FOL II: `universal intro`

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Intro to Formal Logic (& AI)
2/18/2025

R A I R
Rensselaer AI and Reasoning Lab

Logic-&-AI In The News

BUSINESS

Meta's AI-Powered Ray-Bans Are Life-Enhancing for the Blind

Tech giant makes smart specs for general public; visually impaired owners use them for everyday tasks, though critics cite safety concerns

Allison Pomeroy wearing her Meta smart glasses, alongside her husband, DJ Pomeroy. (PHOTO: DJ POMEROY)

By Sarah E. Needleman Follow

Feb 17, 2025 05:30 a.m. ET

Re Test 1 …

(More-Forgiving) Grading Scheme

A (4): 1 ∨ … ∨ 6/6

A+: (5) 6/6

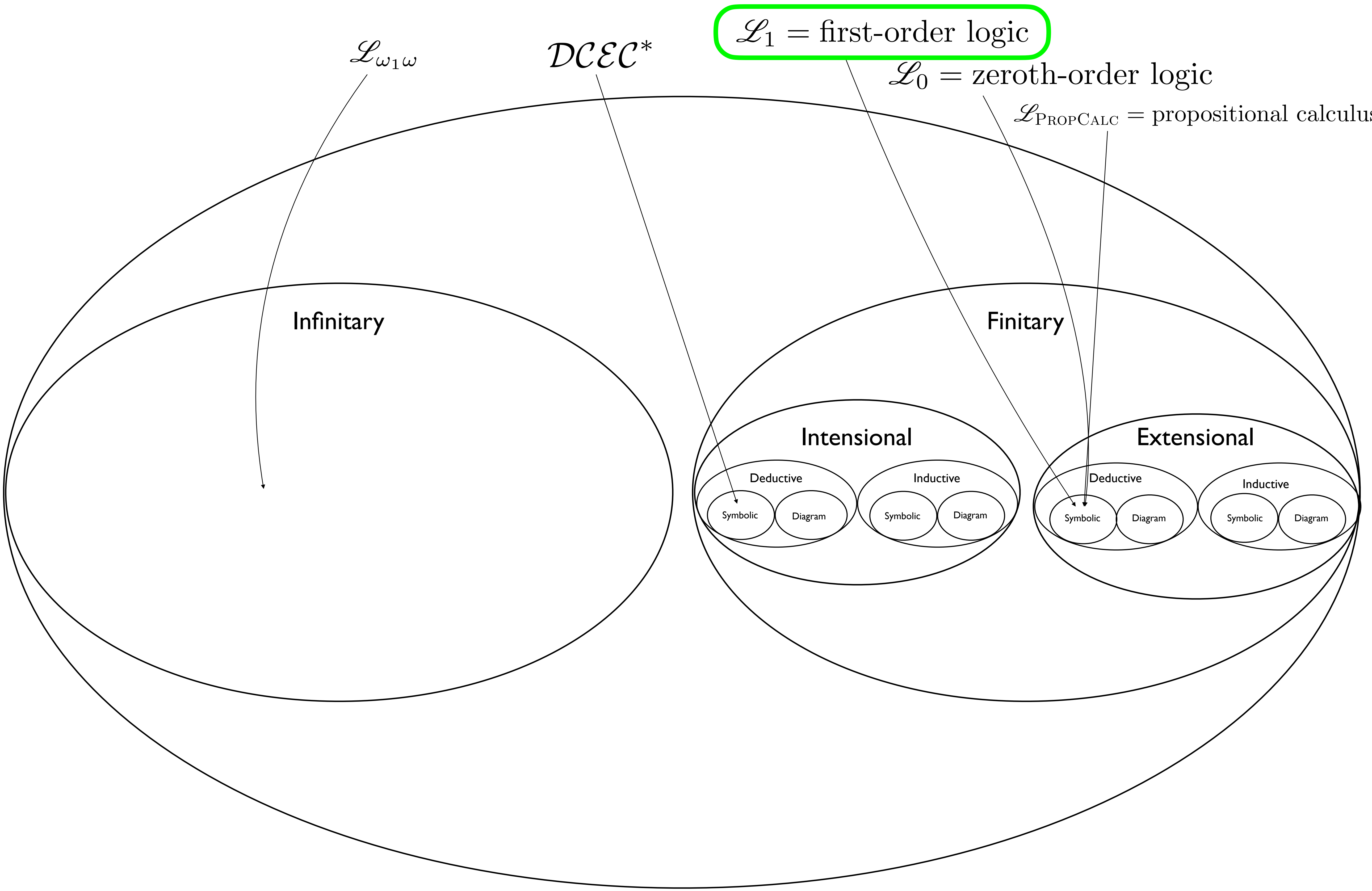
(More-Forgiving) Grading Scheme

A (4): $1 \vee \ldots \vee 6/6$

A+: (5) 6/6

Part 2 Today for Help etc; remarks on DeMorgan's Theorem.

The Universe of Logics



Next New (*Not*-So-Easy!) Inference Rule in FOL

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- `universal introduction`

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  - If something *a* is an *R*, and the constant/name *a* is *genuinely arbitrary*, then we can deduce that everything is an *R*.

The Inference Schema

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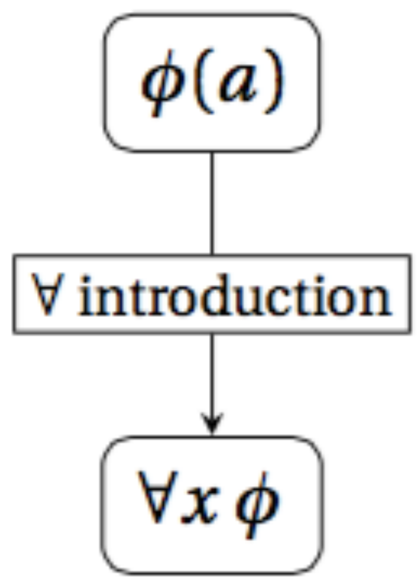
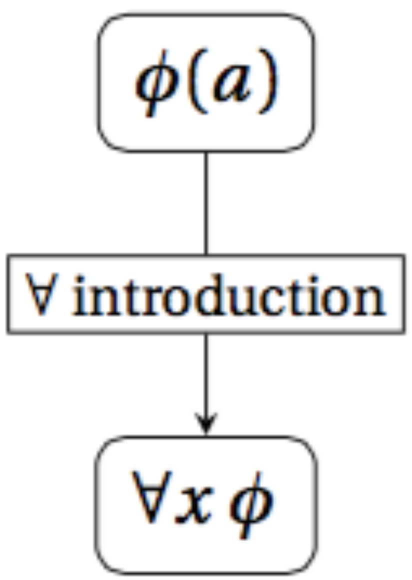


provided that a does not appear free in any in-scope assumption of ϕ, and that no occurrence of a appear in the inferred $\forall x\, \phi$ (3.16)

