# Quantifiers; FOL I; "Proving" God's Existence

#### **Selmer Bringsjord**

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Formal Logic (& AI) 2/13/2025



### Why Amazon is Betting on 'Automated Reasoning' to Reduce AI's Hallucinations

The tech giant says an obscure field that combines Al and math can mitigate—but not completely eliminate—Al's propensity to provide wrong answers



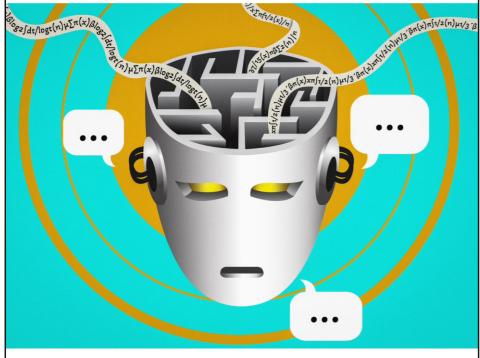
Hallucinations have been a problem for users since AI chatbots hit the mainstream over two years ago. (ILLUSTRATION: THOMAS R. LECHLEITER/WSJ)

By Belle Lin Follow

Feb 05, 2025 07:00 a.m. ET

### Why Amazon is Betting on 'Automated Reasoning' to Reduce AI's Hallucinations

The tech giant says an obscure field that combines Al and math can mitigate—but not completely eliminate—Al's propensity to provide wrong answers



Hallucinations have been a problem for users since AI chatbots hit the mainstream over two years ago. (ILLUSTRATION: THOMAS R. LECHLEITER/WSJ)

By Belle Lin Follow

Feb 05, 2025 07:00 a.m. ET

### Why Do AI Chatbots Have Such a Hard Time Admitting 'I Don't Know'?

Hallucinations are the hottest problem in artificial intelligence, spurring companies and researchers to find new solutions



By Ben Fritz Follow

Feb 11, 2025 05:30 a.m. ET

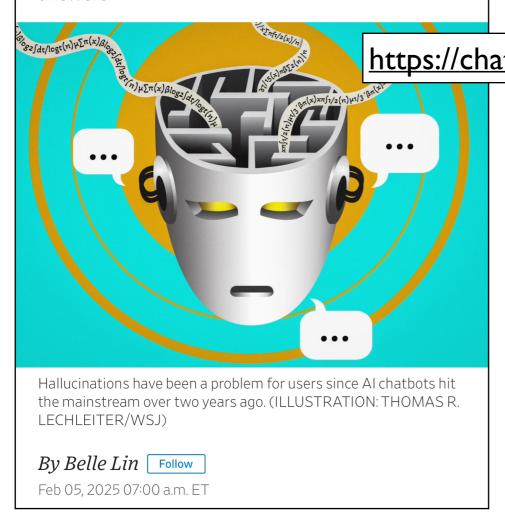
### Why Amazon is Betting on 'Automated Reasoning' to Reduce AI's Hallucinations

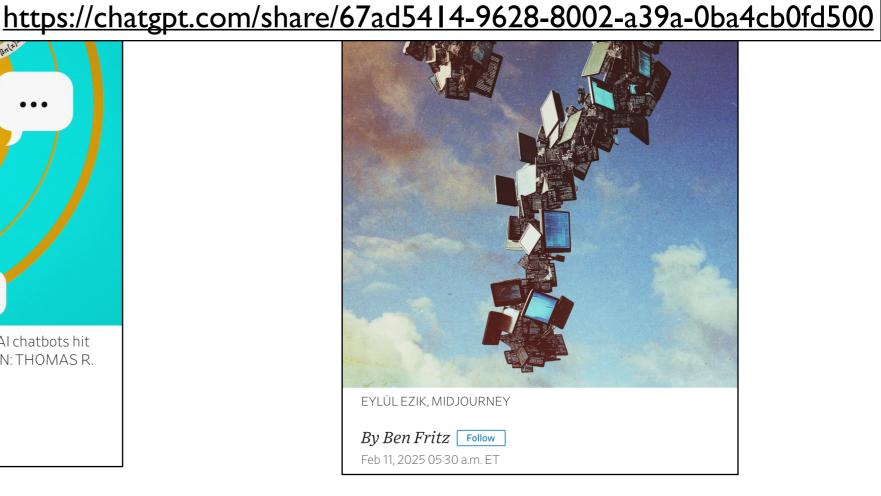
The tech giant says an obscure field that combines
Al and math can mitigate—but not completely
eliminate—Al's propensity to provide wrong
answers

### Why Do AI Chatbots Have Such a Hard Time Admitting 'I Don't Know'?

Hallucinations are the hottest problem in artificial intelligence, spurring companies and researchers to find new solutions

Generated by AI





### Re Test 1...

Re Test 1...

To repeat: any one problem: A

#### Re Test 1...

To repeat: any one problem: A

Use the Als (= oracles), & use them wisely ...

