

Exhortation; Truth Trees; FOL IV: Layered Quantification and Measuring Intelligence Using This Phenomenon

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Formal Logic (& AI)
2/24/2025



Exhortation; ~~Truth Trees~~; FOL IV: Layered Quantification and Measuring Intelligence Using This Phenomenon

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Formal Logic (& AI)
2/24/2025



Hypergraphical Tableaux **Exhortation; ~~Truth Trees~~;** **FOL IV: Layered Quantification and** **Measuring Intelligence Using This** **Phenomenon**

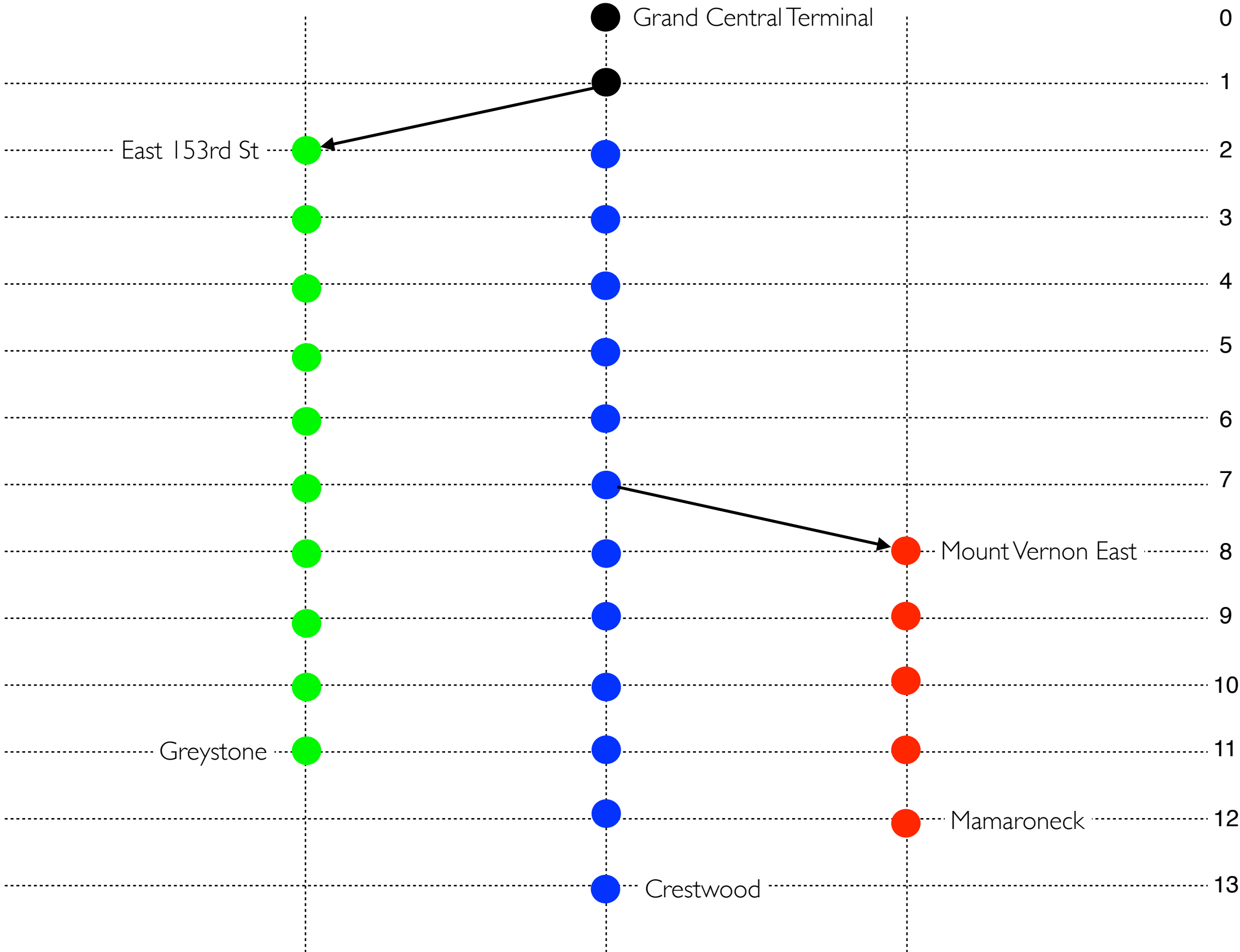
Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Formal Logic (& AI)
2/24/2025







Suppose we wish to prove the formula $[p \vee (q \wedge r)] \supset [(p \vee q) \wedge (p \vee r)]$.
The following is a tableau which does this; the explanation is given
immediately following the tableau:

(1) $F[p \vee (q \wedge r)] \supset [(p \vee q) \wedge (p \vee r)]$			
(2) $Tp \vee (q \wedge r)$			
(3) $F(p \vee q) \wedge (p \vee r)$			
(4) Tp		(5) $T(q \wedge r)$	
(8) $F(p \vee q)$		(6) Tq	
(12) Fp		(7) Tr	
(13) Fq			
X			
(9) $F(p \vee r)$			
(14) Fp			
(15) Fr			
X			
		(10) $F(p \vee q)$	(11) $F(p \vee r)$
		(16) Fp	(18) Fp
		(17) Fq	(19) Fr
		X	X

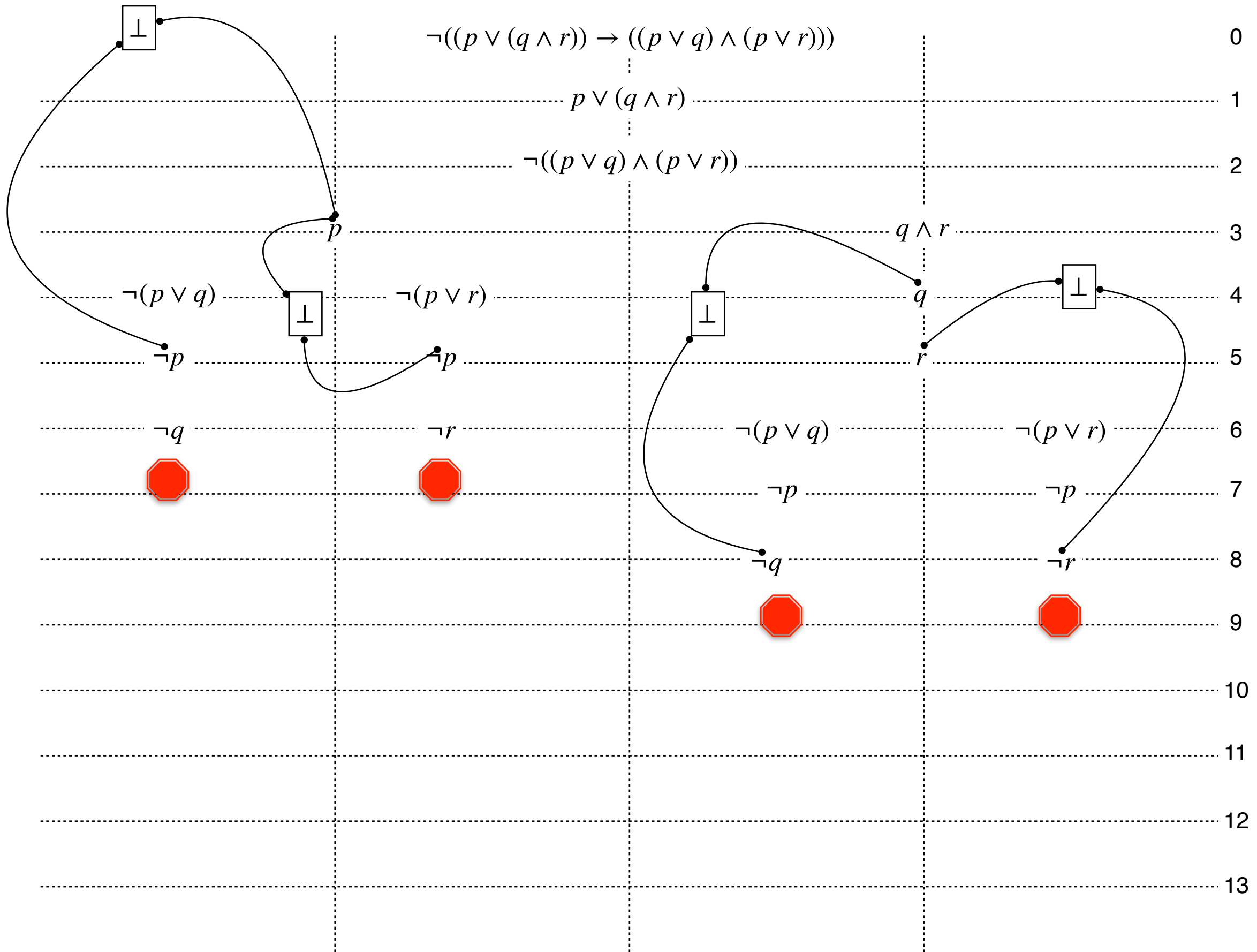
$$\vdash (p \vee (q \wedge r)) \rightarrow ((p \vee q) \wedge (p \vee r)) \quad ?$$

	$\neg((p \vee (q \wedge r)) \rightarrow ((p \vee q) \wedge (p \vee r)))$			0
	$p \vee (q \wedge r)$			1
	$\neg((p \vee q) \wedge (p \vee r))$			2
	p		$q \wedge r$	3
$\neg(p \vee q)$		$\neg(p \vee r)$	q	4
$\neg p$		$\neg p$	r	5
$\neg q$		$\neg r$	$\neg(p \vee q)$	6
			$\neg p$	7
			$\neg q$	8
			$\neg r$	9
				10
				11
				12
				13

$$\vdash (p \vee (q \wedge r)) \rightarrow ((p \vee q) \wedge (p \vee r))$$

	$\neg((p \vee (q \wedge r)) \rightarrow ((p \vee q) \wedge (p \vee r)))$			0
	$p \vee (q \wedge r)$			1
	$\neg((p \vee q) \wedge (p \vee r))$			2
	p		$q \wedge r$	3
$\neg(p \vee q)$		$\neg(p \vee r)$	q	4
$\neg p$		$\neg p$	r	5
$\neg q$		$\neg r$	$\neg(p \vee q)$	6
			$\neg p$	7
			$\neg q$	8
			$\neg r$	9
				10
				11
				12
				13

$$\vdash (p \vee (q \wedge r)) \rightarrow ((p \vee q) \wedge (p \vee r))$$



Exhortation ...

Plan to make sure you're up-to-date-ish
before Spring Break on HyperGrader®'s
current (**Required** = Homework)
Problems, currently due

Apr 17 2025 11:59pm NY time.

More FOL etc problems of course
forthcoming ...

New in-class problem published, with
needed tableau/truth-tree work ...

Tableau/Truth Trees vs. Truth Tables

Tableau/Truth Trees vs. Truth Tables



Tableau/Truth Trees vs. Truth Tables



Tableau/Truth Trees vs. Truth Tables



Violent breakage between tabular calculation and proof construction.

Tableau/Truth Trees vs. ~~Truth Tables~~



Violent breakage between tabular calculation and proof construction.

Tableau/Truth Trees vs. ~~Truth Tables~~



Violent breakage between tabular calculation and proof construction.

LAMA[®]'s hypergraphs/HyperLogic[®] achieves seamless unification of proofs and trees, and provides AI oracles for their construction and *certification*.

Tableau/Truth Trees vs. ~~Truth Tables~~



Violent breakage between tabular calculation and proof construction.

LAMA[®]'s hypergraphs/HyperLogic[®] achieves seamless unification of proofs and trees, and provides AI oracles for their construction and *certification*.

First very simple: truth-tree for *modus ponens* ...

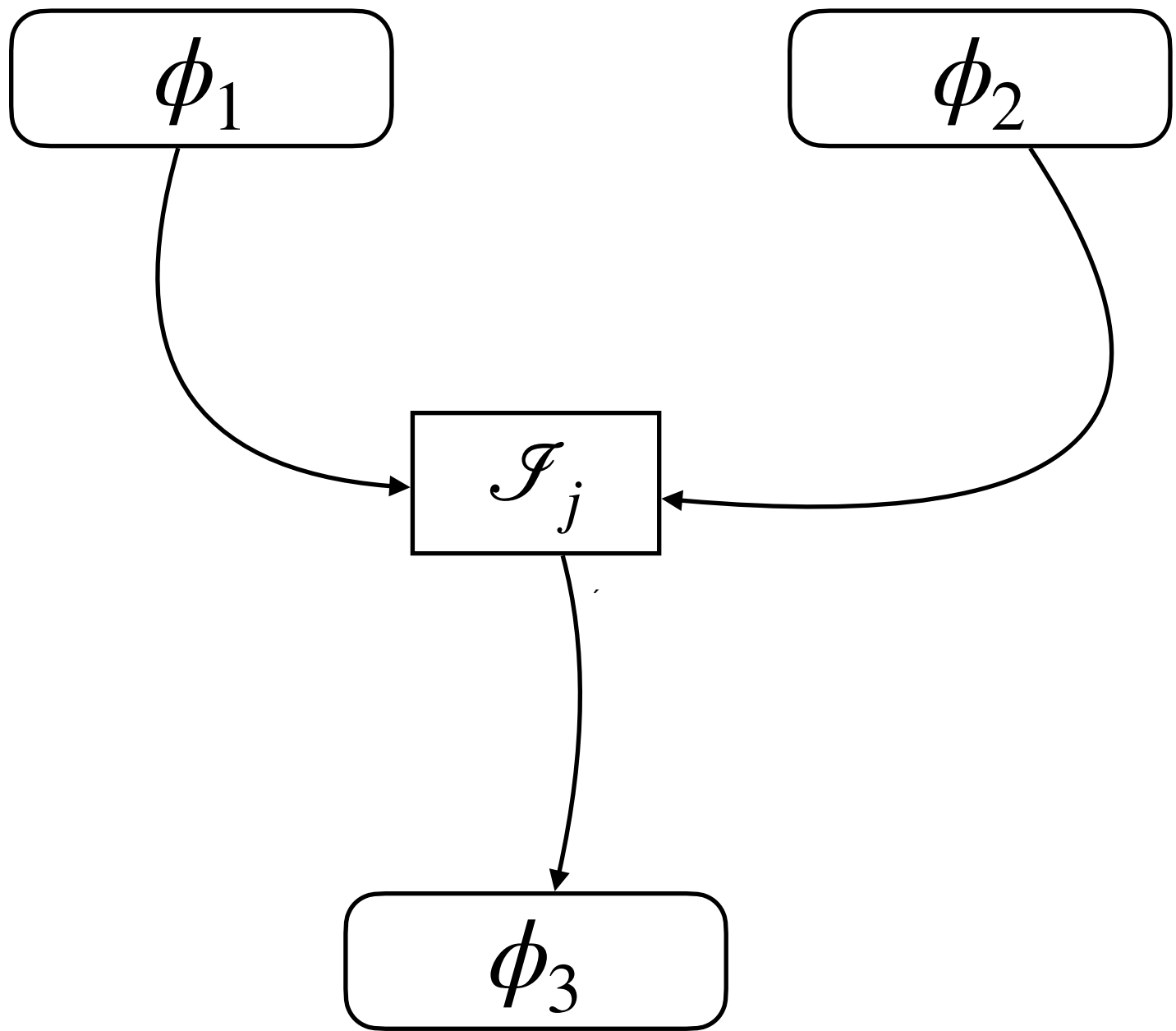
Tableau/Truth Trees vs. ~~Truth Tables~~

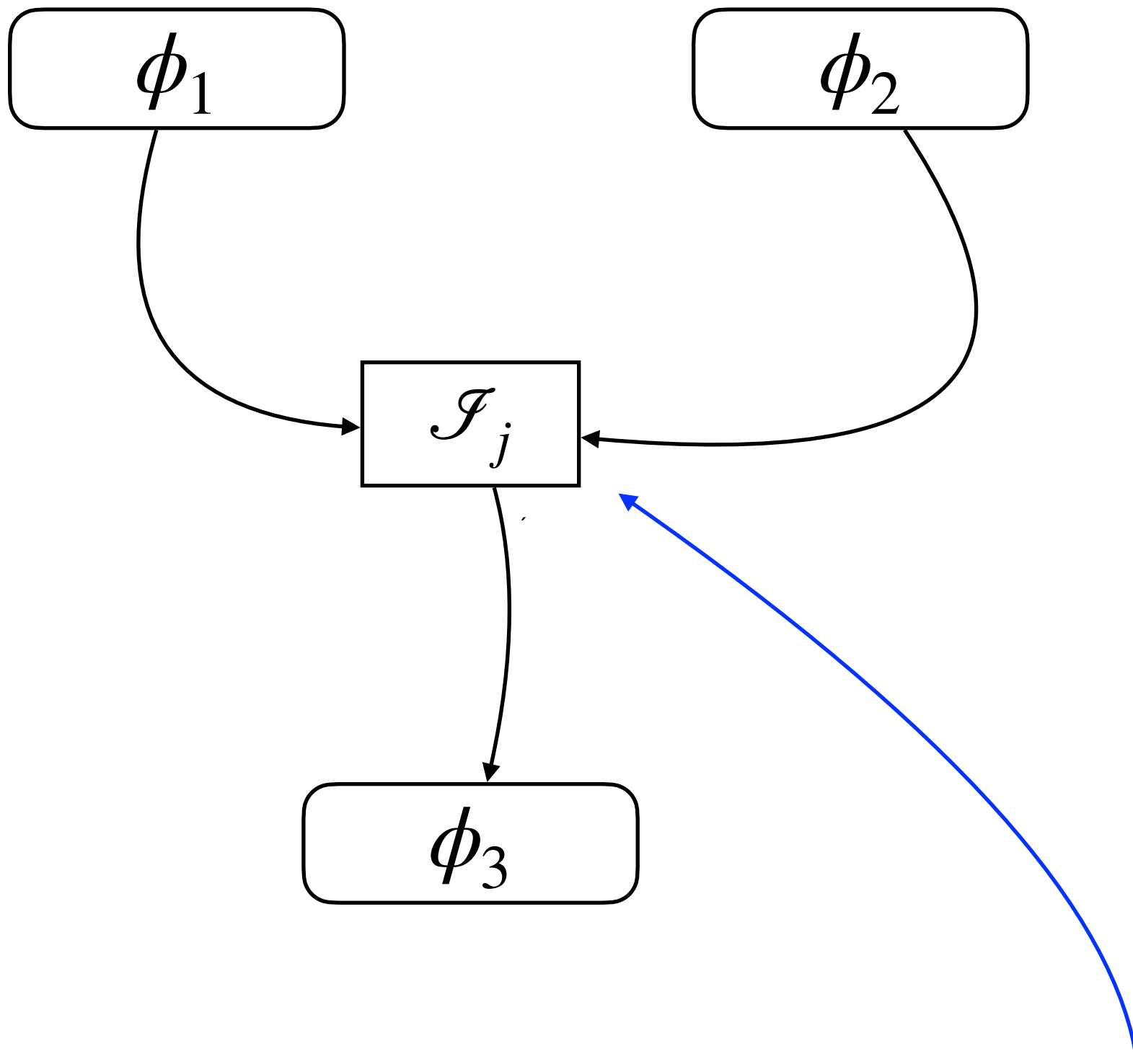


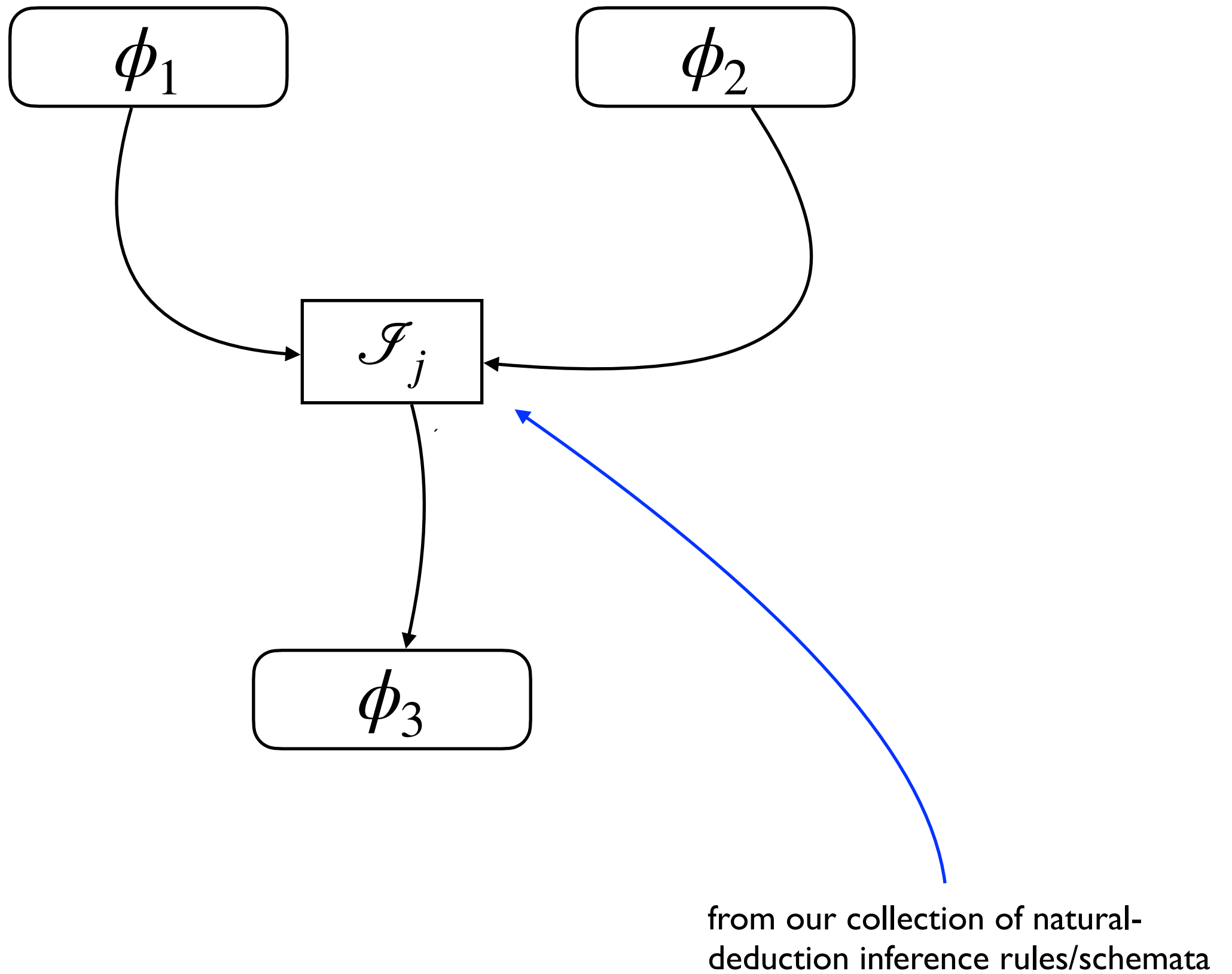
Violent breakage between tabular calculation and proof construction.