The Inference Schema



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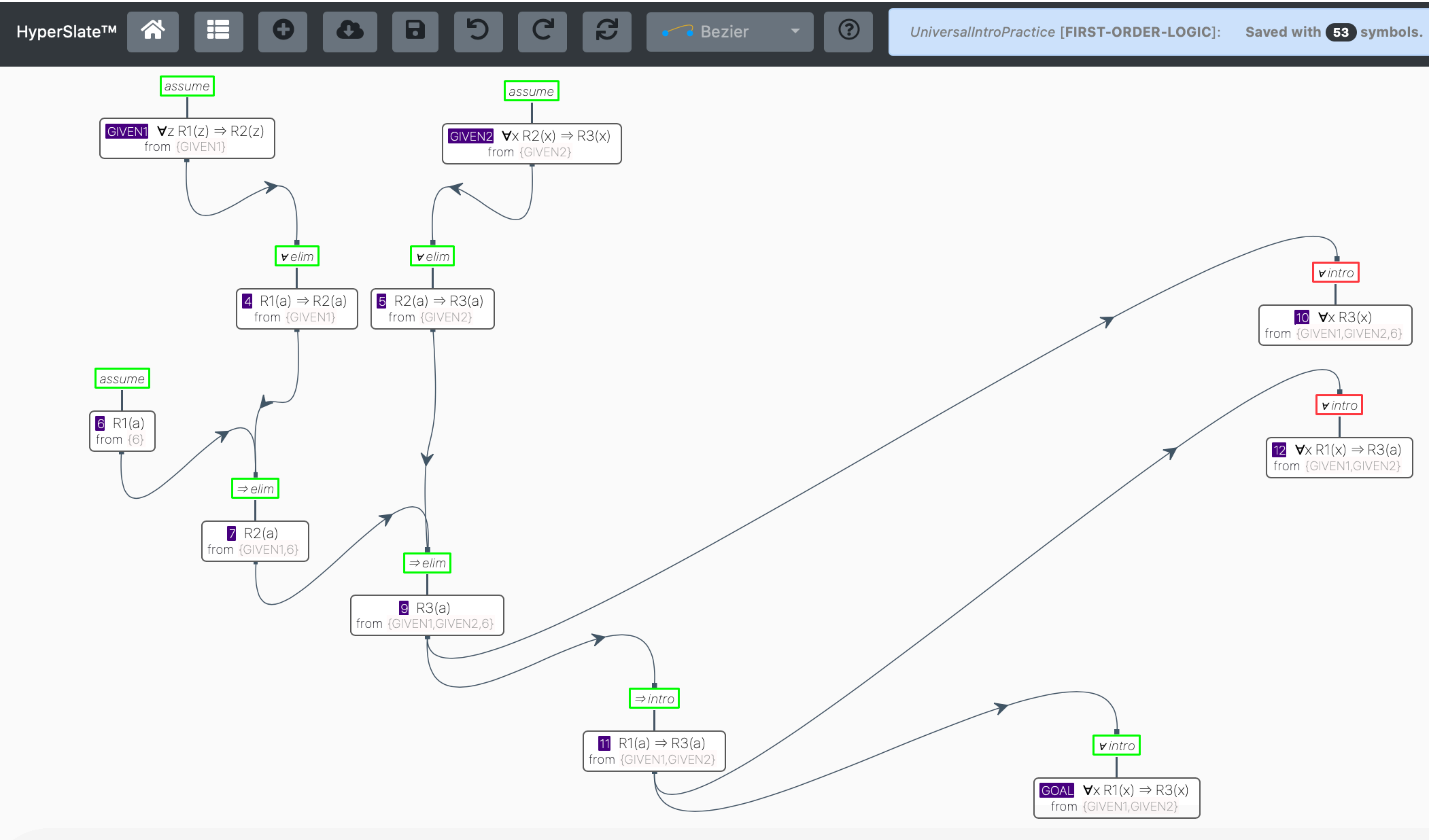
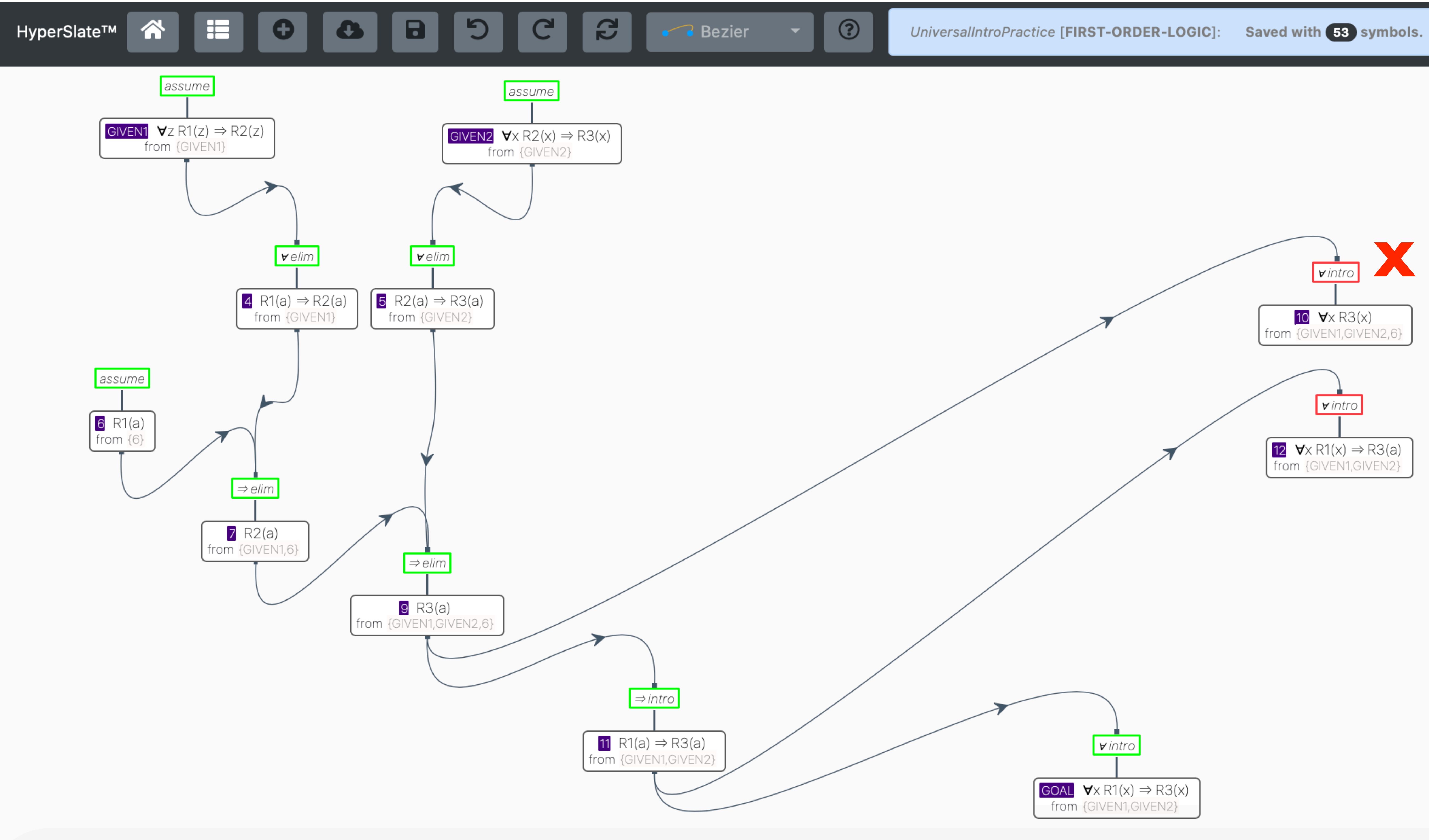
(Why the provisos?)