# HyperGrader® Required Problems: Self-paced, yes! — but interconnected!

BogusBiconditional

tertium\_non\_datur

Disj\_Elim

**Bogus Biconditional** 

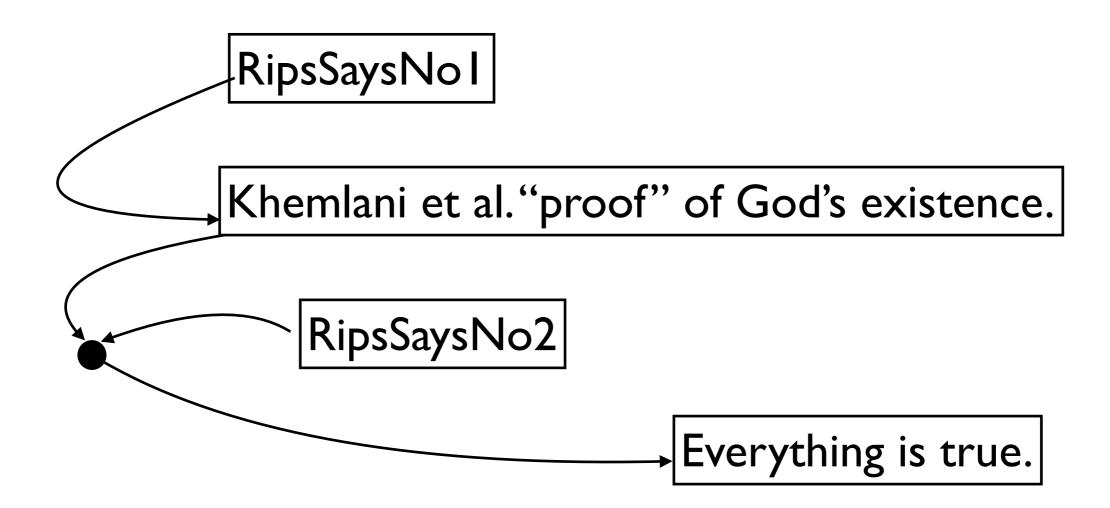
RipsSaysNo1

RipsSaysNo2

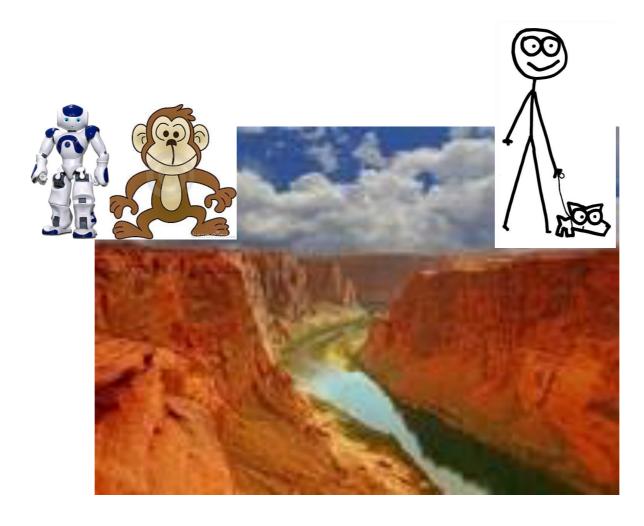
BogusBiconditional

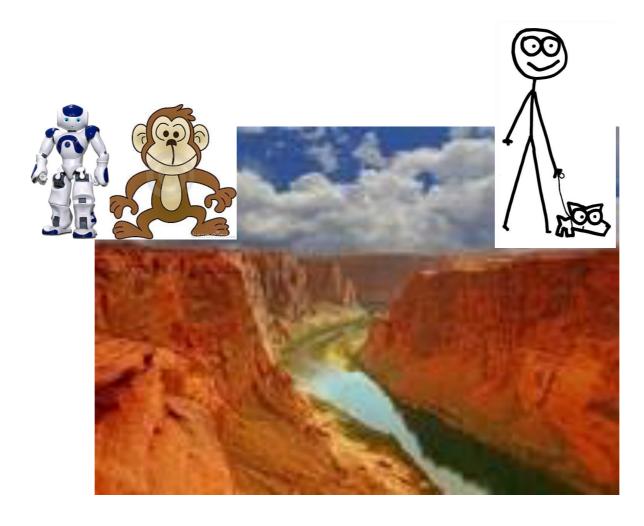
tertium\_non\_datur

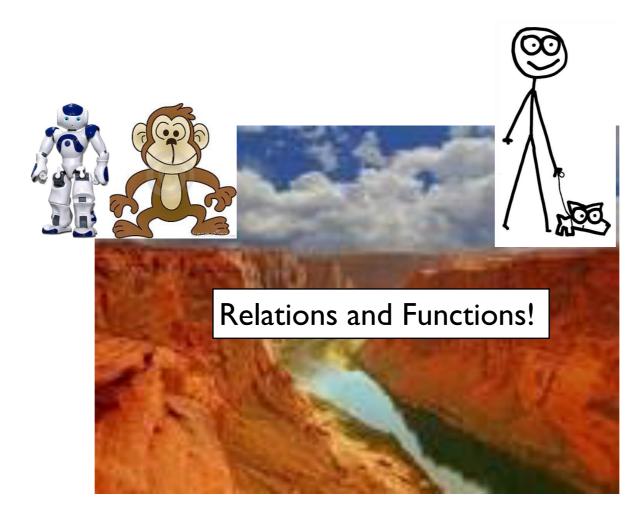
Disj\_Elim

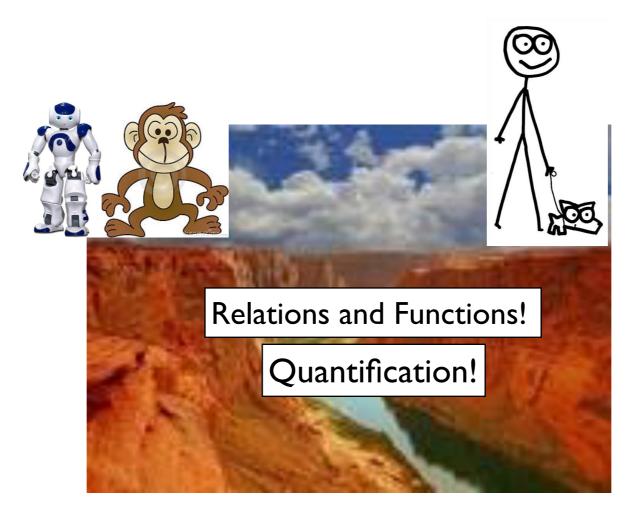


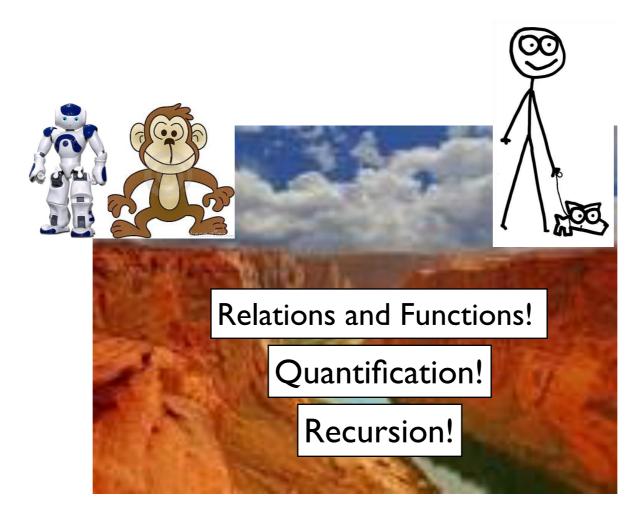
### Quantifiers (etc) ...













### Quantification!



#### Karkooking Problem ...

Everyone karkooks anyone who karkooks someone.

Alvin karkooks Bill.

Can you infer that everyone karkooks Bill?

**ANSWER:** 

JUSTIFICATION:

#### Karkooking Problem ...

Everyone karkooks anyone who karkooks someone.

Alvin karkooks Bill.

Can you infer that everyone karkooks Bill?

**ANSWER:** 

JUSTIFICATION:

Relations and Functions!

Quantification!

Recursion!

- All mammals walk.
- Whales are mammals.
- Therefore:
- Whales walk.

- All of the Frenchmen in the room are winedrinkers.
- Some of the wine-drinkers in the room are gourmets.
- Therefore:
- Some of the Frenchmen in the room are gourmets.

- All mammals walk.
- Whales are mammals.
- Therefore:
- Whales walk.

- All of the Frenchmen in the room are winedrinkers.
- Some of the wine-drinkers in the room are gourmets.
- Therefore:
- Some of the Frenchmen in the room are gourmets.

- All mammals walk.  $\forall x [M(x) \rightarrow W(x)]$
- Whales are mammals.  $\forall x(Wh(x) \rightarrow M(x))$
- Therefore:
- Whales walk.

 $\forall x (Wh(x) \rightarrow W(x))$ 