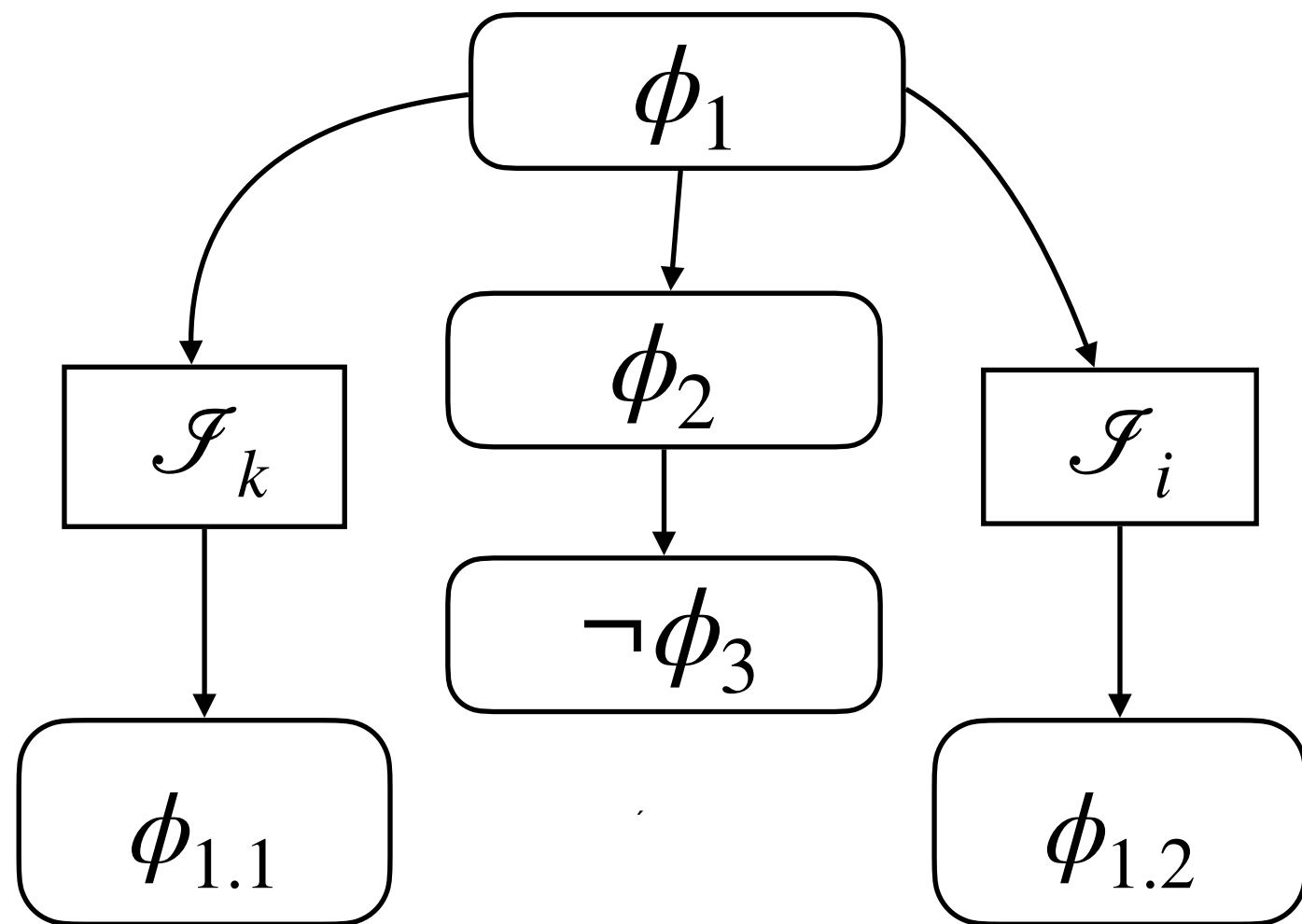
LAMA[®]'s hypergraphs/HyperLogic[®] achieves seamless unification of proofs and trees, and provides AI oracles for their construction and *certification*.

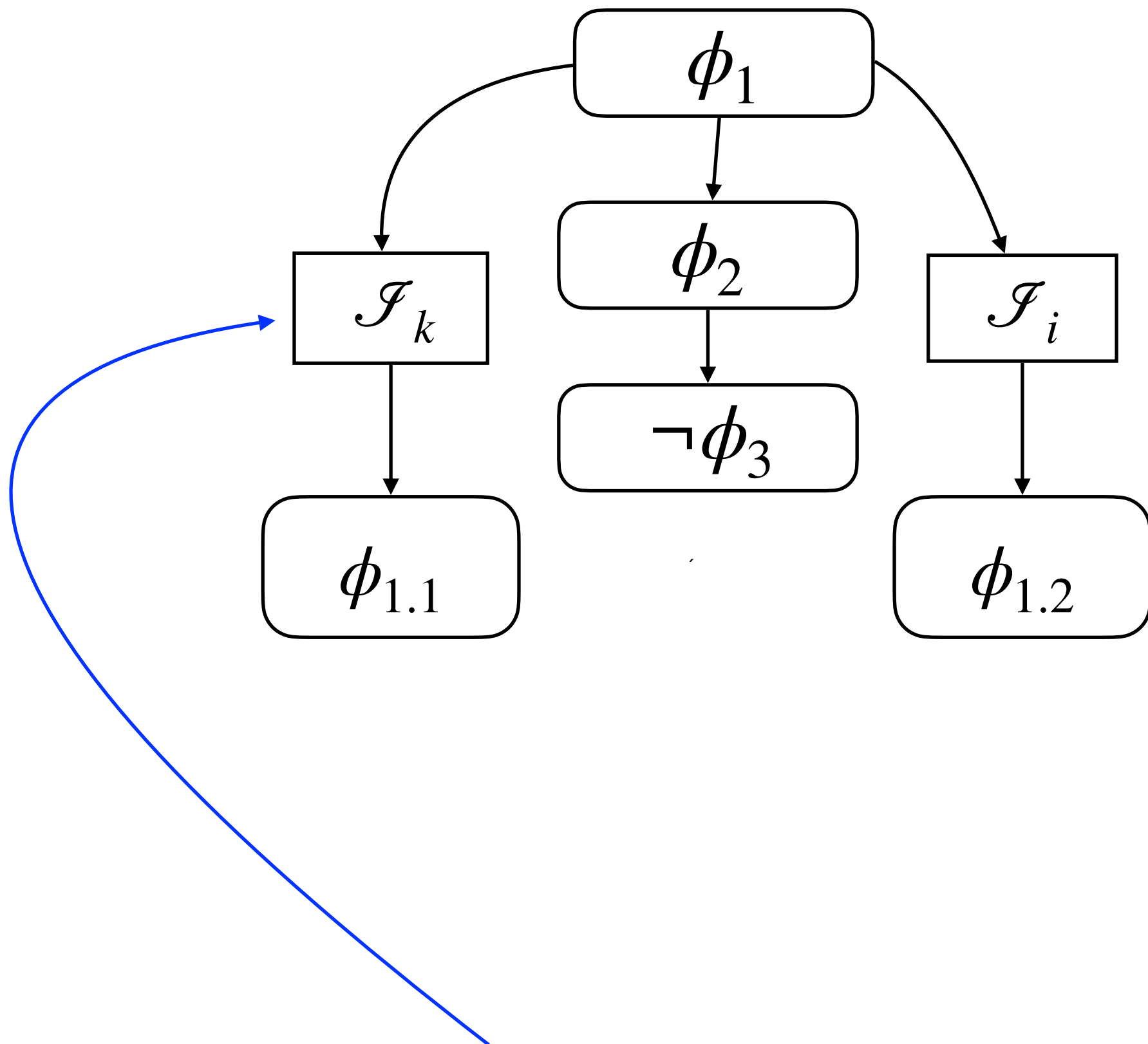
First very simple: truth-tree for *modus ponens* ...

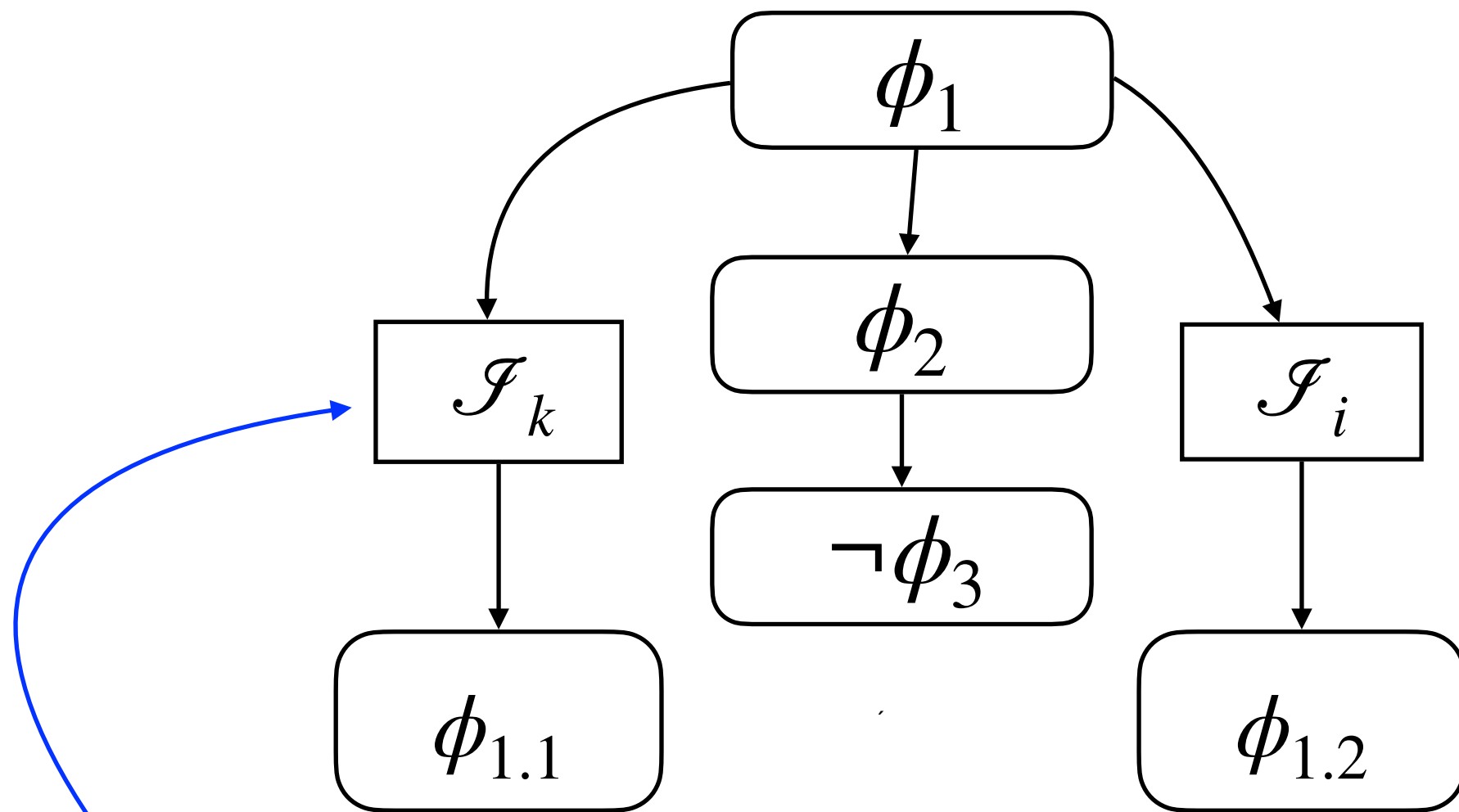




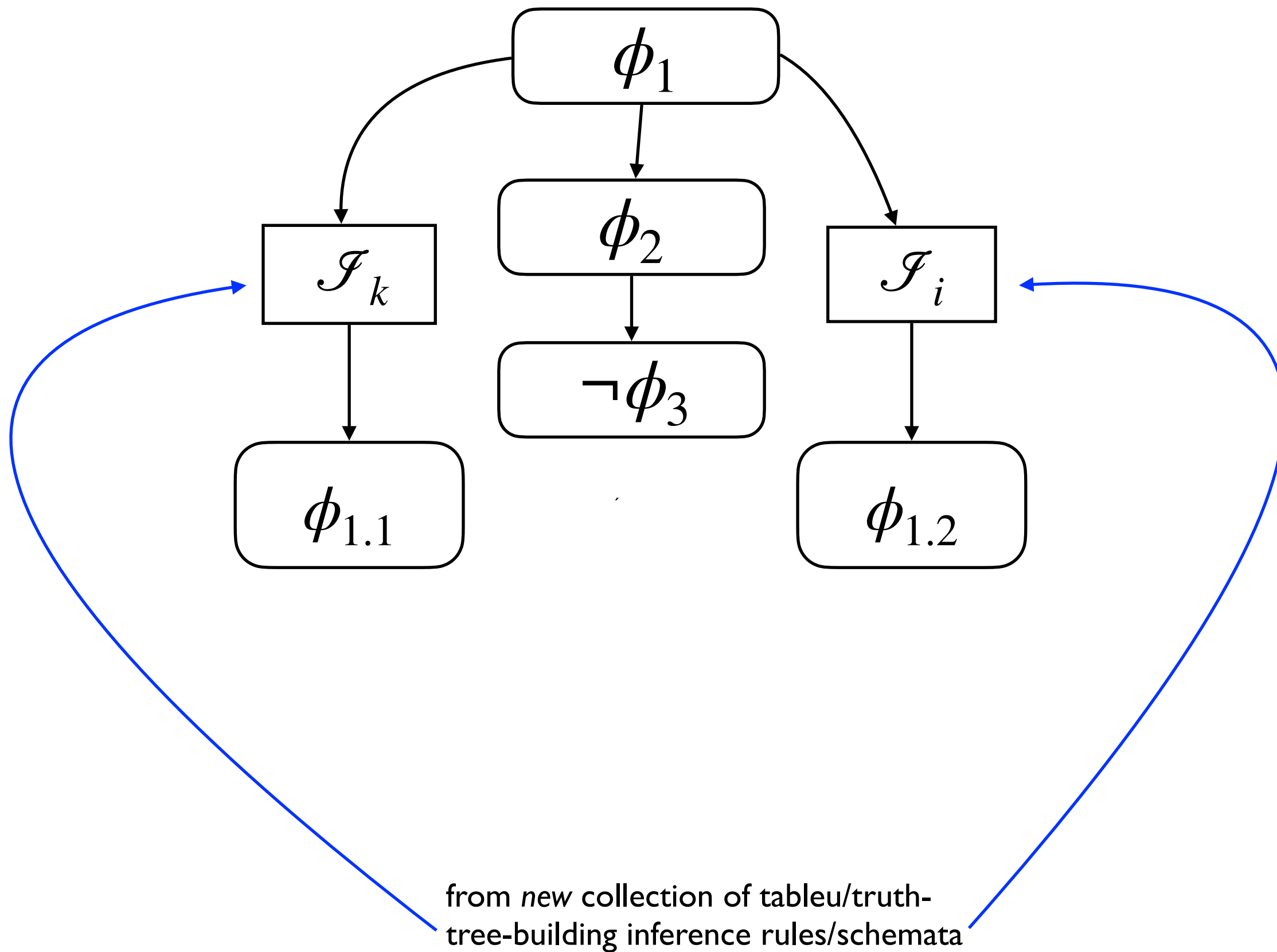




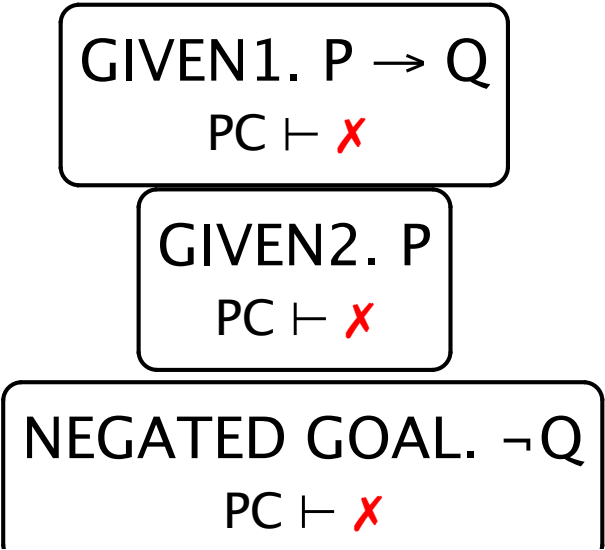




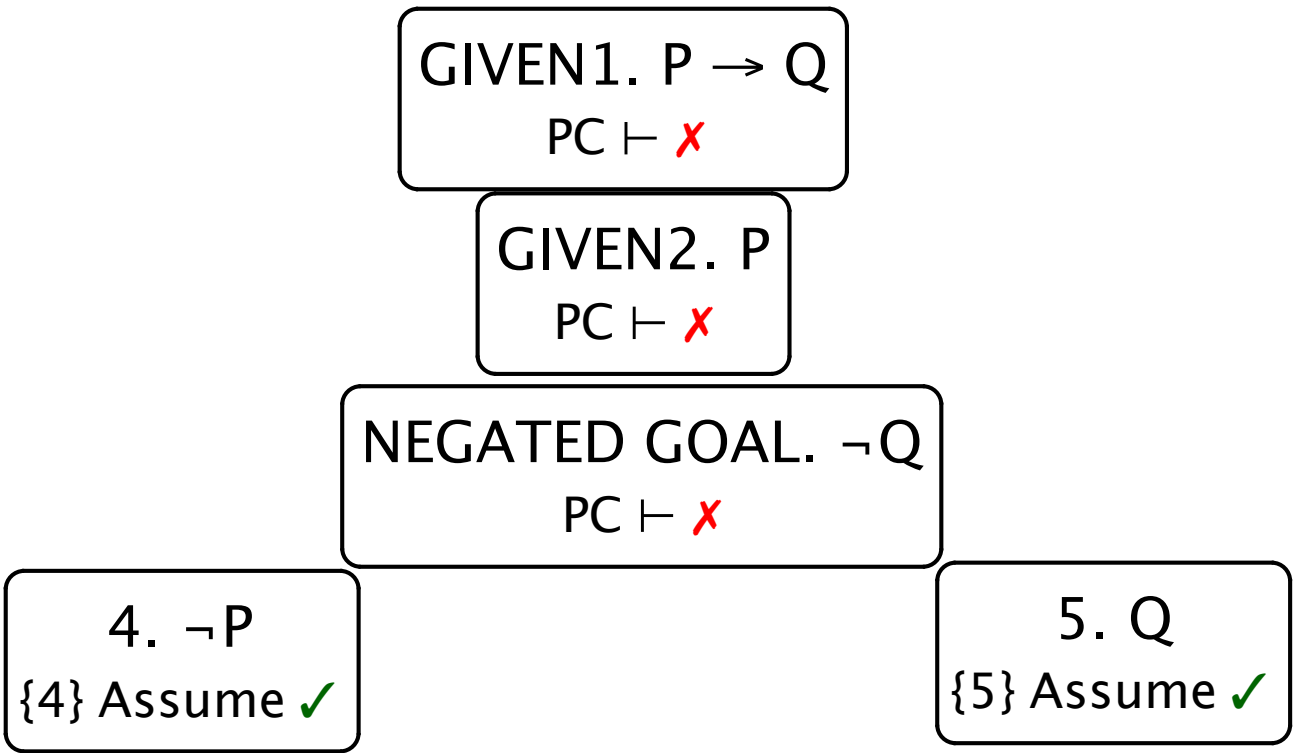
from *new* collection of tableau/truth-
tree-building inference rules/schemata