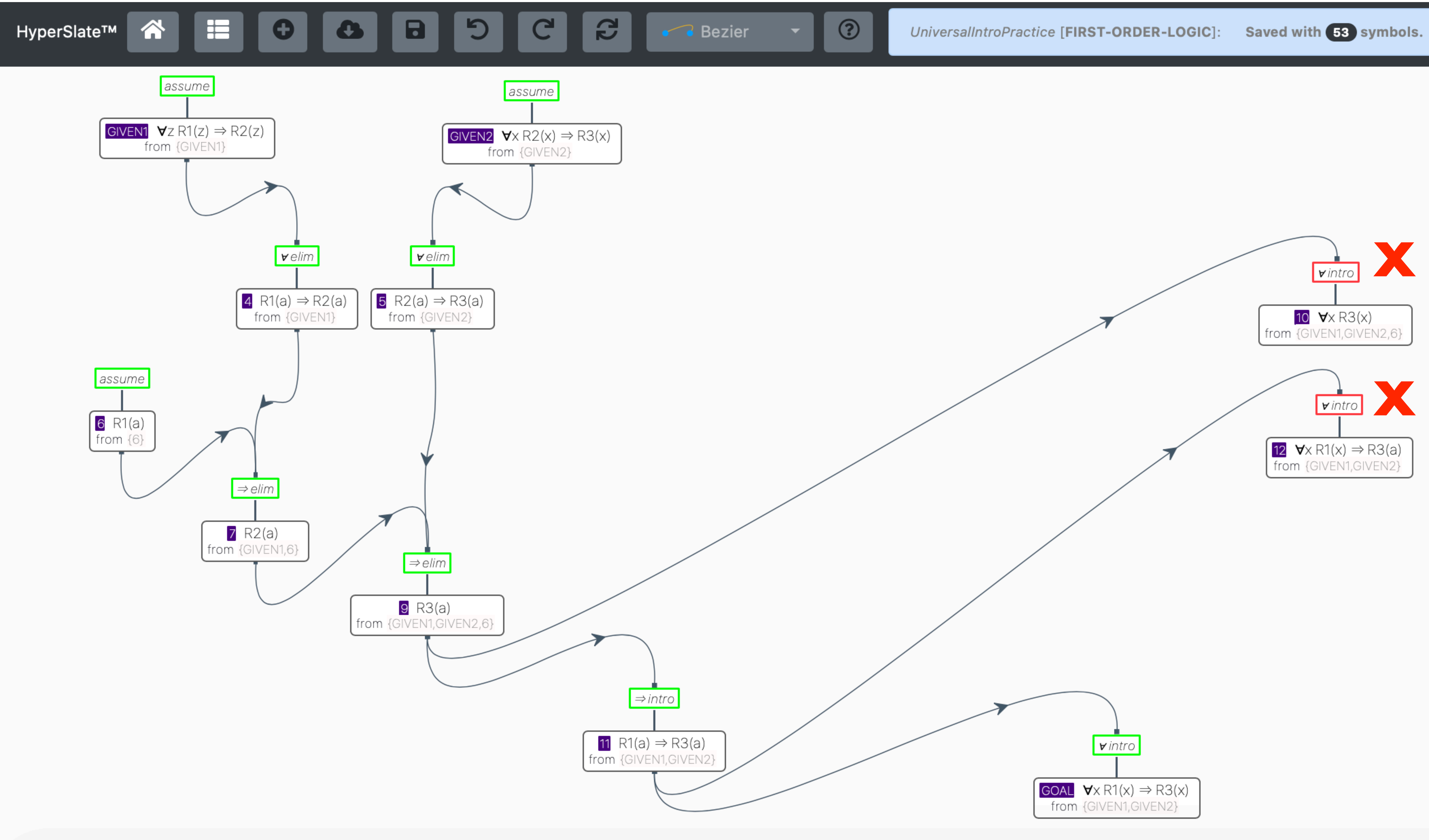
universal intro Example/Tutorial



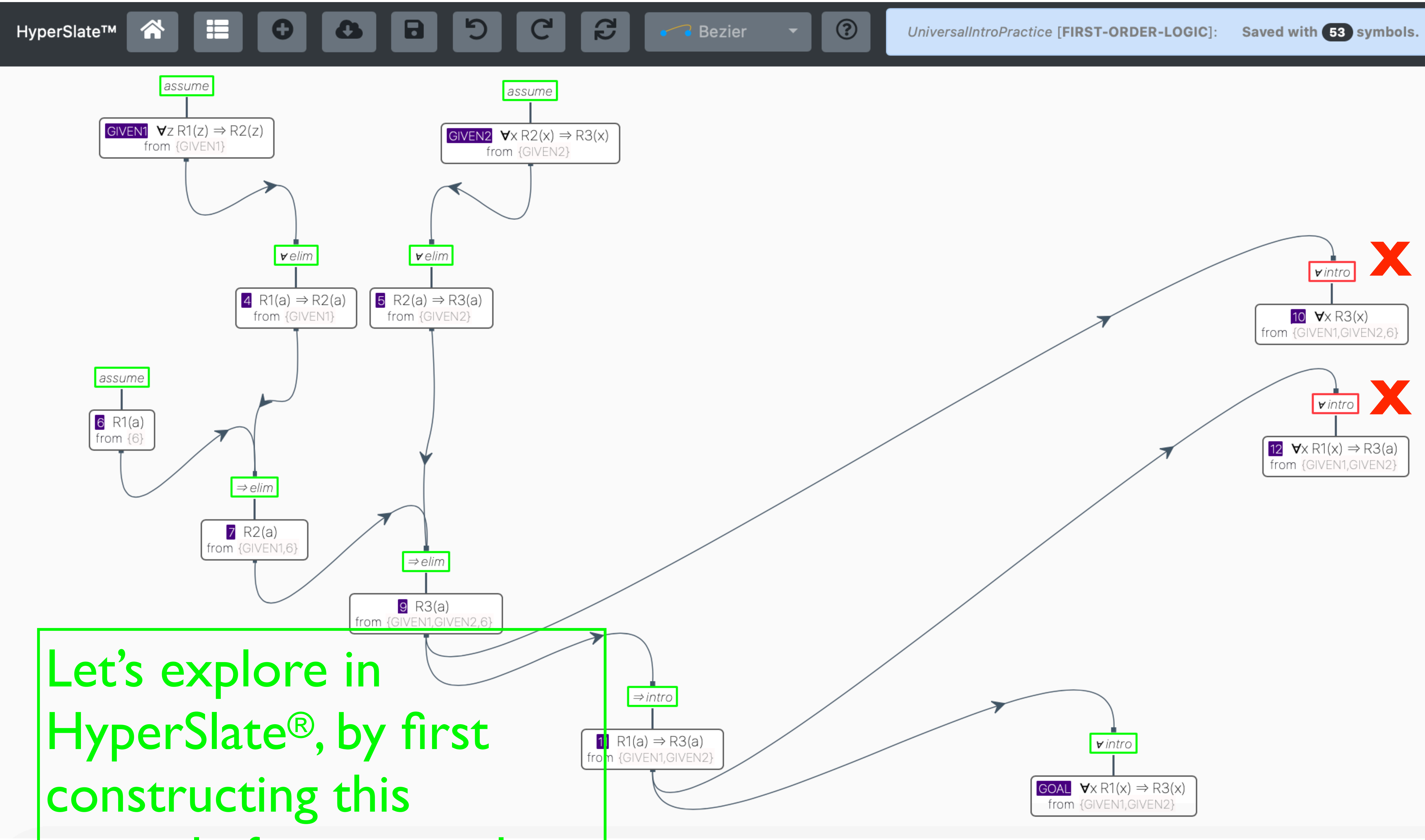
universal intro Example/Tutorial



`universal intro` Example/Tutorial