- All of the Frenchmen in the room are winedrinkers.
- Some of the wine-drinkers in the room are gourmets.
- Therefore:
- Some of the Frenchmen in the room are gourmets.

- All mammals walk.  $\forall x[M(x) \rightarrow W(x)]$
- Whales are mammals.  $\forall x(Wh(x) \rightarrow M(x))$
- Therefore:
- Whales walk.

```
\forall x (Wh(x) \rightarrow W(x))
```

- All of the Frenchmen in the room are winedrinkers.  $\forall x(F(x) \rightarrow W(x))$
- Some of the wine-drinkers in the room are gourmets.

 $\exists x (W(x) \land G(x))$ 

- Therefore:
- Some of the Frenchmen in the room are gourmets.  $\exists x(F(x) \land G(x))$

s-expressions

### Two Proposed Arguments; Valid?

- All mammals walk.  $\forall x[M(x) \rightarrow W(x)]$
- Whales are mammals.  $\forall x(Wh(x) \rightarrow M(x))$
- Therefore:
- Whales walk.

```
\forall x (Wh(x) \rightarrow W(x))
```

- All of the Frenchmen in the room are winedrinkers. ∀x(F(x) → W(x))
- Some of the wine-drinkers in the room are gourmets.

 $\exists x (W(x) \land G(x))$ 

- Therefore:
- Some of the Frenchmen in the room are gourmets.  $\exists x(F(x) \land G(x))$

s-expressions

### Two Proposed Arguments; Valid?

- All mammals walk.  $\forall x[M(x) \rightarrow W(x)]$
- Whales are mammals.  $\forall x(Wh(x) \rightarrow M(x))$
- Therefore:
- Whales walk.

```
\forall x (Wh(x) \rightarrow W(x))
```

 All of the Frenchmen in the room are winedrinkers. ∀x(F(x) → W(x))

```
\forall x (F(x) \to W(x)) \bullet (forall (x) (if (F x) (W x)))
```

 Some of the wine-drinkers in the room are gourmets.

```
\exists x (W(x) \land G(x))
```

• Therefore:

• Some of the Frenchmen in the room are gourmets.  $\exists x(F(x) \land G(x))$ 

- All mammals walk.  $\forall x[M(x) \rightarrow W(x)]$
- Whales are mammals.  $\forall x(Wh(x) \rightarrow M(x))$
- Therefore:
- Whales walk.

```
\forall x (Wh(x) \rightarrow W(x))
```

• All of the Frenchmen in the room are wine-drinkers.  $\forall x(F(x) \rightarrow W(x))$ 

```
\forall x (F(x) \to W(x)) \bullet (forall (x) (if (F x) (W x)))
```

 Some of the wine-drinkers in the room are gourmets.

```
\exists x (W(x) \land G(x)) \exists x (W(x) \land G(x)) \bullet (\text{exists } (x) (\text{and } (\mathbb{W} \ x) (\mathbb{G} \ x)))
```

• Therefore:

• Some of the Frenchmen in the room are gourmets.  $\exists x(F(x) \land G(x))$ 

- All mammals walk.  $\forall x[M(x) \rightarrow W(x)]$
- Whales are mammals.  $\forall x(Wh(x) \rightarrow M(x))$
- Therefore:
- Whales walk.

```
\forall x (Wh(x) \rightarrow W(x))
```

• All of the Frenchmen in the room are wine-drinkers.  $\forall x(F(x) \rightarrow W(x))$ 

```
\forall x (F(x) \to W(x)) \cdot (forall(x)(if(Fx)(Wx)))
```

 Some of the wine-drinkers in the room are gourmets.

```
\exists x (W(x) \land G(x)) \exists x (W(x) \land G(x)) \bullet (\text{exists} (x) (\text{and} (W x) (G x)))
```

• Therefore:

 Some of the Frenchmen in the room are gourmets.

```
\exists x (F(x) \land G(x)) \bullet (\text{exists } (x) (\text{and } (F x) (G x)))
```

# Historically speaking (recall) ...

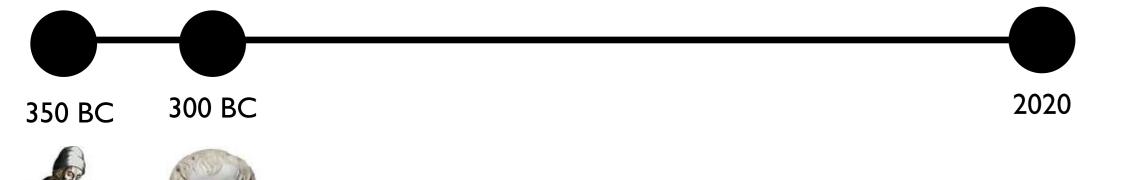


350 BC





Euclid



Euclid

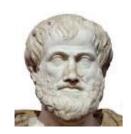


350 BC

300 BC

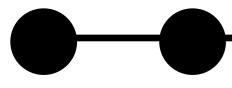






"I don't believe in magic! Why exactly is that so convincing? What the heck is he doing?!? I know! ..."

**Euclid** 



350 BC

300 BC



**Euclid** 



Organon

"I don't believe in magic! Why exactly is that so convincing? What the heck is he doing?!? I know! ..."

2020

"He's using syllogisms!"

E.g.,

All As are Bs.

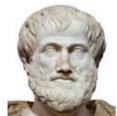
All Bs are Cs.

All As are Cs.



350 BC

300 BC



**Euclid** 



Organon

2020

"I don't believe in magic! Why exactly is that so convincing? What the heck is he doing?!? I know! ..."

"He's using syllogisms!"

E.g.,

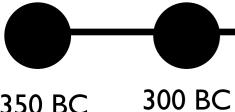
All As are Bs. All Bs are Cs.

All As are Cs.

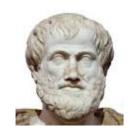


"No. Euclid's proofs are compelling because they are informal versions of proofs in something I've invented: firstorder logic (= FOL =  $\mathcal{L}_1$ )."

2020

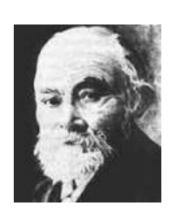


350 BC

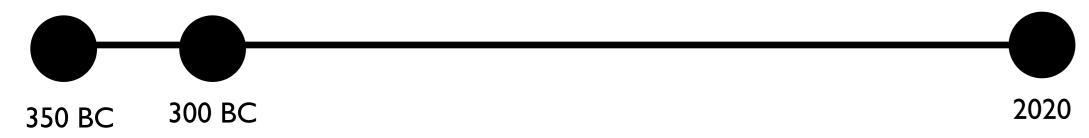


Organon **Euclid** 

"I don't believe in magic! Why exactly is that so convincing? What the heck is he doing?!? I know! ..."

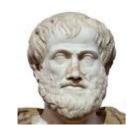


"No. Euclid's proofs are compelling because they are informal versions of proofs in something I've invented: first-order logic (= FOL =  $\mathcal{L}_1$ )."





