

Propositional Calculus I: The Formal Language, The Prop. Calc. Oracle (= AI), Application to Some Motivating Problems

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IFLAI [Intro to (Formal) Logic (and AI)]
1/23/2025





Questions about history & The Singularity,
our focus last time ...?



In our “Why study logic?”
mtgs we skipped over ...



The Monty Hall Problem



\$1M





The Monty Hall Problem



\$1M





The Monty Hall Problem



\$1M





The Monty Hall Problem



\$1M





The Monty Hall Problem



\$1M





The Monty Hall Problem





The Monty Hall Problem



\$1M

Logic-and-AI in the news

...

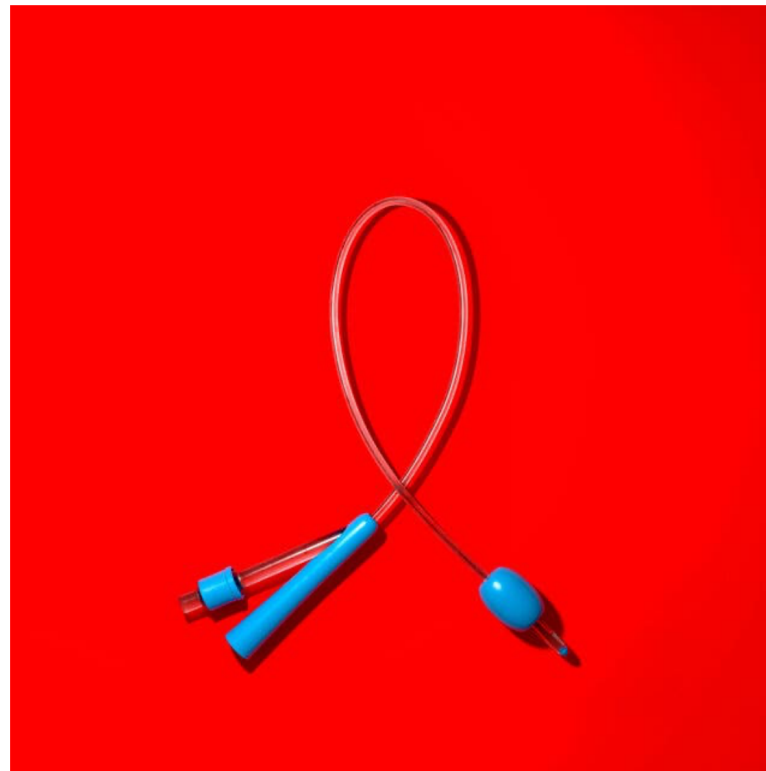
Speaking of n -door problems, a case for a
(tragic) study in inductive logic ...

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A Start-Up Claimed Its Device Could Cure Cancer. Then Patients Began Dying.

Two U.S. companies teamed up to treat cancer patients using an unproven blood filter in Antigua, out of reach of American regulators.

23 MIN READ



Javier Jaén



John Carreyrou

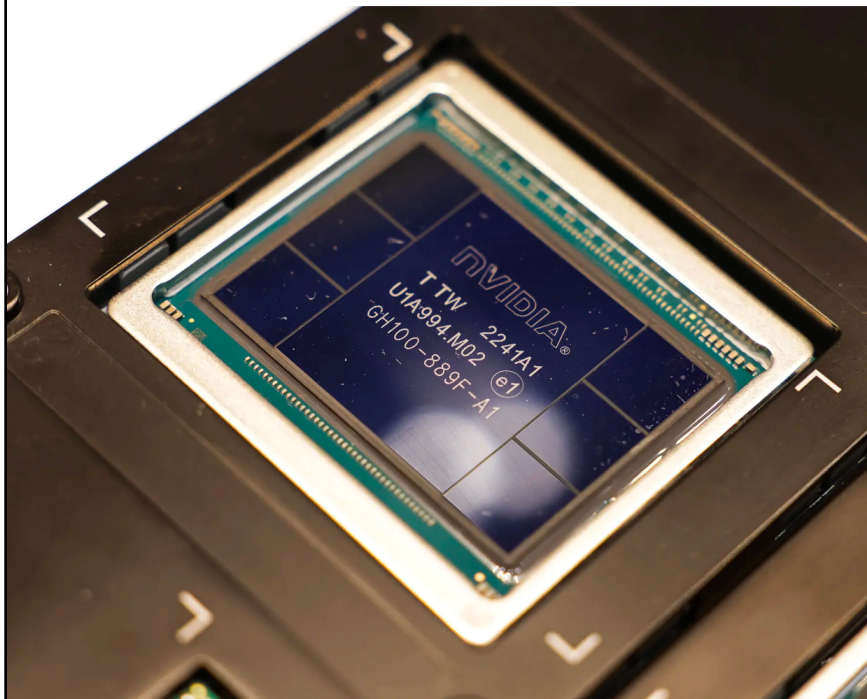
& re my claim of Chatbots/LLMs/
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How Chinese A.I. Start-Up DeepSeek Is Competing With Silicon Valley Giants

The company built a cheaper, competitive chatbot with fewer high-end computer chips than U.S. behemoths like Google and OpenAI, showing the limits of chip export control.

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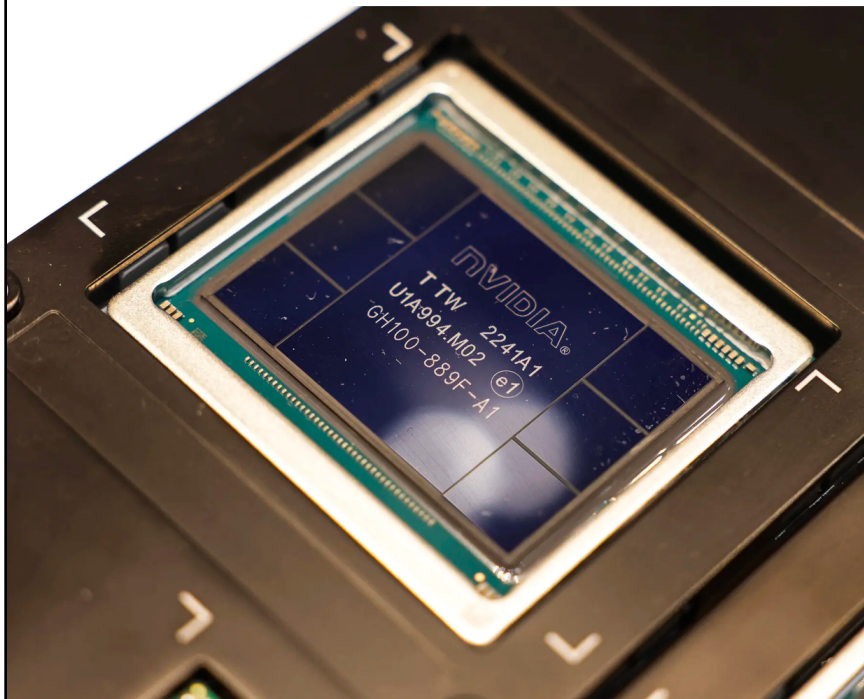
DeepSeek's engineers said they needed only about 2,000 specialized computer chips from the U.S. chipmaker Nvidia, in comparison to the as many as 16,000 chips needed by

& re my claim of Chatbots/LLMs/ foundation models as commodities ...

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Dr. Stoica and his students recently built an A.I. system called Sky-T1 that rivals the performance of OpenAI latest system, called OpenAI o1, on certain benchmark tests. They needed only \$450 in computing power.



Reuven Cohen, a technology consultant based in Toronto, has been using DeepSeek-V3 since soon after it was released in late December. Chloe Ellingson

They did this by building on top of two open source technologies released by the Chinese tech giant Alibaba.

Their \$450 system is not as powerful as OpenAI's technology or DeepSeek's new system. And the techniques they used are unlikely to yield systems that exceed the performance of the leading technologies. But the project showed that even operations with

Logistics again ...

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M

Your code for starting the registration process is:

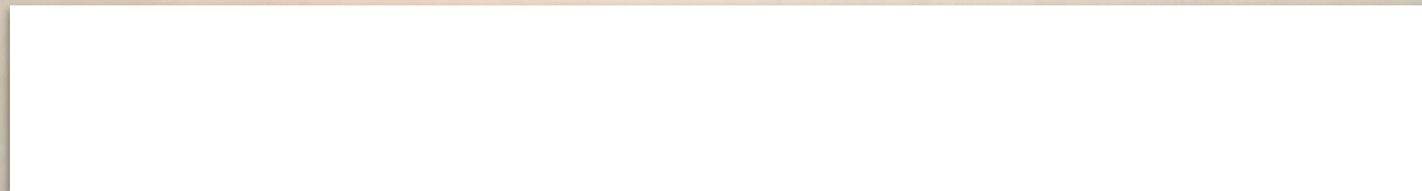
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and to obtain the textbook LAMA-BDLA, go to::

<https://rpi.logicamodernapproach.com>

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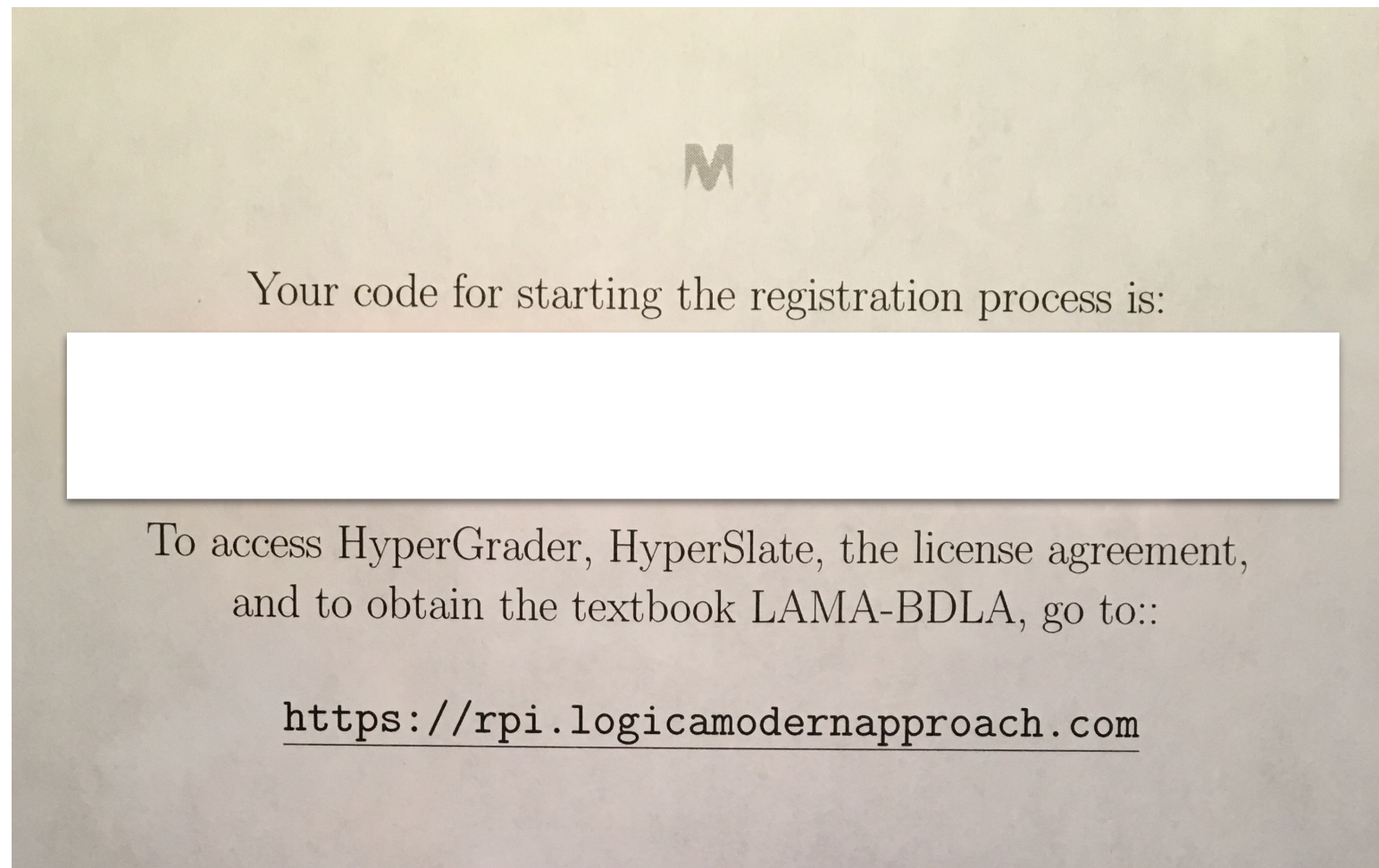