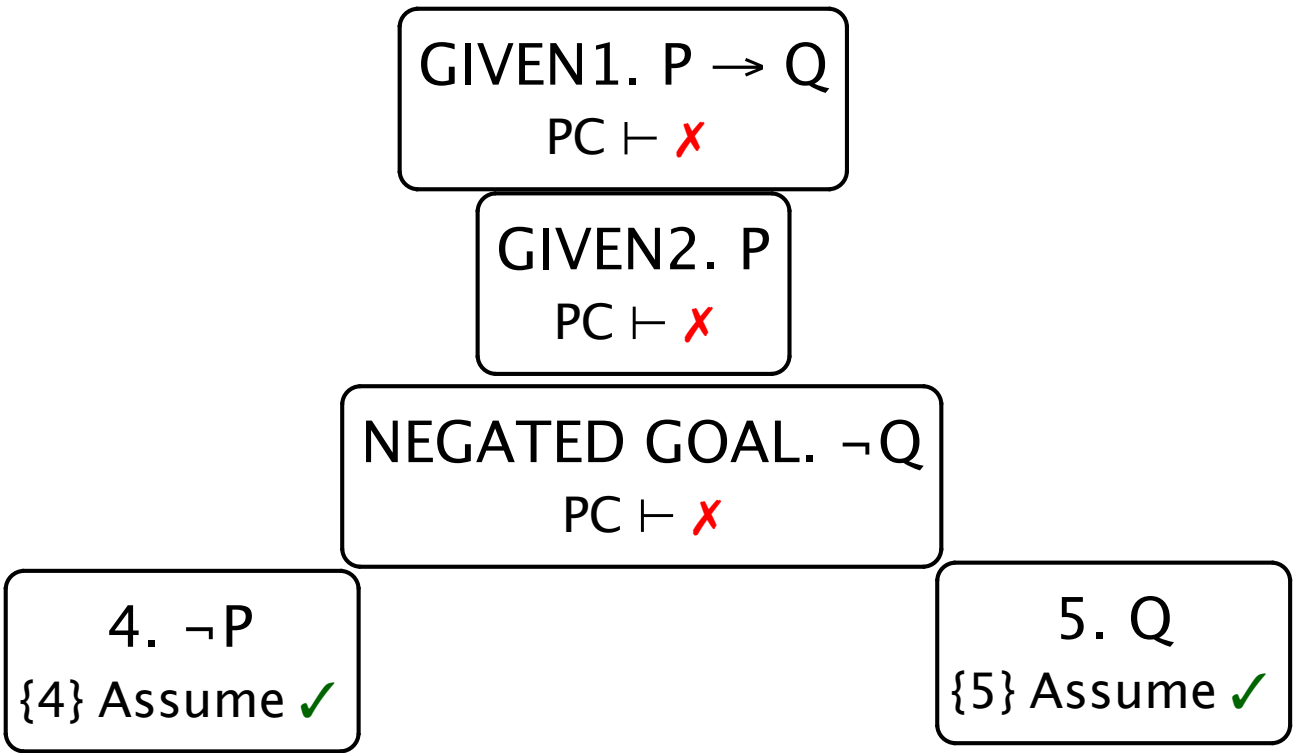
$\{P \rightarrow Q, P\} \vdash Q$



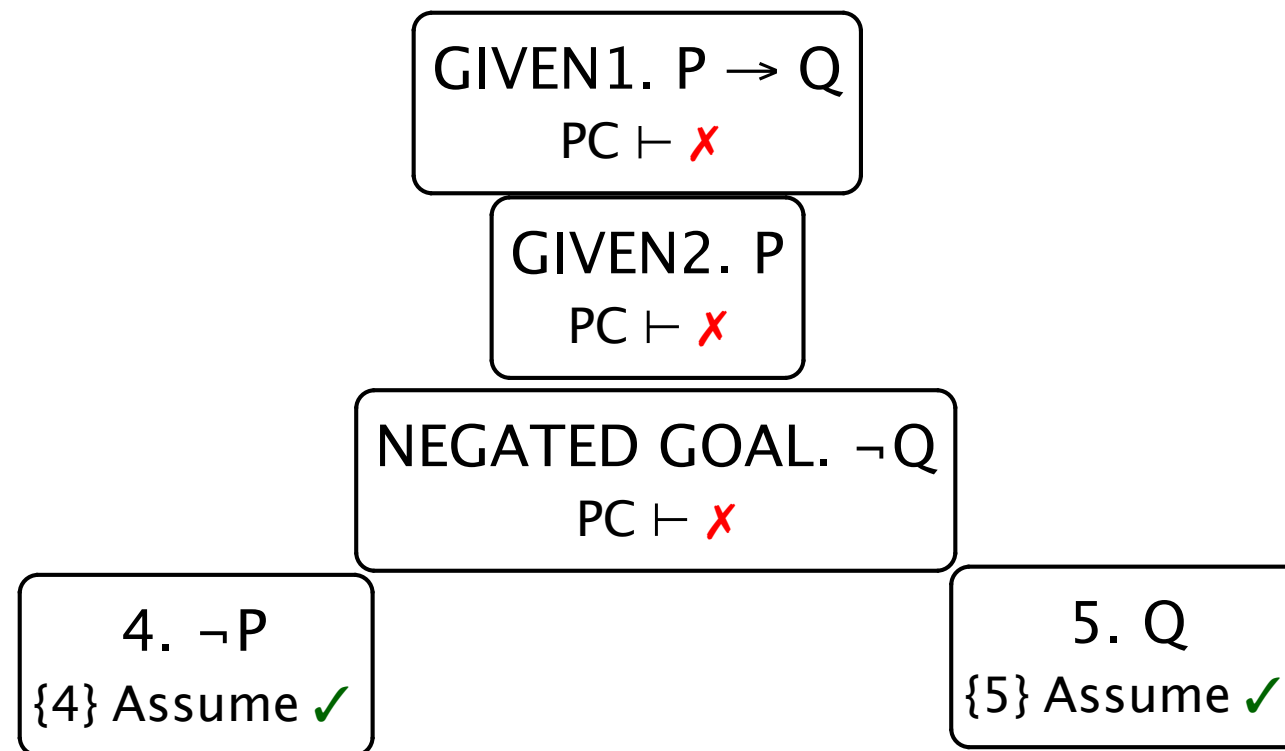
$\{P \rightarrow Q, P\} \vdash Q$



$\{P \rightarrow Q, P\} \vdash Q$

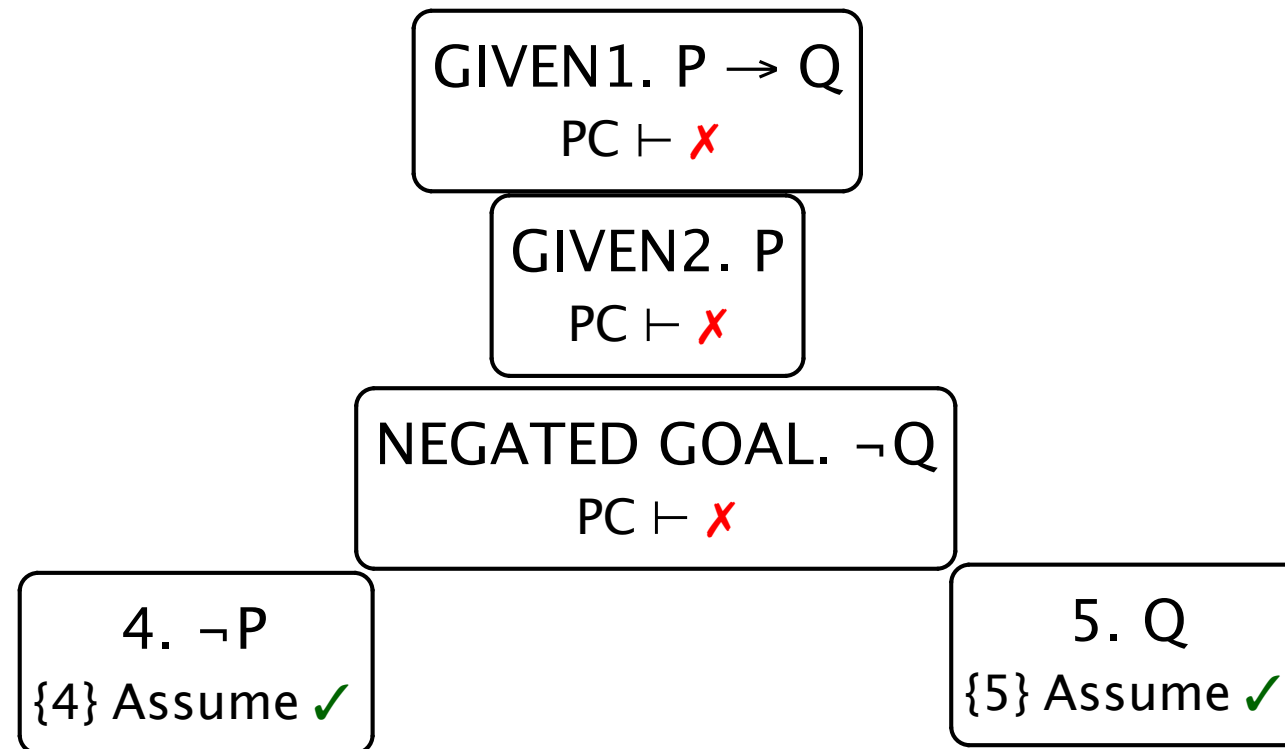


$\{P \rightarrow Q, P\} \vdash Q$



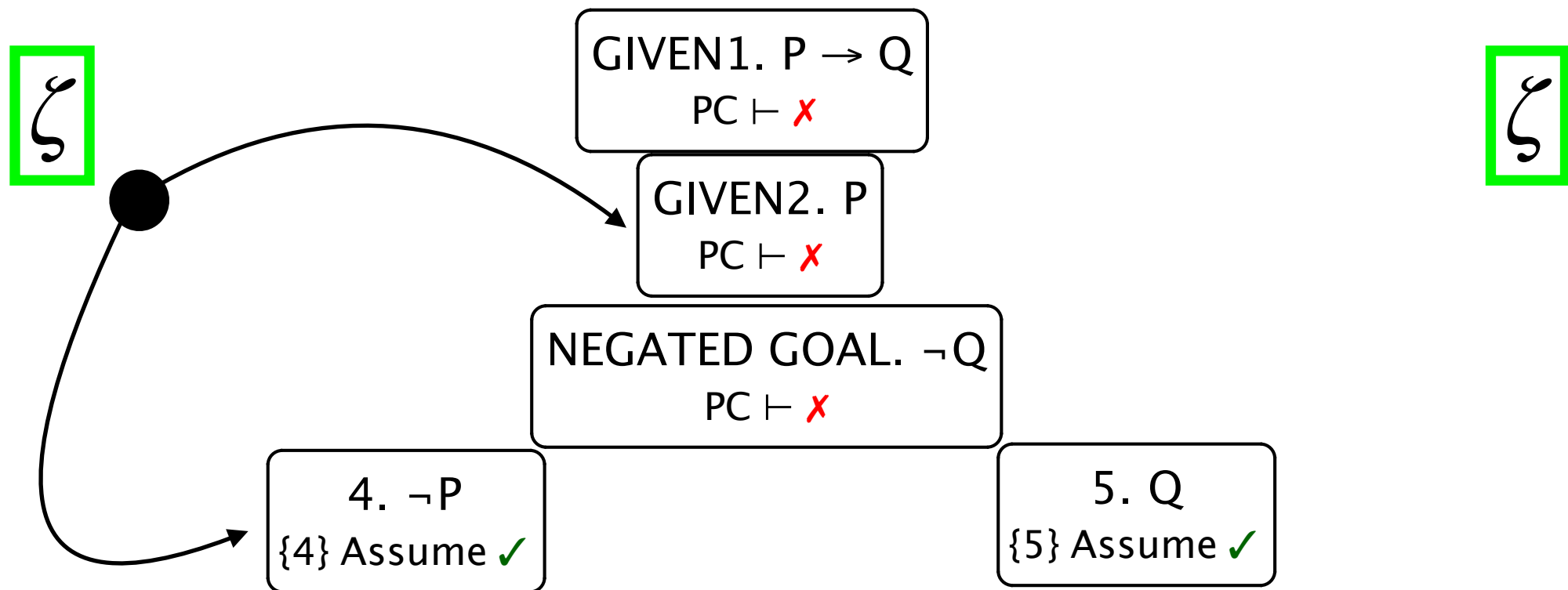
Either way, a contradiction!

$\{P \rightarrow Q, P\} \vdash Q$



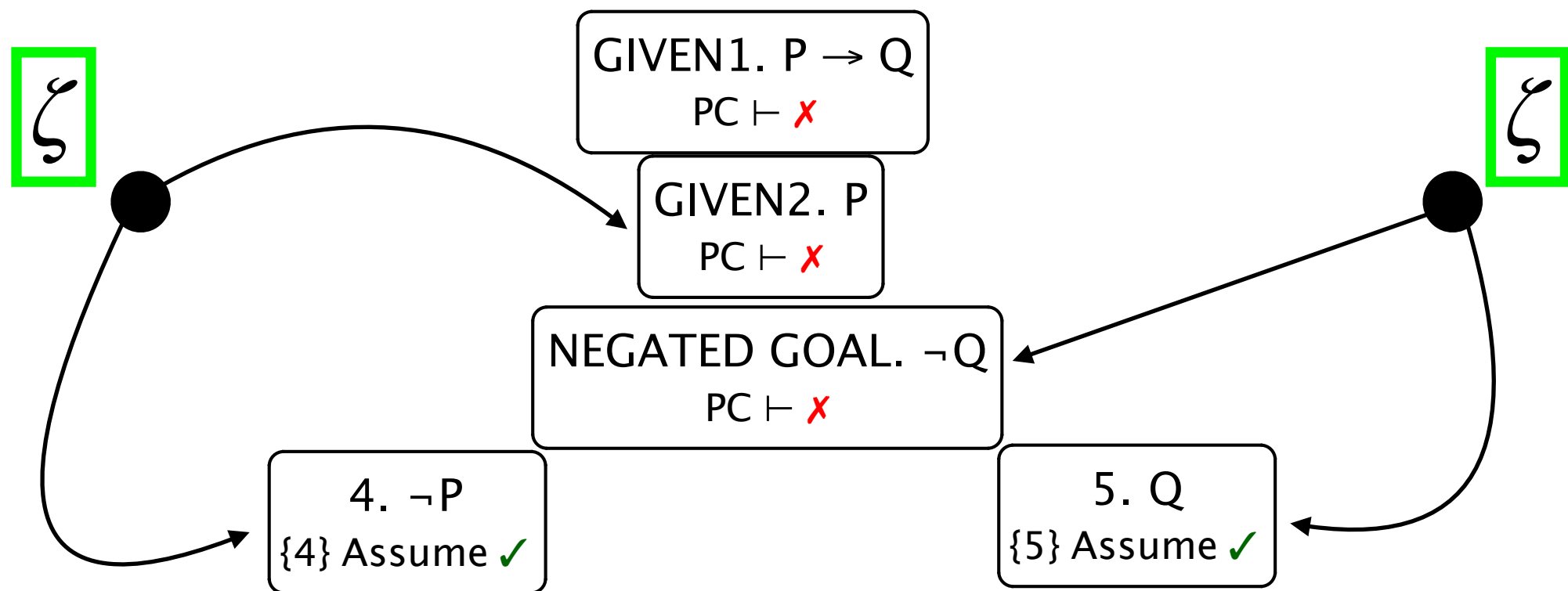
Either way, a contradiction!

$\{P \rightarrow Q, P\} \vdash Q$



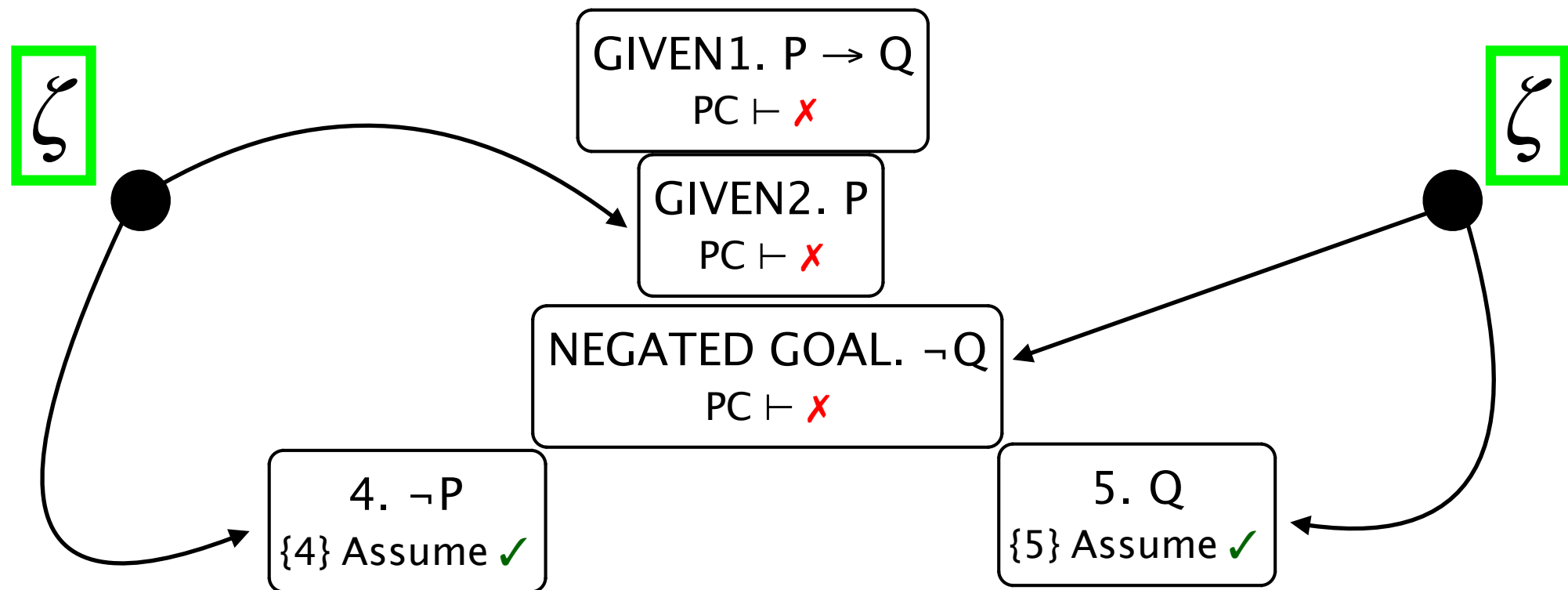
Either way, a contradiction!

$\{P \rightarrow Q, P\} \vdash Q$



Either way, a contradiction!

$\{P \rightarrow Q, P\} \vdash Q$

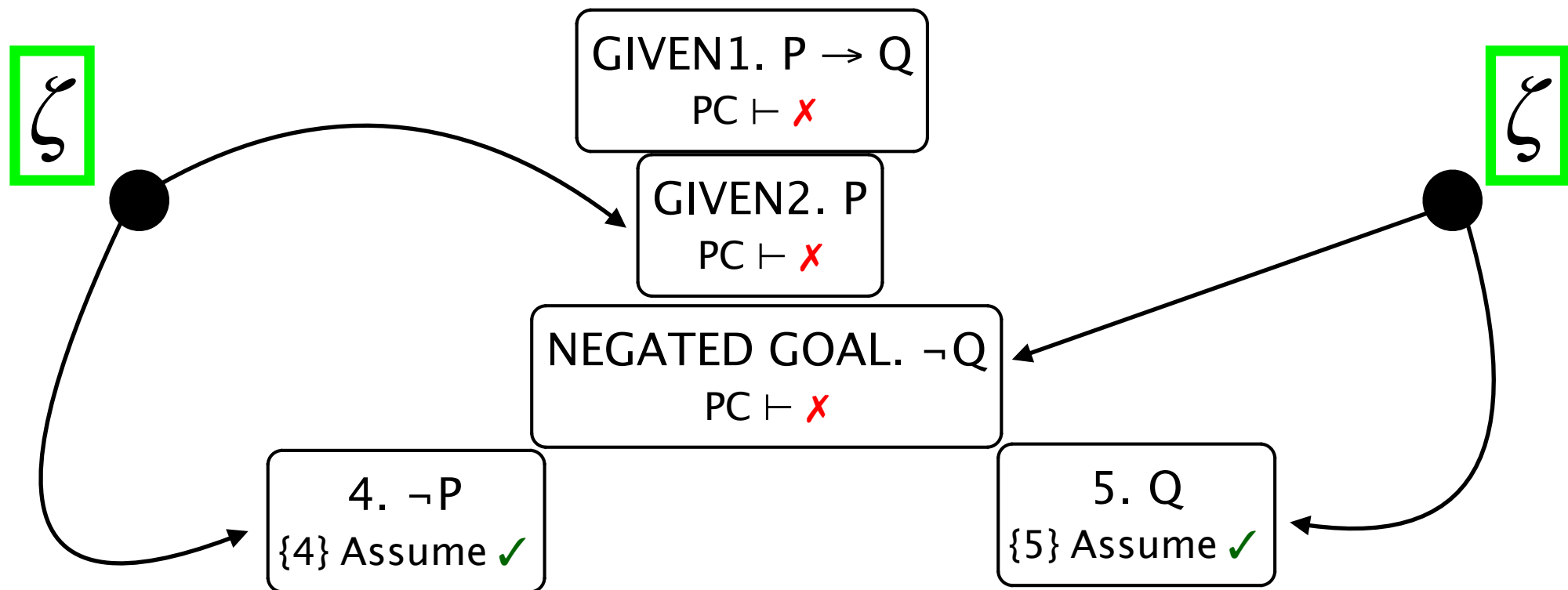


Either way, a contradiction!

Therefore the entailment holds!

$\{P \rightarrow Q, P\} \vdash Q$

This *not* a tree!



Either way, a contradiction!

Therefore the entailment holds!

Slightly Harder Truth Tree

Slightly Harder Truth Tree

$$\vdash (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$$

Slightly Harder Truth Tree

$$\vdash (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$$

(This is the axiom THEN-2 in Frege's (brutal) axiomatization of the propositional calculus.)



Frege

Slightly Harder Truth Tree

$$\vdash (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$$

(This is the axiom THEN-2 in Frege's (brutal) axiomatization of the propositional calculus.)



Frege

https://en.wikipedia.org/wiki/Frege%27s_propositional_calculus

Slightly Harder Truth Tree

$$\vdash (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$$

(This is the axiom THEN-2 in Frege's (brutal) axiomatization of the propositional calculus.)