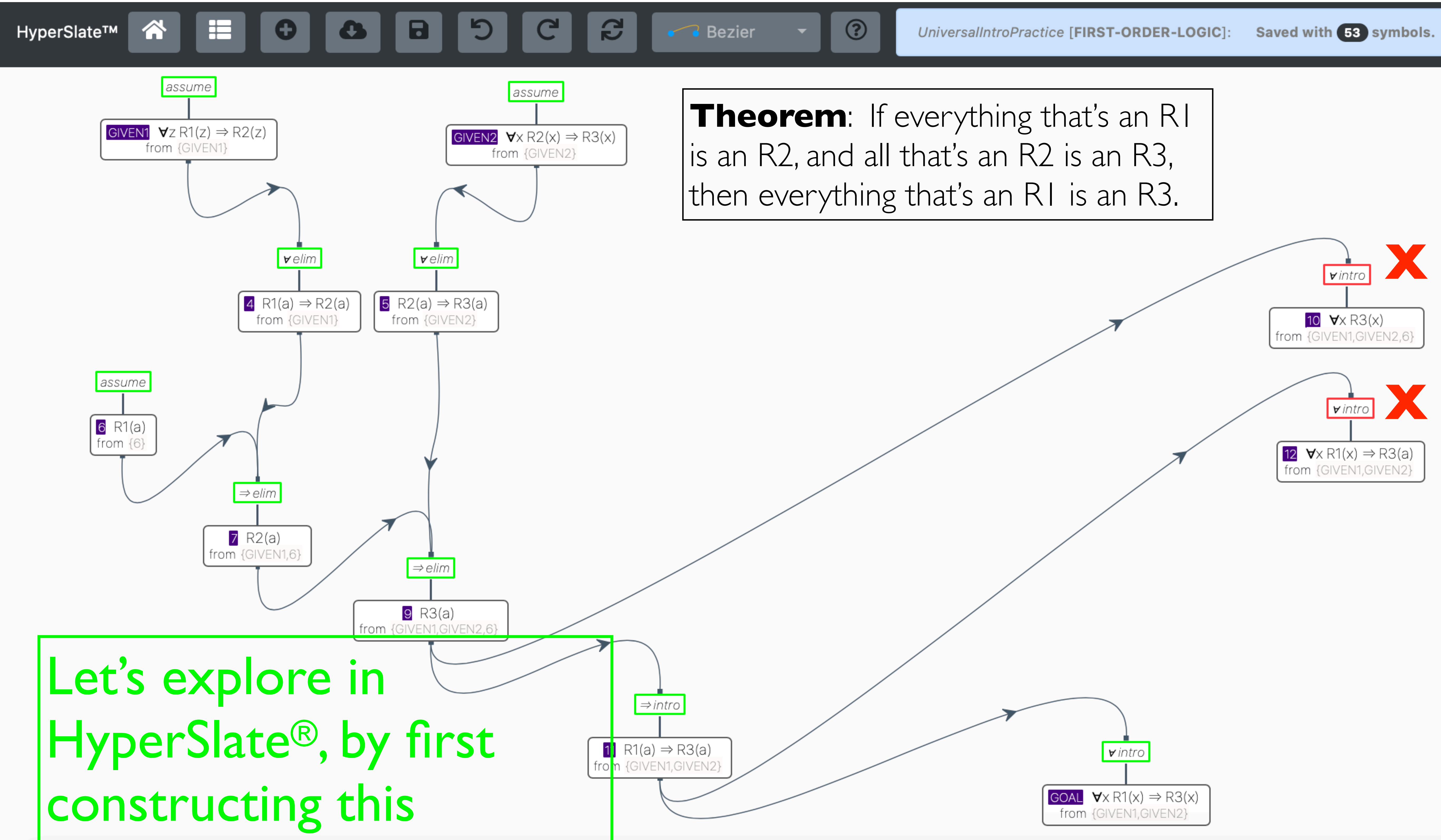


universal intro Example/Tutorial



Let's explore in HyperSlate®, by first constructing this example from scratch …