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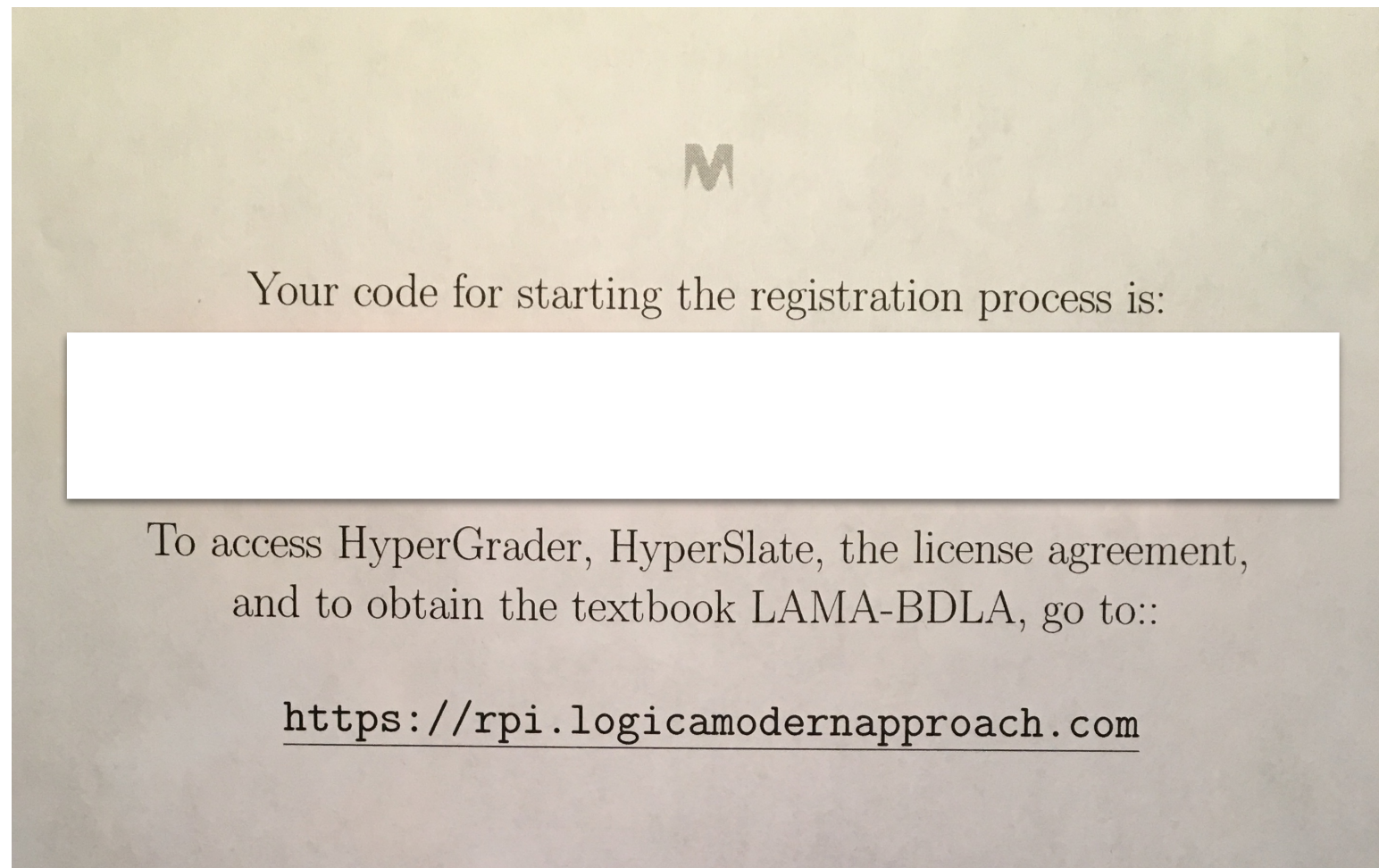
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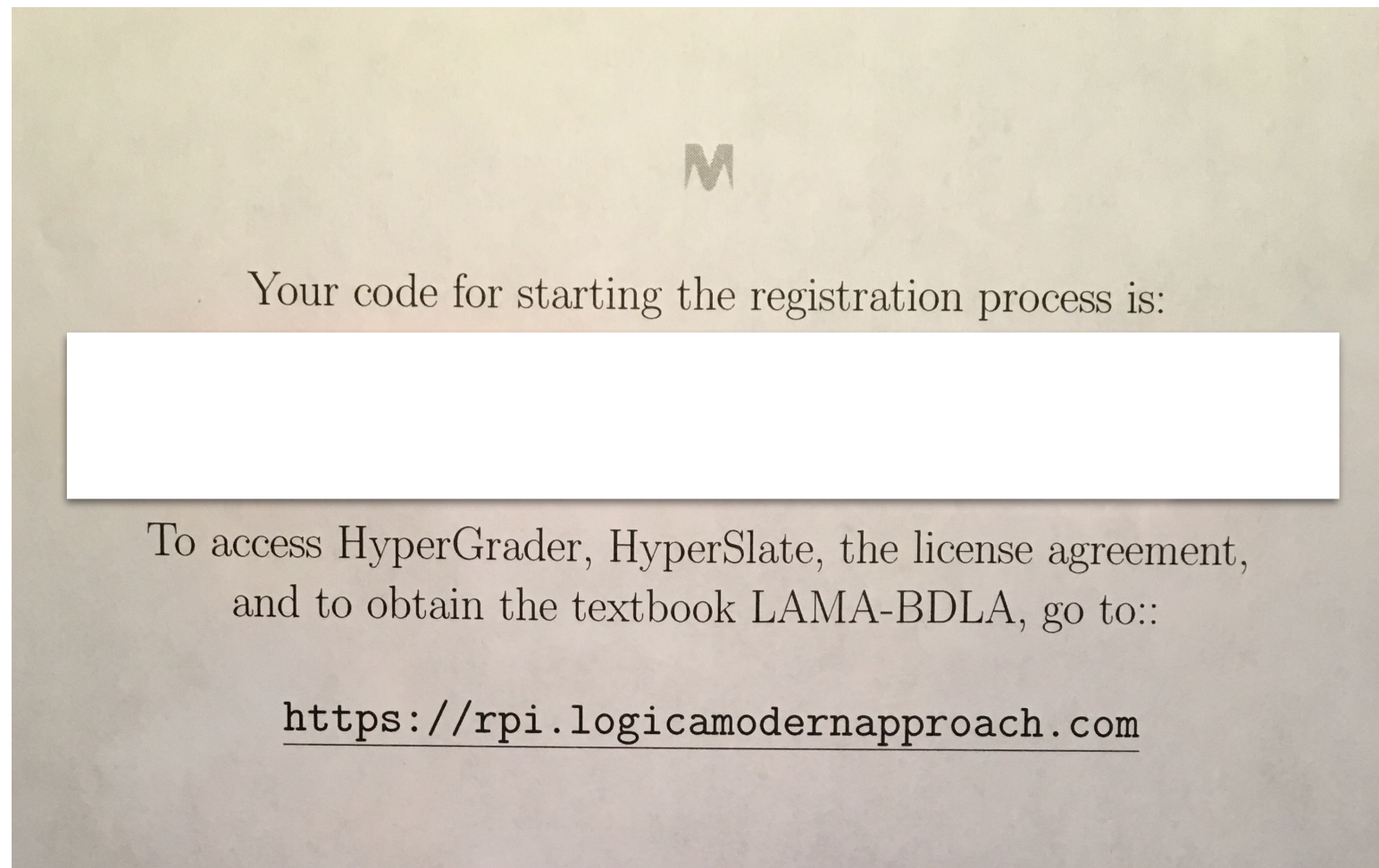


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Watch that the link emailed to you doesn't end up being classified as spam.

Introduction to (Formal) Logic (and AI)

Spring 2021 edition of IFLAI1

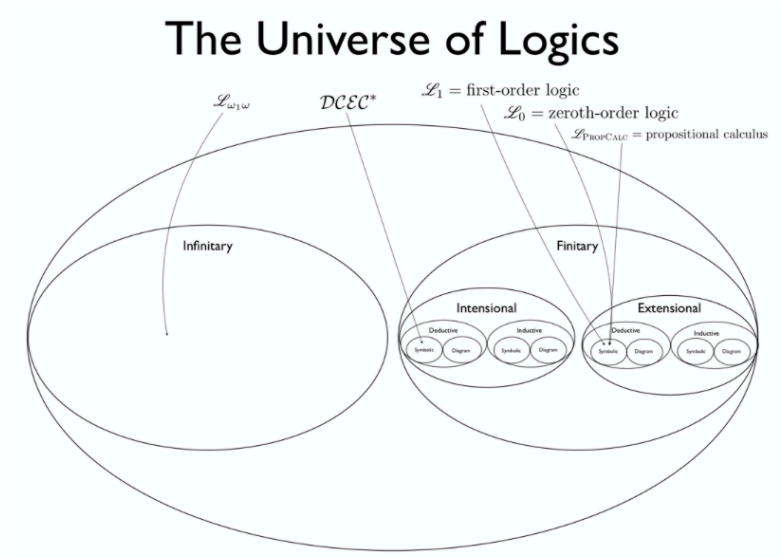
Selmer Bringsjord

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A fully online course, thanks to singular AI technology.

with Naveen Sundar G.
^ KB Foush   ^ Joshua Taylor ^ ...



Micro-homily:

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Having read Chapter 1, now to Chap 2, skipping to ~ p. 34!

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M. Chi: Self-testers end up being self-made.

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M. Chi: Self-testers end up being self-made.

“What category of English sentences does logic focus on?”

The Formal Language

CHAPTER 2. PROPOSITIONAL CALCULUS

Syntax	Formula Type	Sample Representation
$P, P_1, P_2, Q, Q_1, \dots$	Atomic Formulas	"Larry is lucky." as L_l
$\neg\phi$	Negation	"Gary isn't lucky." as $\neg L_g$
$\phi_1 \wedge \dots \wedge \phi_n$	Conjunction	"Both Larry and Carl are lucky." as $L_l \wedge L_c$
$\phi_1 \vee \dots \vee \phi_n$	Disjunction	"Either Billy is lucky or Alvin is." as $L_b \vee L_a$
$\phi \rightarrow \psi$	Conditional (Implication)	"If Ron is lucky, so is Frank." as $L_r \rightarrow L_f$
$\phi \leftrightarrow \psi$	Biconditional (Coimplication)	"Tim is lucky if and only if Kim is." as $L_t \leftrightarrow L_k$

Table 2.1: Syntax of the Propositional Calculus. Note that ϕ , ψ , and ϕ_i stand for arbitrary formulas.

