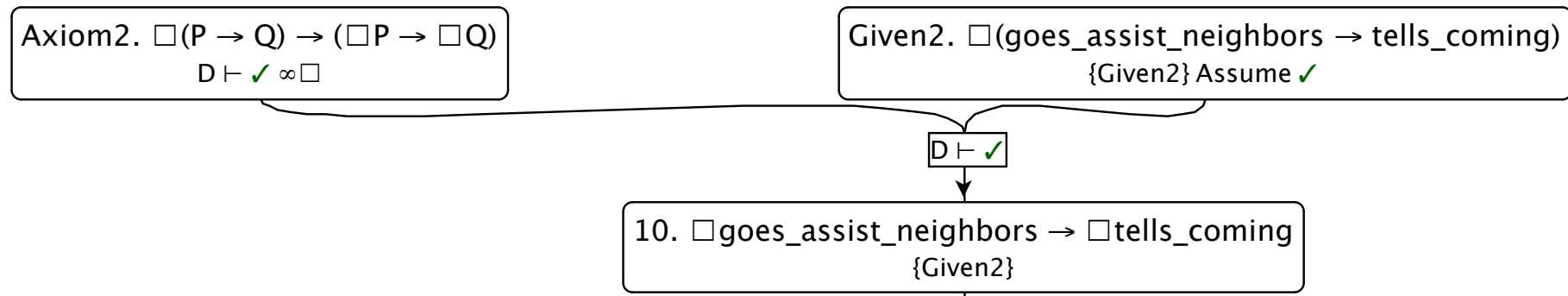


Axiom4. "Modus ponens for provability."
{Axiom4} Assume \checkmark

Axiom5. "Theorems are obligatory."
{Axiom5} Assume \checkmark

Axiom1. "All theorems of the propositional calculus."
{Axiom1} Assume \checkmark

Chisholm's Paradox

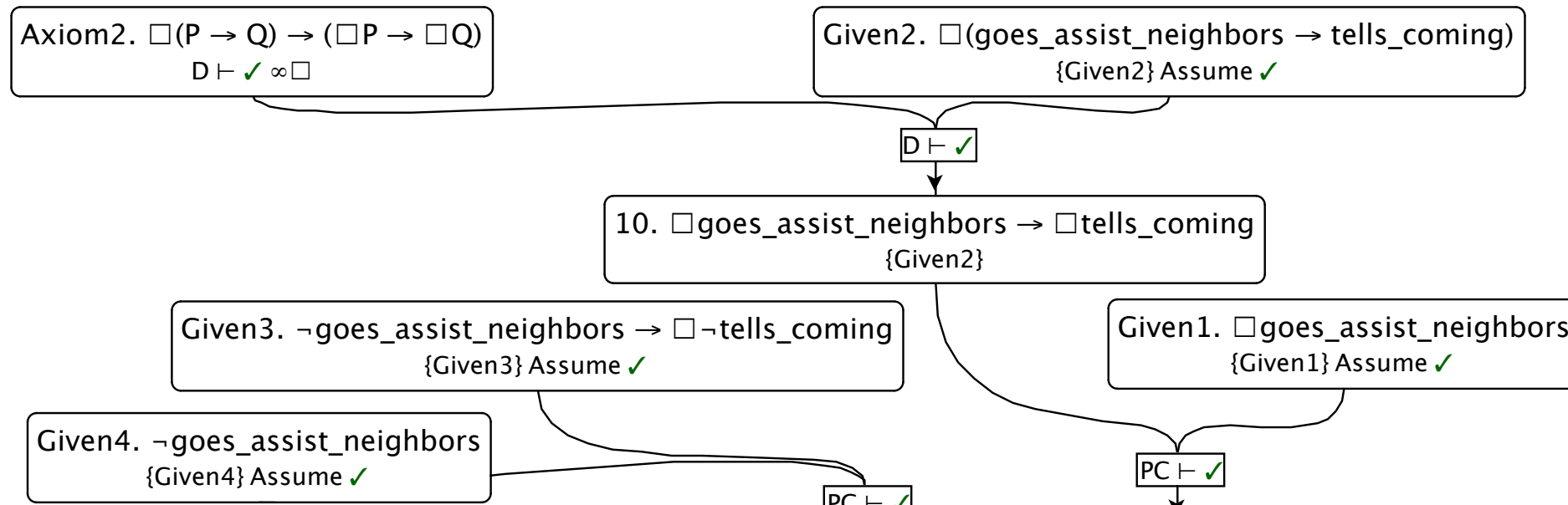


Axiom4. "Modus ponens for provability."
{Axiom4} Assume \checkmark

Axiom5. "Theorems are obligatory."