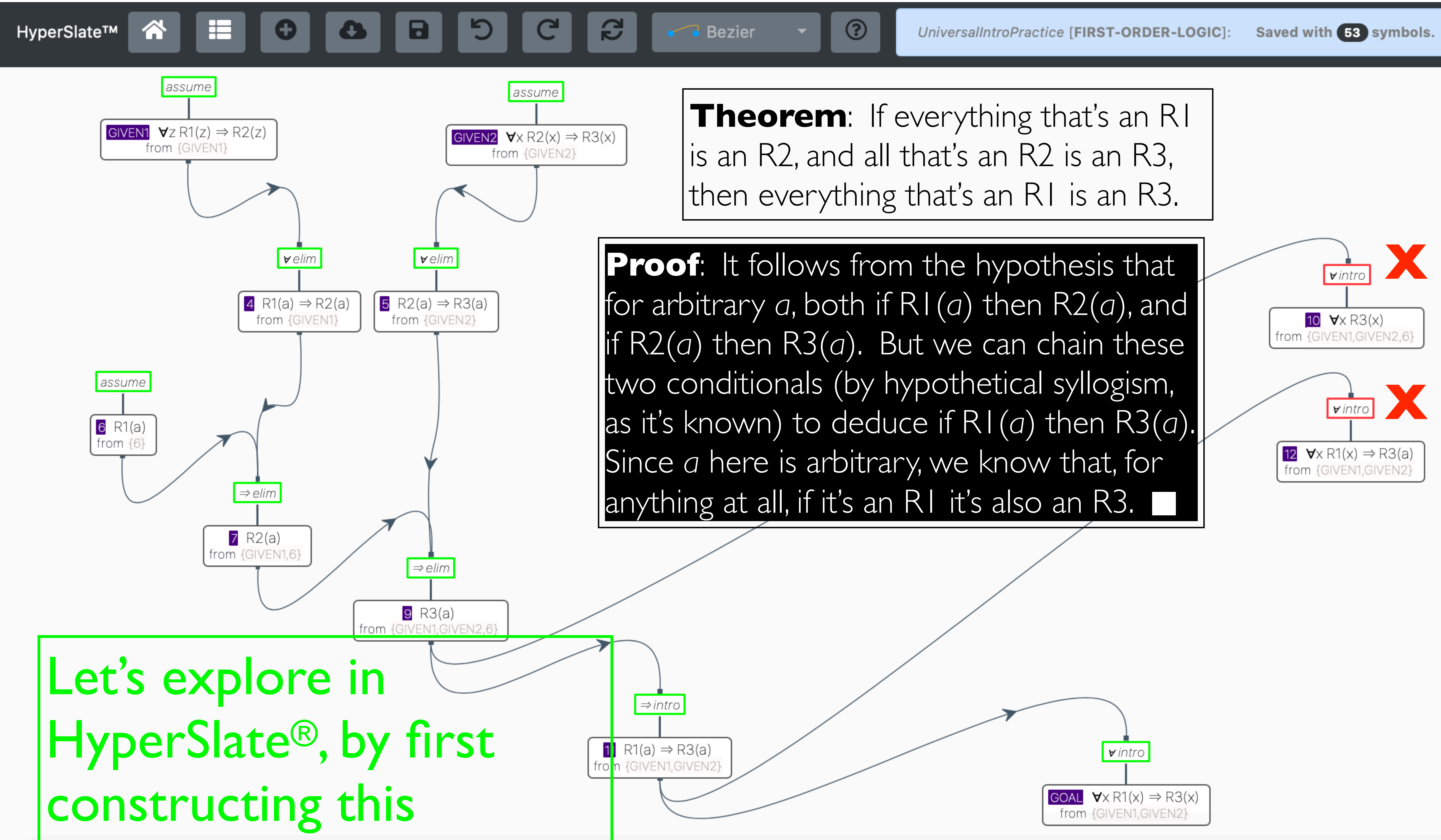
`universal intro` Example/Tutorial



Theorem: If everything that's an R1 is an R2, and all that's an R2 is an R3, then everything that's an R1 is an R3.

