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... is a verb: to live logically, and learn logic, anywhere anytime — and to have fun all along the way.

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The Liar; Russell's Paradox; *Toward* Thoraf's Paradox

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IFLAI
1/27/2025



Logistics ...

Test 2 will be published March
13, with a deadline of March 20.

Test 2 will be published March 13, with a deadline of March 20.

(@ some point over the second half of the break, the HG[®] platform on AWS will be inaccessible for a multi-hour stretch, due to re-versioning and upgrades.)

Types of Paradoxes

- Deductive Paradoxes
- Inductive Paradoxes — coming (e.g. The Lottery Paradox & The St Petersburg Paradox)

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First:

- Deductive Paradoxes
- Inductive Paradoxes — coming (e.g. The Lottery Paradox & The St Petersburg Paradox)

The Liar (Paradox) ...

The (Economical) Liar

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L: This sentence is false.

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If $T(\mathbf{L})$ then $\neg T(\mathbf{L})$

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$T(\mathbf{L})$ iff (i.e., if & only if) $\neg T(\mathbf{L})$

Contradiction!

The (Verbose) Liar — With a Twist

The (Verbose) Liar — With a Twist

Theorem: $2+2 = 5$.

The (Verbose) Liar — With a Twist

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Proof: Set:

The (Verbose) Liar — With a Twist

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Proof: Set:

L: This sentence is false.

The (Verbose) Liar — With a Twist

Theorem: $2+2 = 5$.

Proof: Set:

L: This sentence is false.

L is either true or false. Suppose that it's true. Then since what it says is that it's false, it *is* false; i.e., **L** is false, on this supposition. So we've proved that if **L** is true, **L** is false. Now suppose instead that **L** is false. Then since it says that it's false, it's true; i.e., **L** is true, on our current supposition. We have thus proved that if **L** is false, **L** is true.

Combining the conditionals we've proved yields this: **L** is true if and only if **L** is false, which is a contradiction. (P if and only if $\neg P$ is logically equivalent to P and $\neg P$.) By inference schema *explosion*, it follows that $2+2 = 5$. **QED**

Outlawing Self-Referential Sentences Isn't the Answer!

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- For the following sentences, e.g., are perfectly innocuous, and obviously true or false (only) as the case may be, without complication.

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 - This sentence is a sentence.
 - This sentence contains the letter 'r'.

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- For the following sentences, e.g., are perfectly innocuous, and obviously true or false (only) as the case may be, without complication.
 - This sentence is a sentence.
 - This sentence contains the letter 'r'.
 - This sentence has more than three letters in it.

Outlawing Self-Referential Sentences Isn't the Answer!