{Axiom5} Assume \checkmark

Axiom1. "All theorems of the propositional calculus."
{Axiom1} Assume \checkmark

Chisholm's Paradox

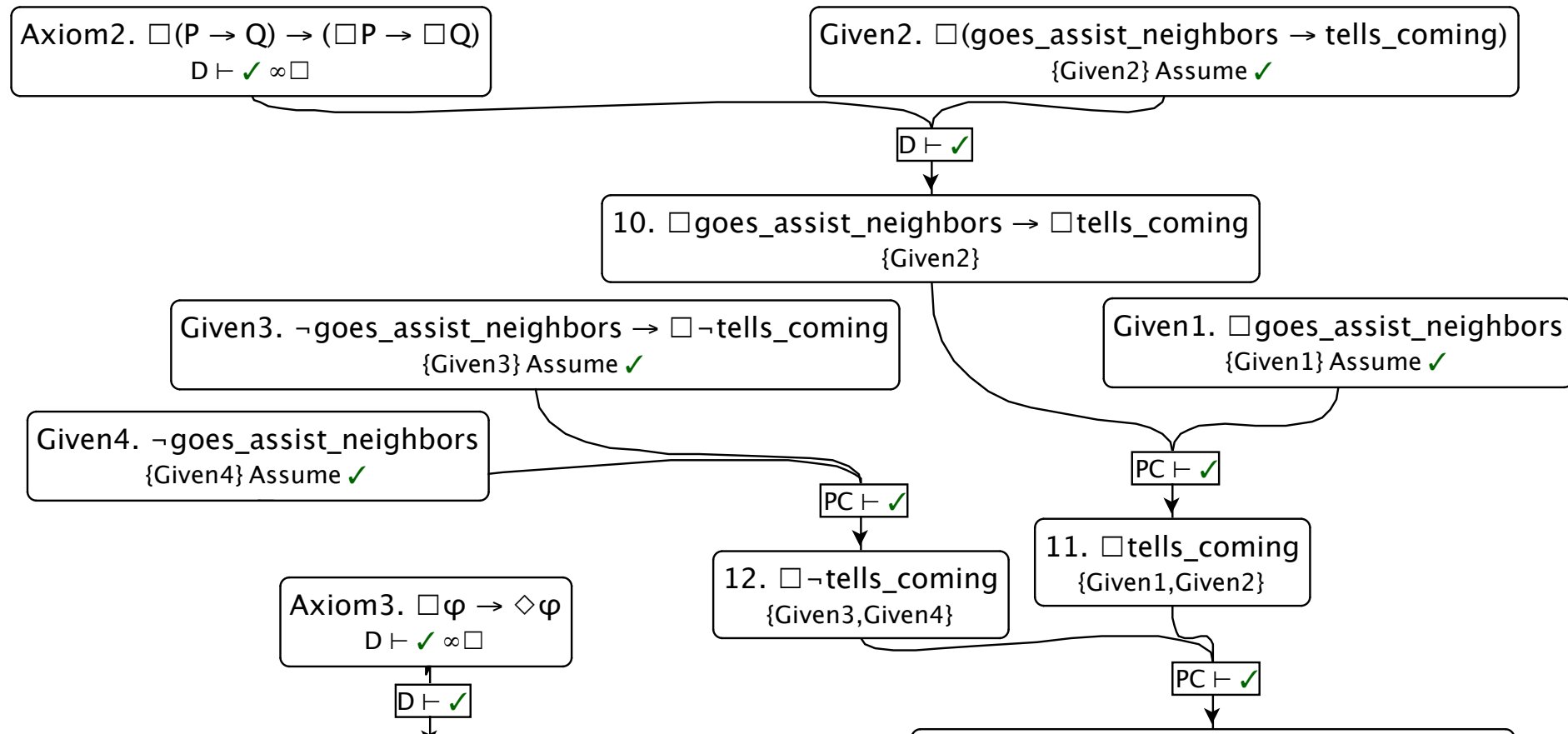


Axiom4. "Modus ponens for provability."
 $\{\text{Axiom4}\} \text{ Assume } \checkmark$

Axiom5. "Theorems are obligatory."
 $\{\text{Axiom5}\} \text{ Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."
 $\{\text{Axiom1}\} \text{ Assume } \checkmark$