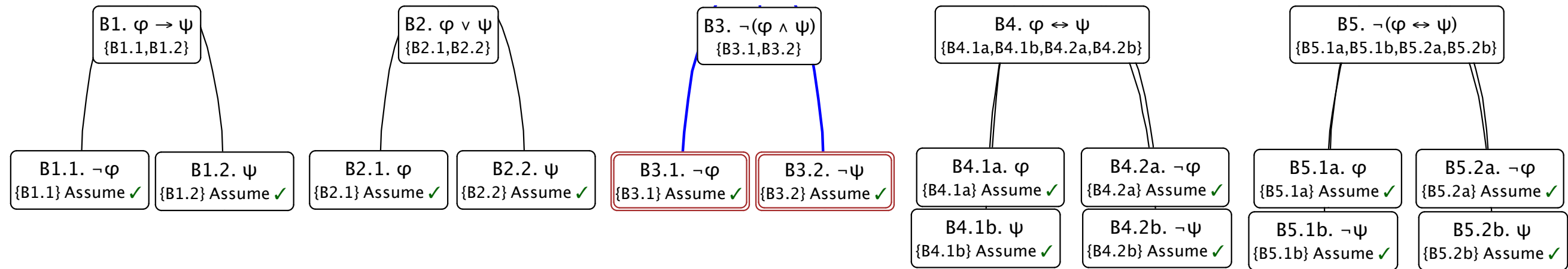


Frege

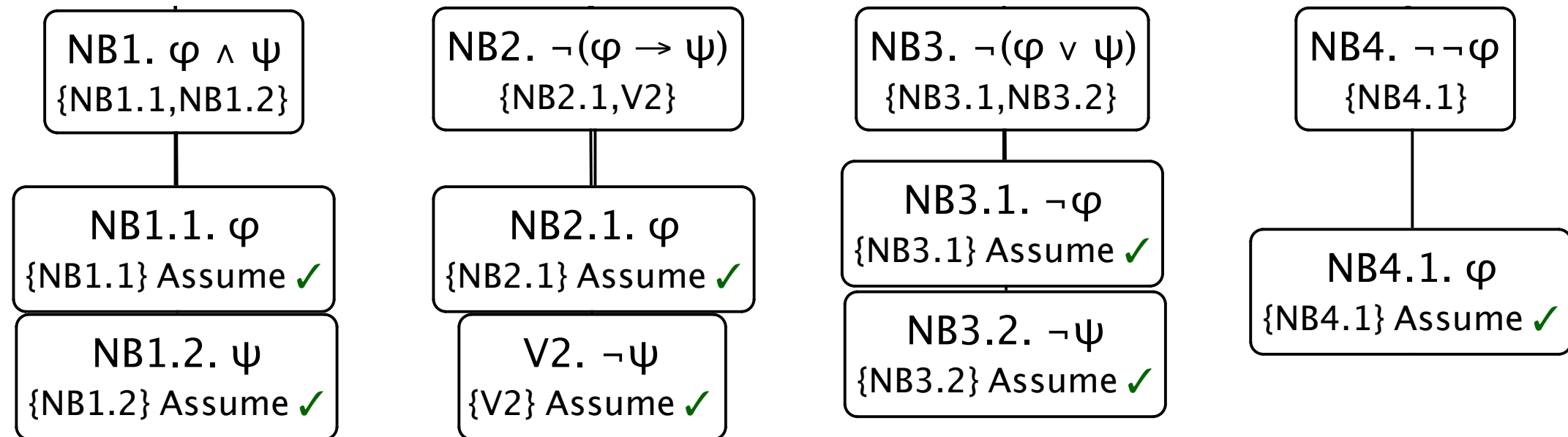
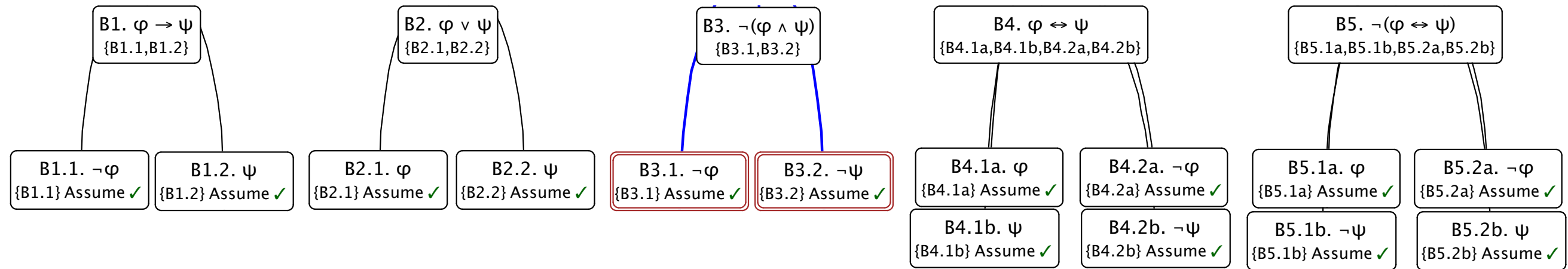
https://en.wikipedia.org/wiki/Frege%27s_propositional_calculus

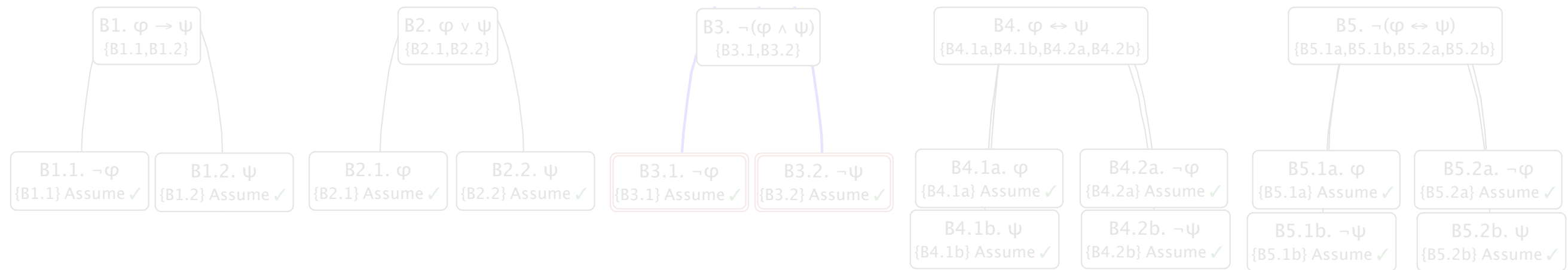
The Rules of the Game!

The Rules of the Game!

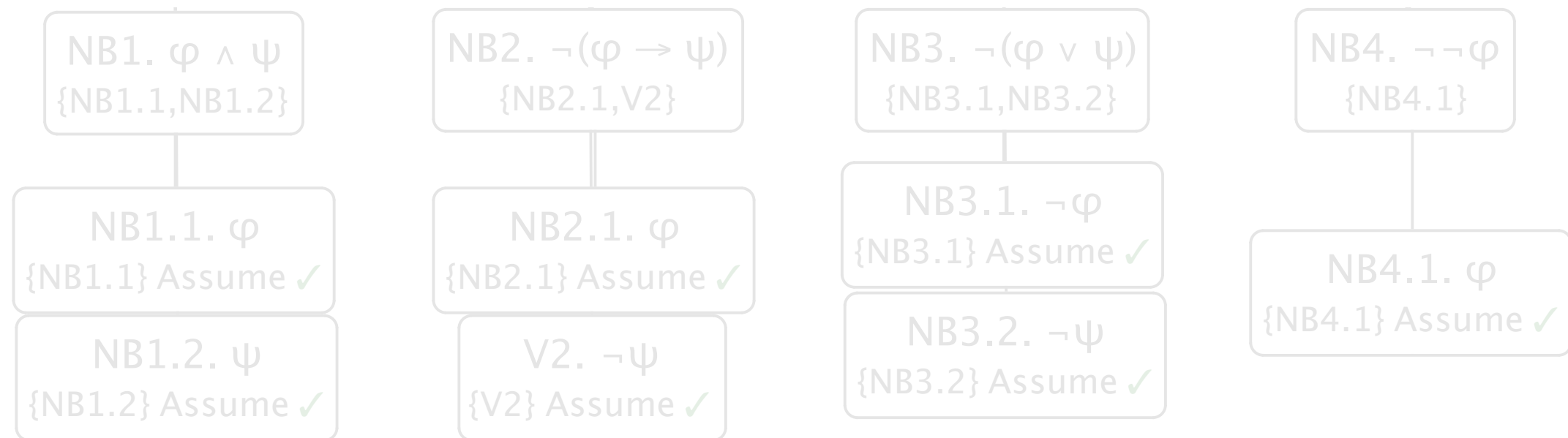


The Rules of the Game!





Questions?



Theorem:

Let ϕ be a theorem in the propositional calculus $= \mathcal{L}_{\text{PC}}$.
Then the truth-tree algorithm will lead to no open branches.

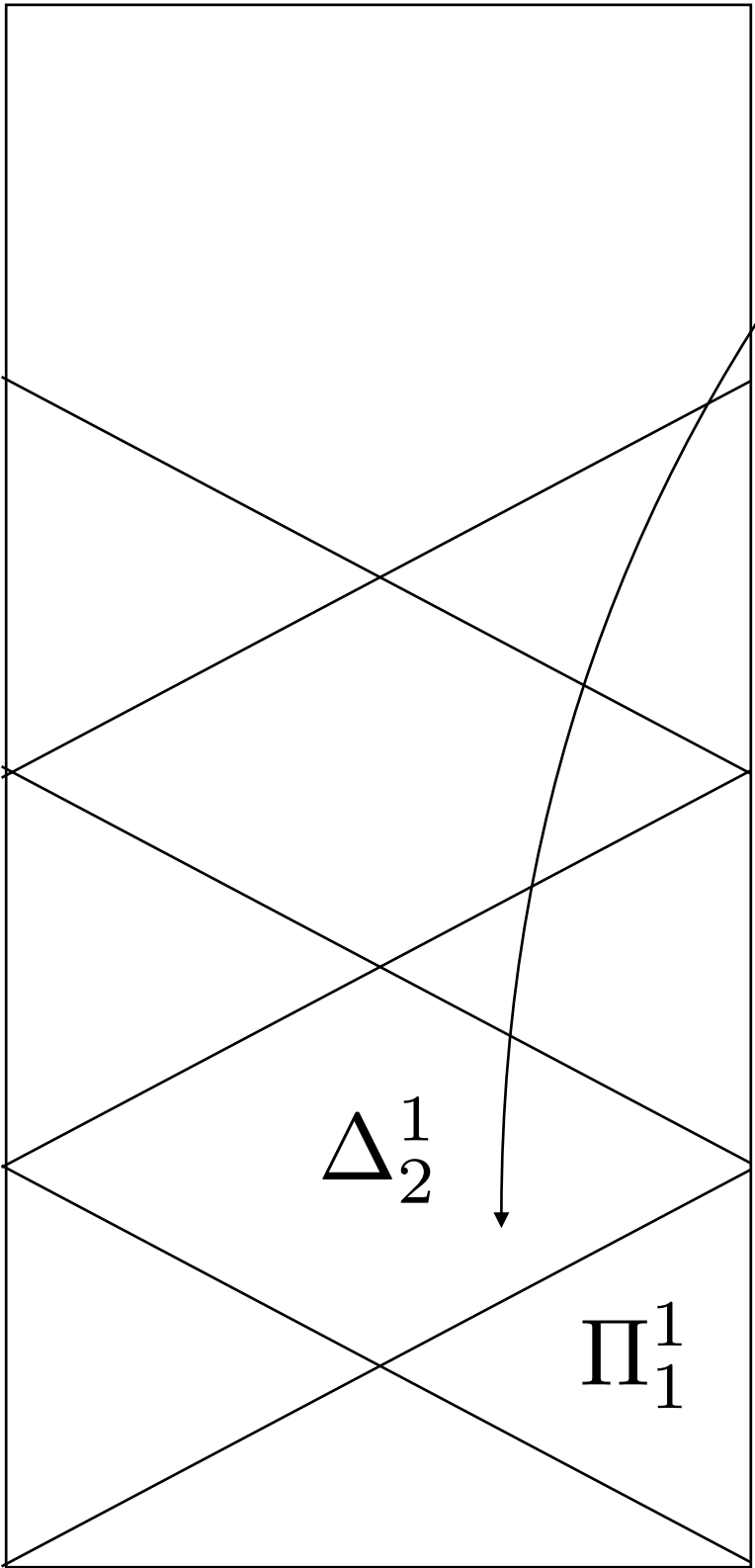
On Measuring The Intelligence of Agents

...

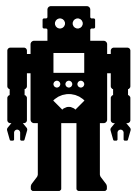
using quantification ...

$\mathcal{EM}$

$\mathcal{A}^n\mathcal{H}$ (Analytic Hierarchy)



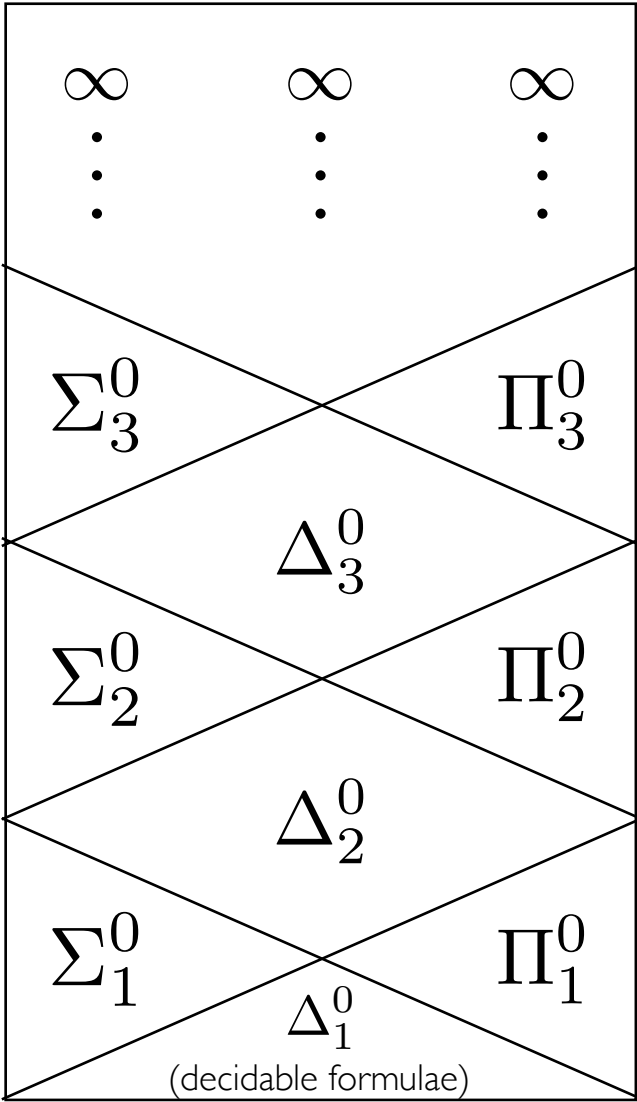
CogSci and AI need to say more about where AI falls/can fall in the landscape.



Infinite Time Turing Machines (ITTMs)

Human Persons
(according to Bringsjord)

$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)



Human Brains
(according to Granger)



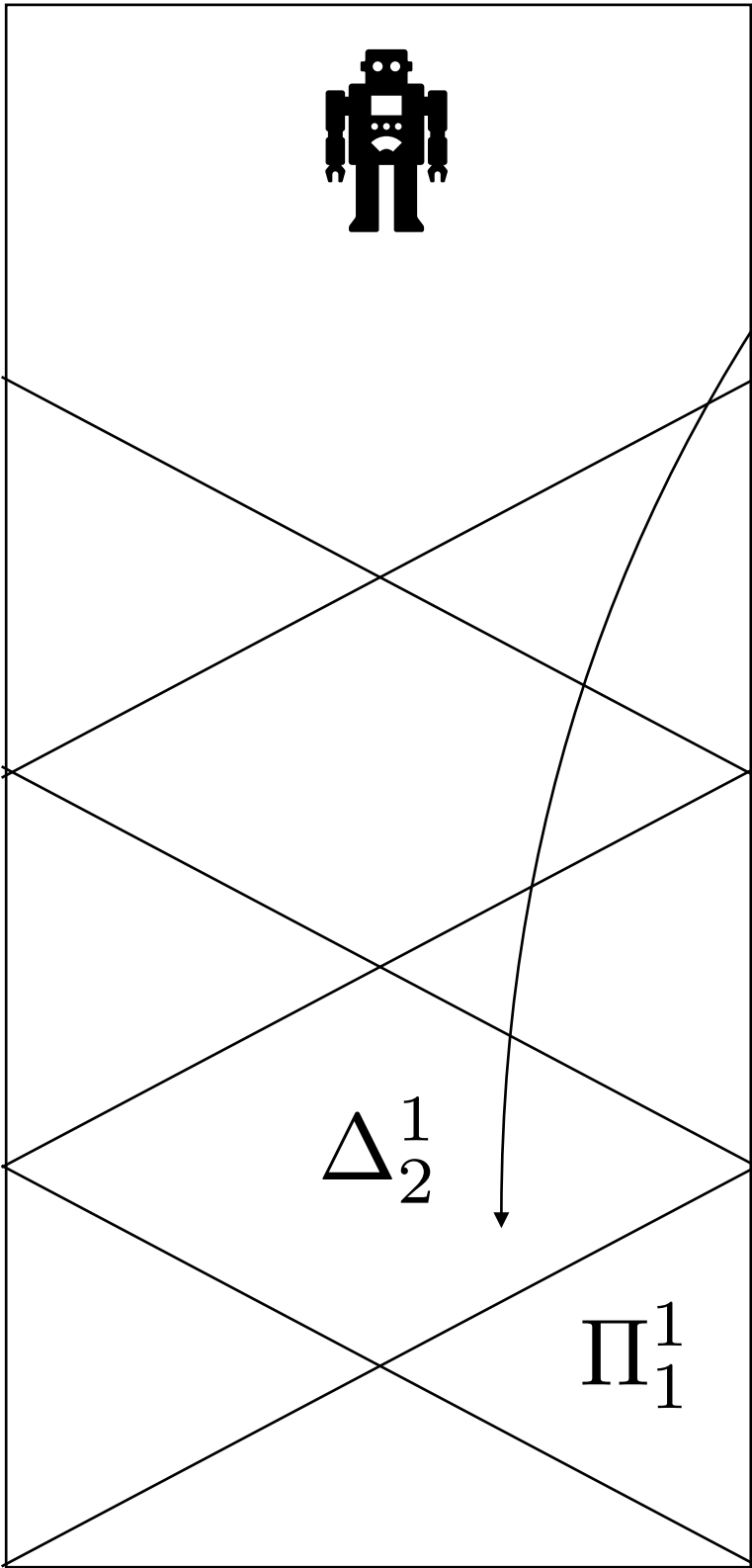
$\mathcal{CH}$ (Chomsky Hierarchy)

Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)



LM

$\mathcal{A}^n\mathcal{H}$ (Analytic Hierarchy)

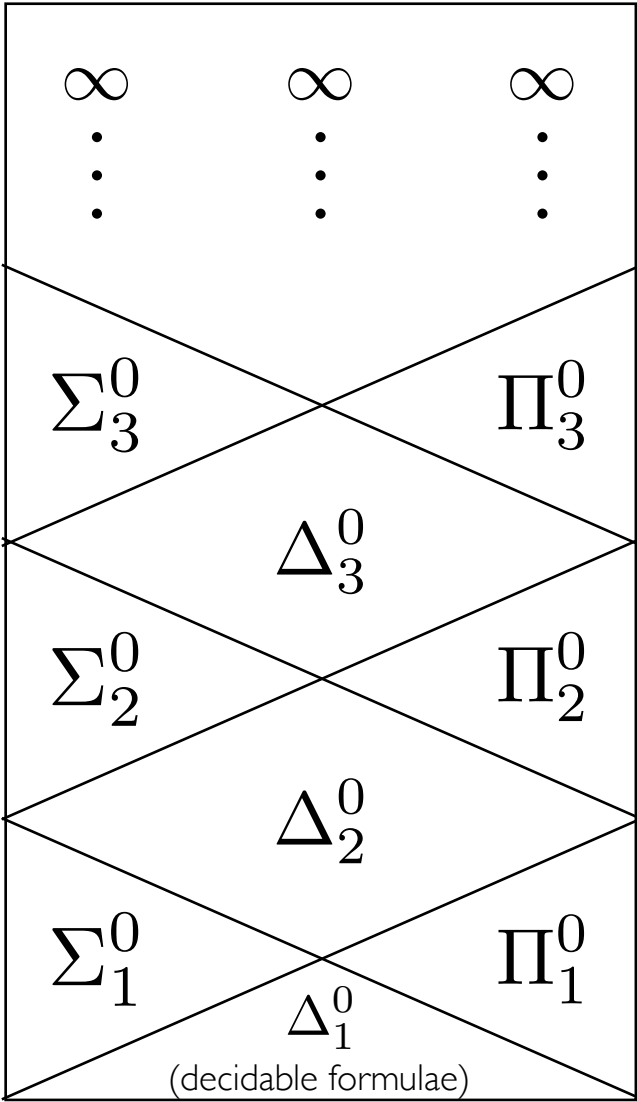


CogSci and AI need to say more about where AI falls/can fall in the landscape.