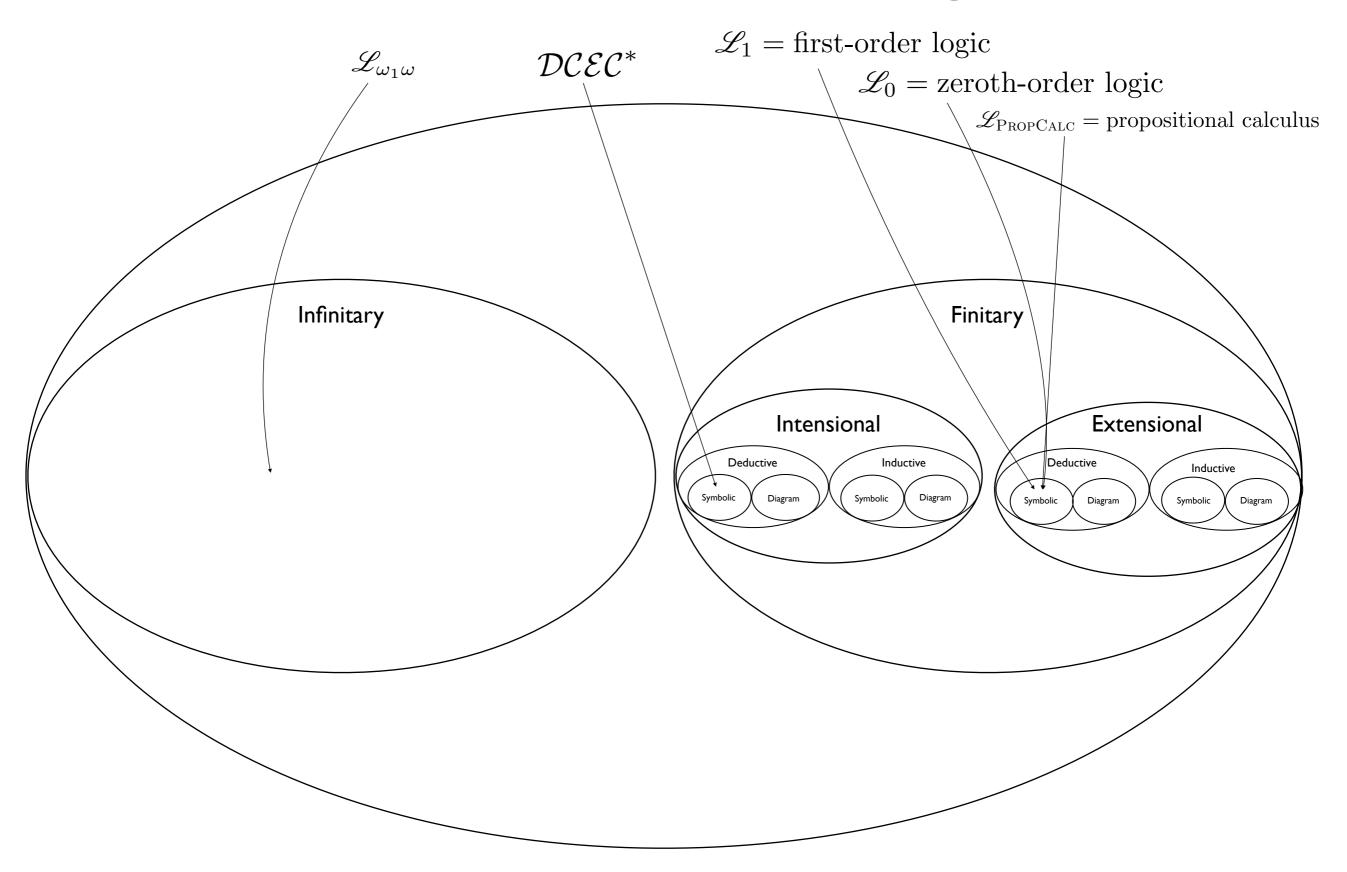
**Euclid** 



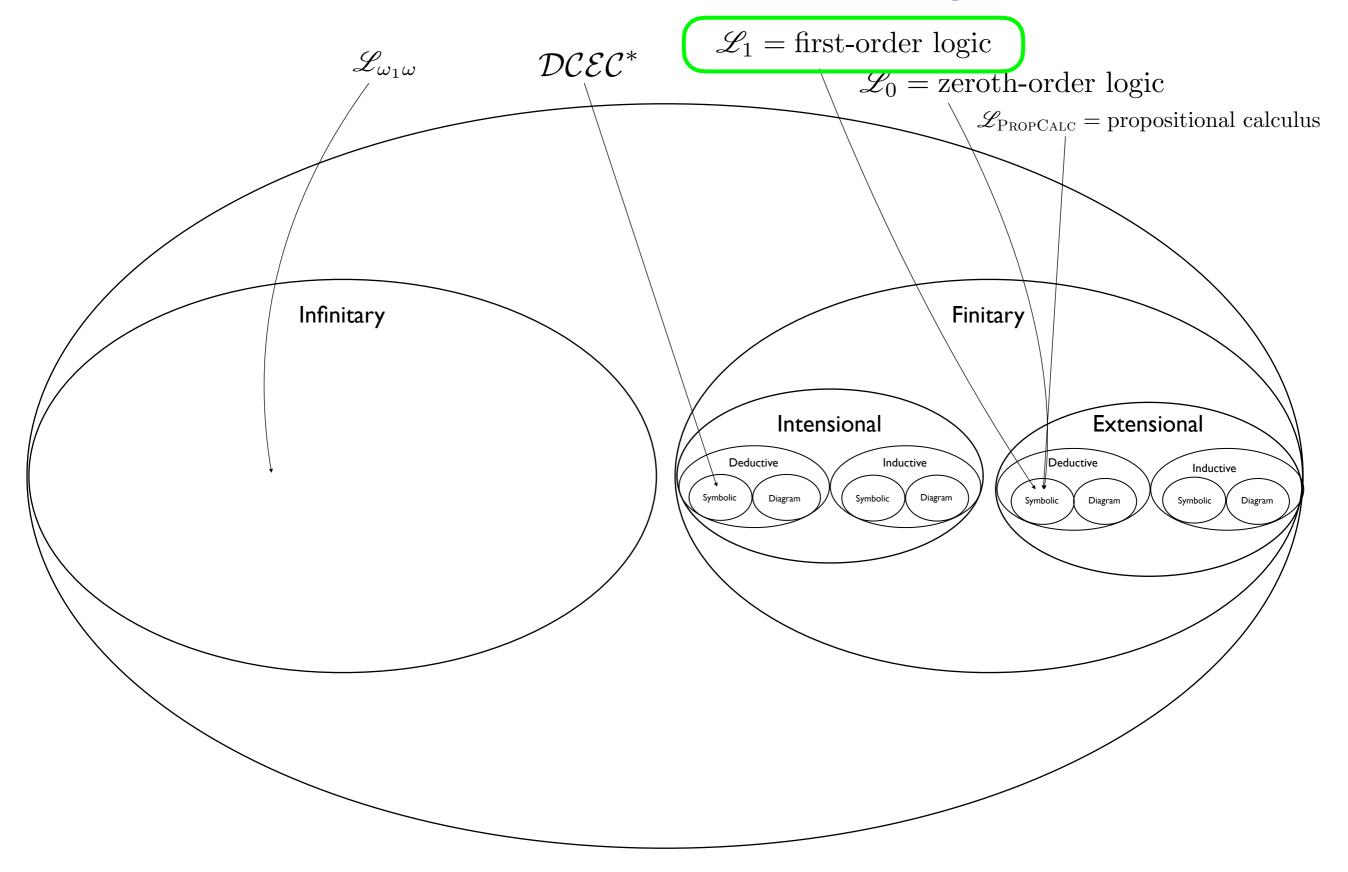
Organon

"I don't believe in magic! Why exactly is that so convincing? What the heck is he doing?!!? I know! ..."

# The Universe of Logics



# The Universe of Logics



universal elimination

- universal elimination
  - If everything is an R, then the particular thing a is an R.

- universal elimination
  - If everything is an R, then the particular thing a is an R.
- existential introduction

- universal elimination
  - If everything is an R, then the particular thing a is an R.
- existential introduction
  - If a is an R, then at least one thing is an R.

- universal elimination
  - If everything is an R, then the particular thing a is an R.
- existential introduction
  - If a is an R, then at least one thing is an R.
- And now we have enough to "prove" that God exists in HyperSlate:)!

- universal elimination
  - If everything is an R, then the particular thing a is an R.
- existential introduction
  - If a is an R, then at least one thing is an R.
- And now we have enough to "prove" that God exists in HyperSlate:)!
  - My apologies to:

- universal elimination
  - If everything is an R, then the particular thing a is an R.
- existential introduction
  - If a is an R, then at least one thing is an R.
- And now we have enough to "prove" that God exists in HyperSlate:)!
  - My apologies to:





- universal elimination
  - If everything is an R, then the particular thing a is an R.
- existential introduction
  - If a is an R, then at least one thing is an R.
- And now we have enough to "prove" that God exists in HyperSlate:)!
  - My apologies to:





Scott's Version of Gödel's Proof, Verified by AI

- universal elimination
  - If everything is an R, then the particular thing a is an R.
- existential introduction