Chisholm's Paradox

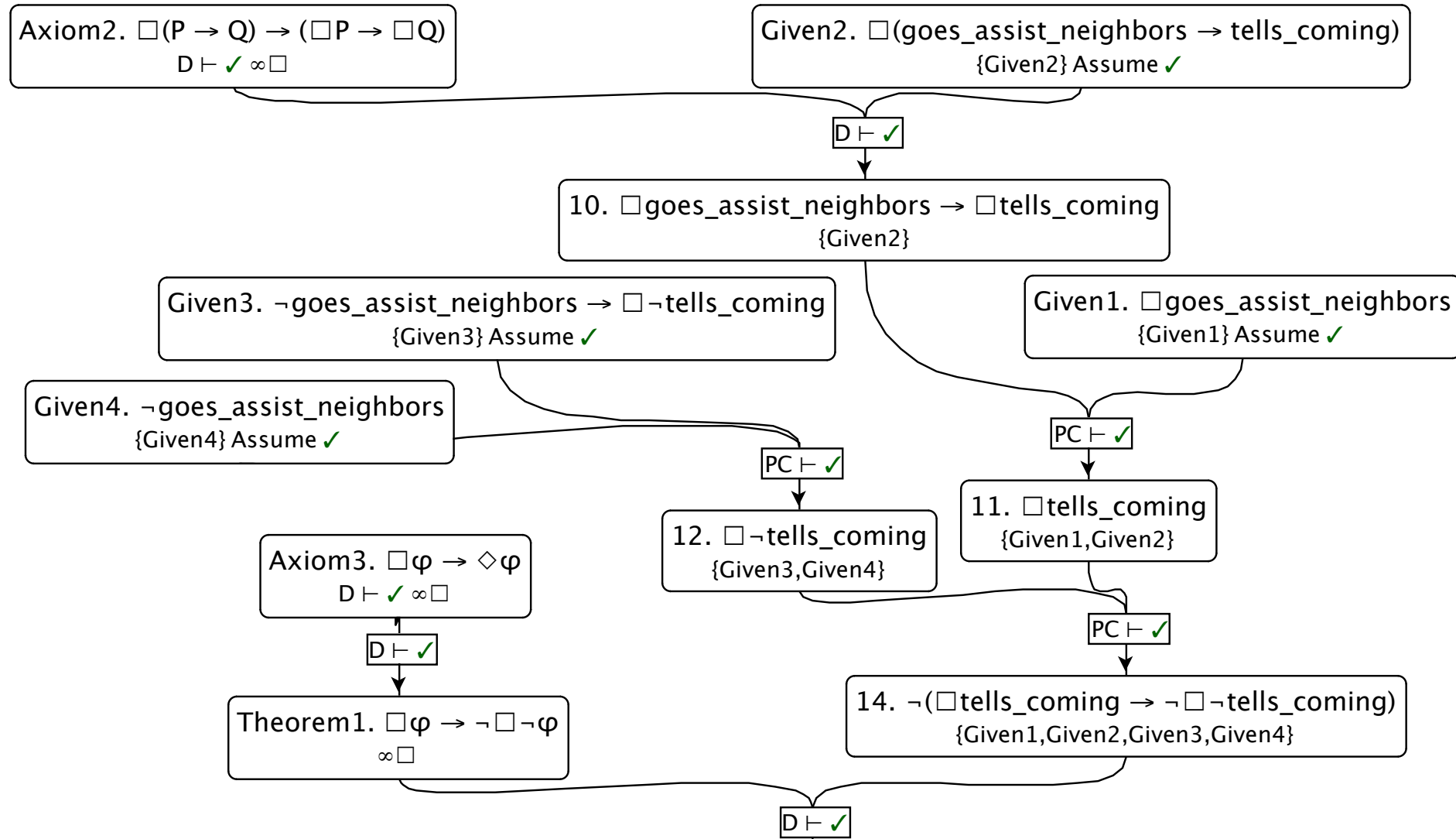


Axiom4. "Modus ponens for provability."
 $\{\text{Axiom4}\} \text{ Assume } \checkmark$

Axiom5. "Theorems are obligatory."
 $\{\text{Axiom5}\} \text{ Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."
 $\{\text{Axiom1}\} \text{ Assume } \checkmark$

Chisholm's Paradox

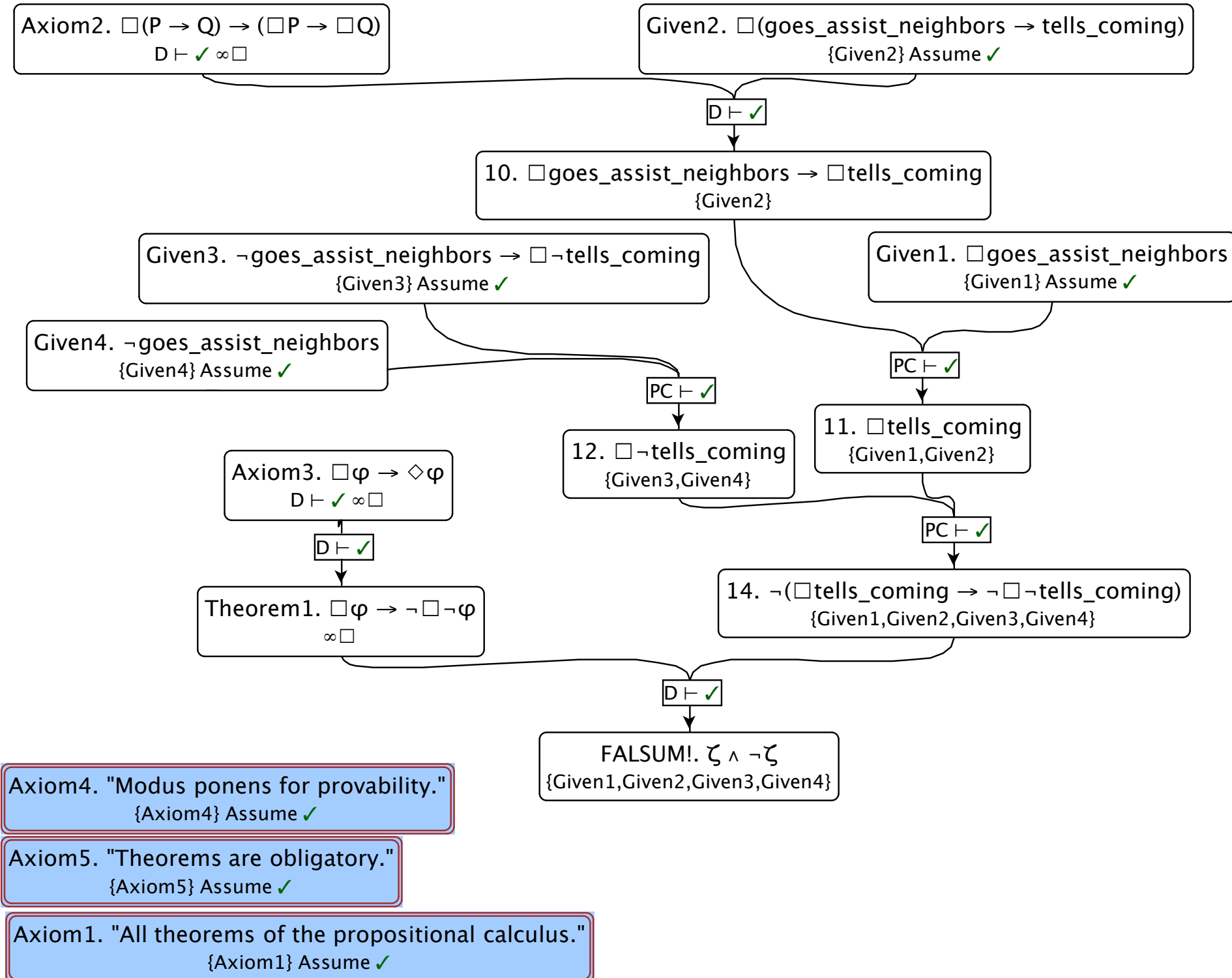


Axiom4. "Modus ponens for provability."
 $\{\text{Axiom4}\} \text{ Assume } \checkmark$