Infinite Time Turing Machines (ITTMs)

Human Persons
(according to Bringsjord)

$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)



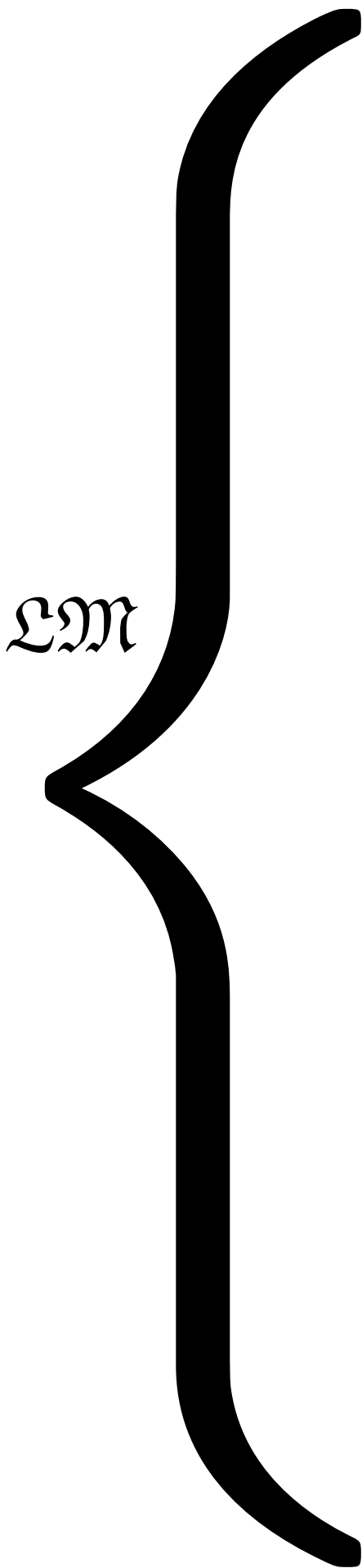
Human Brains
(according to Granger)



$\mathcal{CH}$ (Chomsky Hierarchy)

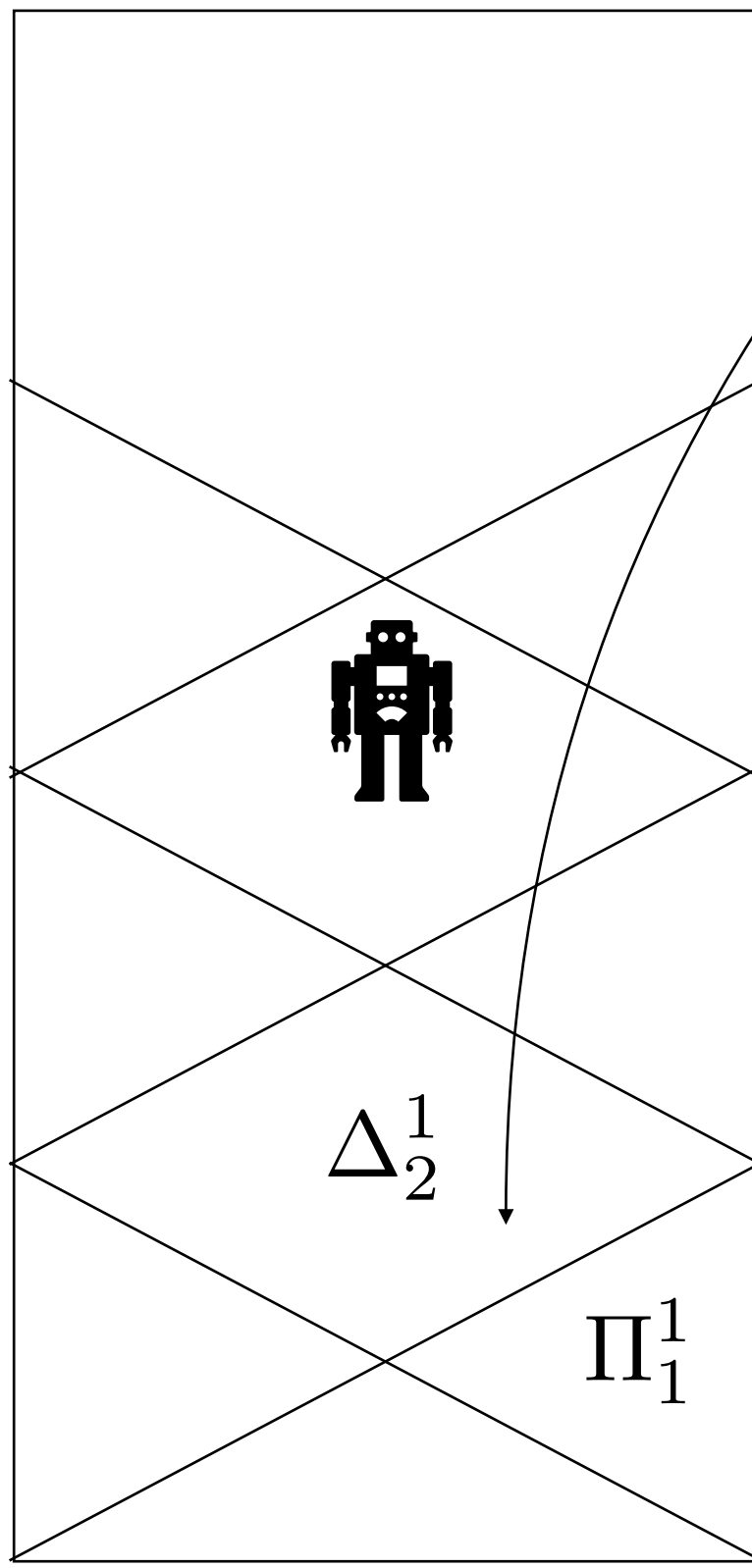
Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)





$\mathfrak{M}$

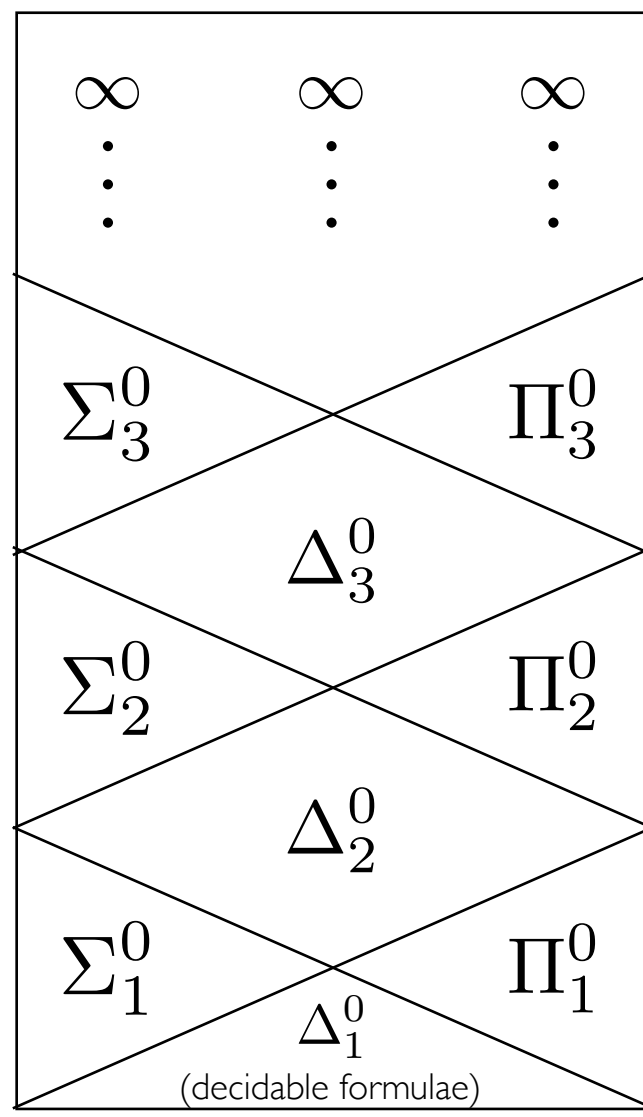
$\mathcal{A}^n \mathcal{H}$ (Analytic Hierarchy)



CogSci and AI need to say more about where AI falls/can fall in the landscape.

Infinite Time Turing Machines (ITTMs)

$\mathcal{A}^r \mathcal{H}$ (Arithmetic Hierarchy)



Human Persons
(according to Bringsjord)

Human Brains
(according to Granger)

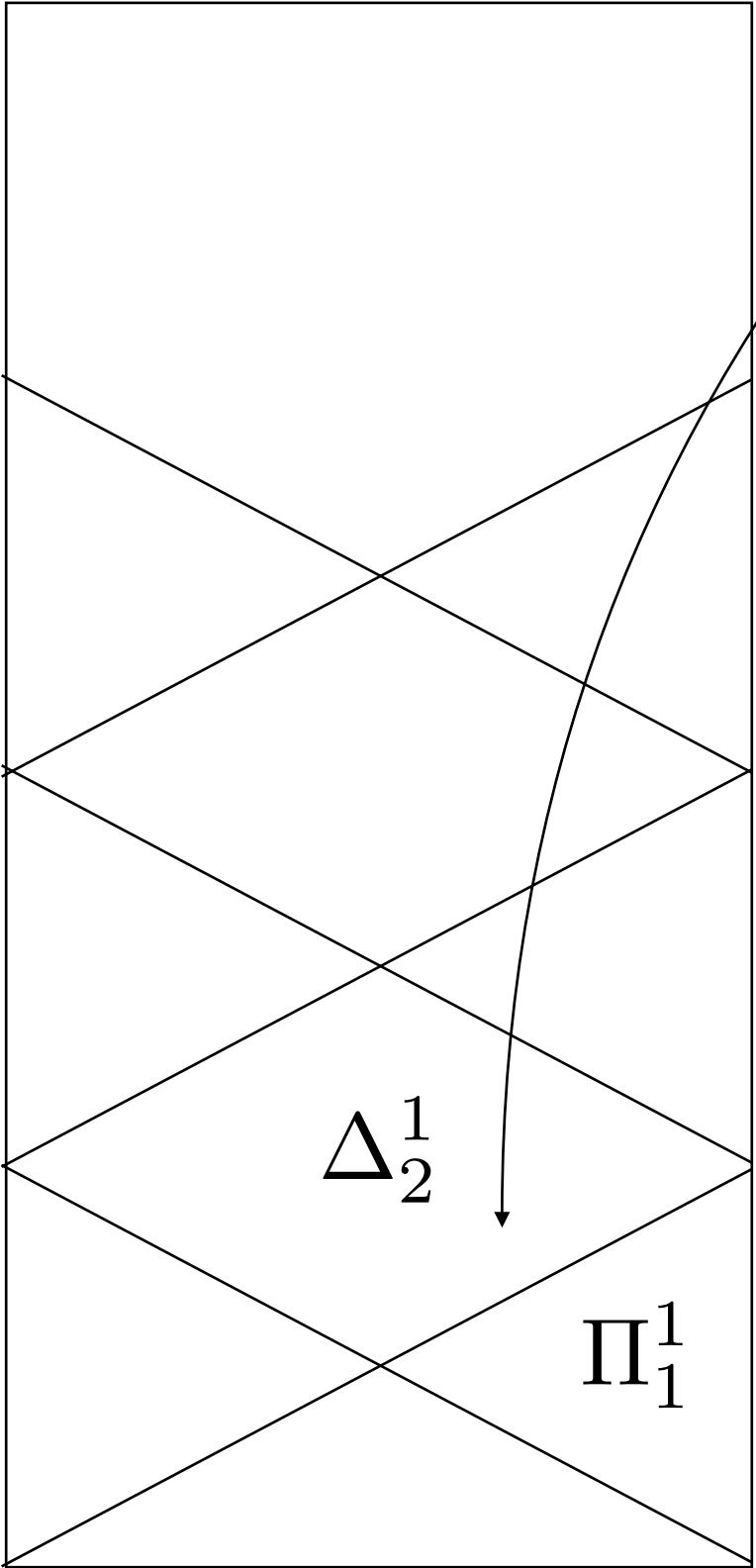


$\mathcal{CH}$ (Chomsky Hierarchy)

Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)

$\mathcal{LM}$

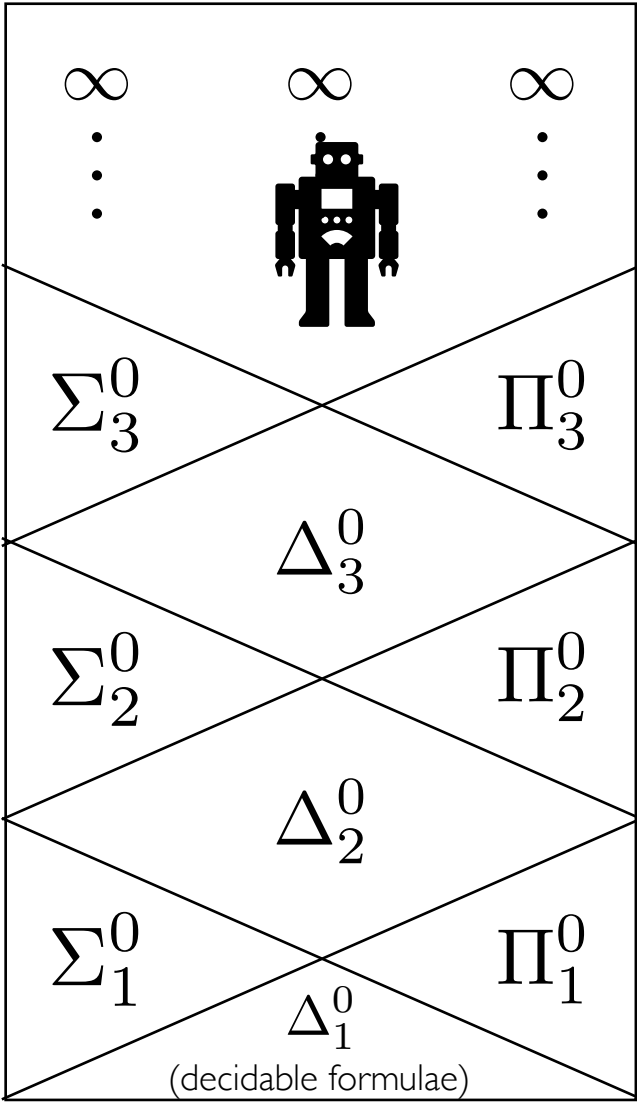
$\mathcal{A}^n\mathcal{H}$ (Analytic Hierarchy)



CogSci and AI need to say more about where AI falls/can fall in the landscape.

Infinite Time Turing Machines (ITTMs)

$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)



Human Persons
(according to Bringsjord)

Human Brains
(according to Granger)



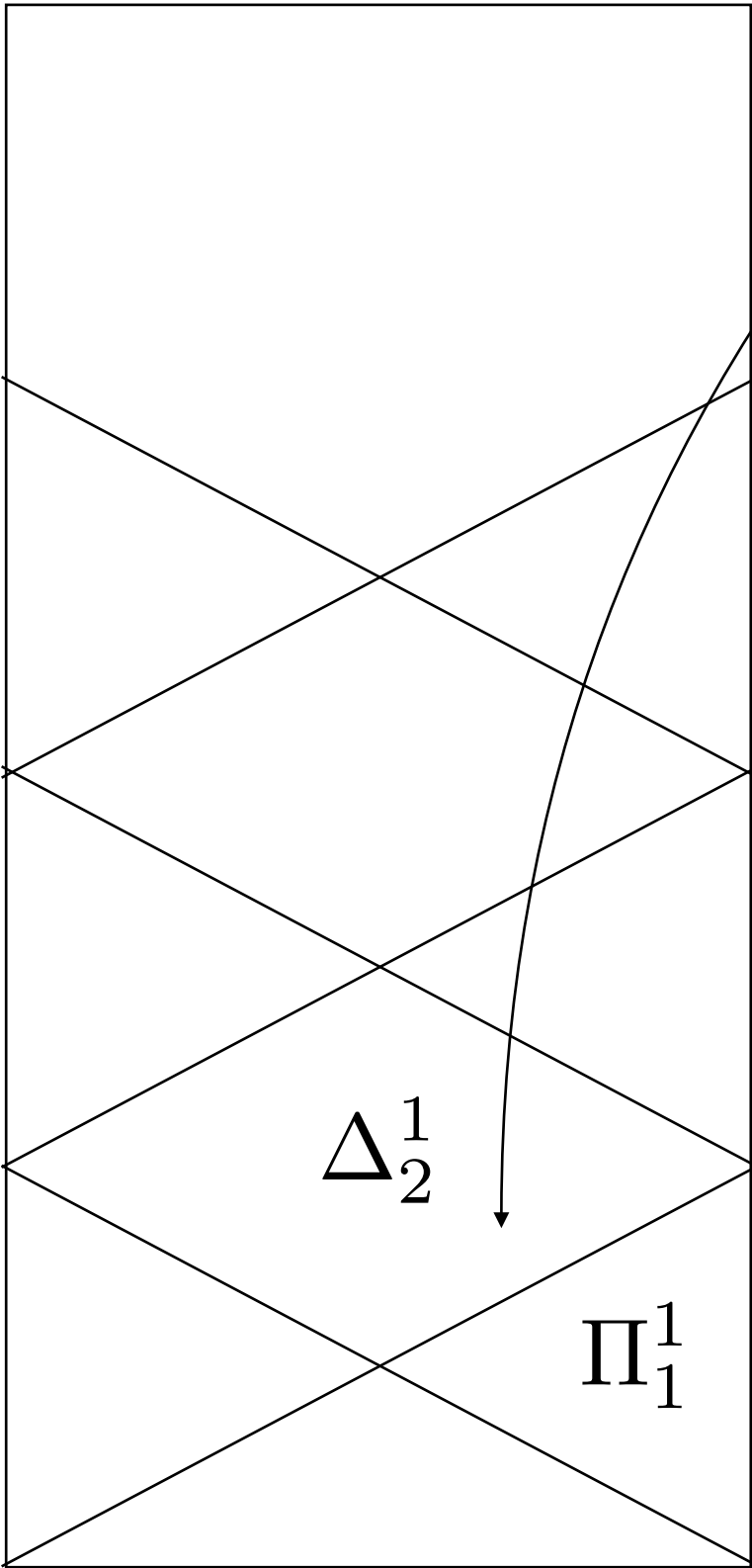
$\mathcal{CH}$ (Chomsky Hierarchy)

Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)



$\mathcal{LM}$

$\mathcal{A}^n\mathcal{H}$ (Analytic Hierarchy)

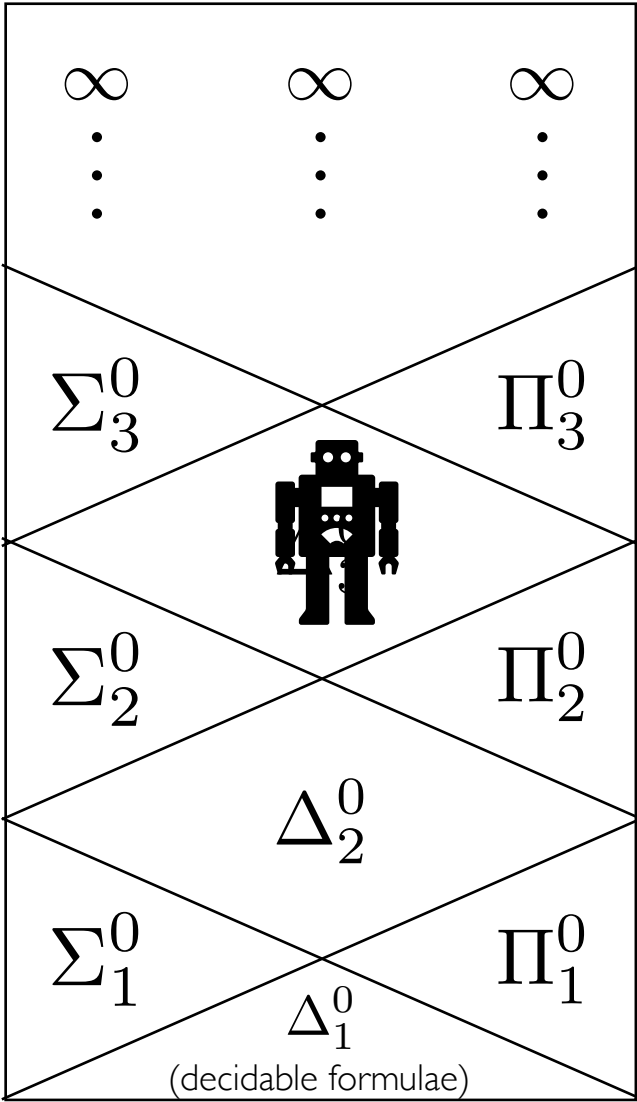


CogSci and AI need to say more about where AI falls/can fall in the landscape.