  - If a is an R, then at least one thing is an R.
- And now we have enough to "prove" that God exists in HyperSlate:)!
  - My apologies to:





Scott's Version of Gödel's Proof, Verified by AI

- universal elimination
  - If everything is an R, then the particular thing a is an R.
- existential introduction
  - If a is an R, then at least one thing is an R.
- And now we have enough to "prove" that God exists in HyperSlate:)!
  - My apologies to:





Scott's Version of Gödel's Proof, Verified by AI

## universal elimination

 $\forall \phi [P(\neg \phi) \leftrightarrow \neg P(\phi)]$ **A1** Either a property or its negation is positive, but not both: **A2** A property necessarily implied by a positive property is positive:  $\forall \phi \forall \psi [(P(\phi) \land \Box \forall x [\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$  $\forall \varphi [P(\varphi) \to \Diamond \exists x \varphi(x)]$ **T1** Positive properties are possibly exemplified:  $G(x) \leftrightarrow \forall \phi [P(\phi) \to \phi(x)]$ **D1** A God-like being possesses all positive properties: **A3** The property of being God-like is positive:  $\Diamond \exists x G(x)$ Possibly, God exists:  $\forall \phi [P(\phi) \rightarrow \Box P(\phi)]$ **A4** Positive properties are necessarily positive: **D2** An essence of an individual is a property possessed by it and necessarily implying any of its properties:  $\phi$  ess.  $x \leftrightarrow \phi(x) \land \forall \psi(\psi(x) \rightarrow \Box \forall y(\phi(y) \rightarrow \psi(y)))$  $\forall x [G(x) \to G \ ess. \ x]$ **T2** Being God-like is an essence of any God-like being: D3 Necessary existence of an individual is the necessary exemplification of all its essences:  $NE(x) \leftrightarrow \forall \phi [\phi \ ess. \ x \rightarrow \Box \exists y \phi(y)]$ **A5** Necessary existence is a positive property: P(NE)