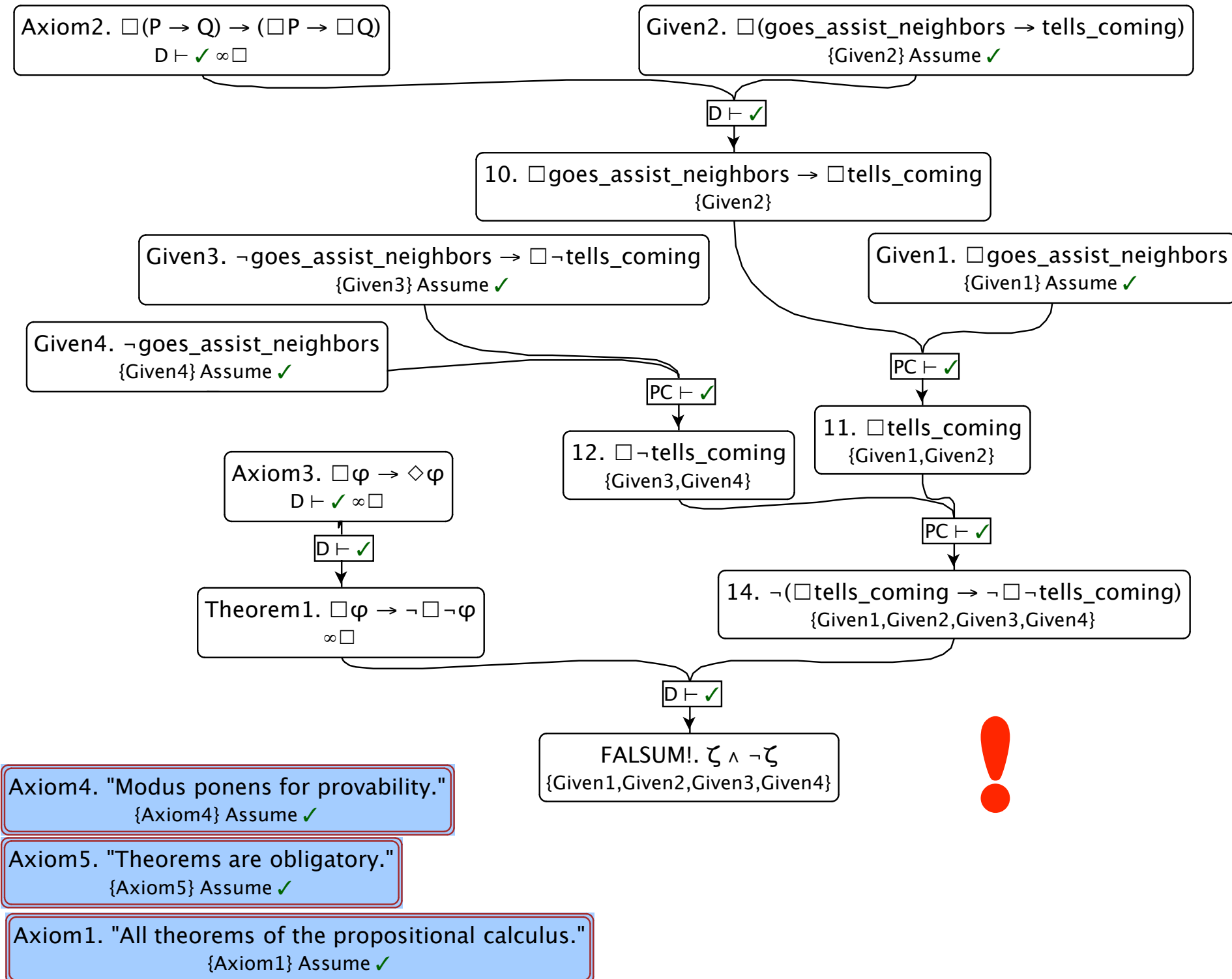
Axiom5. "Theorems are obligatory."
 $\{\text{Axiom5}\} \text{ Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."
 $\{\text{Axiom1}\} \text{ Assume } \checkmark$

Chisholm's Paradox



Chisholm's Paradox



Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot displays five windows from the HyperSlate application, each showing a set of modal logic formulas and their status in different systems. The windows are titled 'Slate - K.slt', 'Slate - T.slt', 'Slate - D.slt', 'Slate - S4.slt', and 'Slate - S5.slt'.

- Slate - K.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
- Slate - T.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
- Slate - D.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$
- Slate - S4.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$
- Slate - S5.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $\{D\} \text{ Assume } \checkmark$
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $\{4\} \text{ Assume } \checkmark$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot displays five modal logic windows in the HyperSlate interface:

- Slate - K.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
- Slate - T.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
- Slate - D.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$
- Slate - S4.slt** (highlighted with a red border)
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$
- Slate - S5.slt** (highlighted with a red border)
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $\{D\} \text{ Assume } \checkmark$
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $\{4\} \text{ Assume } \checkmark$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot displays the HyperSlate interface with several windows showing logic calculi and their axioms. A 'Create file' dialog is open in the center, and a red box highlights the S4 and S5 calculi windows.