Infinite Time Turing Machines (ITTMs)

Human Persons
(according to Bringsjord)

$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)



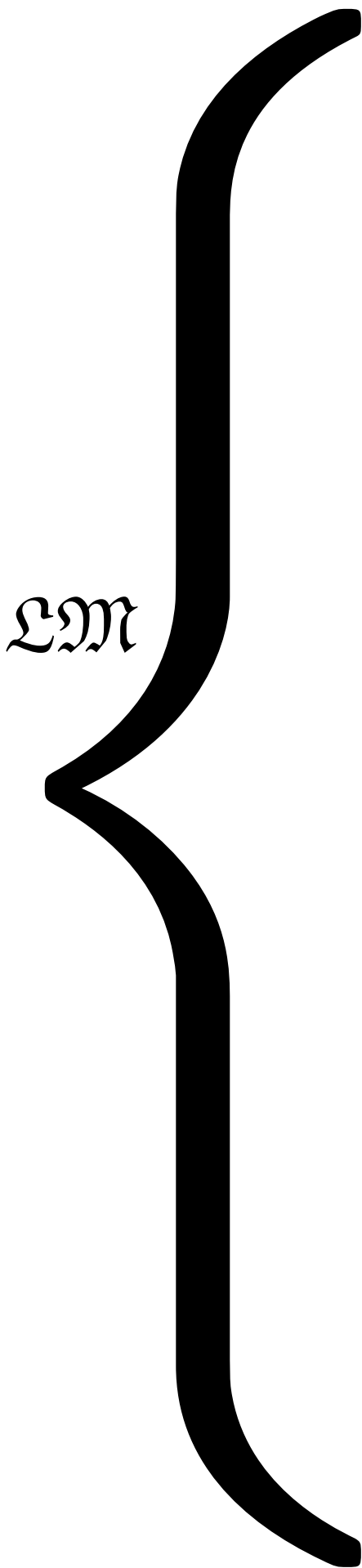
Human Brains
(according to Granger)



$\mathcal{CH}$ (Chomsky Hierarchy)

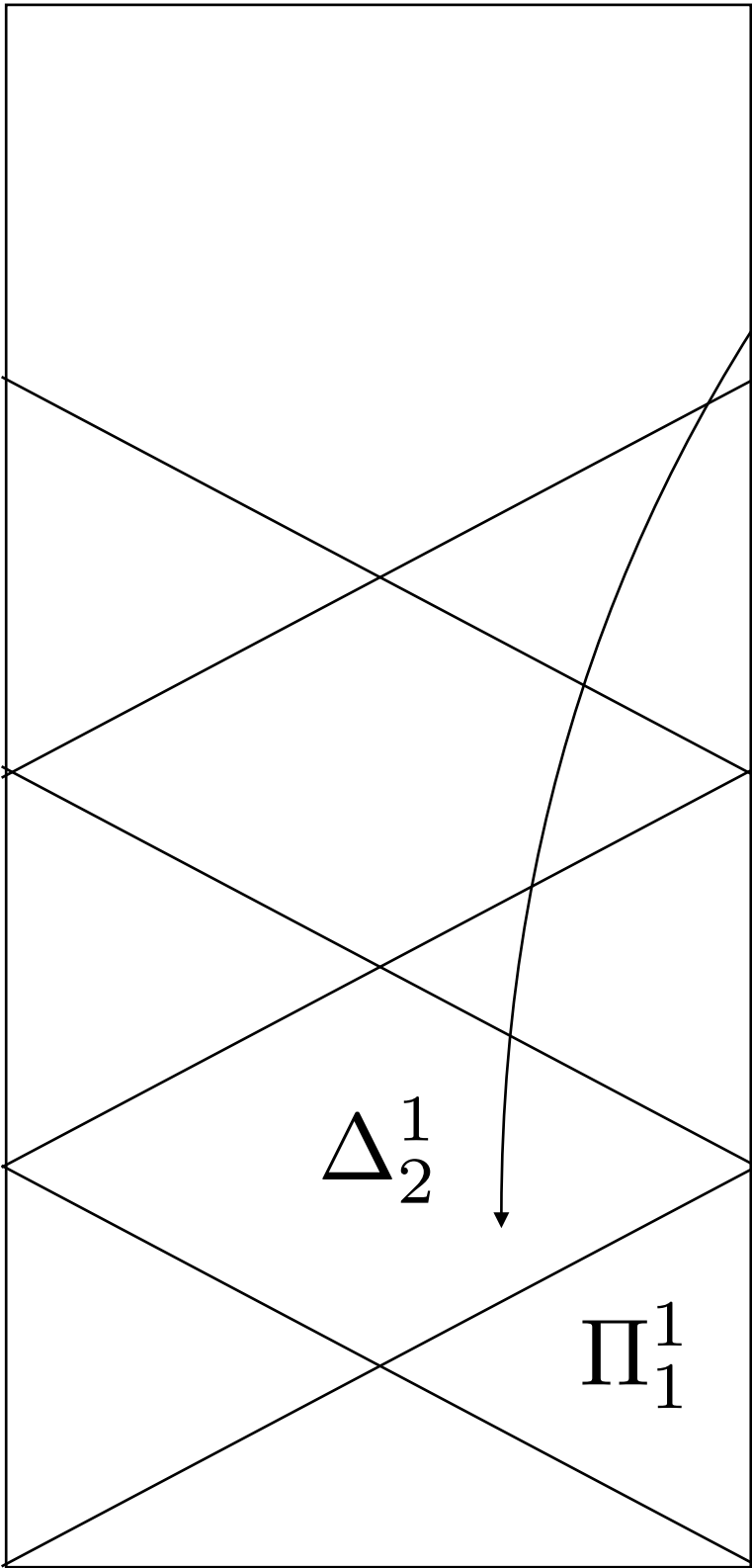
Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)





$\mathfrak{LH}$

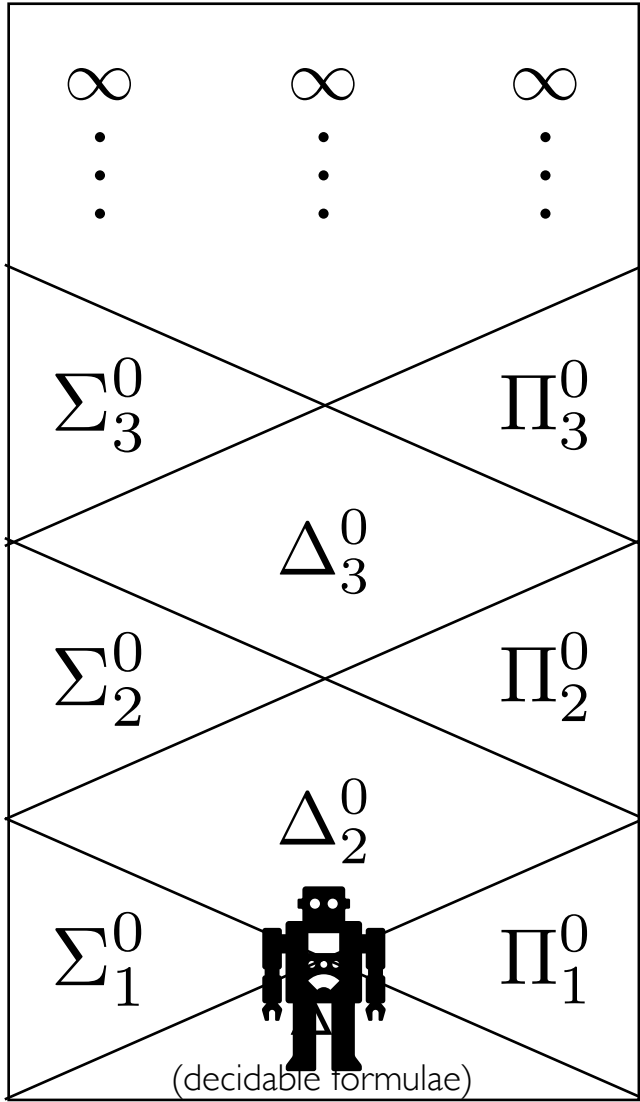
$\mathcal{A}^n \mathcal{H}$ (Analytic Hierarchy)



CogSci and AI need to say more about where AI falls/can fall in the landscape.

Infinite Time Turing Machines (ITTMs)

$\mathcal{A}^r \mathcal{H}$ (Arithmetic Hierarchy)



Human Persons
(according to Bringsjord)

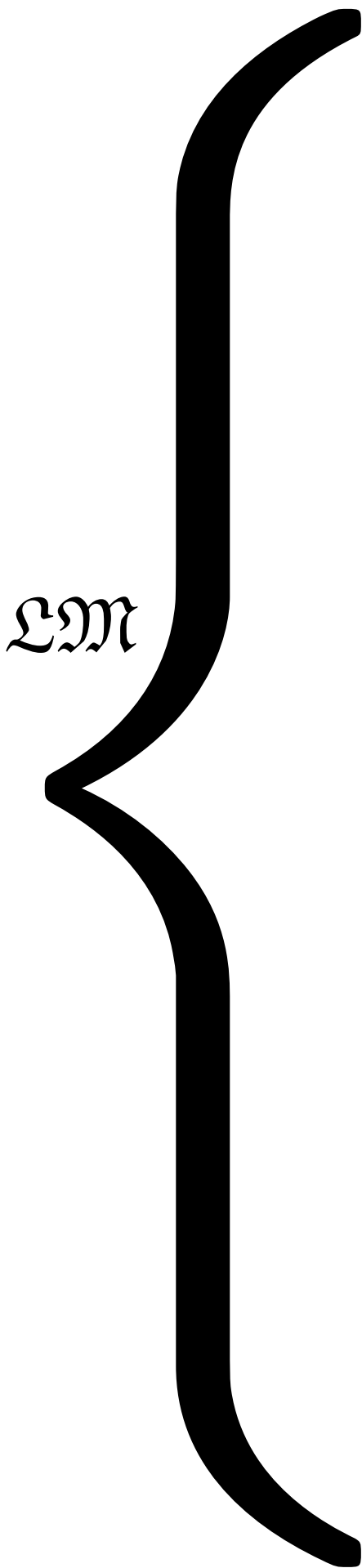
Human Brains
(according to Granger)



$\mathcal{CH}$ (Chomsky Hierarchy)

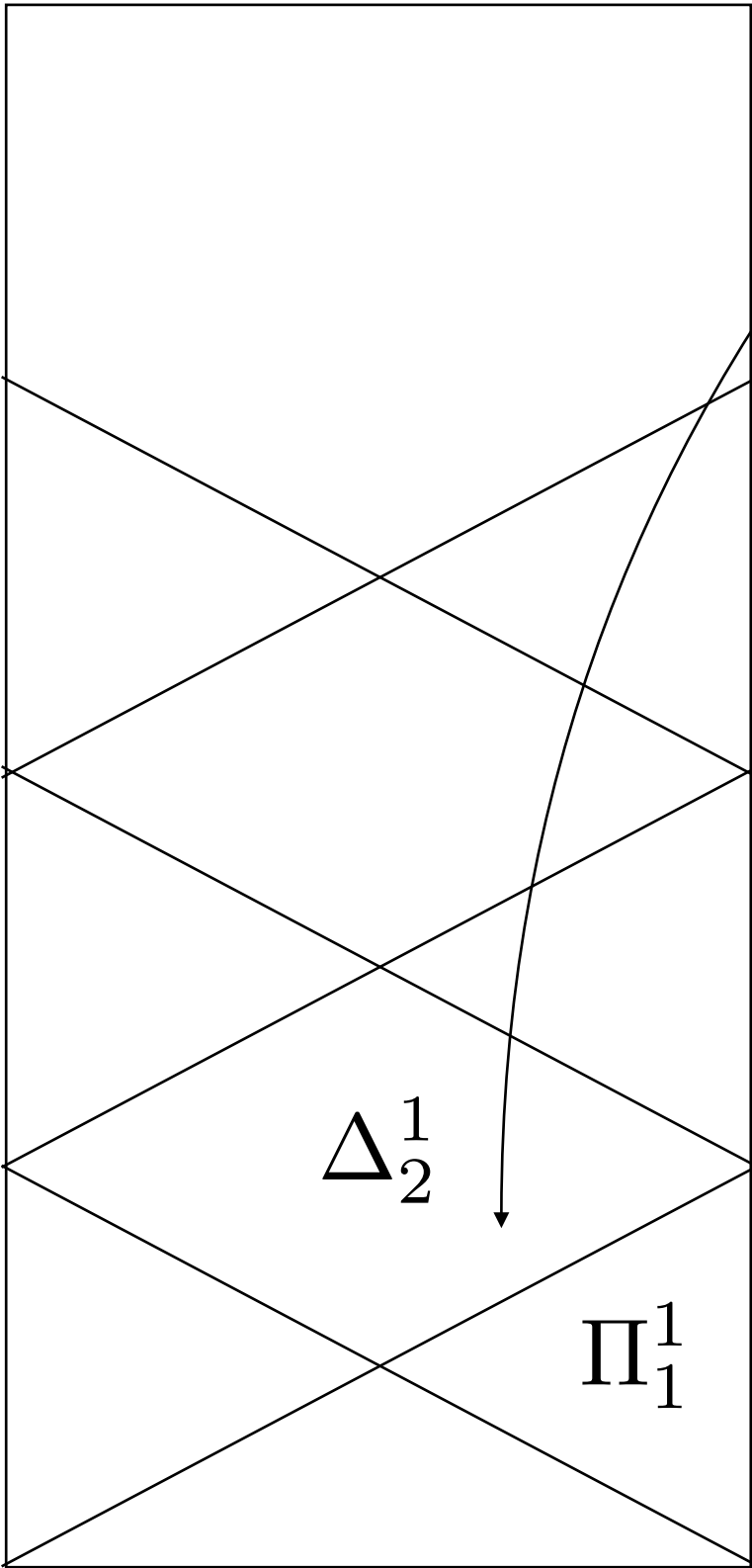
Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)





$\mathcal{LM}$

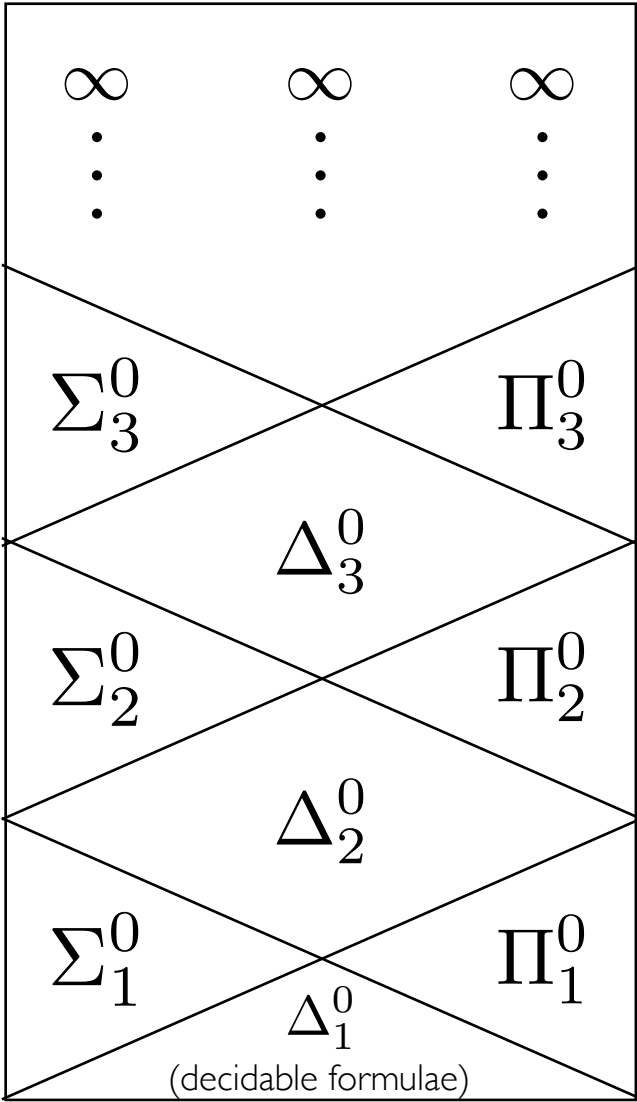
$\mathcal{A}^n\mathcal{H}$ (Analytic Hierarchy)



CogSci and AI need to say more about where AI falls/can fall in the landscape.

Infinite Time Turing Machines (ITTMs)

$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)



Human Persons
(according to Bringsjord)

Human Brains
(according to Granger)



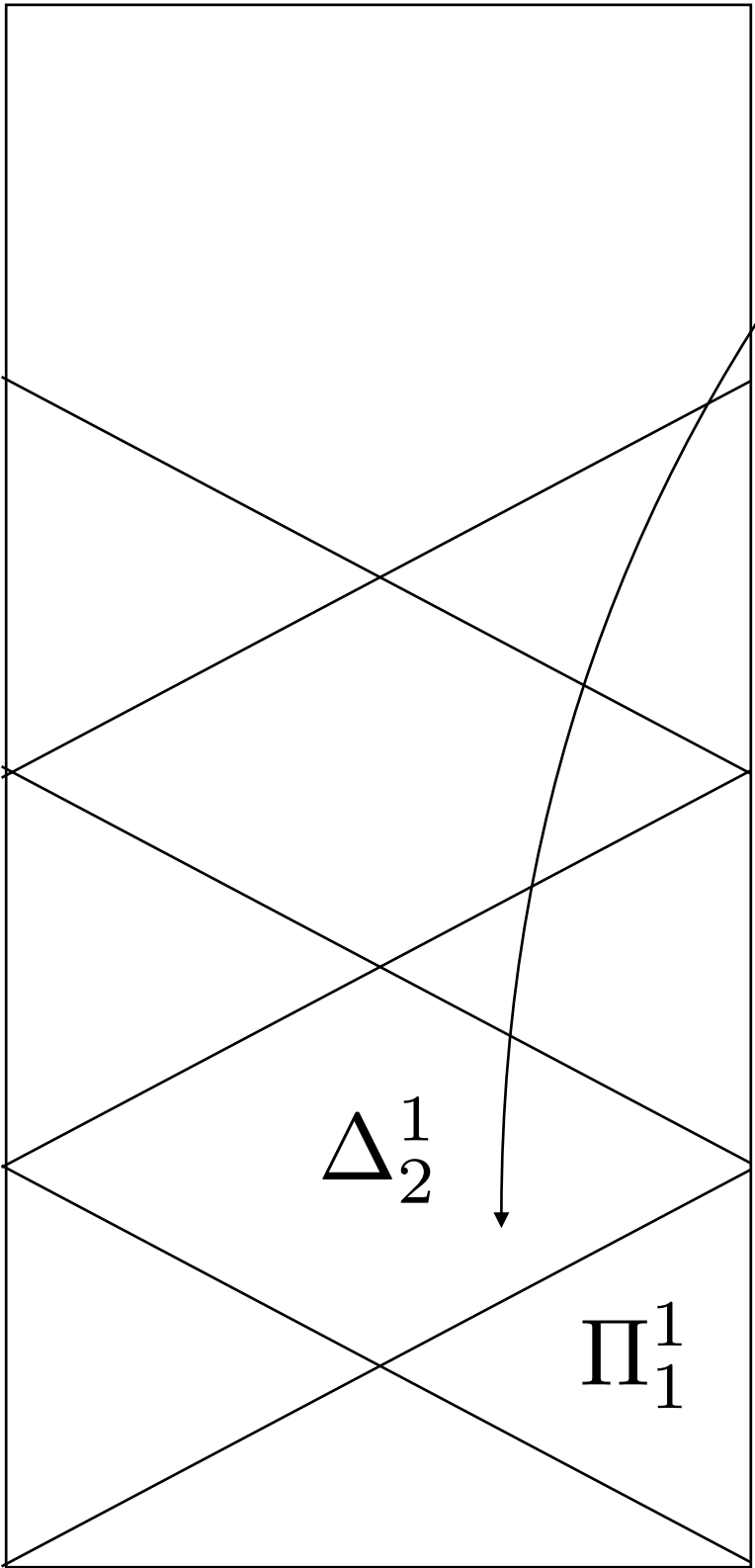
$\mathcal{CH}$ (Chomsky Hierarchy)

Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)

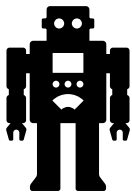


$\mathcal{EM}$

$\mathcal{A}^n\mathcal{H}$ (Analytic Hierarchy)



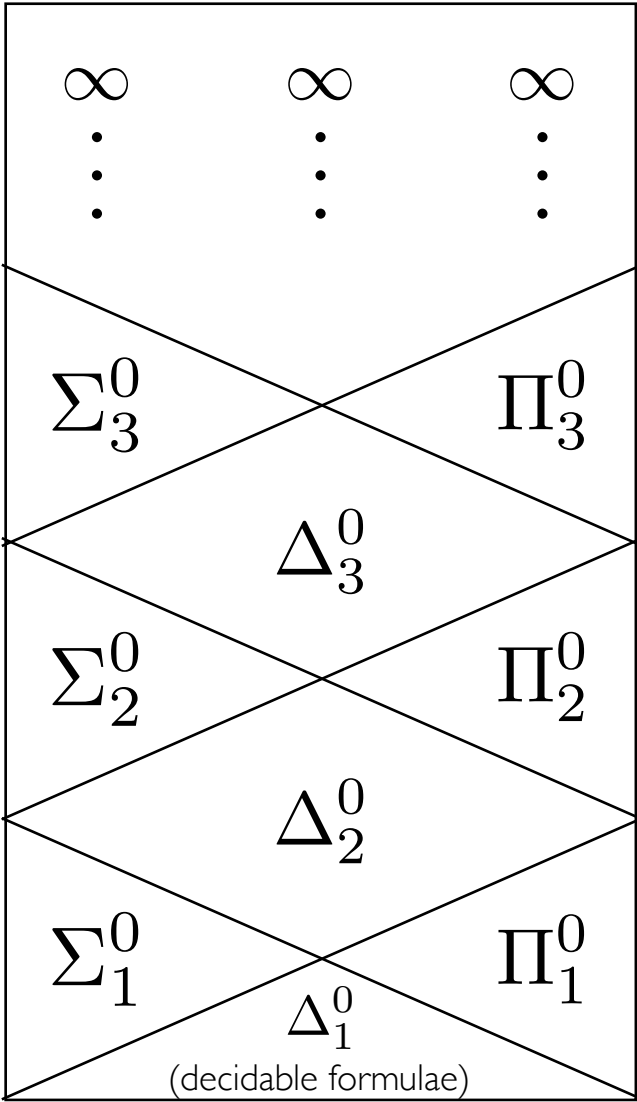
CogSci and AI need to say more about where AI falls/can fall in the landscape.