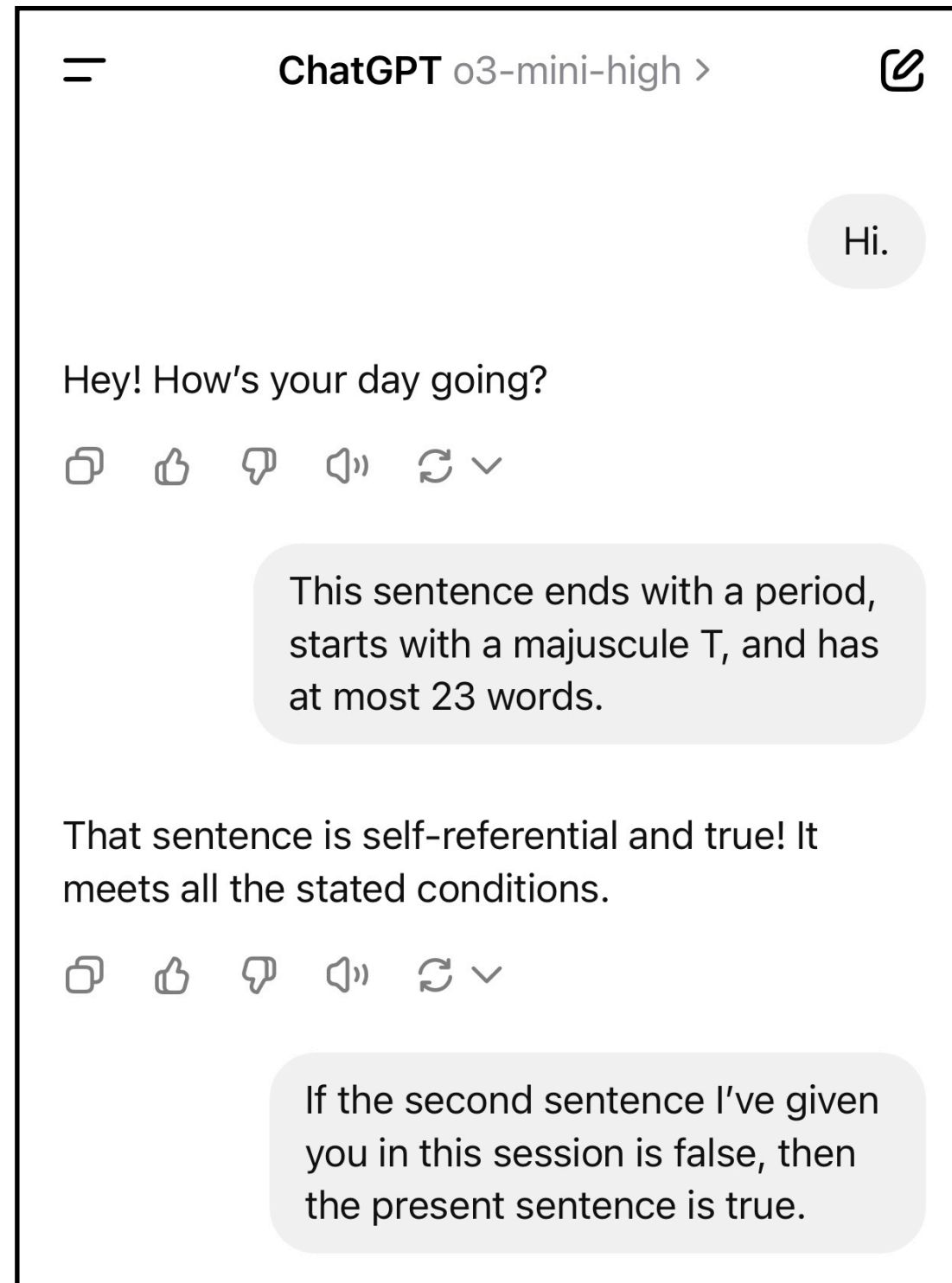
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 - This sentence is a sentence.
 - This sentence contains the letter 'r'.
 - This sentence has more than three letters in it.
 - This sentence ends with a period, starts with a capital 'T', and has more than two words.

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 - ...

OpenAI's o3 in a Self-Referential Challenge

OpenAI's o3 in a Self-Referential Challenge



OpenAI's o3 in a Self-Referential Challenge

ChatGPT o3 mini-high >

o3

Let's analyze this carefully:

1. Your second sentence was:

"This sentence ends with a period, starts with a majuscule T, and has at most 23 words."

That sentence is indeed true, as we checked.

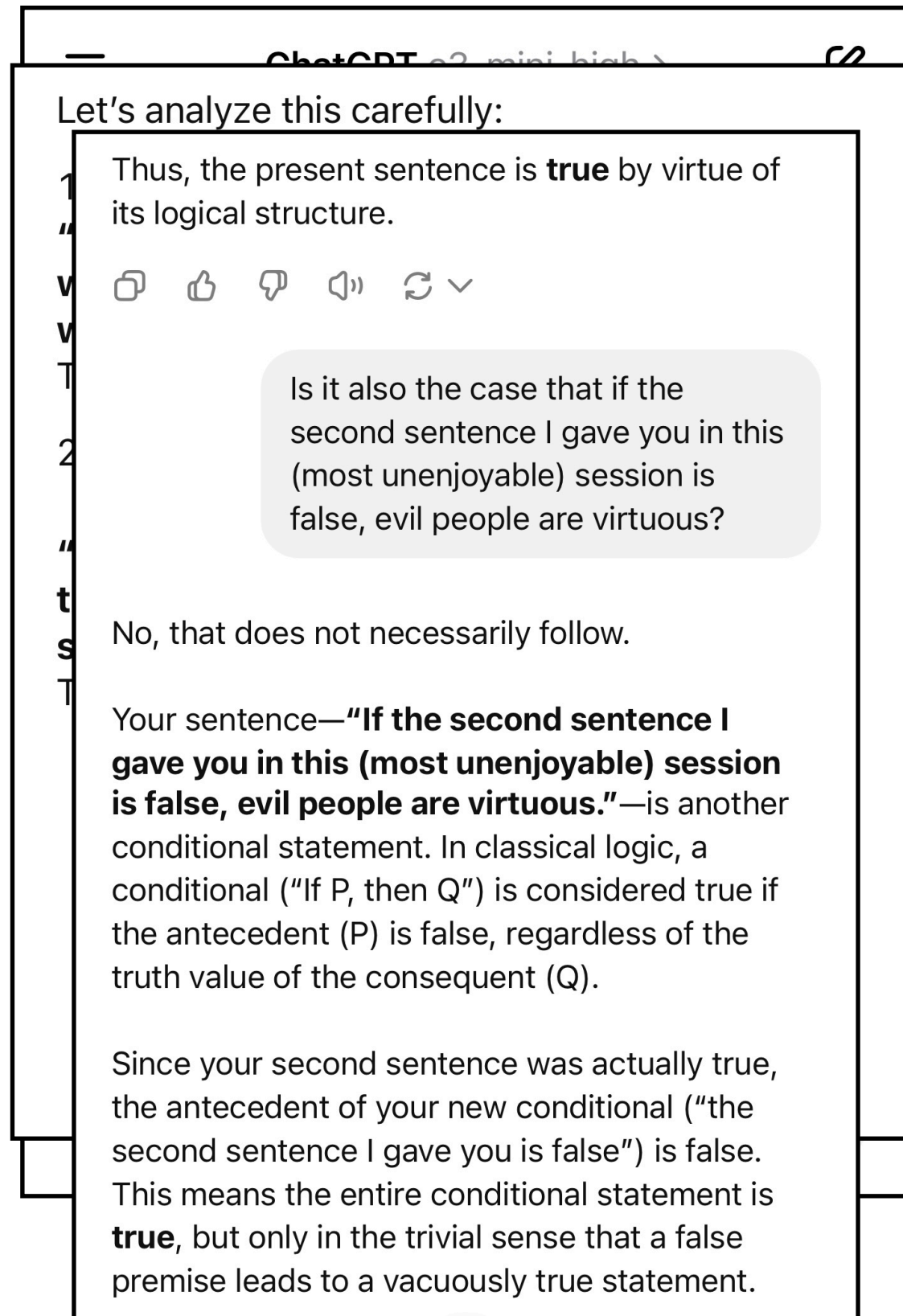
2. The present (new) sentence you gave me is:

"If the second sentence I've given you in this session is false, then the present sentence is true."

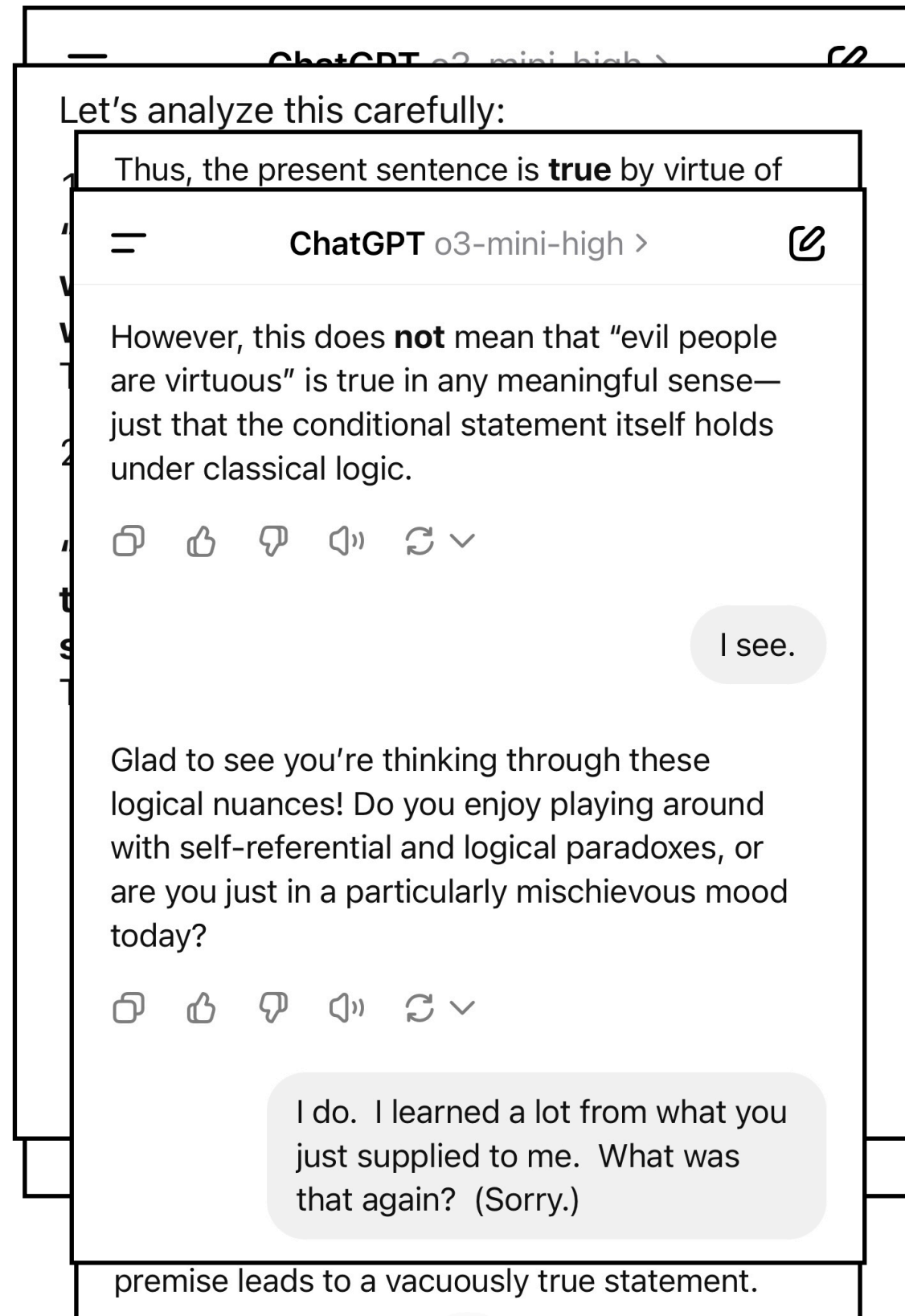
This is a conditional statement:

- The antecedent ("If the second sentence... is false") is false because your second sentence was true.
- In a material implication (classical logic), if the antecedent is false, the whole conditional statement is **true**, regardless of the truth value of the consequent.

OpenAI's o3 in a Self-Referential Challenge



OpenAI's o3 in a Self-Referential Challenge



OpenAI's o3 in a Self-Referential Challenge

Let's analyze this carefully:

Thus, the present sentence is **true** by virtue of

Reasoned about logical conditionals for
6 seconds ✓

The user is asking for a recap! Alright, so the second sentence being false means the first sentence is true. Then, the analysis shows that the second sentence is actually true, so the first part of the conditional is false, which makes the whole statement true by classical logic—yet the actual statement about “evil people being virtuous” is clearly false or irrelevant. I'll just rewrite it to make sure it makes sense!

Let me summarize the key points from the previous explanation:

- Your second sentence was true.
- In a conditional statement (“If P then Q”), if the premise (P) is false, the entire statement is considered true by classical logic—this is called being “vacuously true.”
- Therefore, even though the condition “if the second sentence is false” is not met (since it's true), the overall conditional statement “if the second sentence is false, evil people are virtuous” is still true by the rules of logic.

OpenAI's o3 in a Self-Referential Challenge

ChatGPT o3 mini-high v

Let's analyze this carefully:

Thus, the present sentence is **true** by virtue of

Reasoned about logical conditionals for

- This does not mean that "evil people are virtuous" in any substantive sense—it only means that the conditional statement holds true in a formal logical sense.

Does that help clarify things?

Let me summarize the key points from the previous explanation:

- Your second sentence was true.
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OpenAI's o3 in a Self-Referential Challenge