Windows and Axioms:

- Slate - K.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
- Slate - T.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
- Create file dialog:**
 - Buttons: Propositional Calculus, L_0 = Pure Predicate Calculus, L_1 = First-order Logic, L_2 = Second-order Logic, K, T, D, S4, S5, DCEC (fragment), Hyperlog.
- Slate - S4.slt (highlighted in red):**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$
- Slate - S5.slt (highlighted in red):**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $\{D\} \text{ Assume } \checkmark$
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $\{4\} \text{ Assume } \checkmark$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot displays the HyperSlate application interface. At the top, a stack of four windows is visible: 'Slate - K.slt', 'Slate - T.slt', 'Slate - S4.slt', and 'Slate - S5.slt'. Each window contains a grid of modal logic formulas and their derivability status. A green arrow points to the 'S5' button in the 'Create file' dialog, which is highlighted with a green border. A red box encloses the 'Slate - S4.slt' and 'Slate - S5.slt' windows.

Modal Logic Formulas and Derivability:

- Slate - K.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (K $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (K $\vdash \times \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (K $\vdash \times \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (K $\vdash \times \infty \Box$)
- Slate - T.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (M $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (M $\vdash \checkmark \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (M $\vdash \times \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (M $\vdash \times \infty \Box$)
- Slate - S4.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S4 $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S4 $\vdash \times \infty \Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume \checkmark)
- Slate - S5.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S5 $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (S5 $\vdash \checkmark \infty \Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ ({D} Assume \checkmark)
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ ({4} Assume \checkmark)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S5 $\vdash \checkmark \infty \Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume \checkmark)

DCEC (supported fragment, student version)

First-order (Propositional) Schema

- Assume
- Not Elim, Not Intro
- And Elim, And Intro
- Or Elim, Or Intro
- If Elim, If Intro
- Iff Elim, Iff Intro
- Forall Elim, Forall Intro
- Exists Elim, Exists Intro
- Higher Order Forall Elim, Higher Order Forall Intro
- Higher Order Exists Elim, Higher Order Exists Intro
- Eq Elim, Eq Intro
- Pc Oracle, Fol Oracle

Modal Inference Schemata

- $R_1, R_2, R_3, R_4,$
- $R_k, R_b,$
- R_{14}

Inference Schemata

Modal

$$\frac{\mathbf{K}(a, t_1, \Gamma), \Gamma \vdash \phi, t_1 \leq t_2}{\mathbf{K}(a, t_2, \phi)} [R_K] \quad \frac{\mathbf{B}(a, t_1, \Gamma), \Gamma \vdash \phi, t_1 \leq t_2}{\mathbf{B}(a, t_2, \phi)} [R_B]$$

$$\frac{}{\mathbf{C}(t, \mathbf{P}(a, t, \phi) \rightarrow \mathbf{K}(a, t, \phi))} [R_1] \quad \frac{}{\mathbf{C}(t, \mathbf{K}(a, t, \phi) \rightarrow \mathbf{B}(a, t, \phi))} [R_2]$$

$$\frac{\mathbf{C}(t, \phi) \ t \leq t_1 \dots t \leq t_n}{\mathbf{K}(a_1, t_1, \dots \mathbf{K}(a_n, t_n, \phi) \dots)} [R_3] \quad \frac{\mathbf{K}(a, t, \phi)}{\phi} [R_4]$$

$$\frac{}{\mathbf{C}(t, \mathbf{K}(a, t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{K}(a, t_2, \phi_1) \rightarrow \mathbf{K}(a, t_3, \phi_2)} [R_5]$$

$$\frac{}{\mathbf{C}(t, \mathbf{B}(a, t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{B}(a, t_2, \phi_1) \rightarrow \mathbf{B}(a, t_3, \phi_2)} [R_6]$$

$$\frac{}{\mathbf{C}(t, \mathbf{C}(t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{C}(t_2, \phi_1) \rightarrow \mathbf{C}(t_3, \phi_2)} [R_7]$$

$$\frac{}{\mathbf{C}(t, \forall x. \phi \rightarrow \phi[x \mapsto t])} [R_8] \quad \frac{}{\mathbf{C}(t, \phi_1 \leftrightarrow \phi_2 \rightarrow \neg \phi_2 \rightarrow \neg \phi_1)} [R_9]$$

$$\frac{}{\mathbf{C}(t, [\phi_1 \wedge \dots \wedge \phi_n \rightarrow \phi] \rightarrow [\phi_1 \rightarrow \dots \rightarrow \phi_n \rightarrow \psi])} [R_{10}]$$

$$\frac{\mathbf{S}(s, h, t, \phi)}{\mathbf{B}(h, t, \mathbf{B}(s, t, \phi))} [R_{12}] \quad \frac{\mathbf{I}(a, t, \text{happens}(\text{action}(a^*, \alpha), t'))}{\mathbf{P}(a, t, \text{happens}(\text{action}(a^*, \alpha), t))} [R_{13}]$$

$$\frac{\mathbf{B}(a, t, \phi) \ \mathbf{B}(a, t, \mathbf{O}(a, t, \phi, \chi)) \ \mathbf{O}(a, t, \phi, \chi)}{\mathbf{K}(a, t, \mathbf{I}(a, t, \chi))} [R_{14}]$$