Infinite Time Turing Machines (ITTMs)

Human Persons
(according to Bringsjord)

$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)



Human Brains
(according to Granger)



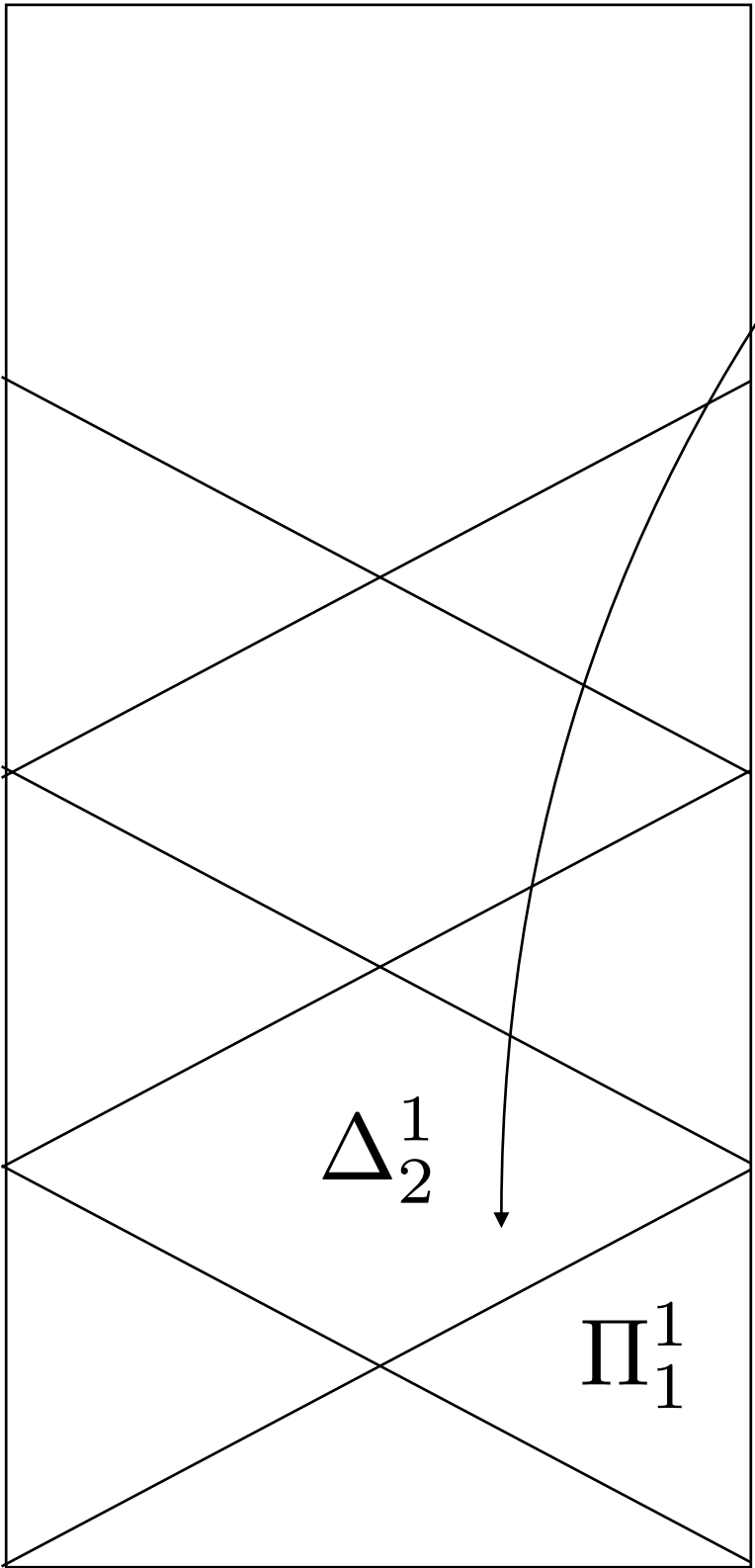
$\mathcal{CH}$ (Chomsky Hierarchy)

Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)

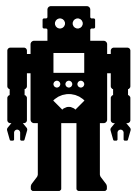


$\mathcal{EM}$

$\mathcal{A}^n\mathcal{H}$ (Analytic Hierarchy)



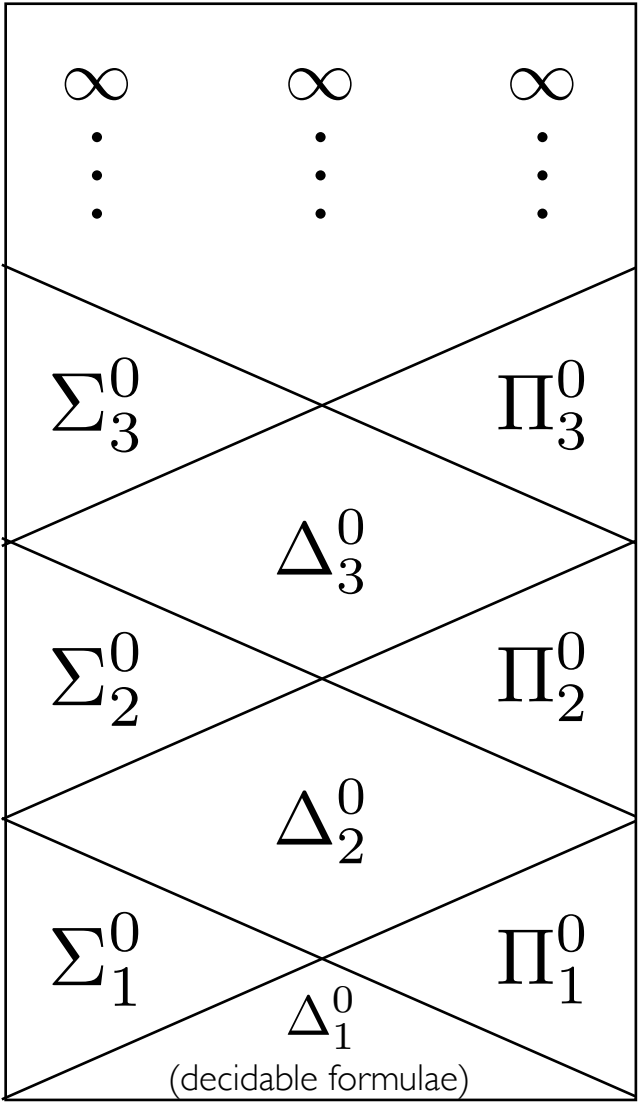
CogSci and AI need to say more about where AI falls/can fall in the landscape.



Infinite Time Turing Machines (ITTMs)

Human Persons
(according to Bringsjord)

$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)



Human Brains
(according to Granger)



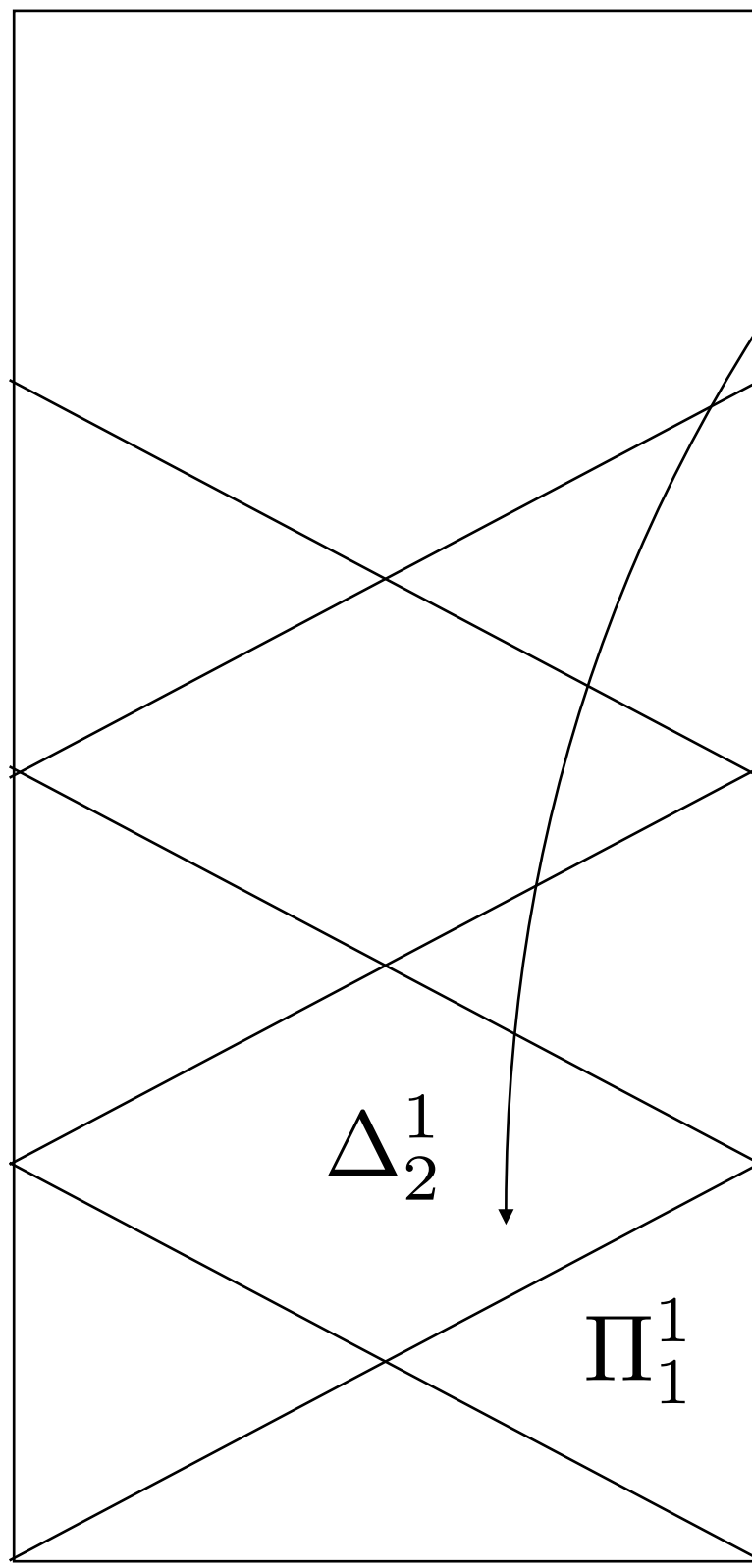
$\mathcal{CH}$ (Chomsky Hierarchy)

Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)

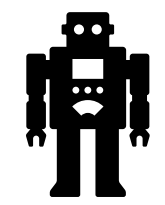


LM ?

$\mathcal{A}^n \mathcal{H}$ (Analytic Hierarchy)

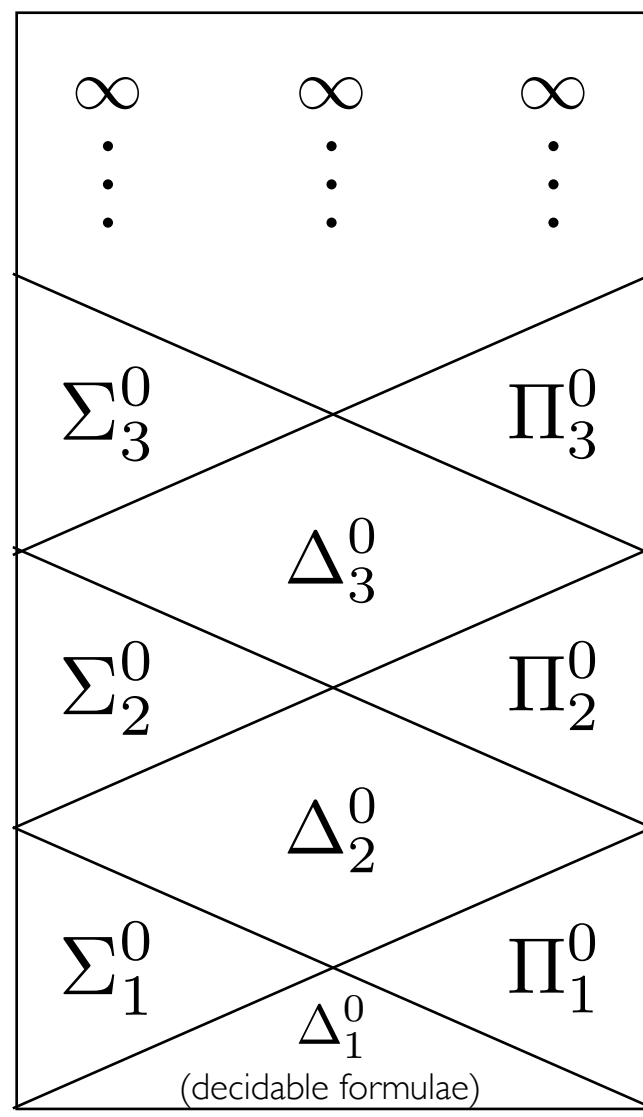


CogSci and AI need to say more about where AI falls/can fall in the landscape.



Infinite Time Turing Machines (ITTMs)

$\mathcal{A}^r \mathcal{H}$ (Arithmetic Hierarchy)



Human Persons (according to Bringsjord)

Human Brains (according to Granger)



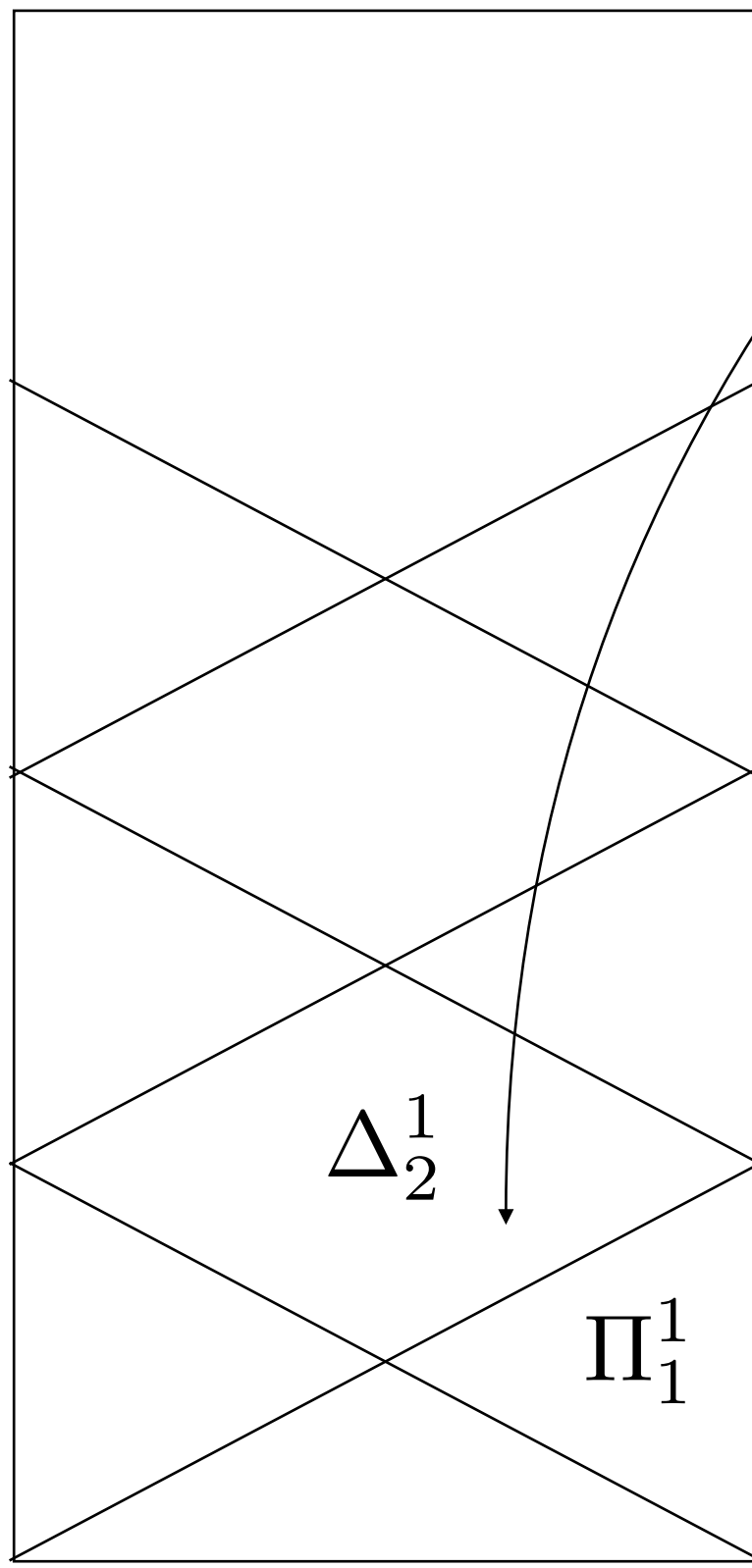
$\mathcal{CH}$ (Chomsky Hierarchy)

Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)

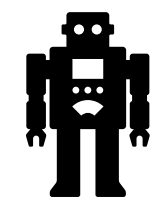


LM ?

$\mathcal{A}^n\mathcal{H}$ (Analytic Hierarchy)



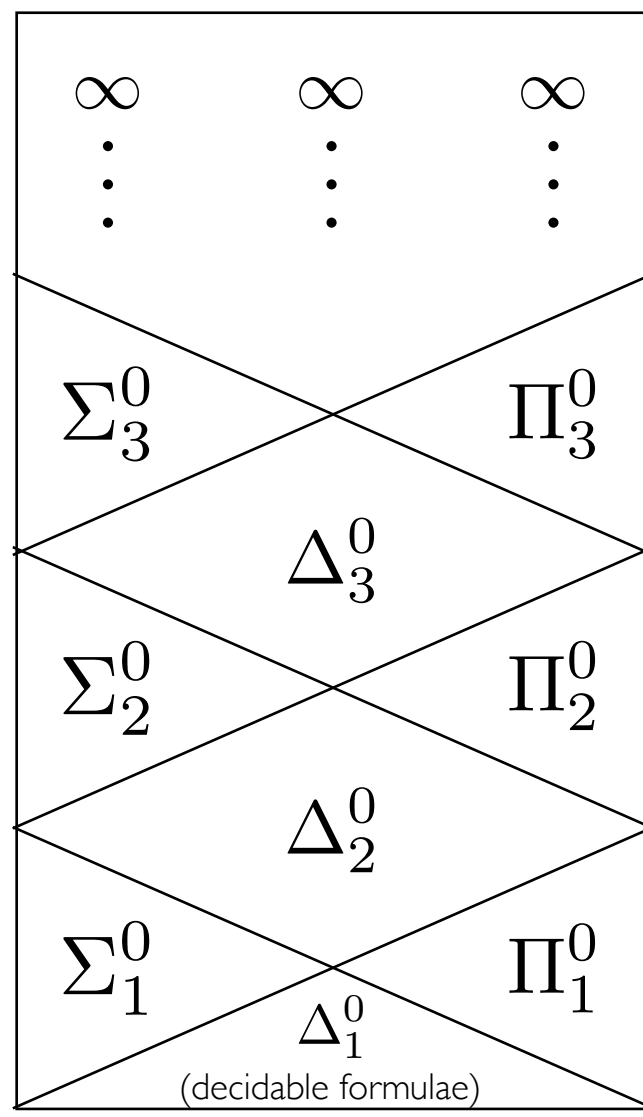
CogSci and AI need to say more about where AI falls/can fall in the landscape.



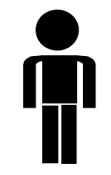
Infinite Time Turing Machines (ITTMs)

Human Persons
(according to Bringsjord)

$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)



Human Brains
(according to Granger)



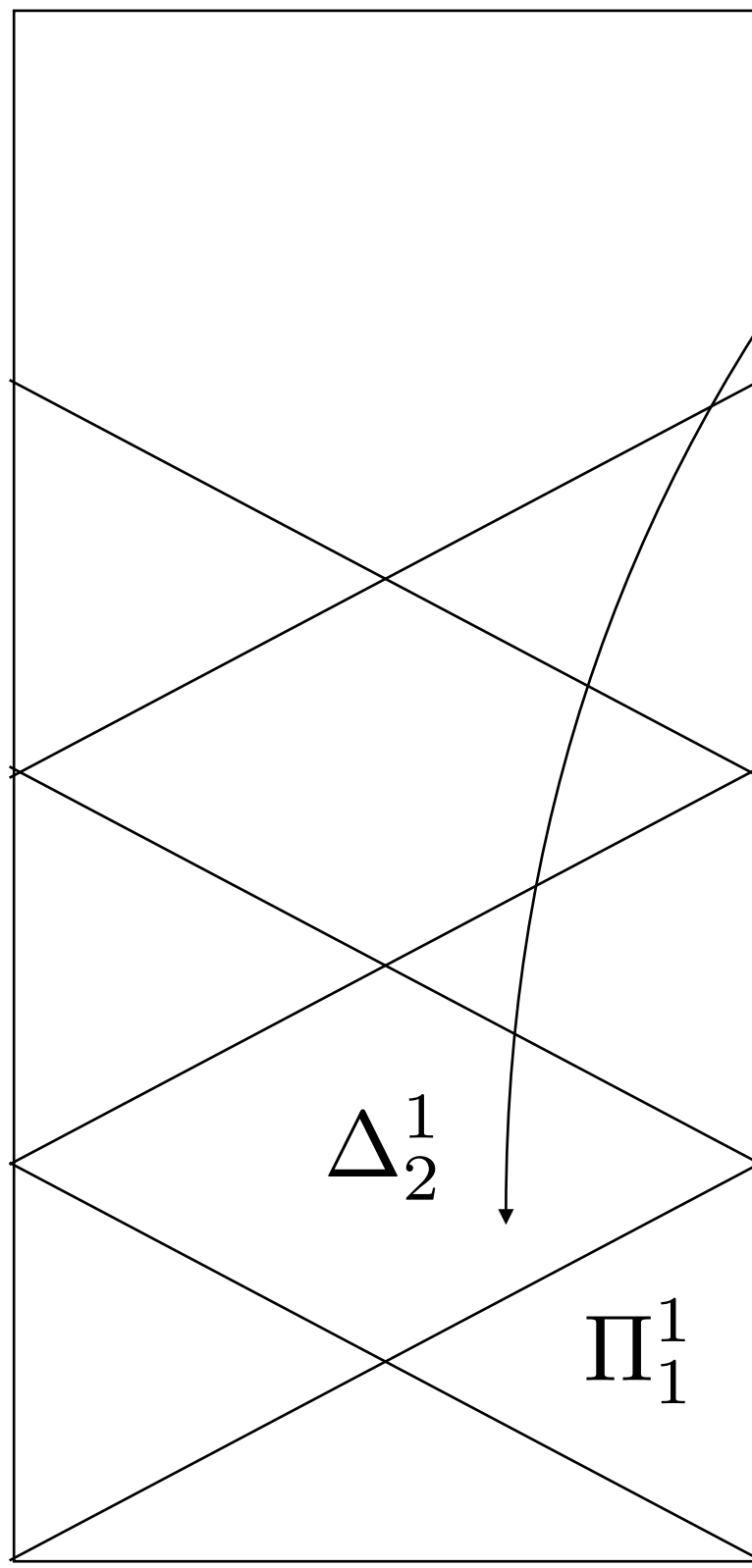
$\mathcal{CH}$ (Chomsky Hierarchy)

Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)



LM ?