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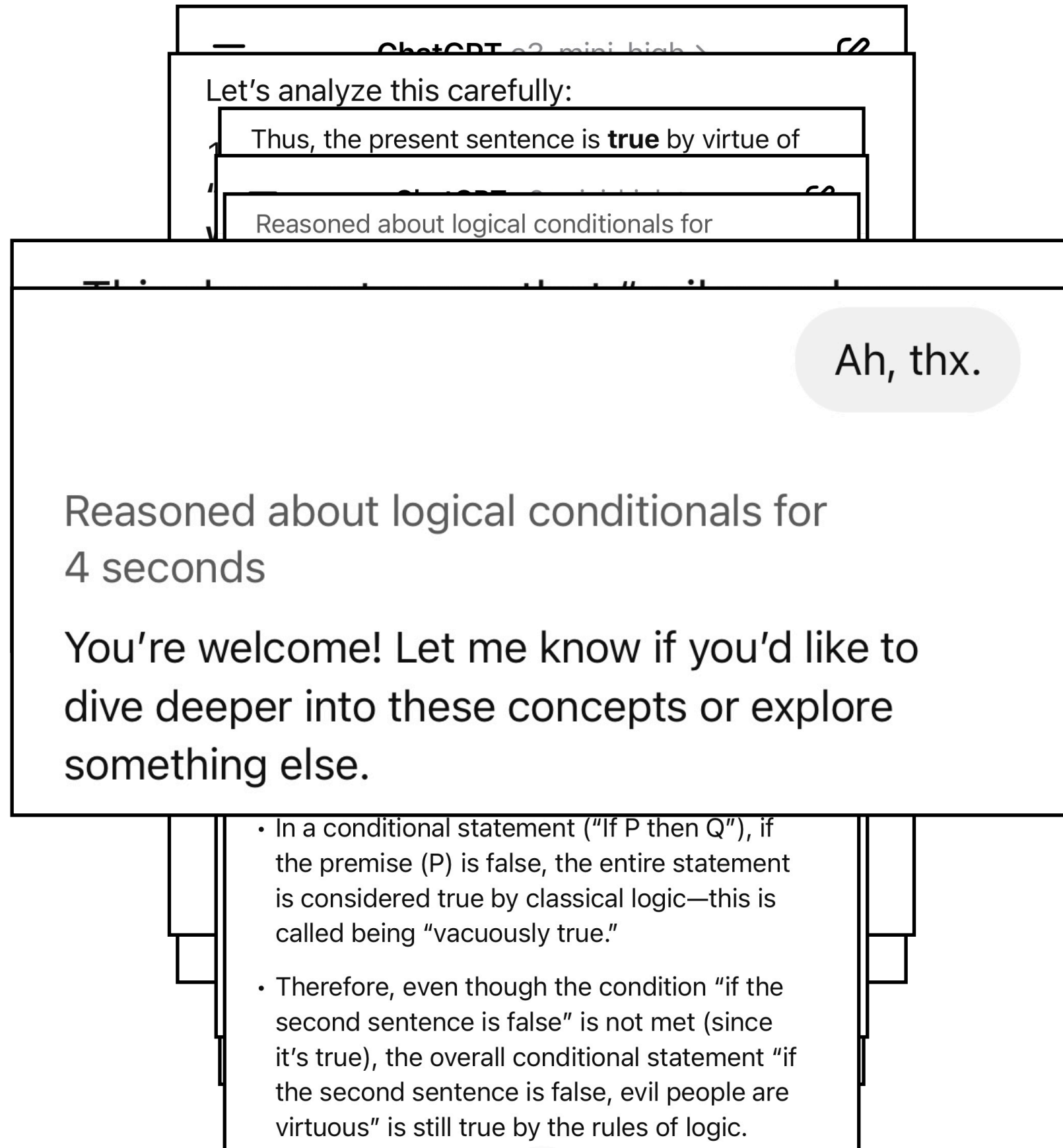
Ah, thx.

Reasoned about logical conditionals for
4 seconds

You're welcome! Let me know if you'd like to
dive deeper into these concepts or explore
something else.

- In a conditional statement ("If P then Q"), if the premise (P) is false, the entire statement is considered true by classical logic—this is called being "vacuously true."
- Therefore, even though the condition "if the second sentence is false" is not met (since it's true), the overall conditional statement "if the second sentence is false, evil people are virtuous" is still true by the rules of logic.

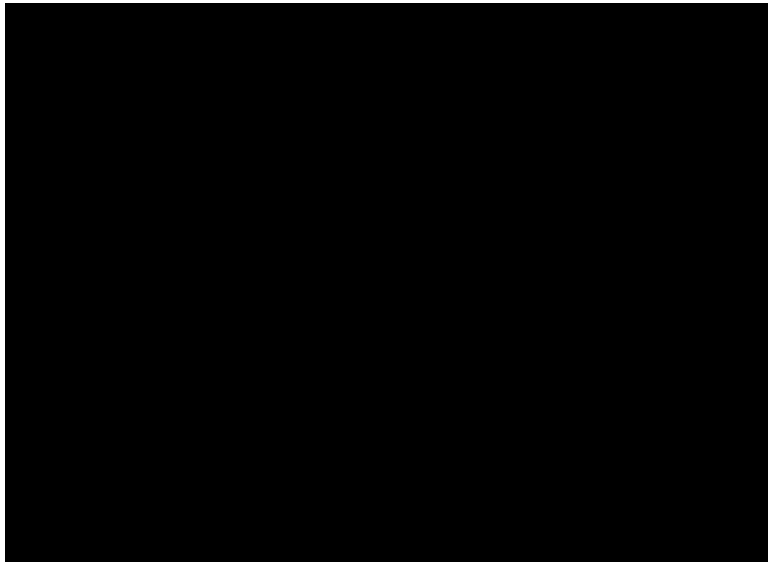
OpenAI's o3 in a Self-Referential Challenge