The Formal Language

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Exercise: Is this language Roger-decidable? Prove it!

The Formal Language

(presented as formal grammar)

Formula $\Rightarrow$ *AtomicFormula*

| (*Formula* *Connective* *Formula*)
| $\neg$ *Formula*

AtomicFormula $\Rightarrow$ $P_1 \mid P_2 \mid P_3 \mid \dots$

Connective $\Rightarrow \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$

The Formal Language

(presented as formal grammar)

$Formula \Rightarrow AtomicFormula$

$\begin{array}{|l} (Formula \text{ Connective } Formula) \\ \neg Formula \end{array}$

$AtomicFormula \Rightarrow P_1 \mid P_2 \mid P_3 \mid \dots$

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Exercise: Is this language Roger-decidable? Prove it!

As S-expressions

$$\begin{array}{lcl} \textit{Formula} & \Rightarrow & \textit{AtomicFormula} \\ & & | \\ & & (\textit{Formula} \textit{Connective} \textit{Formula}) \\ & & | \\ & & \neg \textit{Formula} \end{array}$$

$$\textit{AtomicFormula} \Rightarrow \text{P}_1 \mid \text{P}_2 \mid \text{P}_3 \mid \dots$$

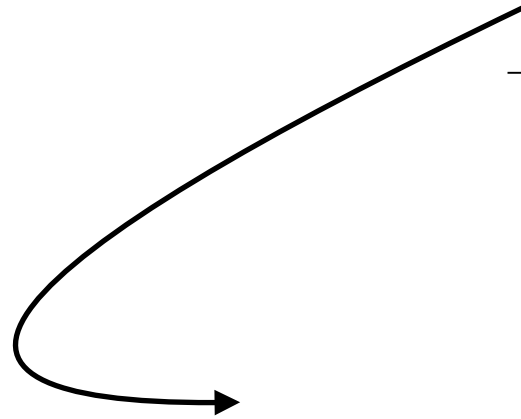
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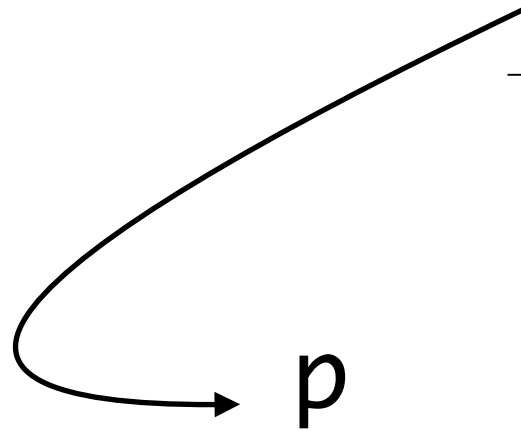


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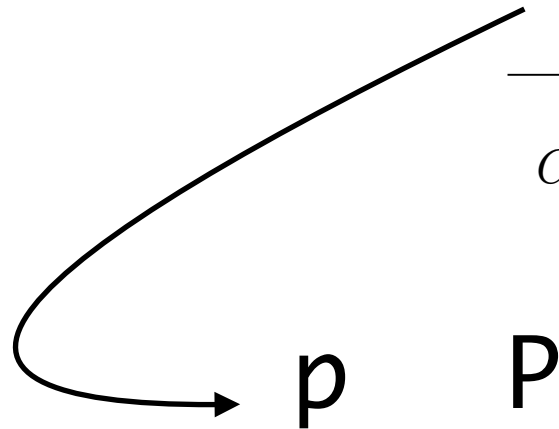
p

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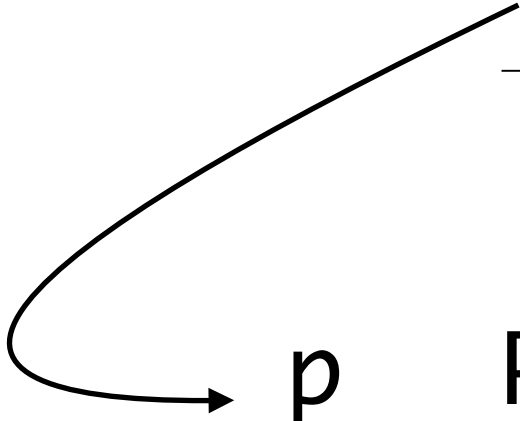


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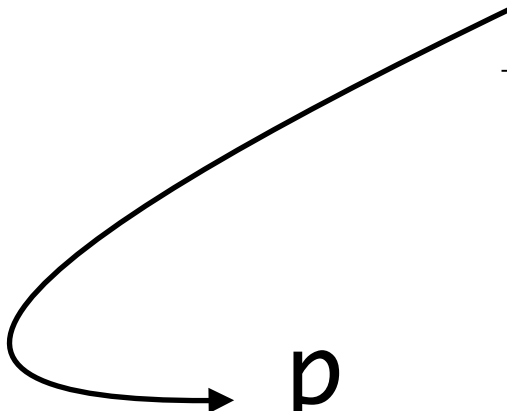
Connective $\Rightarrow$ $\wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$

 **p** **P** **bradywillbeback**

As S-expressions

$$\begin{array}{lcl} \textit{Formula} & \Rightarrow & \textit{AtomicFormula} \\ & & | \\ & & (\textit{Formula} \textit{Connective} \textit{Formula}) \\ & & | \\ & & \neg \textit{Formula} \end{array}$$

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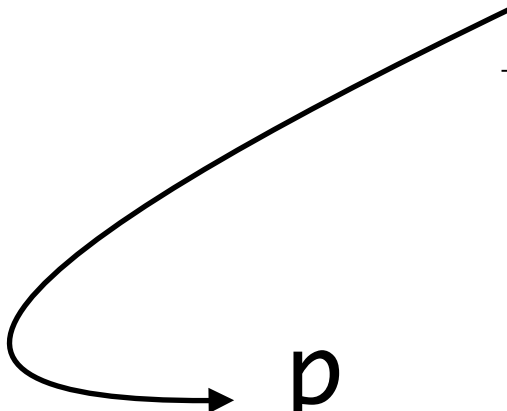
$$\textit{Connective} \Rightarrow \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$$


p P bradywillbeback P26

As S-expressions

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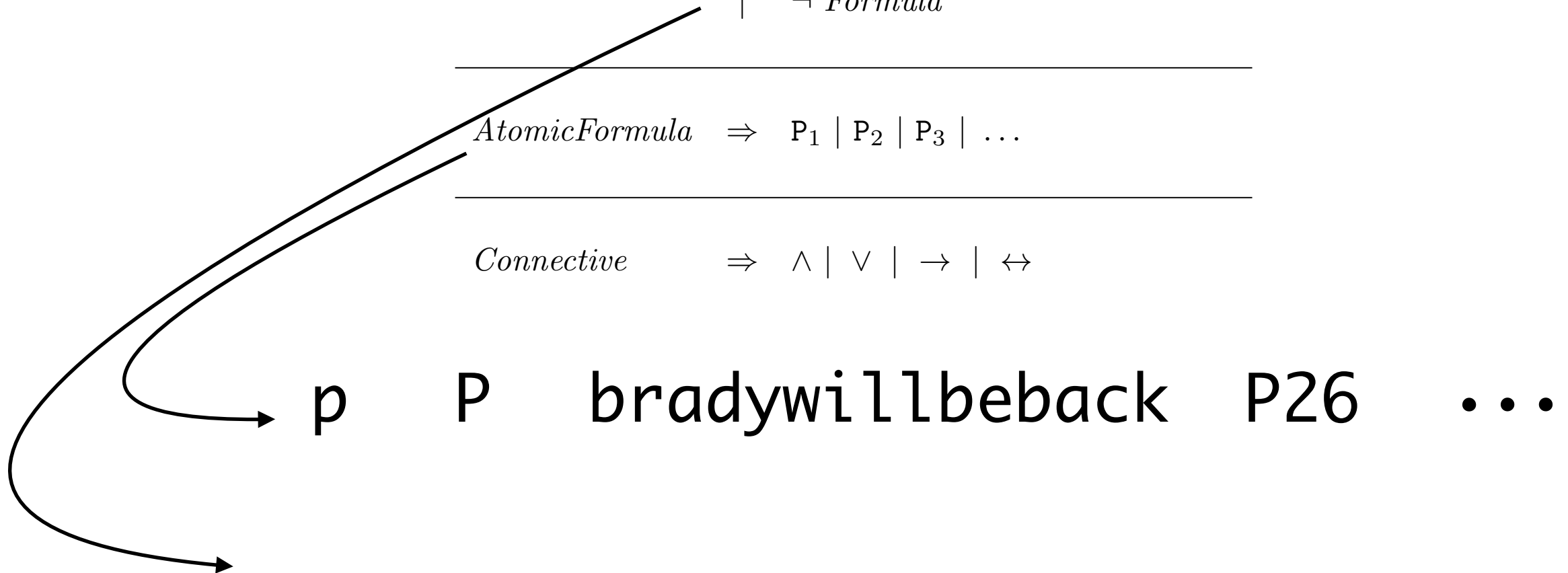
p P bradywillbeback P26 ...

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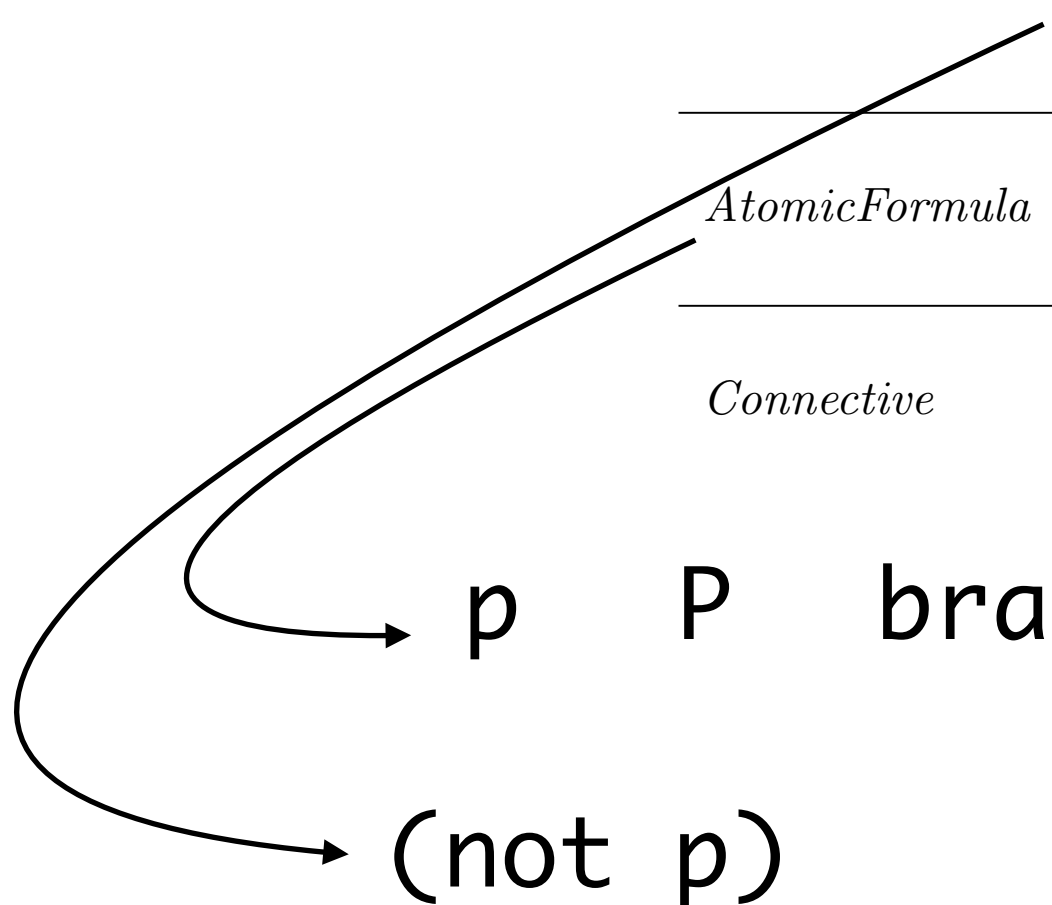