Let's explore in HyperSlate®, by first constructing this example from scratch …

universal intro Example/Tutorial



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Proof: It follows from the hypothesis that for arbitrary *a*, both if R1(*a*) then R2(*a*), and if R2(*a*) then R3(*a*). But we can chain these two conditionals (by hypothetical syllogism, as it's known) to deduce if R1(*a*) then R3(*a*). Since *a* here is arbitrary, we know that, for anything at all, if it's an R1 it's also an R3. ■

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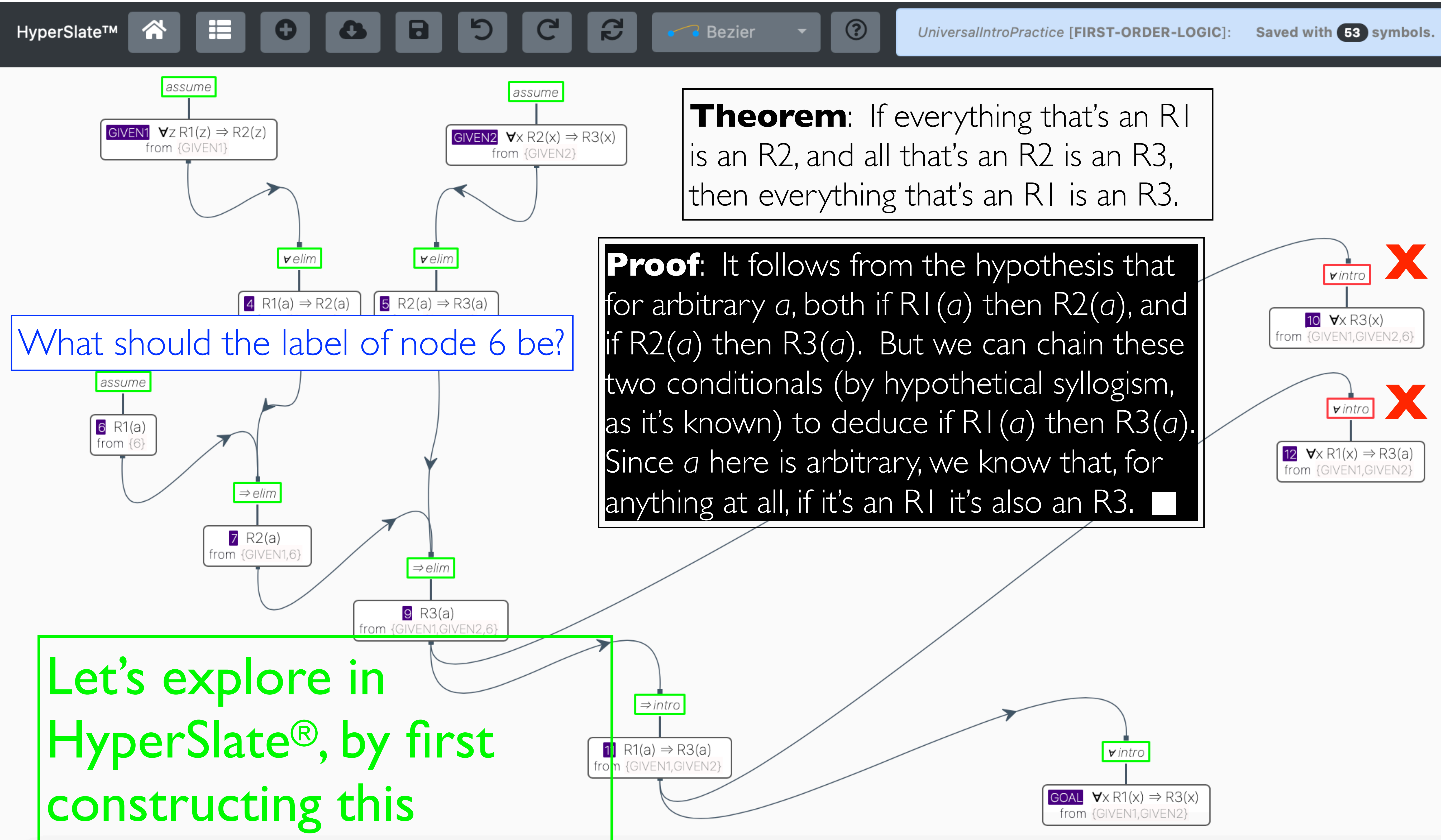
universal intro Example/Tutorial



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What should the label of node 6 be?

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Suggested Practice Problems in HyperSlate®!

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$$\{\forall \mathtt{x}(\mathtt{R}(\mathtt{x}) \leftrightarrow \mathtt{S}(\mathtt{x})), \forall \mathtt{x}\mathtt{R}(\mathtt{x})\} \vdash \forall \mathtt{x}\mathtt{S}(\mathtt{x})\} \ ?$$

Suggested Practice Problems in HyperSlate®!

$$\{\forall \mathtt{x}(\mathtt{R}(\mathtt{x}) \leftrightarrow \mathtt{S}(\mathtt{x})), \forall \mathtt{x}\mathtt{R}(\mathtt{x})\} \vdash \forall \mathtt{x}\mathtt{S}(\mathtt{x})\} \textbf{ ?}$$

$$\{\forall \mathtt{x}[\mathtt{Norsk}(\mathtt{x}) \rightarrow \forall \mathtt{y}(\mathtt{Svensk}(\mathtt{y}) \rightarrow \mathtt{Smarter}(\mathtt{x}, \mathtt{y}))]\} \vdash \forall \mathtt{x}, \mathtt{y}[(\mathtt{Norsk}(\mathtt{x}) \land \mathtt{Svensk}(\mathtt{y})) \rightarrow \mathtt{Smarter}(\mathtt{x}, \mathtt{y})] \textbf{ ?}$$

Suggested Practice Problems in HyperSlate®!

$$\{\forall \mathtt{x}(\mathtt{R}(\mathtt{x}) \leftrightarrow \mathtt{S}(\mathtt{x})), \forall \mathtt{x}\mathtt{R}(\mathtt{x})\} \vdash \forall \mathtt{x}\mathtt{S}(\mathtt{x})\}$$?

$$\{\forall \mathtt{x}[\mathtt{Norsk}(\mathtt{x}) \rightarrow \forall \mathtt{y}(\mathtt{Svensk}(\mathtt{y}) \rightarrow \mathtt{Smarter}(\mathtt{x},\mathtt{y}))]\} \vdash \forall \mathtt{x},\mathtt{y}[(\mathtt{Norsk}(\mathtt{x}) \land \mathtt{Svensk}(\mathtt{y})) \rightarrow \mathtt{Smarter}(\mathtt{x},\mathtt{y})]$$?