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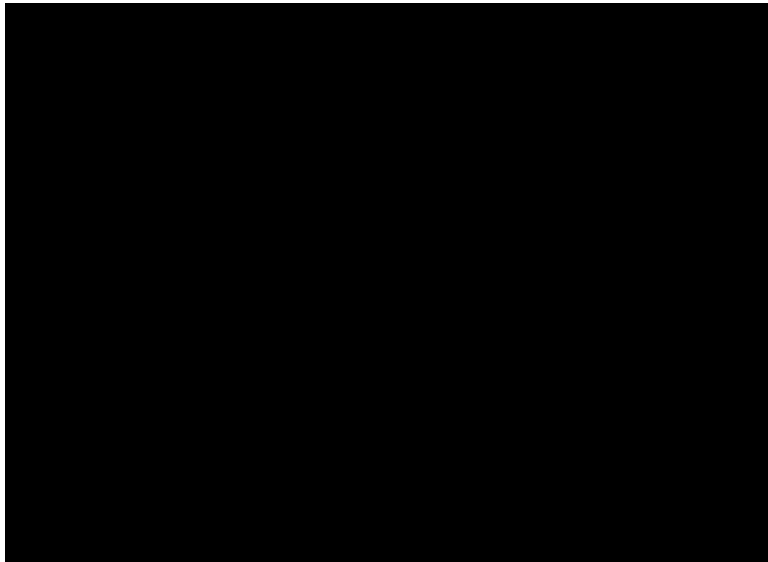
Outlawing Self-Referential Sentences Isn't the Answer!

Box 1

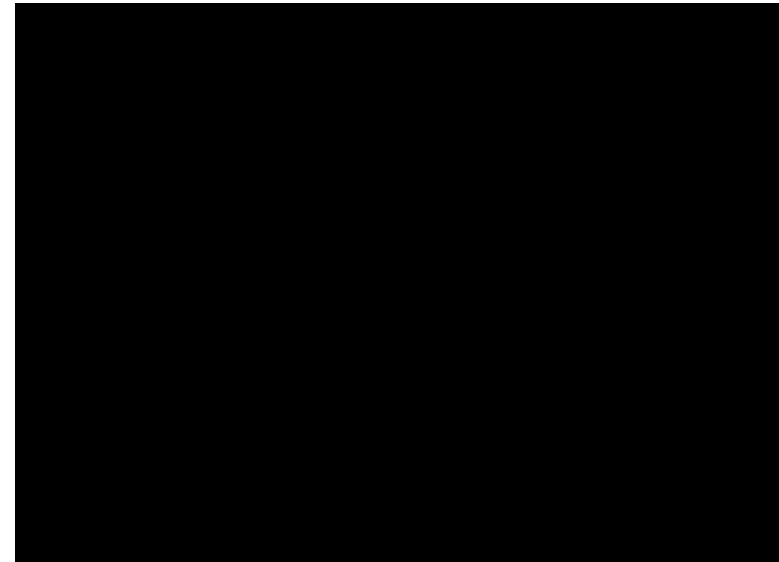


Outlawing Self-Referential Sentences Isn't the Answer!

Box 1



Box 2



Outlawing Self-Referential Sentences Isn't the Answer!

Box 1

The sentence in
Box 2 is true.

Box 2

The sentence in
Box 1 is false.

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1

The sentence in
Box 2 is true.

Neither
sentence is
self-referential.

Box 2

The sentence in
Box 1 is false.

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1

The sentence in
Box 2 is true.

Neither
sentence is
self-referential.

Box 2

The sentence in
Box 1 is false.

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1

The sentence in
Box 2 is true.

Neither
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Box 2

The sentence in
Box 1 is false.

Suppose that the sentence in Box 1 is true. Then the sentence in Box 2 is true (because the sentence in Box 1 says that that sentence is true). But then the sentence in Box 1 is false (because the sentence in Box 2 says that that sentence is false). So, if the sentence in Box 1 is true, it's false. On the other hand, by parallel deduction, if the sentence in Box 1 is false, the sentence in Box 1 is true. (Make sure you work out and verify the reasoning that establishes the previous sentence.) We thus have again a contradiction: The sentence in Box 1 is true if and only if it's not true.

Well do you have a solution, Selmer?

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Of course :). But ...

Well do you have a solution, Selmer?