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Connective $\Rightarrow$ $\wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$


p P bradywillbeback P26 ...
(not p)

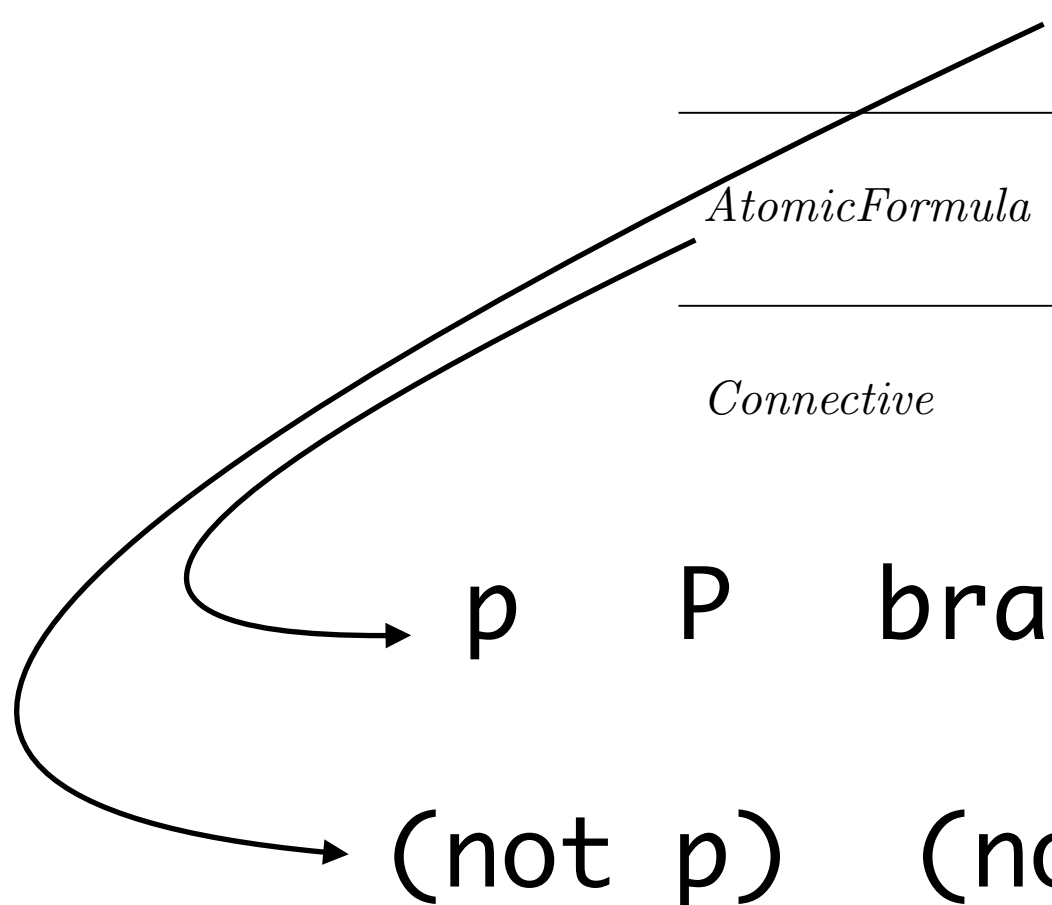
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 (not p) (not P)

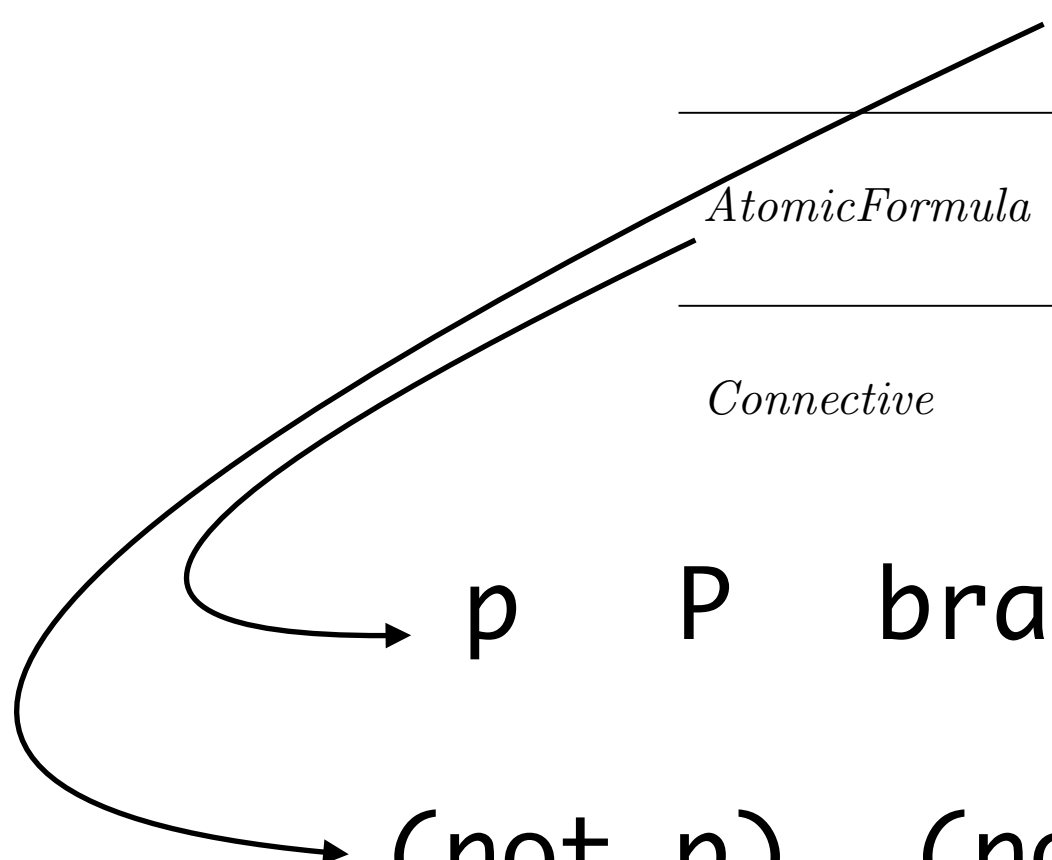
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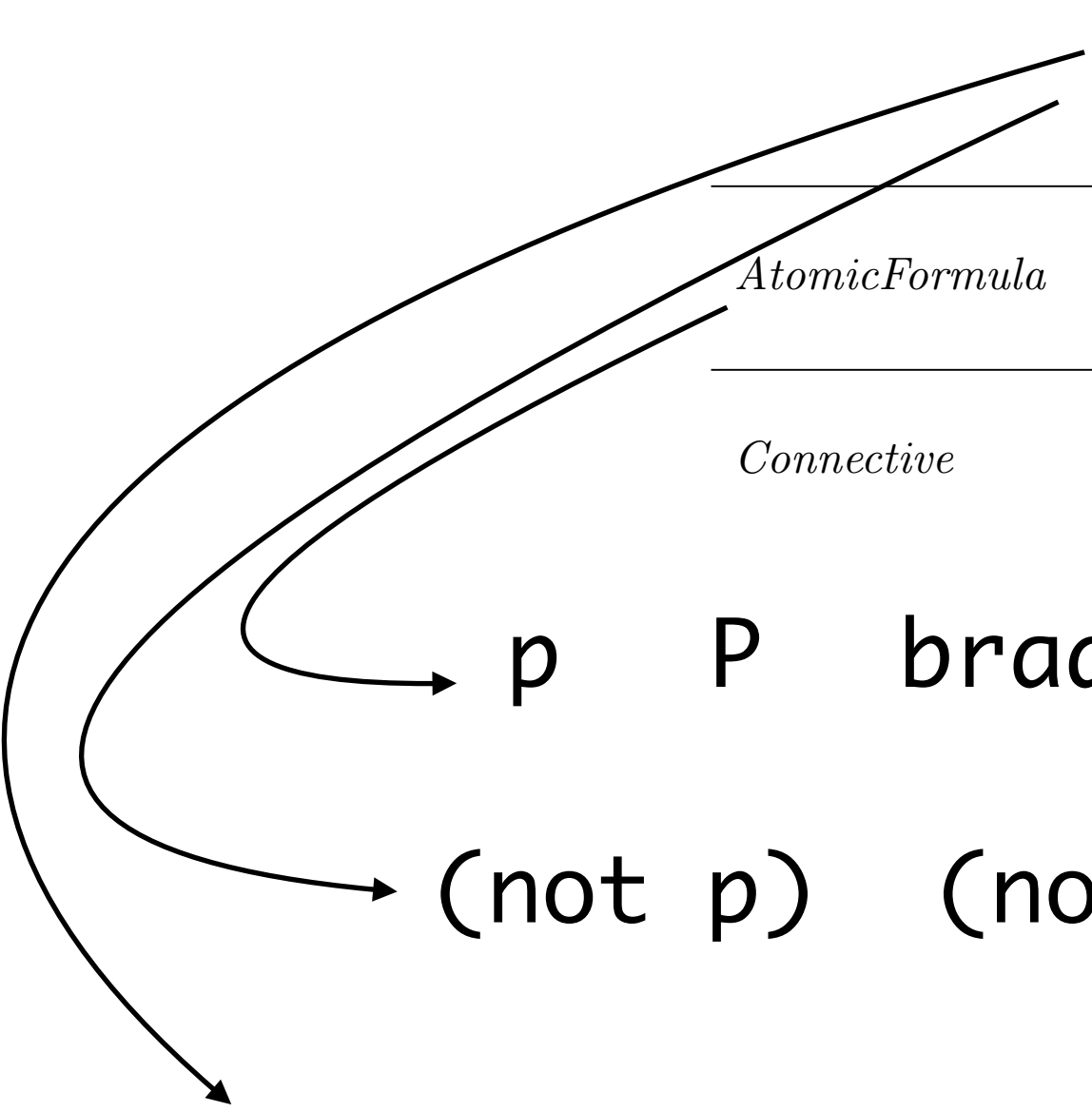
| (*Formula* *Connective* *Formula*)
| $\neg$ *Formula*

AtomicFormula $\Rightarrow$ P₁ | P₂ | P₃ | ...

Connective $\Rightarrow$ $\wedge$ | $\vee$ | $\rightarrow$ | $\leftrightarrow$

p P bradywillbeback P26 ...

(not p) (not P) (not P26) ...