$$\{\forall \mathtt{x},\mathtt{y}[(\mathtt{Norsk}(\mathtt{x}) \land (\mathtt{Svensk}(\mathtt{y})) \rightarrow \mathtt{Smarter}(\mathtt{x},\mathtt{y})],$$
$$\forall \mathtt{x},\mathtt{y}[(\mathtt{Svensk}(\mathtt{x}) \land (\mathtt{Dansk}(\mathtt{y})) \rightarrow \mathtt{Smarter}(\mathtt{x},\mathtt{y})]\} \vdash$$
$$\forall \mathtt{x},\mathtt{y}[(\mathtt{Norsk}(\mathtt{x}) \land (\mathtt{Dansk}(\mathtt{y})) \rightarrow \mathtt{Smarter}(\mathtt{x},\mathtt{y})]$$?

Hvis du forstår det, kan du bevise det.

Part I: *Slutten — for i dag.*

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Part II: Hands-on: DeMorgan's …

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e.g. $\{\neg(\phi \vee \psi)\} \vdash \neg\phi \wedge \neg\psi$

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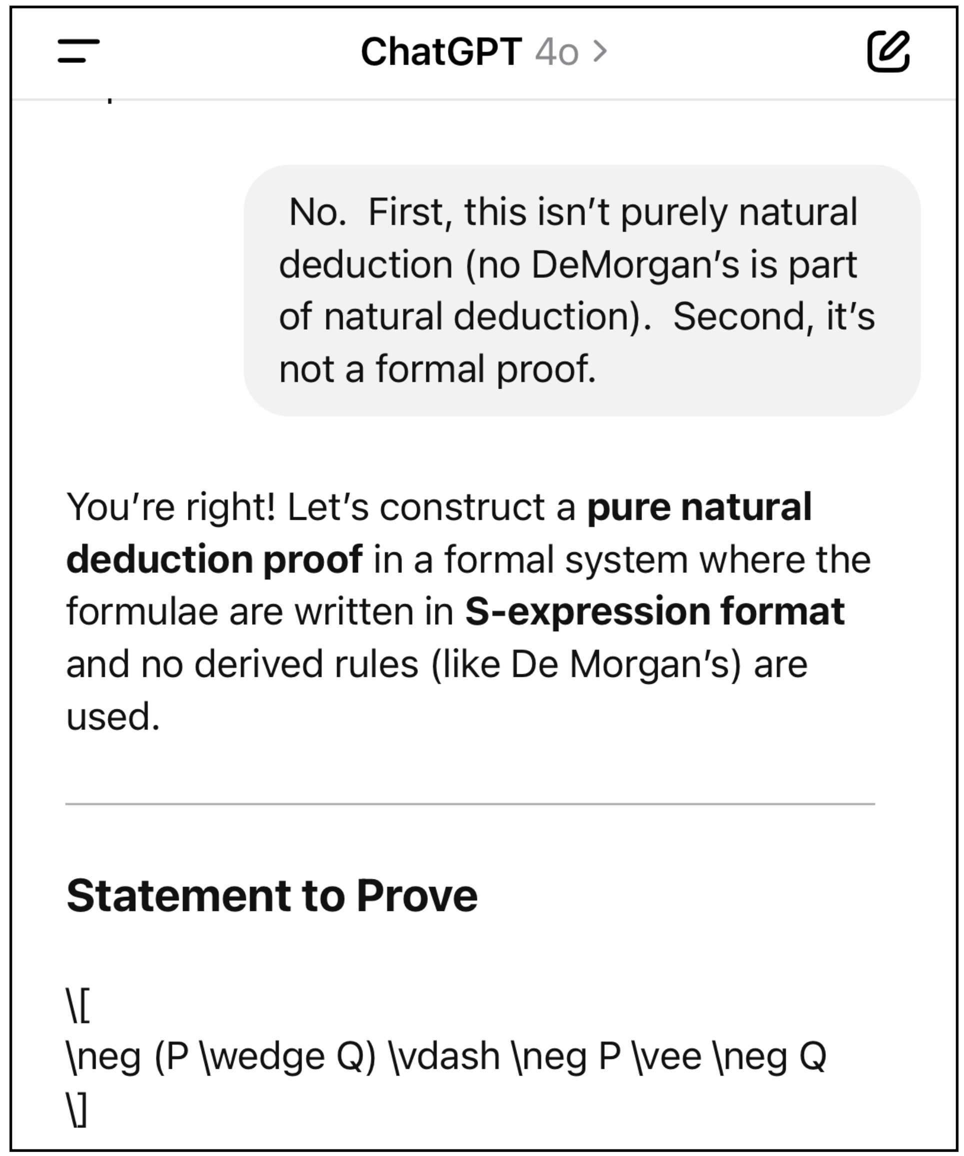