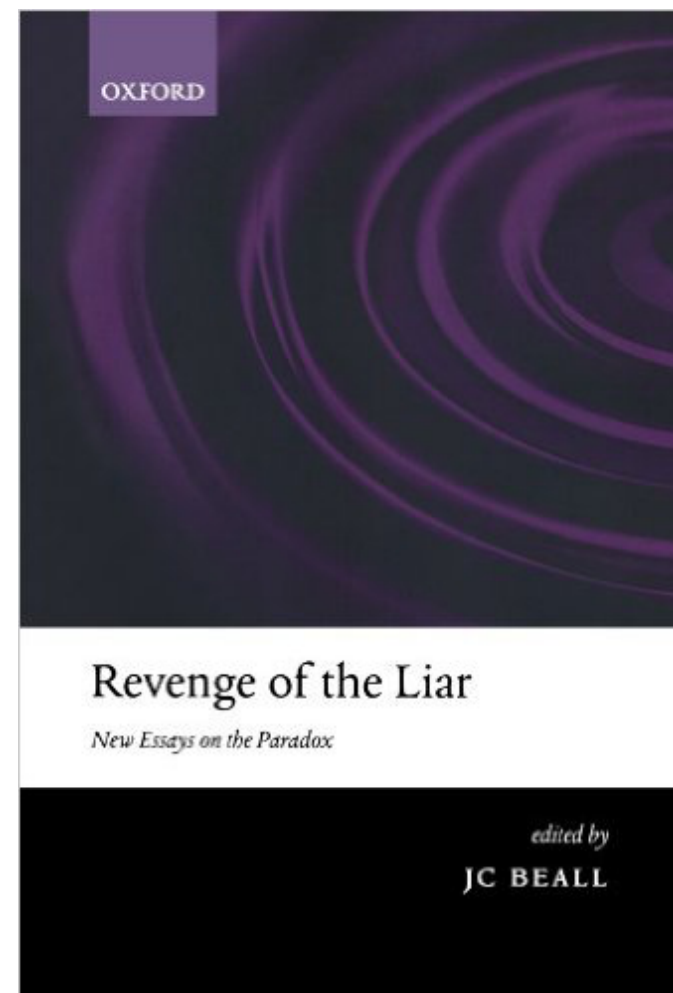
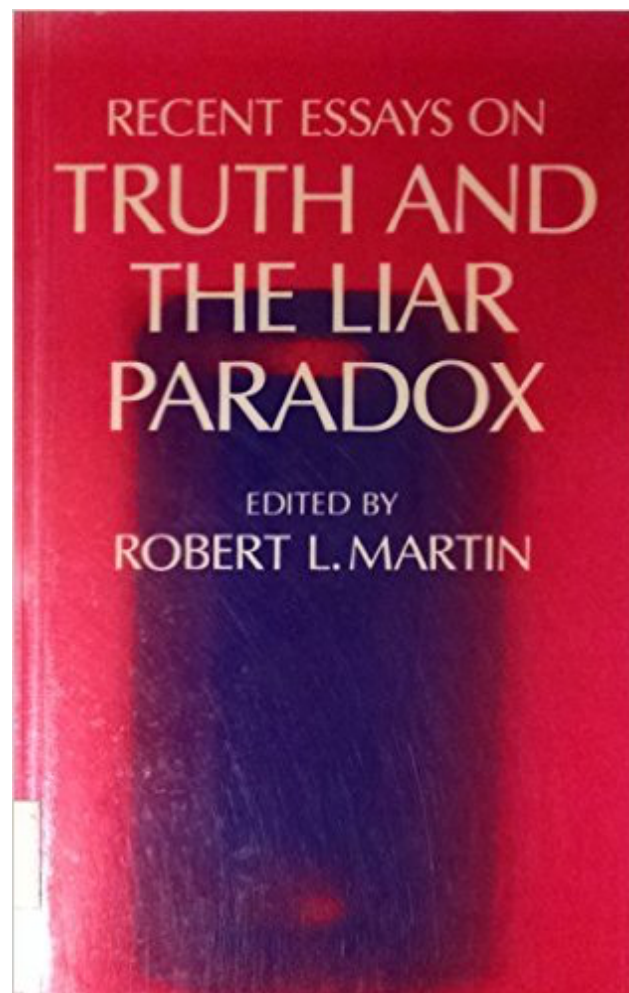
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Further Reading ...

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Further Reading ...



Russell's Paradox ...

Friday's Hill, Haslemere, 16 June 1902

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I am on the point of finishing a book on the principles of mathematics and in it I should like to discuss your work very thoroughly.¹ I already have your books or shall buy them soon, but I would be very grateful to you if you could send me reprints of your articles in various periodicals. In case this should be impossible, however, I will obtain them from a library.

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The above contradiction, when expressed in Peano's ideography, reads as follows:

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The Foundation Crumbles

The Rest of Math,
Engineering, etc.

Foundation

Axiom V etc.

The Foundation Crumbles

The Rest of Math,
Engineering, etc.

Foundation

Axiom V etc.

Axiom V $\exists x \forall y [y \in x \leftrightarrow \phi(y)]$

The Foundation Crumbles

The Rest of Math,
Engineering, etc.