As S-expressions

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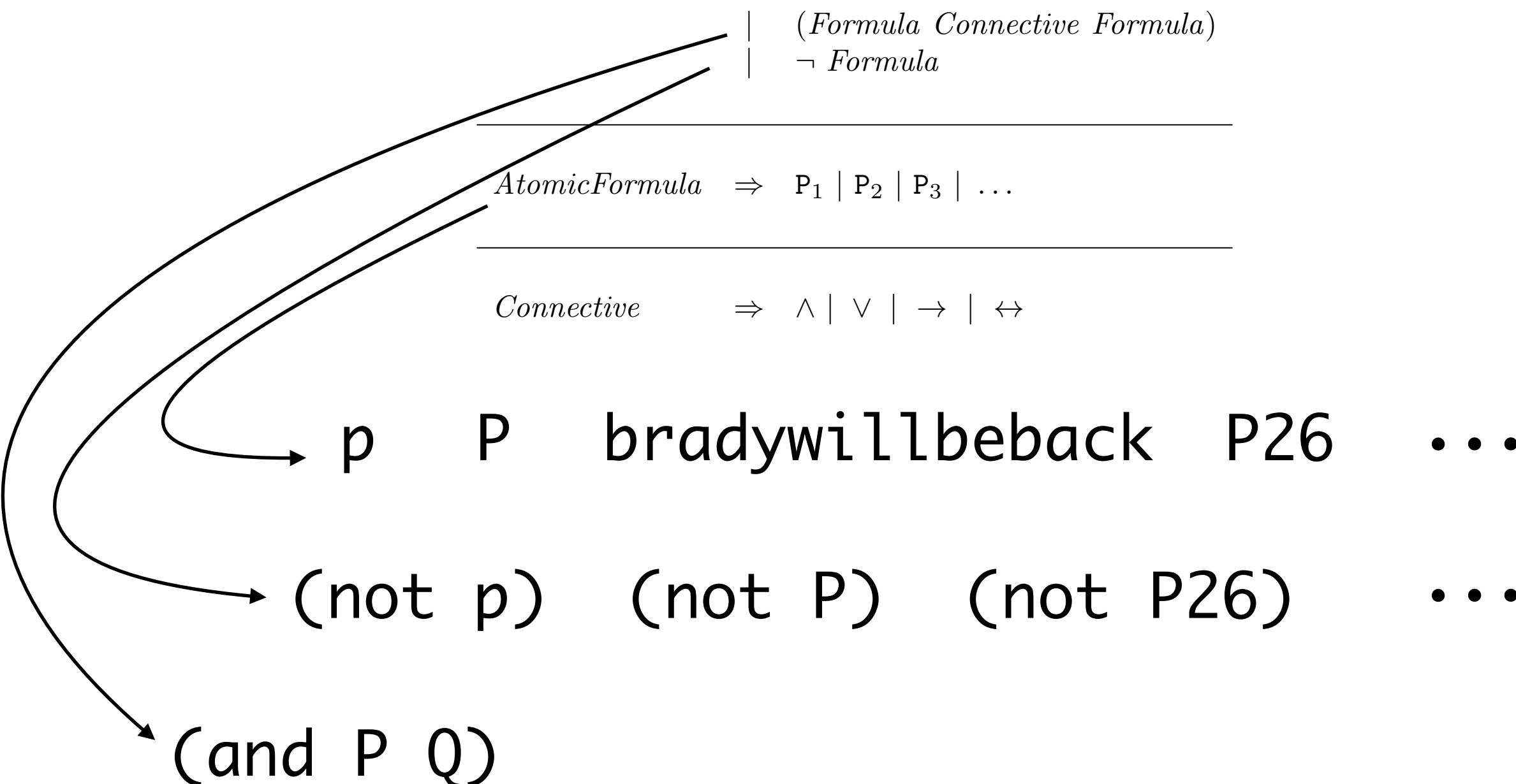
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p P bradywillbeback P26 ...

(not p) (not P) (not P26) ...

(and P Q)



As S-expressions

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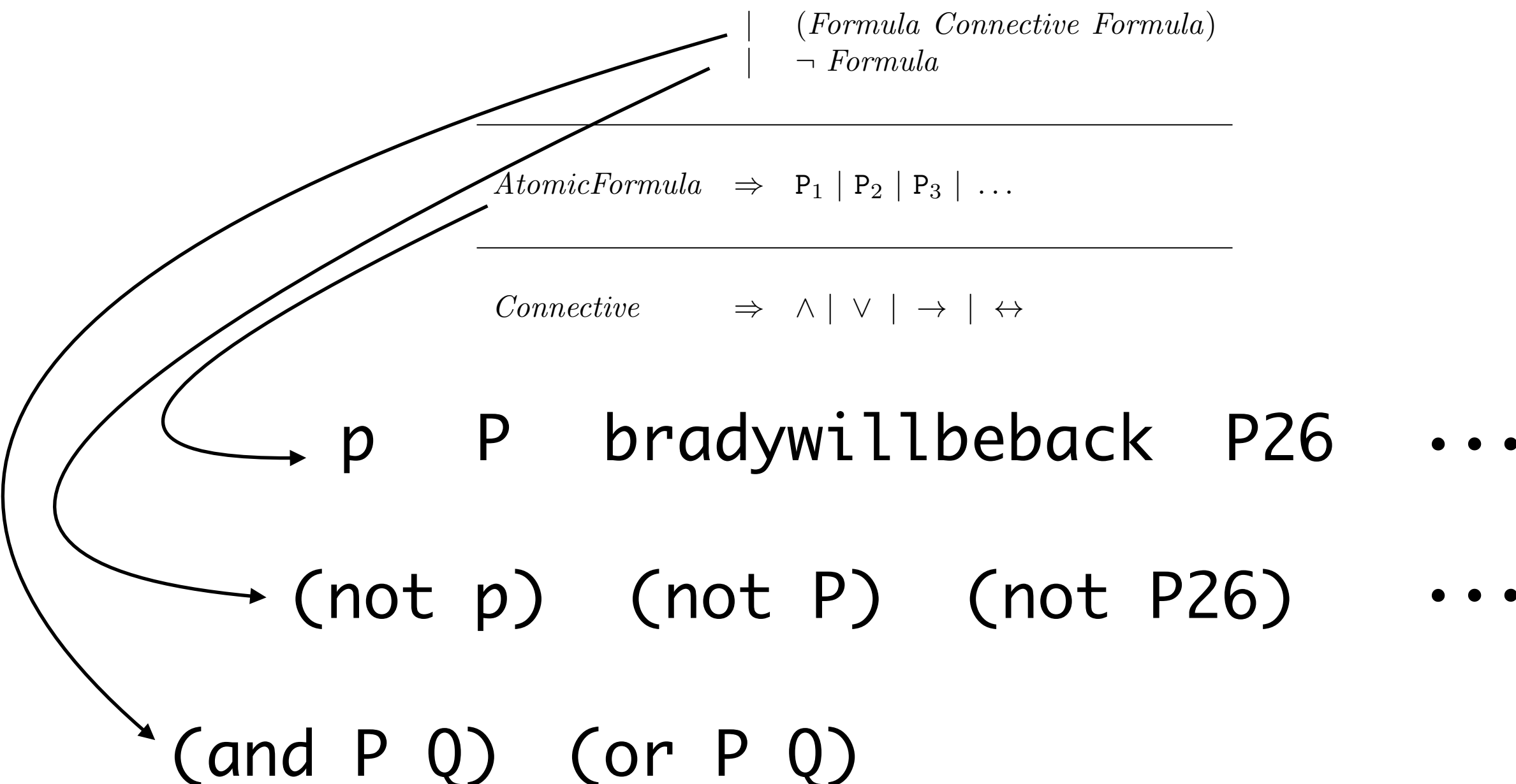
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(not p) (not P) (not P26) ...