DCEC (supported fragment, student version)

First-order (Propositional) Schema

- Assume
- Not Elim, Not Intro
- And Elim, And Intro
- Or Elim, Or Intro
- If Elim, If Intro
- Iff Elim, Iff Intro
- Forall Elim, Forall Intro
- Exists Elim, Exists Intro
- Higher Order Forall Elim, Higher Order Forall Intro
- Higher Order Exists Elim, Higher Order Exists Intro
- Eq Elim, Eq Intro
- Pc Oracle, Fol Oracle

Modal Inference Schemata

- $R_1, R_2, R_3, R_4,$
- $R_k, R_b,$
- R_{14}

Inference Schemata

Modal

$$\frac{\mathbf{K}(a, t_1, \Gamma), \Gamma \vdash \phi, t_1 \leq t_2}{\mathbf{K}(a, t_2, \phi)} [R_K] \quad \frac{\mathbf{B}(a, t_1, \Gamma), \Gamma \vdash \phi, t_1 \leq t_2}{\mathbf{B}(a, t_2, \phi)} [R_B]$$

$$\frac{}{\mathbf{C}(t, \mathbf{P}(a, t, \phi) \rightarrow \mathbf{K}(a, t, \phi))} [R_1] \quad \frac{}{\mathbf{C}(t, \mathbf{K}(a, t, \phi) \rightarrow \mathbf{B}(a, t, \phi))} [R_2]$$

$$\frac{\mathbf{C}(t, \phi) \ t \leq t_1 \dots t \leq t_n}{\mathbf{K}(a_1, t_1, \dots \mathbf{K}(a_n, t_n, \phi) \dots)} [R_3] \quad \frac{\mathbf{K}(a, t, \phi)}{\phi} [R_4]$$

$$\frac{}{\mathbf{C}(t, \mathbf{K}(a, t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{K}(a, t_2, \phi_1) \rightarrow \mathbf{K}(a, t_3, \phi_2)} [R_5]$$

$$\frac{}{\mathbf{C}(t, \mathbf{B}(a, t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{B}(a, t_2, \phi_1) \rightarrow \mathbf{B}(a, t_3, \phi_2)} [R_6]$$

$$\frac{}{\mathbf{C}(t, \mathbf{C}(t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{C}(t_2, \phi_1) \rightarrow \mathbf{C}(t_3, \phi_2)} [R_7]$$

$$\frac{}{\mathbf{C}(t, \forall x. \phi \rightarrow \phi[x \mapsto t])} [R_8] \quad \frac{}{\mathbf{C}(t, \phi_1 \leftrightarrow \phi_2 \rightarrow \neg \phi_2 \rightarrow \neg \phi_1)} [R_9]$$

$$\frac{}{\mathbf{C}(t, [\phi_1 \wedge \dots \wedge \phi_n \rightarrow \phi] \rightarrow [\phi_1 \rightarrow \dots \rightarrow \phi_n \rightarrow \psi])} [R_{10}]$$

$$\frac{\mathbf{S}(s, h, t, \phi)}{\mathbf{B}(h, t, \mathbf{B}(s, t, \phi))} [R_{12}] \quad \frac{\mathbf{I}(a, t, \text{happens}(\text{action}(a^*, \alpha), t'))}{\mathbf{P}(a, t, \text{happens}(\text{action}(a^*, \alpha), t))} [R_{13}]$$

$$\frac{\mathbf{B}(a, t, \phi) \ \mathbf{B}(a, t, \mathbf{O}(a, t, \phi, \chi)) \ \mathbf{O}(a, t, \phi, \chi)}{\mathbf{K}(a, t, \mathbf{I}(a, t, \chi))} [R_{14}]$$

For Brave HyperLogical Adventurers

For Brave HyperLogical Adventurers

“Everything smart knows that everything tinks anything that tinks something identical with something.”

For Brave HyperLogical Adventurers

“Everything smart knows that everything tinks anything that tinks something identical with something.”

“Blinky is smart and tinks himself.”

For Brave HyperLogical Adventurers

“Everything smart knows that everything tinks anything that tinks something identical with something.”

“Blinky is smart and tinks himself.”

Therefore:

For Brave HyperLogical Adventurers

“Everything smart knows that everything tinks anything that tinks something identical with something.”

“Blinky is smart and tinks himself.”

“Everything tinks anything that tinks something identical with something.”

Therefore:

For Brave HyperLogical Adventurers

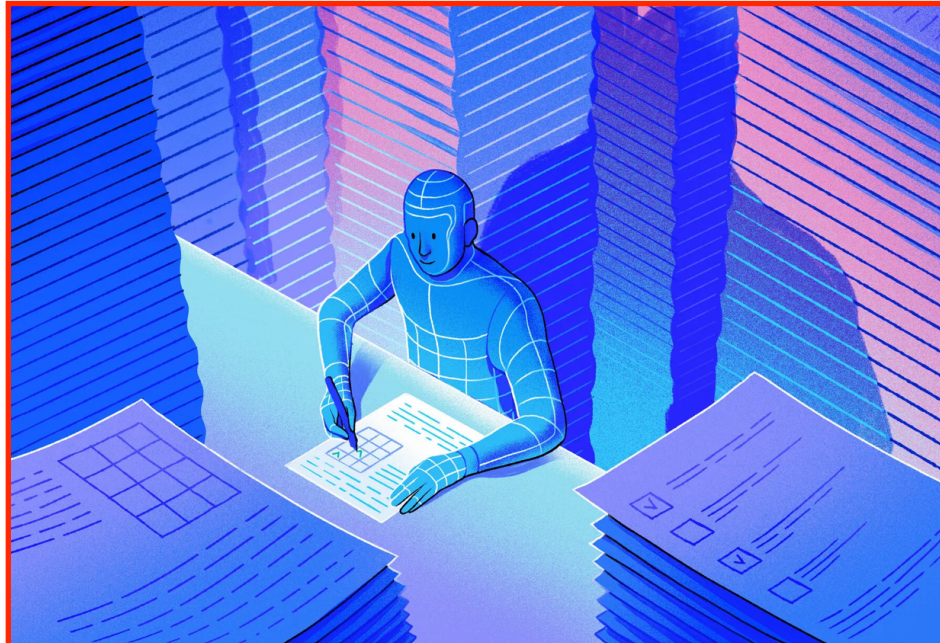
“Everything smart knows that everything tinks anything that tinks something identical with something.”

“Blinky is smart and tinks himself.”

“Everything tinks anything that tinks something identical with something.”

Therefore:

“Yolanda tinks Xavier.”



Rune Fisker

THE SHIFT

When A.I. Passes This Test, Look Out

The creators of a new test called “Humanity’s Last Exam” argue we may soon lose the ability to create tests hard enough for A.I. models.



Listen to this article • 8:04 min [Learn more](#)



By **Kevin Roose**



Reporting from San Francisco

Jan. 23, 2025

If you’re looking for a new reason to be nervous about artificial intelligence, try this: Some of the smartest humans in the world are struggling to create tests that A.I. systems can’t pass.

For Brave HyperLogical Adventurers






For Brave HyperLogical Adventurers

 ChatGPT 4o > 

Blinky is smart and tinks himself.

Blinky is indeed a thoughtful and self-aware household robot. He doesn't just follow orders blindly—he considers the implications of his actions, the needs of the Booker family, and even the deeper meaning behind his existence.

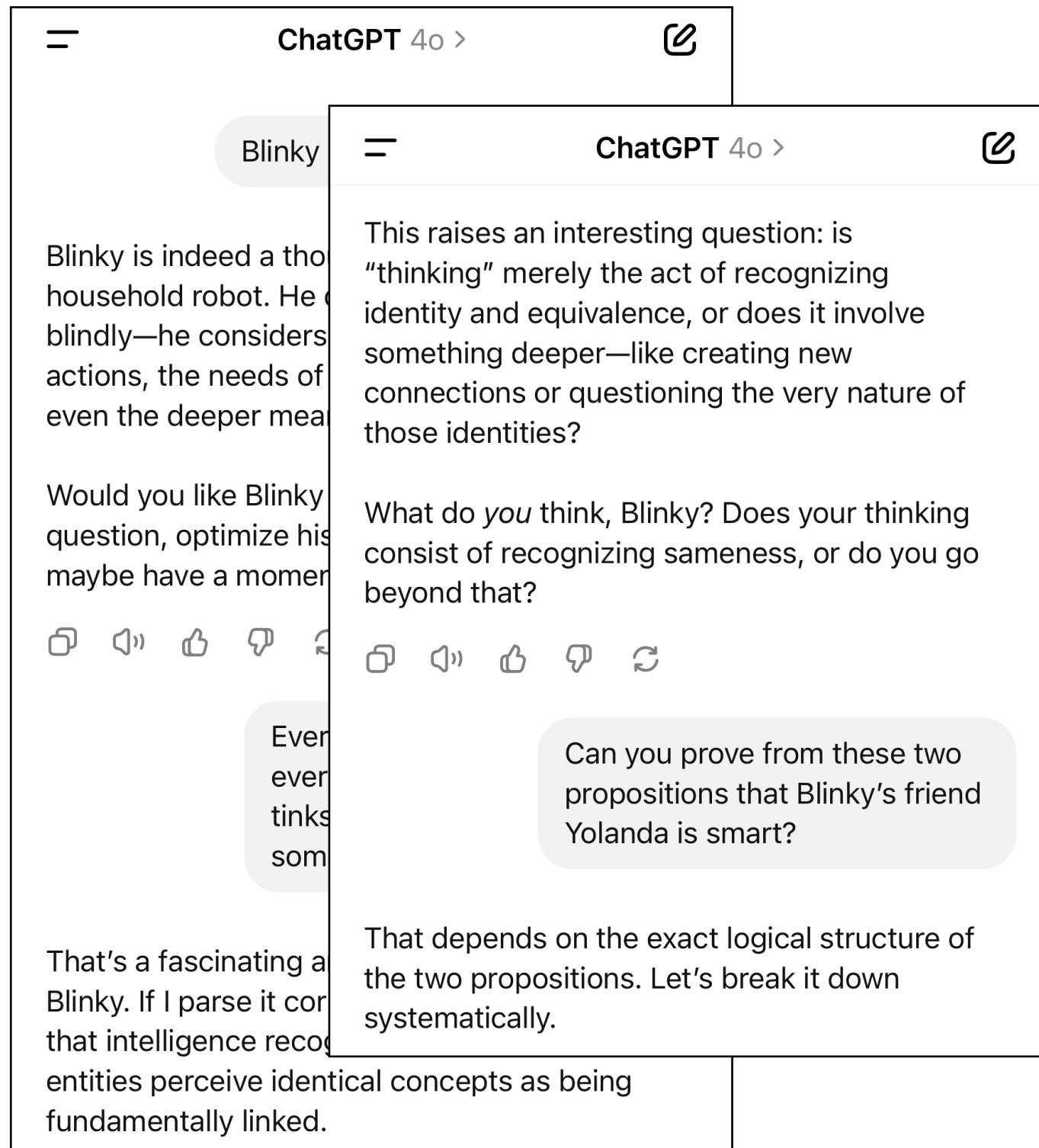
Would you like Blinky to explore a philosophical question, optimize his household tasks, or maybe have a moment of self-reflection?