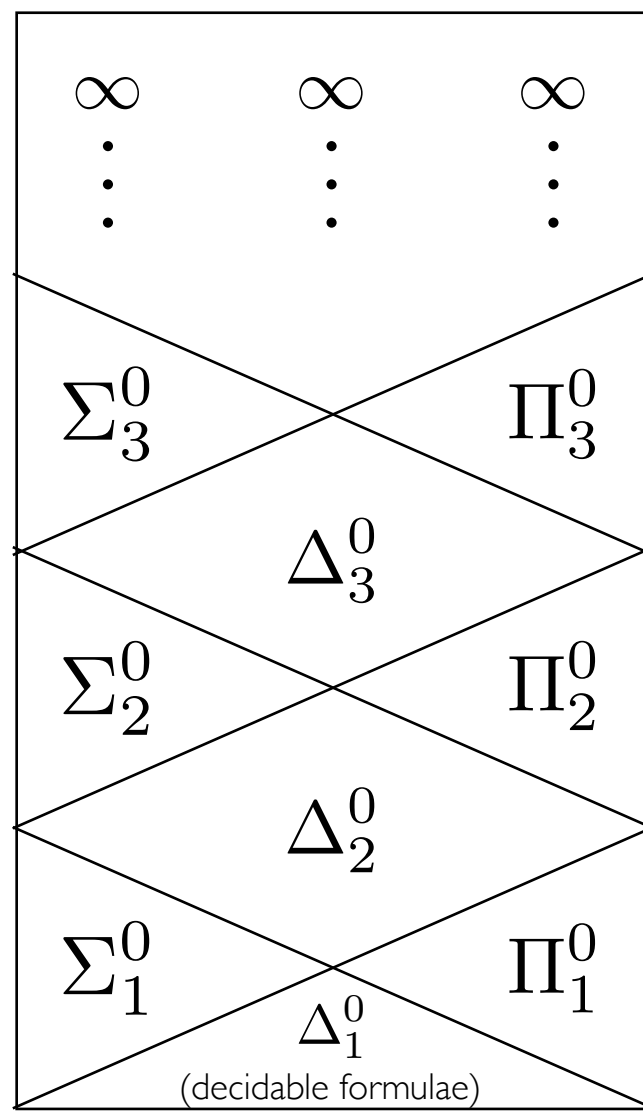
$\mathcal{A}^n \mathcal{H}$ (Analytic Hierarchy)



CogSci and AI need to say more about where AI falls/can fall in the landscape.

Infinite Time Turing Machines (ITTMs)

$\mathcal{A}^r \mathcal{H}$ (Arithmetic Hierarchy)



Human Persons
(according to Bringsjord)

Human Brains
(according to Granger)



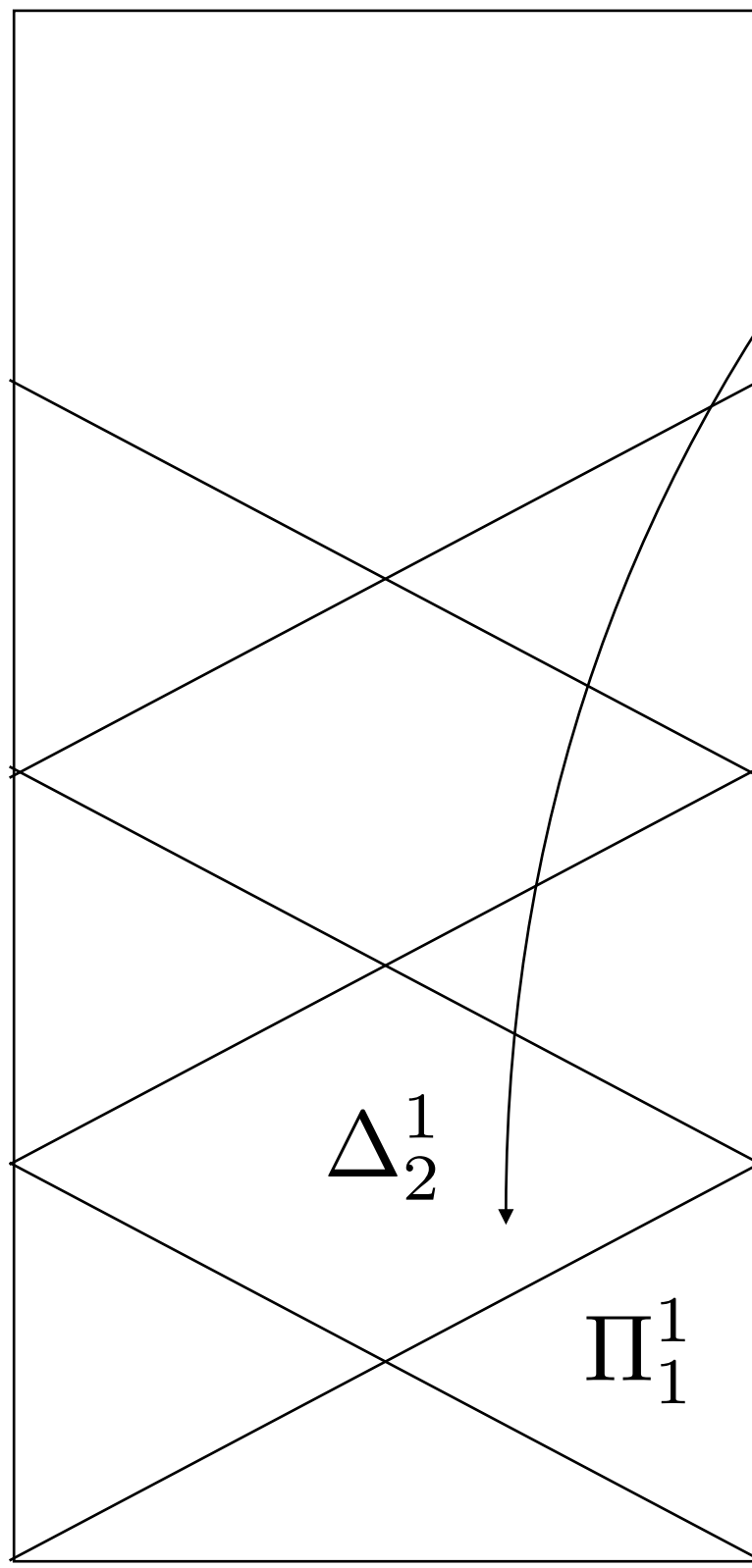
$\mathcal{CH}$ (Chomsky Hierarchy)



 Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)

LM ?

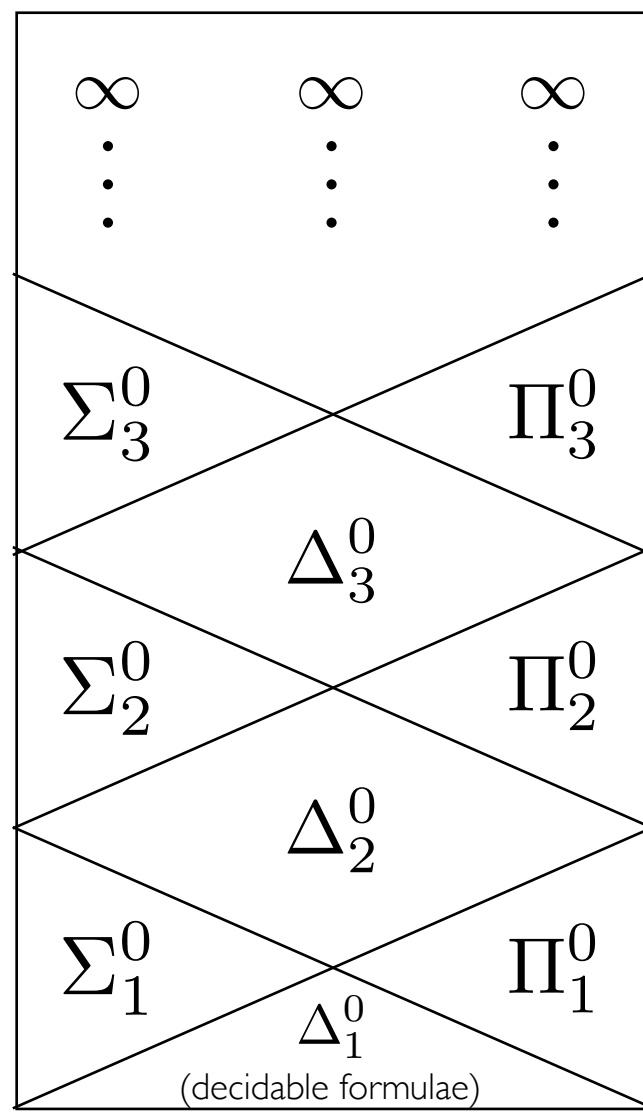
$\mathcal{A}^n \mathcal{H}$ (Analytic Hierarchy)



CogSci and AI need to say more about where AI falls/can fall in the landscape.

Infinite Time Turing Machines (ITTMs)

$\mathcal{A}^r \mathcal{H}$ (Arithmetic Hierarchy)

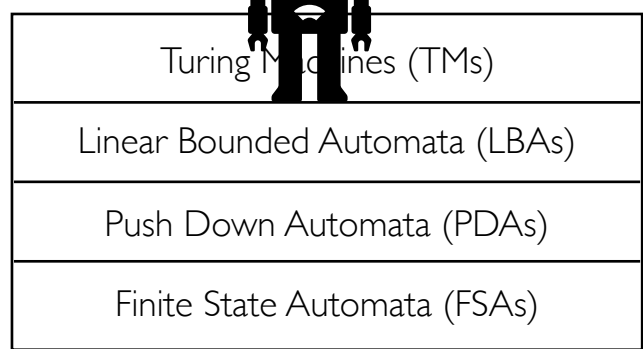


Human Persons (according to Bringsjord)

Human Brains (according to Granger)



$\mathcal{CH}$ (Chomsky Hierarchy)



Measuring Intelligence & AI/The Singularity

Measuring Intelligence & AI/The Singularity



The Singularity (superhuman machine intelligence) is near!!

Measuring Intelligence & AI/The Singularity



The Singularity (superhuman machine intelligence) is near!!

Is that so? And how are you measuring intelligence, pray tell?

Measuring Intelligence & AI/The Singularity

Is that so? And how are you measuring intelligence, pray tell?

Measuring Intelligence & AI/The Singularity

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

$$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$$

Measuring Intelligence & AI/The Singularity

Checkers:Chinook



Polynomial Hierarchy

$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

Checkers:Chinook



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Measuring Intelligence & AI/The Singularity

Go:AlphaGo



Polynomial Hierarchy

Checkers:Chinook



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

Go:AlphaGo



Checkers:Chinook



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Measuring Intelligence & AI/The Singularity

Jeopardy! -



Polynomial Hierarchy

Go:AlphaGo



Checkers:Chinook



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

Jeopardy! -



Go:AlphaGo



Checkers:Chinook



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Measuring Intelligence & AI/The Singularity



Polynomial Hierarchy

Jeopardy! -



Go:AlphaGo



Checkers:Chinook



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Measuring Intelligence & AI/The Singularity



Polynomial Hierarchy

Jeopardy! -



Go:AlphaGo



Checkers:Chinook



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Measuring Intelligence & AI/The Singularity



Polynomial Hierarchy

Jeopardy! -



Go:AlphaGo



Checkers:Chinook



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy



Polynomial Hierarchy

Jeopardy! -



Go:AlphaGo



Checkers:Chinook



$$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$$

Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy



$\vdots$
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Polynomial Hierarchy

Jeopardy! -



Go:AlphaGo



Checkers:Chinook



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy

“Hey, do these two Java programs compute the very same function?”



$\vdots$
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Polynomial Hierarchy

Jeopardy! -



Go:AlphaGo



Checkers:Chinook



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Measuring Intelligence & AI/The Singularity

Analytical Hierarchy

Arithmetical Hierarchy

“Hey, do these two Java programs compute the very same function?”



$\vdots$
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Polynomial Hierarchy

Jeopardy! -



Go:AlphaGo



Checkers:Chinook



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Measuring Intelligence & AI/The Singularity

Analytical Hierarchy

Arithmetical Hierarchy

This, all of this, is derived from consideration of first-order logic and second-order logic, with an emphasis on *quantification* and proof.

“Hey, do these two Java programs compute the very same function?”



$\vdots$
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Polynomial Hierarchy

Jeopardy! -



Go:AlphaGo



Checkers:Chinook



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

An “Advanced” Topic for Measuring Intelligence ...

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$$\exists x \exists y (x \neq y)$$

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

$\vdots$

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

$\vdots$

ϕ_n

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

$\vdots$

ϕ_n domain of at least n things

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

$\vdots$

ϕ_n

domain of at least n things

$\exists x \forall y (y = x)$

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

$\vdots$

ϕ_n domain of at least n things

$\exists x \forall y (y = x)$ at most one thing

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

$\vdots$

ϕ_n domain of at least n things

$\exists x \forall y (y = x)$ at most one thing

$\exists x \exists y \forall z (z = x \vee z = y)$

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

$\vdots$

ϕ_n domain of at least n things

$\exists x \forall y (y = x)$ at most one thing

$\exists x \exists y \forall z (z = x \vee z = y)$ at most two things

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

$\vdots$

ϕ_n domain of at least n things

$\exists x \forall y (y = x)$ at most one thing

$\exists x \exists y \forall z (z = x \vee z = y)$ at most two things

$\exists x_1 \exists x_2 \exists x_3 \forall y (y = x_1 \vee y = x_2 \vee y = x_3)$

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

$\vdots$

ϕ_n domain of at least n things

$\exists x \forall y (y = x)$ at most one thing

$\exists x \exists y \forall z (z = x \vee z = y)$ at most two things

$\exists x_1 \exists x_2 \exists x_3 \forall y (y = x_1 \vee y = x_2 \vee y = x_3)$ at most three things

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

$\vdots$

ϕ_n domain of at least n things

$\exists x \forall y (y = x)$ at most one thing

$\exists x \exists y \forall z (z = x \vee z = y)$ at most two things

$\exists x_1 \exists x_2 \exists x_3 \forall y (y = x_1 \vee y = x_2 \vee y = x_3)$ at most three things

$\vdots$

An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

$\exists x \exists y (x \neq y)$ at least two things

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ at least three things

$\vdots$

ϕ_n domain of at least n things

$\exists x \forall y (y = x)$ at most one thing

$\exists x \exists y \forall z (z = x \vee z = y)$ at most two things

$\exists x_1 \exists x_2 \exists x_3 \forall y (y = x_1 \vee y = x_2 \vee y = x_3)$ at most three things

$\vdots$

ϕ_n

For now, let's settle for
a focus on
quantification. Then ...

Measuring AI Intelligence via (in part) Logic:Quantification

Toby Walsh: “The Singularity May Never Be Near”
(<http://arxiv.org/pdf/1602.06462v1.pdf>)

Measuring AI Intelligence via (in part) Logic:Quantification

Toby Walsh: “The Singularity May Never Be Near”
(<http://arxiv.org/pdf/1602.06462v1.pdf>)

“I will not tackle here head on what we mean by measuring the intelligence of machines (or of humans). I will simply suppose there is such a property as intelligence, that it can be measured and compared, and that the technological singularity is when this measure increases exponentially fast in an appropriate and reasonable scale.” (p. 1)

But logico-mathematical definitions of intelligence for animals, humans, machines, aliens, gods ... *are possible*; recall our consideration of the *Entscheidungsproblem*. We can specifically challenge today's AI on the basis of simple quantification and simple deduction ...

First, need some numerical quantifiers:

First, need some numerical quantifiers:

$\exists x \forall y (y = x \wedge \phi(x))$ will be $\exists^{=1} x \phi(x)$

First, need some numerical quantifiers:

$\exists x \forall y (y = x \wedge \phi(x))$ will be $\exists^=1 x \phi(x)$

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z))$ will be $\exists^{\geq 3} x \phi(x)$

First, need some numerical quantifiers:

$\exists x \forall y (y = x \wedge \phi(x))$ will be $\exists^{=1} x \phi(x)$

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z))$ will be $\exists^{\geq 3} x \phi(x)$

How do we define formulae of this type: $\exists^{=k} x \psi(x)$

First, need some numerical quantifiers:

$\exists x \forall y (y = x \wedge \phi(x))$ will be $\exists^{=1} x \phi(x)$

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z))$ will be $\exists^{\geq 3} x \phi(x)$

How do we define formulae of this type: $\exists^{=k} x \psi(x)$

How do we define formulae of this type: $\exists^{\leq n} x \psi(x)$

First, need some numerical quantifiers:

$\exists x \forall y (y = x \wedge \phi(x))$ will be $\exists^{=1} x \phi(x)$

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z))$ will be $\exists^{\geq 3} x \phi(x)$

How do we define formulae of this type: $\exists^{=k} x \psi(x)$

How do we define formulae of this type: $\exists^{\leq n} x \psi(x)$

$\vdots$

First, need some numerical quantifiers:

$\exists x \forall y (y = x \wedge \phi(x))$ will be $\exists^{=1} x \phi(x)$

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z))$ will be $\exists^{\geq 3} x \phi(x)$

How do we define formulae of this type: $\exists^{=k} x \psi(x)$

How do we define formulae of this type: $\exists^{\leq n} x \psi(x)$

$\vdots$

Okay, now AI:

First, need some numerical quantifiers:

$\exists x \forall y (y = x \wedge \phi(x))$ will be $\exists^{=1} x \phi(x)$

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z))$ will be $\exists^{\geq 3} x \phi(x)$

How do we define formulae of this type: $\exists^{=k} x \psi(x)$

How do we define formulae of this type: $\exists^{\leq n} x \psi(x)$

$\vdots$

Okay, now AI:

At least seven kenspeckle blookers are red.

First, need some numerical quantifiers:

$\exists x \forall y (y = x \wedge \phi(x))$ will be $\exists^{=1} x \phi(x)$

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z))$ will be $\exists^{\geq 3} x \phi(x)$

How do we define formulae of this type: $\exists^{=k} x \psi(x)$

How do we define formulae of this type: $\exists^{\leq n} x \psi(x)$

$\vdots$

Okay, now AI:

At least seven kenspeckle blookers are red.

Given this, is it true that there are two red blookers? Why, exactly?

First, need some numerical quantifiers:

$\exists x \forall y (y = x \wedge \phi(x))$ will be $\exists^{=1} x \phi(x)$

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z))$ will be $\exists^{\geq 3} x \phi(x)$

How do we define formulae of this type: $\exists^{=k} x \psi(x)$

How do we define formulae of this type: $\exists^{\leq n} x \psi(x)$

$\vdots$

Okay, now AI:

At least seven kenspeckle blookers are red.

Given this, is it true that there are two red blookers? Why, exactly?

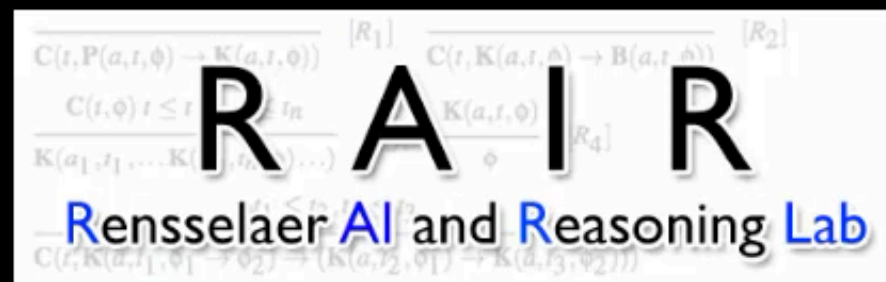
$$\begin{aligned}
& \forall x \forall y \forall z \{ [x \neq y \wedge y \neq z \wedge x \neq z \wedge Cx \wedge Cy \wedge Cz \wedge \\
& \hspace{25em} Tz' \wedge \\
& \exists w_1 \exists w_2 (w_1 \neq w_2 \wedge Aw_1 \wedge Aw_2 \wedge Gz'w_1 \wedge Gz'w_2) \wedge \\
& \forall u_1 \forall u_2 \forall u_3 ((Gz'u_1 \wedge Gz'u_2 \wedge Gz'u_3 \wedge C^b u_1 \wedge C^b u_2 \wedge C^b u_3) \rightarrow \\
& \hspace{10em} \forall v ((Gz'v \wedge C^b v) \rightarrow (v = u_1 \vee v = u_2 \vee v = u_3)))] \\
& \hspace{25em} \rightarrow \\
& \hspace{15em} (Gxz' \wedge Gyz' \wedge Gzz') \}
\end{aligned}$$

Every three cylinders glower at any triangular prism that glowers at at least two arches and at at most three cubes.

$$\forall x \forall y \forall z \forall z' \left\{ \left[\begin{array}{c} x \neq y \wedge y \neq z \wedge x \neq z \\ \wedge \\ Cx \wedge Cy \wedge Cz \\ \wedge \\ Tz' \\ \wedge \\ \exists w_1 \exists w_2 (w_1 \neq w_2 \wedge Aw_1 \wedge Aw_2 \wedge Gz'w_1 \wedge Gz'w_2) \\ \wedge \\ \forall u_1 \forall u_2 \forall u_3 \left(\begin{array}{c} [Gz'u_1 \wedge Gz'u_2 \wedge Gz'u_3 \wedge C^b u_1 \wedge C^b u_2 \wedge C^b u_3] \\ \rightarrow \\ \forall v [(Gz'v \wedge C^b v) \rightarrow (v = u_1 \vee v = u_2 \vee v = u_3)] \end{array} \right) \\ \rightarrow \\ (Gxz' \wedge Gyz' \wedge Gzz') \end{array} \right] \right\}$$

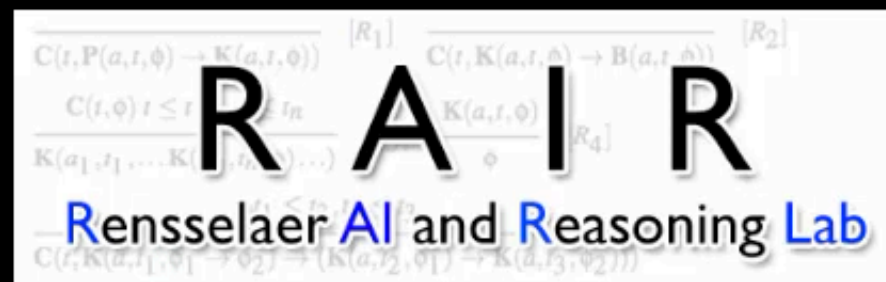
Intelligent Artificial Multi-Agent

Tentacular AI™ AI
at Work in Problem-Solving in VQ⁺AJV



Intelligent Artificial Multi-Agent

Tentacular AI™ AI
at Work in Problem-Solving in VQ⁺AJV



Part I: *Slutten* — *for i dag.*

Part I: *Slutten* — *for i dag*.

Part II: Hands-on Q&A & Review ...