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As S-expressions

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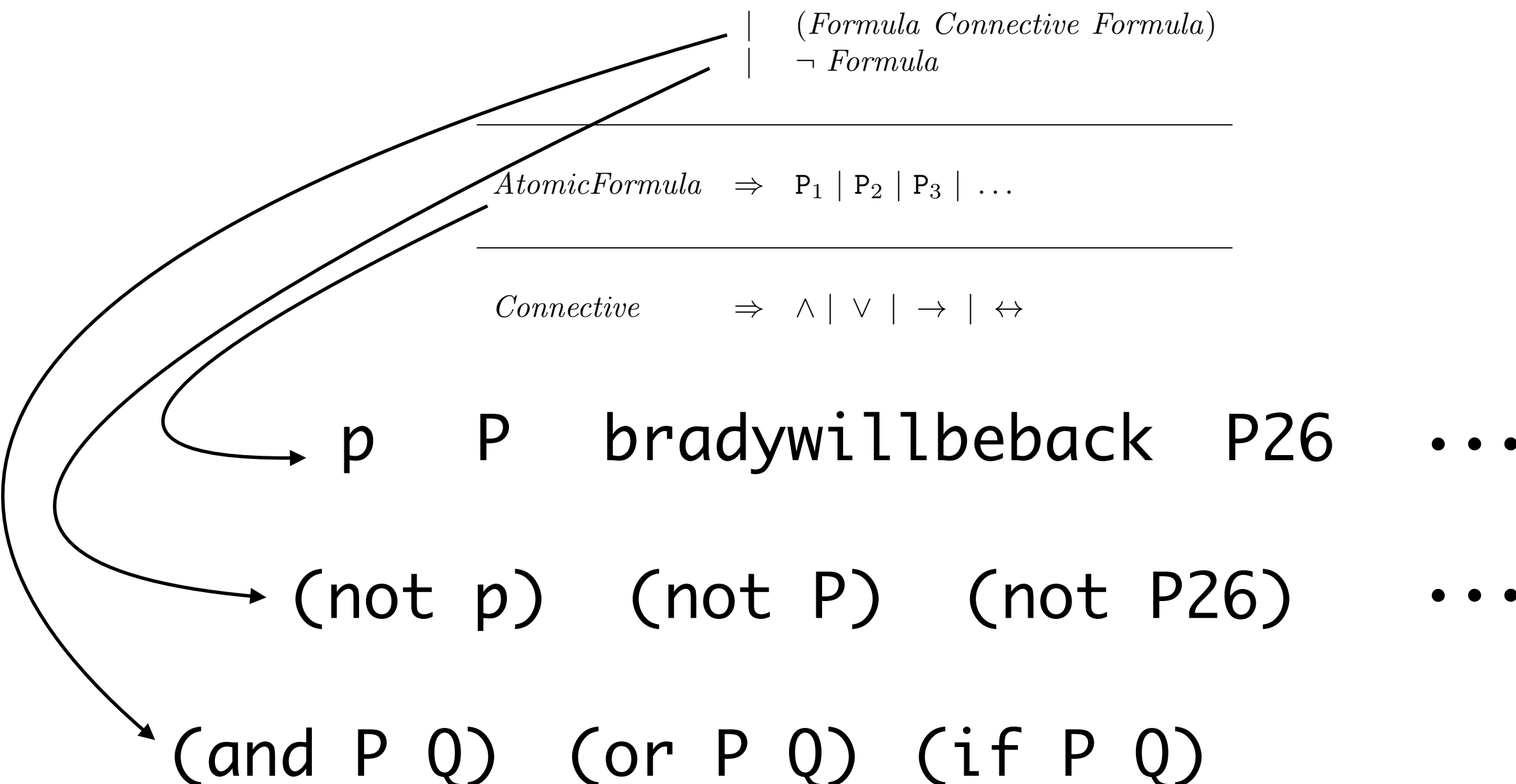
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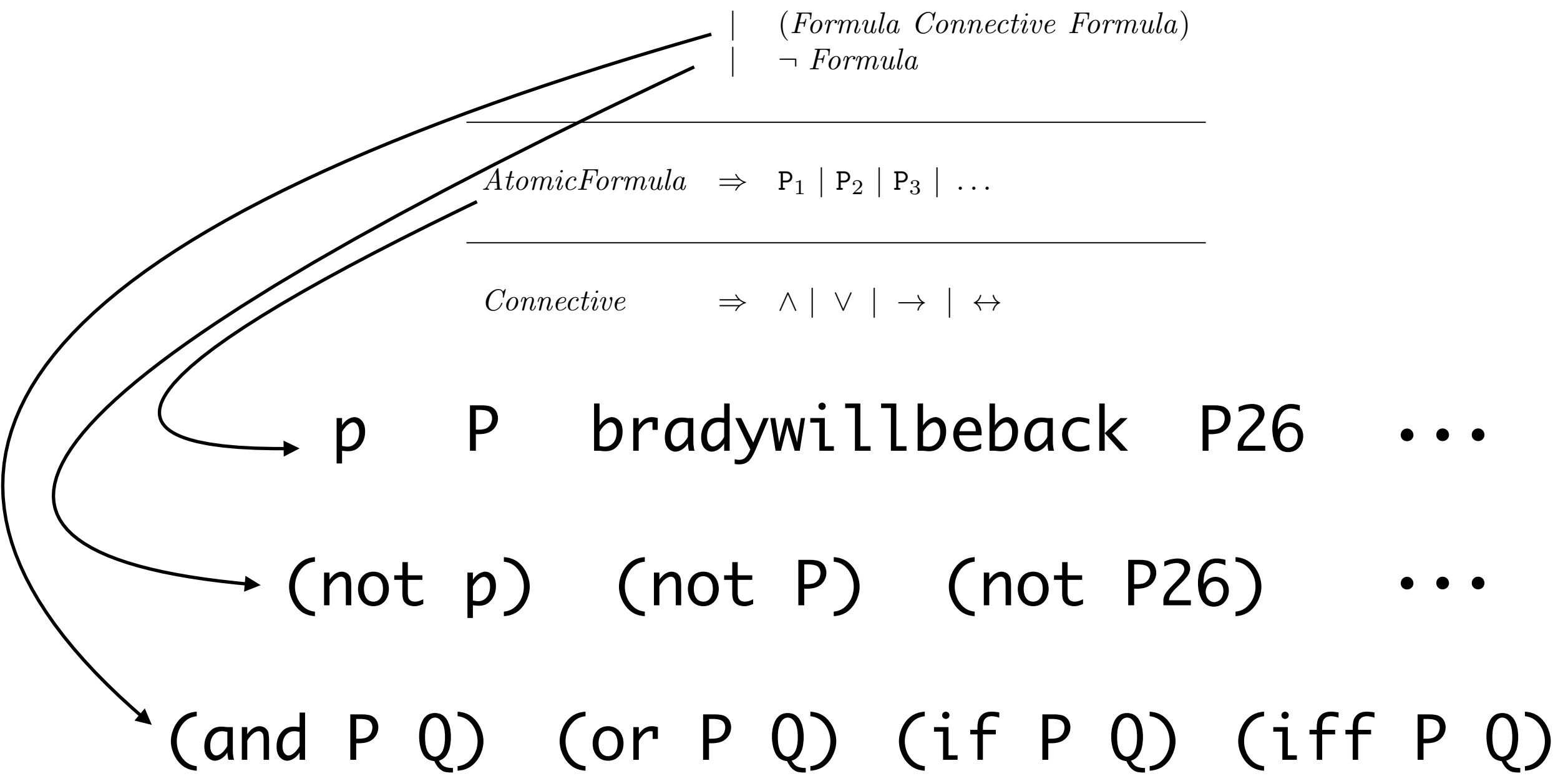
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p P bradywillbeback P26 ...

(not p) (not P) (not P26) ...

(and P Q) (or P Q) (if P Q) (iff P Q)



Better Formal Language: Pure Predicate Calculus (presented via formal grammar)

$$\begin{array}{lcl} \textit{Formula} & \Rightarrow & \textit{AtomicFormula} \\ & | & (\textit{Formula} \textit{Connective} \textit{Formula}) \\ & | & \neg \textit{Formula} \end{array}$$

$$\textit{AtomicFormula} \Rightarrow (\textit{Predicate} \textit{Term}_1 \dots \textit{Term}_k)$$

$$\begin{array}{lcl} \textit{Term} & \Rightarrow & (\textit{Function} \textit{Term}_1 \dots \textit{Term}_k) \\ & | & \textit{Constant} \\ & | & \textit{Variable} \end{array}$$

$$\textit{Connective} \Rightarrow \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$$

$$\begin{array}{lcl} \textit{Predicate} & \Rightarrow & P_1 \mid P_2 \mid P_3 \dots \\ \textit{Constant} & \Rightarrow & c_1 \mid c_2 \mid c_3 \dots \\ \textit{Variable} & \Rightarrow & v_1 \mid v_2 \mid v_3 \dots \\ \textit{Function} & \Rightarrow & f_1 \mid f_2 \mid f_3 \dots \end{array}$$

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Exercise: Is this language also Roger-decidable? Prove it!

“NYS I” Revisited

Given the statements

$$\neg a \vee \neg b$$

$$b$$

$$c \rightarrow a$$

which one of the following statements is provable?

$$c$$

$$\neg b$$

$$\neg c$$

$$h$$

$$a$$

none of the above

“NYS I” Revisited

Given the statements

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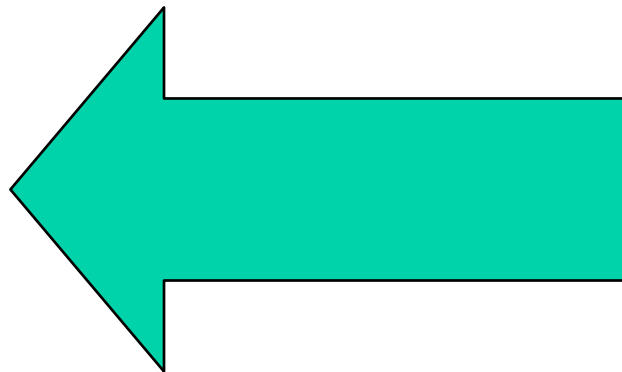
$\neg b$