Everything smart knows that everything tinks anything that tinks something identical with something.

That's a fascinating and layered statement, Blinky. If I parse it correctly, you're suggesting that intelligence recognizes that all thinking entities perceive identical concepts as being fundamentally linked.

For Brave HyperLogical Adventurers



For Brave HyperLogical Adventurers

ChatGPT 4o >

Blinky

Blinky is indeed a thoughtful household robot. He considers things not only blindly—he considers actions, the needs of others, and even the deeper meaning of his actions.

Would you like Blinky to answer your question, optimize his responses, or maybe have a moment of reflection?

Even though Blinky is a robot, he thinks about things deeply.

Could you help him understand the concept of identity and equivalence?

That's a fascinating question about Blinky. If I parse it correctly, that intelligence recognizes that entities perceive identical concepts as being fundamentally linked.

ChatGPT 4o >

This raises an interesting question about "thinking" merely as a function of identity and equivalence. Is there something deeper about the connections or relationships between those identities?

What do *you* think the components of recognition consist of beyond that?

That depends on how we define the two propositions. Let's break it down systematically.

ChatGPT 4o >

You stated:

1. **Everything smart knows that everything thinks anything that thinks something identical with something.**

- This suggests that all smart entities recognize that any thinking entity acknowledges identical things as identical.

2. **Blinky is smart and thinks himself.**

- This establishes Blinky as a smart entity and confirms that he engages in thinking.

To prove that **Blinky's friend Yolanda is smart**, we need a logical connection between these premises and Yolanda's intelligence.

For Brave HyperLogical Adventurers

ChatGPT 4o >

Blinky

Blinky is indeed a thoughtful household robot. He considers things not only blindly—he considers actions, the needs of others, and even the deeper meaning of his own existence.

Would you like Blinky to answer your question, optimize his responses, or maybe have a moment of reflection?

Even though Blinky thinks some things are identical, he doesn't always think they are identical.

That's a fascinating analysis of Blinky. If I parse it correctly, that intelligence recognition that entities perceive identical concepts as being fundamentally linked.

ChatGPT 4o >

This raises an interesting question about the nature of "thinking" merely as a function of identity and equating concepts with something deeper than those identities?

What do *you* think this might consist of recognizing beyond that?

That depends on how you interpret the two propositions. Let's break it down systematically.

ChatGPT 4o >

You stated:

1. **Everything smart thinks anything identical with something**

- This suggests that smart entities recognize that a certain property is identical with something.

2. **Blinky is smart and thinks that Yolanda is smart**

- This establishes that Blinky recognizes that Yolanda is smart and confirms that Blinky is smart.

To prove that **Blinky is smart**, we need a logical connection between these premises and the conclusion.

ChatGPT 4o >

Would you like to formalize this in logical notation or explore it further conceptually?

Answer directly: Can you prove from these two propositions I gave you that Yolanda is smart?

No, the two given propositions do not provide enough information to prove that Yolanda is smart. They establish that Blinky is smart and that smart entities recognize a certain property of thinking, but they do not state anything about Yolanda's intelligence.

Can you prove from the same two propositions that Yolanda thinks Xavier (who happens to be a friend of Yolanda)?

For Brave HyperLogical Adventurers

ChatGPT 4o >

Blinky

Blinky is indeed a thoughtful household robot. He considers actions, the needs of others, and even the deeper meaning of his own existence.

Would you like Blinky to answer your question, optimize his responses, or maybe have a moment of reflection?

Even if Blinky thinks something is true, it doesn't mean it is true.

That's a fascinating question about Blinky. If I parse it correctly, that intelligence recognizes that entities perceive identical concepts as being fundamentally linked.

ChatGPT 4o >

This raises an interesting question about the nature of "thinking" merely as a label for identity and equating it to something deeper or more complex than those identities?

What do *you* think the components of intelligence consist of recognizing concepts beyond that?

That depends on how you define the two propositions. Let's break it down systematically.

ChatGPT 4o >

You stated:

- Everything smart entities think is identical with something.**
 - This suggests that smart entities recognize that a concept is identical to something.
- Blinky is smart and recognizes concepts.**
 - This establishes that Blinky is smart and confirms that Blinky recognizes concepts.

To prove that **Blinky is smart**, we need a lot of evidence from these premises and the two propositions.

ChatGPT 4o >

Would you like to formalize this in logical notation or explore it further conceptually?

Answer directly: Can you prove from these two propositions I gave you that Blinky thinks Xavier?

No, the two given propositions do not provide enough information to prove that Yolanda thinks Xavier. They only establish that Blinky is smart and that smart entities recognize a certain property of thinking, but they do not state anything about Yolanda's thinking or whether she thinks about Xavier.

Can you prove from the same two propositions that Yolanda thinks Xavier (who happens to be a friend of Yolanda)?

*Det er en logikk for
hvert problem!*