## • My apologies to:

**T3** Necessarily, God exists:









 $\Box \exists x G(x)$ 

Scott's Version of Gödel's Proof, Verified by AI

 $\mathcal{L}_3 + \text{modal logic } \mathbf{S5}$ 

## universal elimination

**A1** Either a property or its negation is positive, but not both:

 $\forall \phi [P(\neg \phi) \leftrightarrow \neg P(\phi)]$ 

**A2** A property necessarily implied

by a positive property is positive:

 $\forall \phi \forall \psi [(P(\phi) \land \Box \forall x [\phi(x) \to \psi(x)]) \to P(\psi)]$ 

**T1** Positive properties are possibly exemplified:

 $\forall \varphi [P(\varphi) \to \Diamond \exists x \varphi(x)]$  $G(x) \leftrightarrow \forall \phi [P(\phi) \to \phi(x)]$ 

**D1** A *God-like* being possesses all positive properties:

 $f(x) \leftrightarrow \forall \psi[1 \ (\psi) \rightarrow \psi(x)]$  P(G)

A3 The property of being God-like is positive:C Possibly, God exists:

 $\Diamond \exists x G(x)$ 

A4 Positive properties are necessarily positive:

 $\forall \phi [P(\phi) \rightarrow \Box P(\phi)]$ 

**D2** An essence of an individual is

a property possessed by it and

necessarily implying any of its properties:  $\phi$  ess.  $x \leftrightarrow \phi(x) \land \forall \psi(\psi(x) \rightarrow \Box \forall y(\phi(y) \rightarrow \psi(y)))$ 

**T2** Being God-like is an essence of any God-like being:

 $\forall x[G(x) \to G \ ess. \ x]$ 

**D3** Necessary existence of an individual is

the necessary exemplification of all its essences:

 $NE(x) \leftrightarrow \forall \phi [\phi \ ess. \ x \rightarrow \Box \exists y \phi(y)]$ 

**A5** Necessary existence is a positive property:

P(NE)

**T3** Necessarily, God exists:

 $\Box \exists x G(x)$ 

## I'vy apologies to:









Scott's Version of Gödel's Proof, Verified by AI

### **COGNITIVE SCIENCE**

A Multidisciplinary Journal



Cognitive Science 42 (2018) 1887–1924 © 2018 Cognitive Science Society, Inc. All rights reserved. ISSN: 1551-6709 online

DOI: 10.1111/cogs.12634

#### Facts and Possibilities: A Model-Based Theory of Sentential Reasoning

Sangeet S. Khemlani, a Ruth M. J. Byrne, Philip N. Johnson-Laird C,d

<sup>a</sup>Navy Center for Applied Research in Artificial Intelligence, US Naval Research Laboratory

<sup>b</sup>School of Psychology and Institute of Neuroscience, Trinity College Dublin, University of Dublin

<sup>c</sup>Department of Psychology, Princeton University

<sup>d</sup>Department of Psychology, New York University

Received 8 April 2017; received in revised form 17 April 2018; accepted 3 May 2018

#### Abstract

This article presents a fundamental advance in the theory of mental models as an explanation of reasoning about facts, possibilities, and probabilities. It postulates that the meanings of compound assertions, such as conditionals (*if*) and disjunctions (*or*), unlike those in logic, refer to conjunctions of epistemic possibilities that hold in default of information to the contrary. Various factors such as general knowledge can modulate these interpretations. New information can always override sentential inferences; that is, reasoning in daily life is defeasible (or nonmonotonic). The theory is a dual process one: It distinguishes between intuitive inferences (based on system 1) and deliberative inferences (based on system 2). The article describes a computer implementation of the theory, including its two systems of reasoning, and it shows how the program simulates crucial predictions that evidence corroborates. It concludes with a discussion of how the theory contrasts with those based on logic or on probabilities.

Keywords: Deduction; Logic; Mental models; Nonmonotonicity; Reasoning; Possibility

#### 1. Introduction

People reason about facts, possibilities, and probabilities. Psychologists have carried out many studies of factual inferences, such as:

If the card is an ace then it is a heart.
 The card is an ace.
 Therefore, the card is a heart.

Correspondence should be sent to Sangeet Khemlani, Navy Center for Applied Research in Artificial Intelligence, Naval Research Laboratory, 4555 Overlook Drive, Washington, DC 20375. E-mail: skhemlani@gmail.com

#### COCNITIVE SCIENCE



S. S. Khemlani, R. M. J. Byrne, P. N. Johnson-Laird/Cognitive Science 42 (2018)

1917

seem true a priori and those that are contingent is "an unempirical dogma of empiricism." Not anymore. The empirical studies we have described show that individuals innocent of philosophical niceties judged that assertions can be true (or false) a priori as a result of their meaning.

In logic, if a material conditional is false then its *if*-clause is true. So a very short proof for the existence of God is sound in logic:

38. It is not the case that if God exists then atheism is correct. Therefore, God exists.

Its premise is true, and it implies both that God exists and that atheism is not correct. It therefore follows from this conjunction that God exists. In the model theory, a conditional's meaning is not a material implication, not a conditional probability, not a set of possible worlds, and not an inferential relation. It is instead a conjunction of possibilities, each of which is assumed in default of information to the contrary. And so the falsity of a conditional does not imply that its *if*-clause is true, which renders the "proof" in (38) invalid. Individuals judge that the following assertion is false:

39. If Sonia has pneumonia then she is healthy.

But its falsity does not imply that Sonia has pneumonia, and indeed individuals judge that it is possible that Sonia does not have pneumonia (Quelhas et al., 2016). Only one case is impossible:

Sonia has pneumonia Sonia is healthy

That is why (39) is false. The modulation algorithm we described mirrors these evaluations. Yet a complex sort of modulation is at present beyond the program. As Byrne (1989) showed, individuals draw their own conclusion from premises, such as:

If she meets her friend then she will go to a play.
 She meets her friend.