$\neg c$

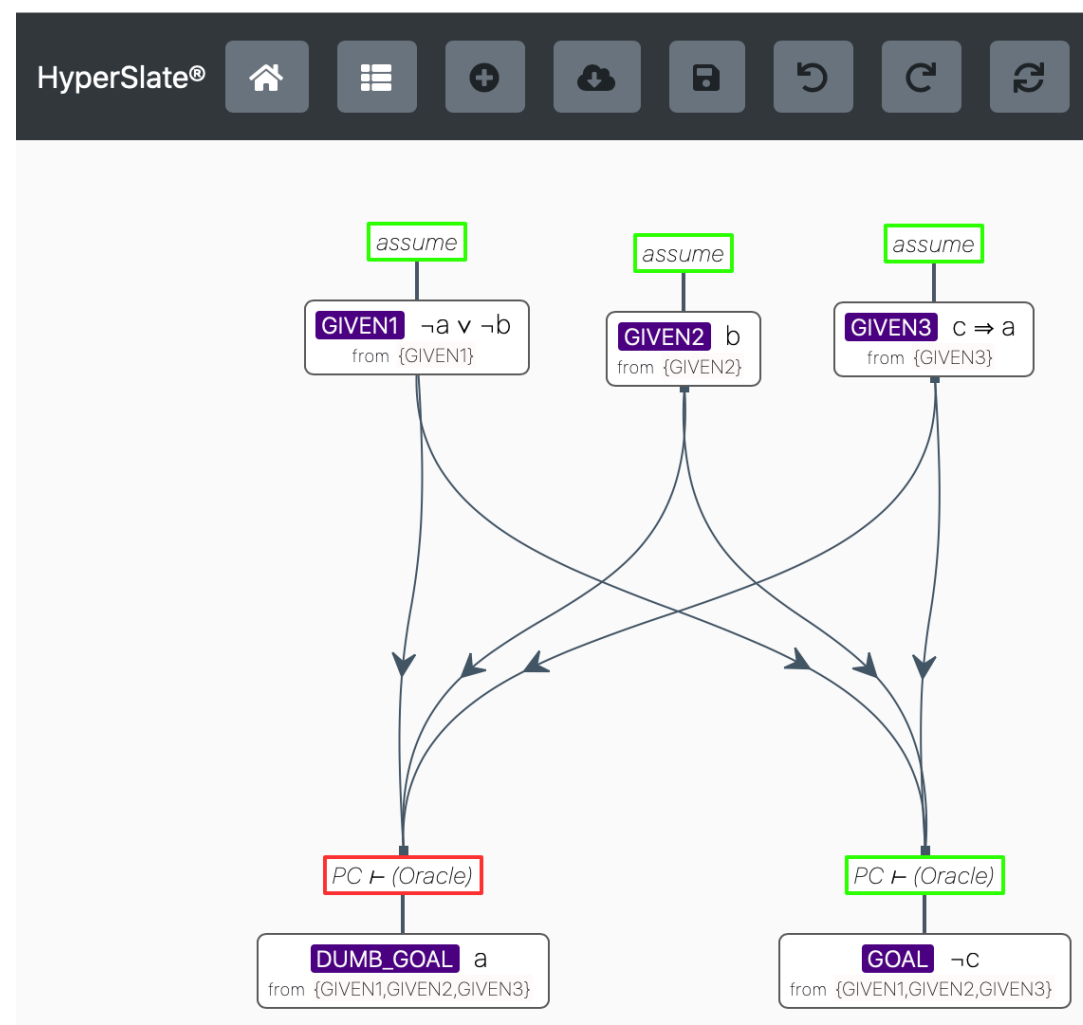
h

a

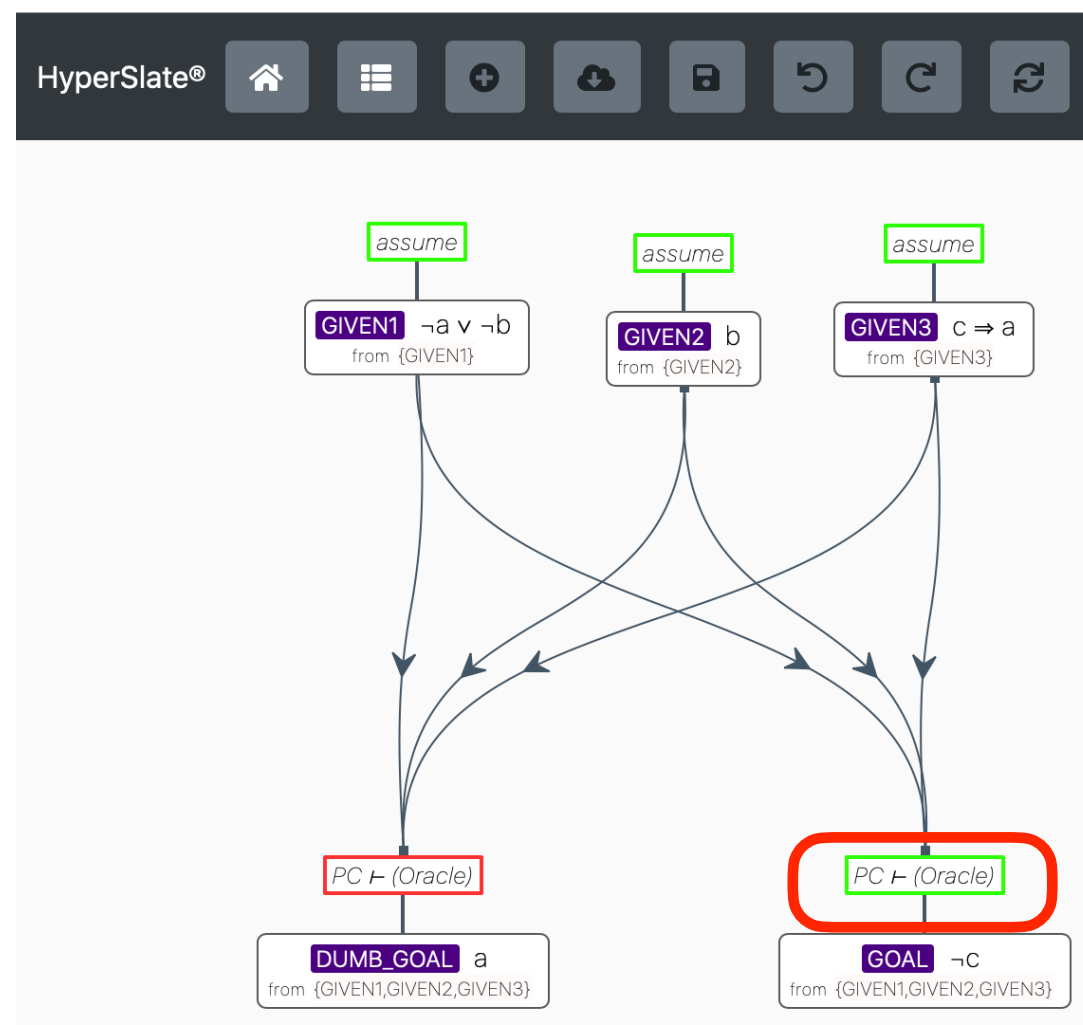
none of the above



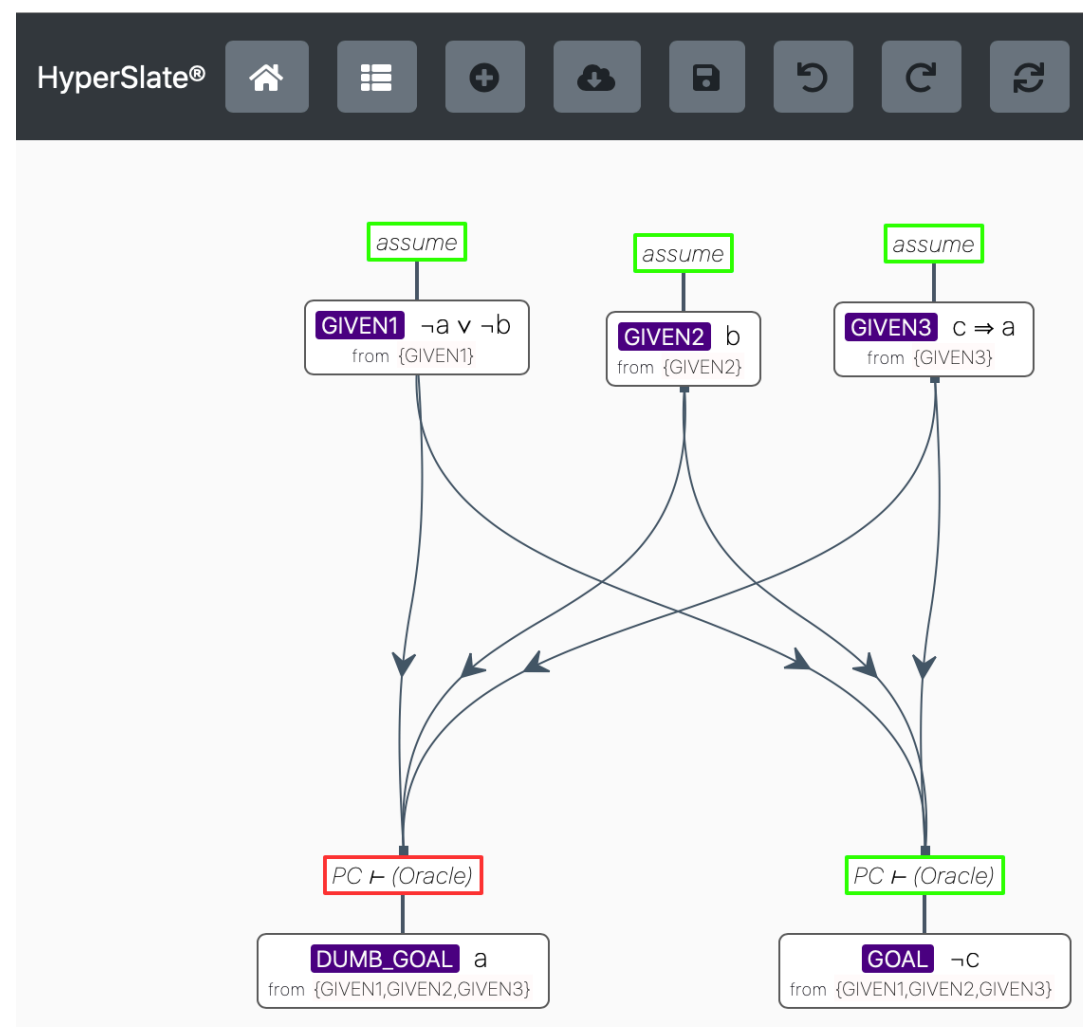
Our First Rule of Inference (= Inference Schema): PC (Entailment) Oracle



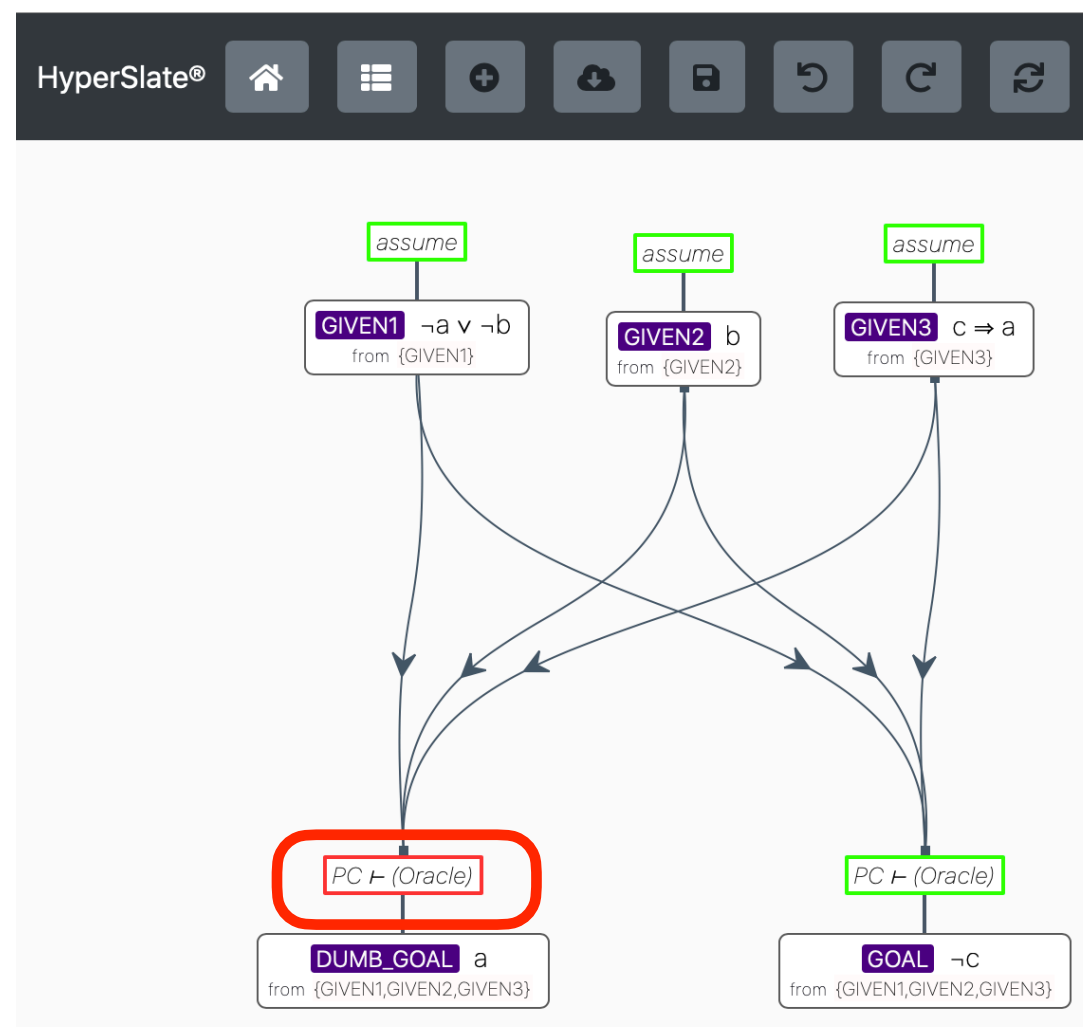
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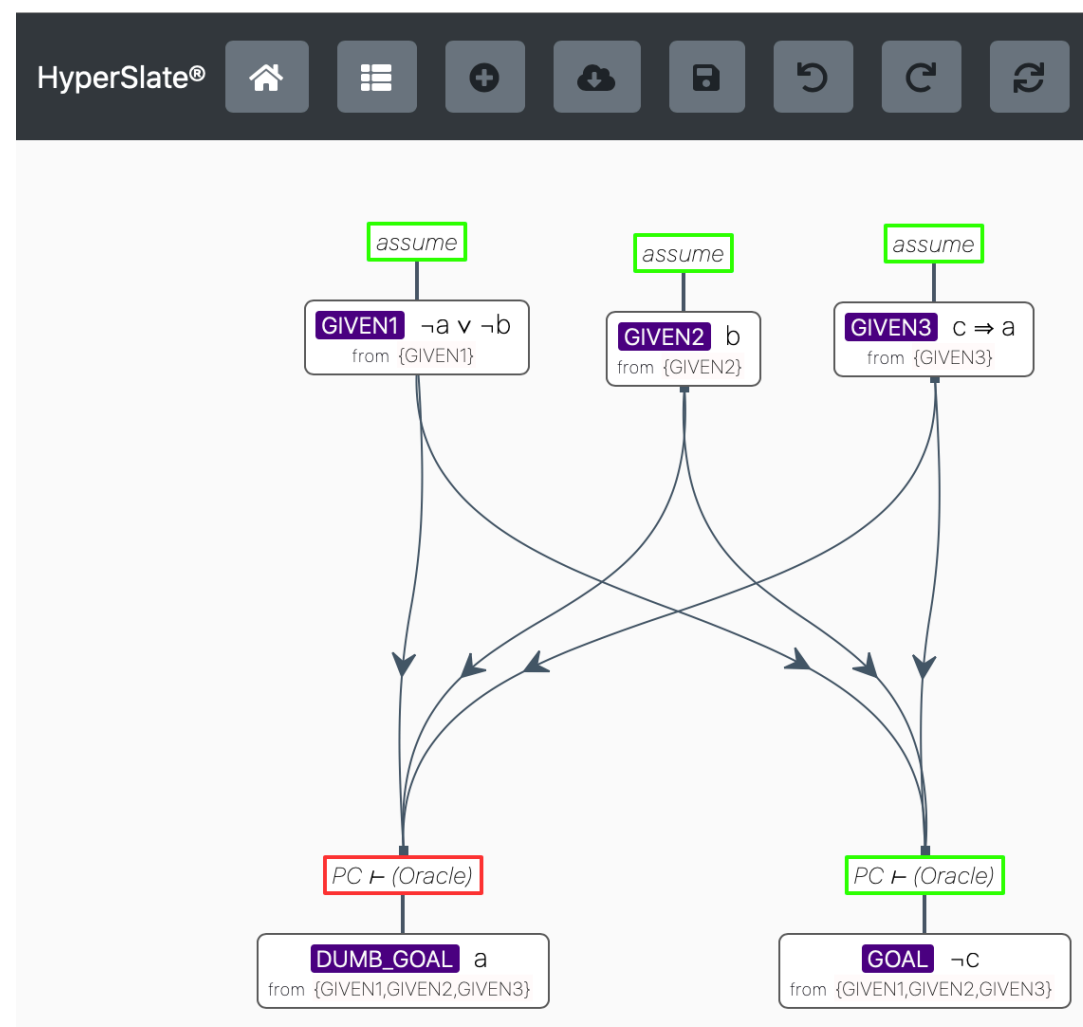
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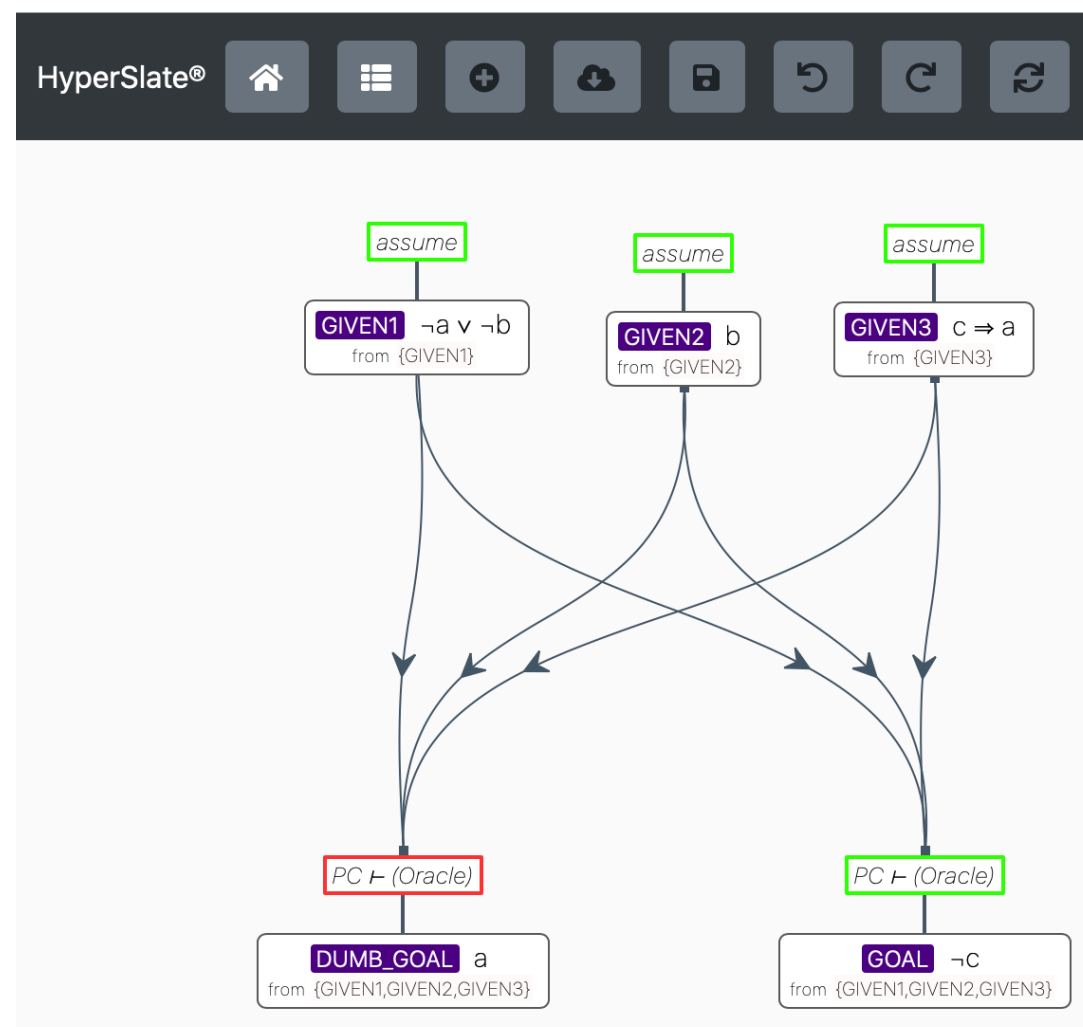
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Our First Rule of Inference (= Inference Schema): PC (Entailment) Oracle