Foundation



Axiom V $\exists x \forall y [y \in x \leftrightarrow \phi(y)]$

a formula of arbitrary size in which the variable y is free; this formula ascribes a property to y

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$$\text{Axiom V} \quad \exists x \forall y [y \in x \leftrightarrow \phi(y)]$$

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There was once a small town in Norway in which there was a barber who shaved all and only the men residing in the town who didn't shave themselves.



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There was once a small town in Norway in which there resided a male barber who shaved all and only the men residing in the town who didn't shave themselves.

Such a situation is impossible!

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Proof: Let's assume for the sake of argument that such a situation can be. Without loss of generality, let the town be Lyngdal and the male Lyngdalian barber be Olaf. Either Olaf shaves himself or he doesn't. But either case leads straight to a contradiction. Therefore the situation is in fact impossible. Here we go ...

Suppose Olaf shaves himself. Then it follows that he doesn't shave himself. Suppose on the other hand that Olaf doesn't shave himself. Then it follows that he does shave himself. Hence, Olaf shaves himself if and only if he doesn't shave himself, which is a contradiction. **QED**

Axiom V $\exists x \forall y [y \in x \leftrightarrow \phi(y)]$

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Russell's Theorem:

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$$\vdash \neg \exists x \forall y (y \in x \leftrightarrow y \notin y)$$

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(Take that, Frege!)

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<http://plato.stanford.edu/entries/russell-paradox/#HOTP>

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What about Thoraf's paradox?

(Skolem's Paradox)

(For a nice overview of Skolem's Paradox, see <https://plato.stanford.edu/entries/paradox-skolem>.)

What about Thoraf's paradox?

(Skolem's Paradox)

3.8.4.1 Can First-Order Logic Capture Infinitude and Finitude?

Does the machinery introduced in the previous section enable us to show that the concepts of finitude and infinitude can be captured by suitable use of first-order logic? If so, how? We should first immediately sharpen this question, which as it stands is somewhat unclear. Let's first target the capturing of infinitude in FOL. Then our initial sharpening move is to stipulate that we are interested specifically in figuring out how we might use FOL to express that a set is countably infinite. (Recall that we defined what it is for a set to be countably infinite in §1.5.3.) In further sharpening of the intuitively expressed question that kicked off the present section, what shall be looking for is how to specify a set Φ that is such that a given interpretation

$$\mathcal{I} \models \Phi \text{ iff domain } \mathcal{D} \text{ in } \mathcal{I} \text{ is countably infinite}$$

where the set Φ contains only formulae in FOL. If we can somehow obtain such a set Φ , then we will have found a way to capture countable infinitude because the domain \mathcal{D} here must be countably infinite. Can you meet this challenge, by drawing upon what was done in the previous section?

Now, what about finitude? Can it be captured by formulae in FOL? The question here can be taken to consist in the challenge to find a set Ψ such that a given interpretation

$$\mathcal{I} \models \Psi \text{ iff domain } \mathcal{D} \text{ in } \mathcal{I} \text{ is finite}$$

where, again, the set in question nce again contains only formulae in FOL.

(For a nice overview of Skolem's Paradox, see <https://plato.stanford.edu/entries/paradox-skolem>.)

3.8.4.1 Can First-Order Logic Capture Infinitude and Finitude?

Does the machinery introduced in the previous section enable us to show that the concepts of finitude and infinitude can be captured by suitable use of first-order logic? If so, how? We should first immediately sharpen this question, which as it stands is somewhat unclear. Let's first target the capturing of infinitude in FOL. Then our initial sharpening move is to stipulate that we are interested specifically in figuring out how we might use FOL to express that a set is countably infinite. (Recall that we defined what it is for a set to be countably infinite in §1.5.3.) In further sharpening of the intuitively expressed question that kicked off the present section, what shall be looking for is how to specify a set Φ that is such that a given interpretation

$$\mathcal{I} \models \Phi \text{ iff domain } \mathcal{D} \text{ in } \mathcal{I} \text{ is countably infinite}$$

where the set Φ contains only formulae in FOL. If we can somehow obtain such a set Φ , then we will have found a way to capture countable infinitude because the domain \mathcal{D} here must be countably infinite. Can you meet this challenge, by drawing upon what was done in the previous section?

Now, what about finitude? Can it be captured by formulae in FOL? The question here can be taken to consist in the challenge to find a set Ψ such that a given interpretation

$$\mathcal{I} \models \Psi \text{ iff domain } \mathcal{D} \text{ in } \mathcal{I} \text{ is finite}$$

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\$20!

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*Hvis du forstår det, kan
du bevise det.*

Part I: *Slutten* — *for i dag.*

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Part II: Hands-on Q&A & Review ...