They infer that she will go to a play. But when the premises have a further conditional of the following sort added to them:

41. If she has enough money then she will go to a play.

reasoners tend not to make the inference (see also Byrne, Espino, & Santamaria, 1999). The additional premise reminds them of a necessary condition for going to a play: One needs money to pay for the tickets. But no premise has established this condition, and so they balk at the inference. The inference is complex, and the modulation algorithm has yet to capture it.

S. S. Khemlani, R. M. J. Byrne, P. N. Johnson-Laird/Cognitive Science 42 (2018)

1917

seem true a priori and those that are contingent is "an unempirical dogma of empiricism." Not anymore. The empirical studies we have described show that individuals innocent of philosophical niceties judged that assertions can be true (or false) a priori as a result of their meaning.

In logic, if a material conditional is false then its *if*-clause is true. So a very short proof for the existence of God is sound in logic:

It is not the case that if God exists then atheism is correct.
 Therefore, God exists.

Its premise is true, and it implies both that God exists and that atheism is not correct. It therefore follows from this conjunction that God exists. In the model theory, a conditional's meaning is not a material implication, not a conditional probability, not a set of possible worlds, and not an inferential relation. It is instead a conjunction of possibilities, each of which is assumed in default of information to the contrary. And so the falsity of a conditional does not imply that its *if*-clause is true, which renders the "proof" in (38) invalid. Individuals judge that the following assertion is false:

39. If Sonia has pneumonia then she is healthy.

But its falsity does not imply that Sonia has pneumonia, and indeed individuals judge that it is possible that Sonia does not have pneumonia (Quelhas et al., 2016). Only one case is impossible:

Sonia has pneumonia Sonia is healthy

That is why (39) is false. The modulation algorithm we described mirrors these evaluations. Yet a complex sort of modulation is at present beyond the program. As Byrne (1989)

S. S. Khemlani, R. M. J. Byrne, P. N. Johnson-Laird/Cognitive Science 42 (2018)

1917

seem true a priori and those that are contingent is "an unempirical dogma of empiricism." Not anymore. The empirical studies we have described show that individuals innocent of philosophical niceties judged that assertions can be true (or false) a priori as a result of their meaning.

In logic, if a material conditional is false then its *if*-clause is true. So a very short proof for the existence of God is sound in logic:

It is not the case that if God exists then atheism is correct.
 Therefore, God exists.

Its premise is true, and it implies both that God exists and that atheism is not correct. It therefore follows from this conjunction that God exists. In the model theory, a conditional's meaning is not a material implication, not a conditional probability, not a set of possible worlds, and not an inferential relation. It is instead a conjunction of possibilities, each of which is assumed in default of information to the contrary. And so the falsity of a conditional does not imply that its *if*-clause is true, which renders the "proof" in (38) invalid. Individuals judge that the following assertion is false:

39. If Sonia has pneumonia then she is healthy.

But its falsity does not imply that Sonia has pneumonia, and indeed individuals judge that it is possible that Sonia does not have pneumonia (Quelhas et al., 2016). Only one case is impossible:

Sonia has pneumonia Sonia is healthy

That is why (39) is false. The modulation algorithm we described mirrors these evaluations. Yet a complex sort of modulation is at present beyond the program. As Byrne (1989)

S. S. Khemlani, R. M. J. Byrne, P. N. Johnson-Laird/Cognitive Science 42 (2018)

1917

seem true a priori and those that are contingent is "an unempirical dogma of empiricism." Not anymore. The empirical studies we have described show that individuals innocent of philosophical niceties judged that assertions can be true (or false) a priori as a result of their meaning.

In logic, if a material conditional is false then its *if*-clause is true. So a very short proof for the existence of God is sound in logic:

It is not the case that if God exists then atheism is correct.
 Therefore, God exists.

Its premise is true, and it implies both that God exists and that atheism is not correct. It therefore follows from this conjunction that God exists. In the model theory, a conditional's meaning is not a material implication, not a conditional probability, not a set of possible worlds, and not an inferential relation. It is instead a conjunction of possibilities, each of which is assumed in default of information to the contrary. And so the falsity of a conditional does not imply that its *if*-clause is true, which renders the "proof" in (38) invalid. Individuals judge that the following assertion is false:

39. If Sonia has pneumonia then she is healthy.

But its falsity does not imply that Sonia has pneumonia, and indeed individuals judge that it is possible that Sonia does not have pneumonia (Quelhas et al., 2016). Only one case is impossible:

Sonia has pneumonia Sonia is healthy

That is why (39) is false. The modulation algorithm we described mirrors these evaluations. Yet a complex sort of modulation is at present beyond the program. As Byrne (1989)

# Part I: Slutten — for i dag.

Part I: Slutten — for i dag.

Part II: Hands-on Q&A & Review ...

# Den rasjonelle delen av menneskesinnet er basert på logikk.