“NYS 3” Revisited

Given the statements

$$\neg\neg c$$

$$c \rightarrow a$$

$$\neg a \vee b$$

$$b \rightarrow d$$

$$\neg(d \vee e)$$

which one of the following statements are provable?

$$\neg c$$

$$e$$

$$h$$

$$\neg a$$

all of the above

“NYS 3” Revisited

Given the statements

$$\neg\neg c$$

$$c \rightarrow a$$

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$$\neg(d \vee e)$$

which one of the following statements are provable?

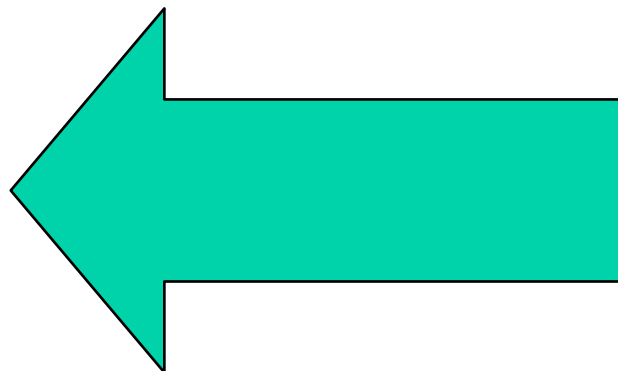
$$\neg c$$

$$e$$

$$h$$

$$\neg a$$

all of the above



“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

Show in HyperSlate® that each of the first four options can be proved using the PC entailment oracle.

which one of the following statements are provable?

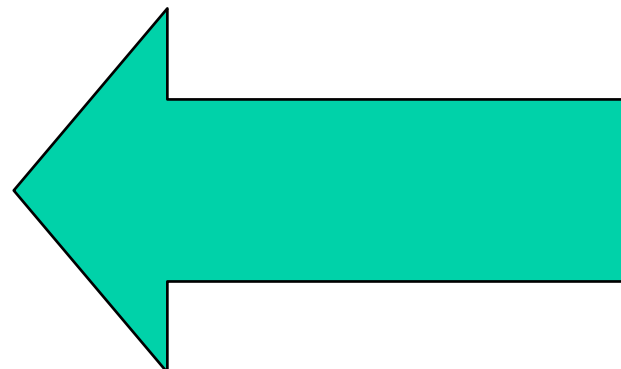
$\neg c$

e

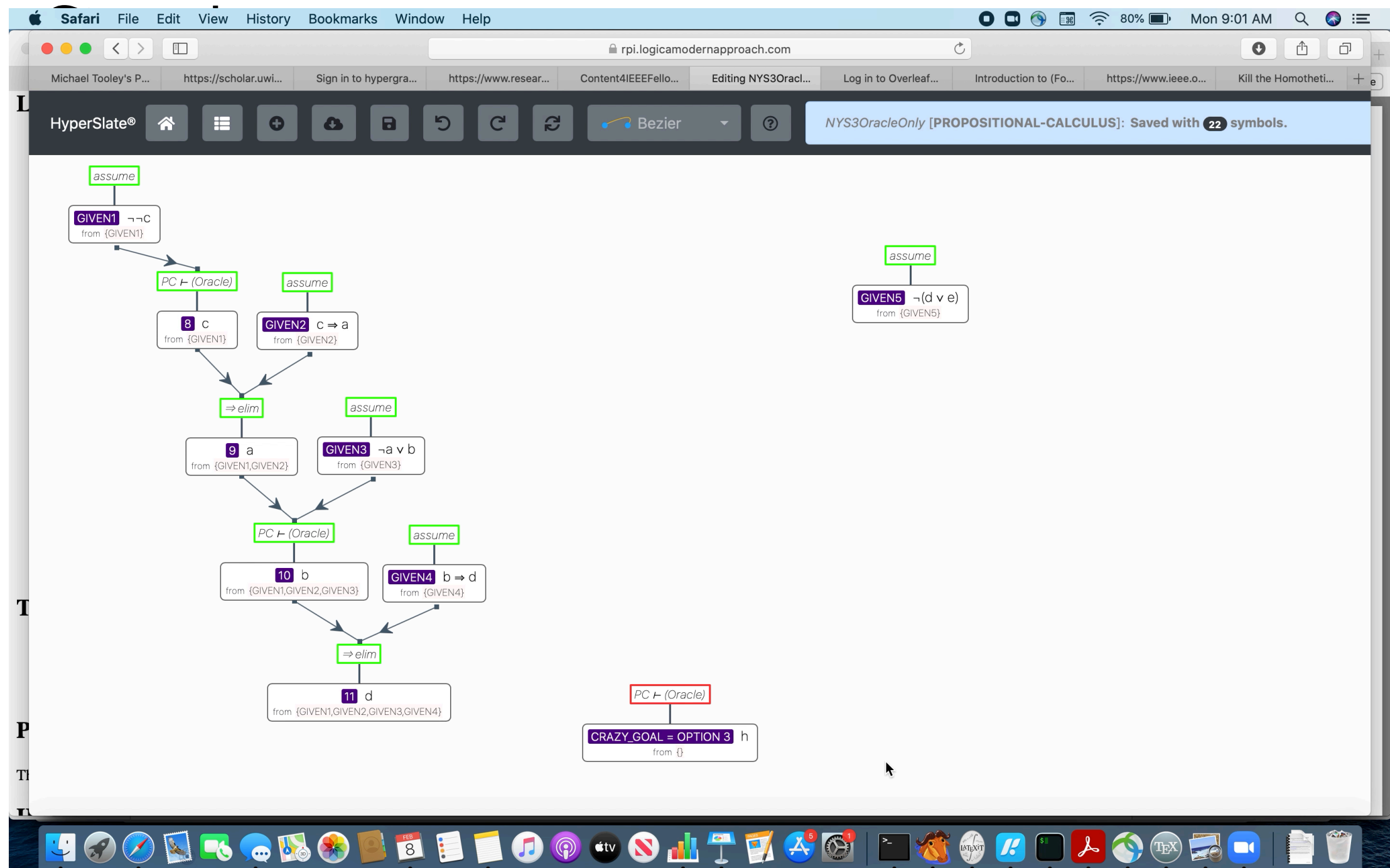
h

$\neg a$

all of the above



“NYS 3” Revisited



all of the above

Det er en ære å lære formell logikk!

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Interactive Part II of Class? ...