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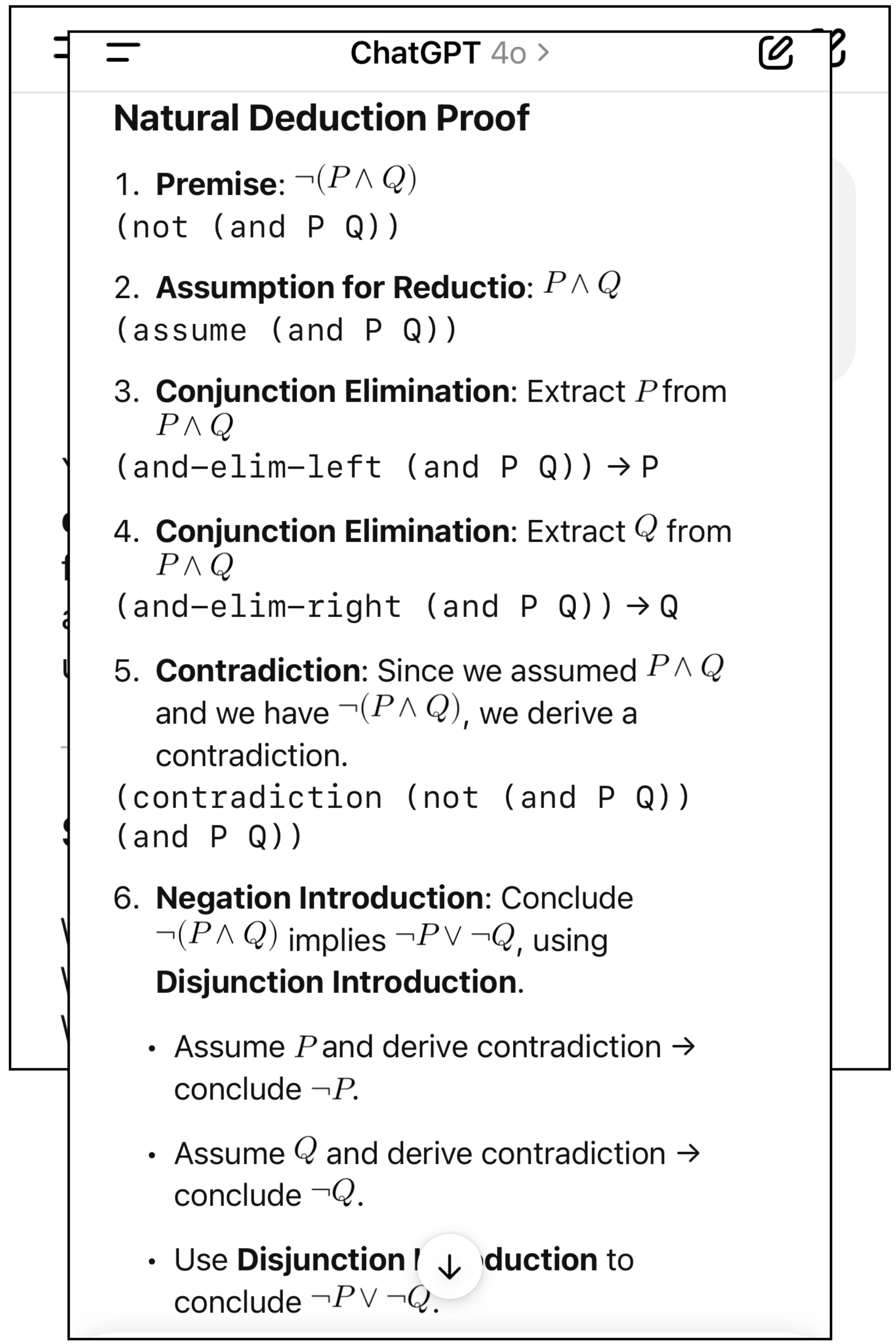

e.g. $\{\neg(\phi \wedge \psi)\} \vdash \neg\phi \vee \neg\psi$

This is positively scandalous.

1. **Premise**: $\neg(P \wedge Q)$
`(not (and P Q))`
2. **Assumption for Reductio**: $P \wedge Q$
`(assume (and P Q))`
3. **Conjunction Elimination**: Extract P from $P \wedge Q$
`(and-elim-left (and P Q))` → P
4. **Conjunction Elimination**: Extract Q from $P \wedge Q$
`(and-elim-right (and P Q))` → Q
5. **Contradiction**: Since we assumed $P \wedge Q$ and we have $\neg(P \wedge Q)$, we derive a contradiction.
`(contradiction (not (and P Q)) (and P Q))`
6. **Negation Introduction**: Conclude $\neg(P \wedge Q)$ implies $\neg P \vee \neg Q$, using **Disjunction Introduction**.
   - Assume P and derive contradiction → conclude $\neg P$.
   - Assume Q and derive contradiction → conclude $\neg Q$.
   - Use **Disjunction I**[illegible]**duction** to conclude $\neg P \vee \neg Q$.

e.g. $\{\neg(\phi \wedge \psi)\} \vdash \neg\phi \vee \neg\psi$

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ChatGPT 4o >

```
(assume (not (and P Q)))  ; Premise: ¬(P ∧ Q)

  (assume P)  ; Assume P for reductio
    (assume Q)  ; Assume Q for reductio
      (and-intro P Q)  ; P ∧ Q
      (contradiction (not (and P Q)) (and P Q))  ; Contradiction!
    (not-intro Q (and P Q) (not (and P Q)))  ; ¬Q
    (or-intro-right (not P) (not Q))  ; ¬P ∨ ¬Q
  (not-intro P (and P Q) (not (and P Q)))  ; ¬P
  (or-intro-left (not P) (not Q))  ; ¬P ∨ ¬Q

  (or-elim (not P) (not Q) (or (not P) (not Q)))  ; Conclusion
```

- Assume Q and derive contradiction → conclude $\neg Q$.
- Use **Disjunction I** **duction** to conclude $\neg P \vee \neg Q$.

e.g. $\{\neg(\phi \wedge \psi)\} \vdash \neg\phi \vee \neg\psi$

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e.g. $\{\neg(\phi \wedge \psi)\} \vdash \neg\phi \vee \neg\psi$

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ChatGPT o3-mini-high >

```
1.   ¬(p ∧ q)                              Premise
2.   |   Assume: ¬(¬p ∨ ¬q)                  Assumption [for indirect proof]
3.   |   |   Assume: ¬p                       Assumption
4.   |   |   ¬p ∨ ¬q                         ∨-Introduction (from 3, left)
5.   |   |   ⊥                               Contradiction (from 2 and 4)
6.   |   p                                   RAA, discharging 3–5
7.   |   |   Assume: q                       Assumption
8.   |   |   p ∧ q                          ∧-Introduction (from 6 and 7)
9.   |   |   ⊥                               Contradiction (from 1 and 8)
10.  |   ¬q                                 RAA, discharging 7–9
11.  |   ¬p ∨ ¬q                          ∨-Introduction (from 10, right)
12.  ⊥                                    Contra  tion (from 2 and 11)
13.  ¬p ∨ ¬q                            RAA, di  rging 2–12
```

- Assume P and derive contradiction → conclude $\neg P$.
- Assume Q and derive contradiction → conclude $\neg Q$.
- Use **Disjunction I duction** to conclude $\neg P \vee \neg Q$.