

Second-Order Logic and the k -order Ladder; Second-Order Axiomatized Arithmetic; Astrologic

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Intro to Formal Logic (& AI)
3/17/2025



FOL

✓ FOL

✓ FOL

Epistemic + FOL

$B_d B_v B_d V v$

✓ FOL

Epistemic + FOL (for coverage of “killer” robots, later)
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TOL

$\exists X[X(j) \wedge \neg X(m) \wedge S(X)]$

✓ FOL

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$B_d B_v B_d V v$

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! $\exists X[X(j) \wedge \neg X(m) \wedge S(X)]$



Double-Minded Man

The Contemporary Craft of Creating Characters Meets Today's Cognitive Architectures: A Case Study in Expressivity*

Selmer Bringsjord • John Licato • Alexander Bringsjord

version of 0121161500NY

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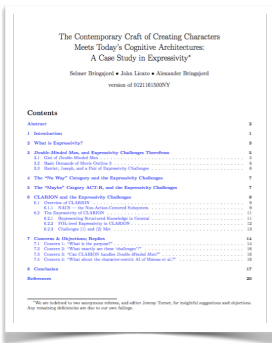
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*We are indebted to two anonymous referees, and editor Jeremy Turner, for insightful suggestions and objections. Any remaining deficiencies are due to our own failings.

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Double-Minded Man

Double-Minded Man



Movie Outline - Double-Minded_Man_010316.mvo

Arial 12 Scene Heading 100%

Outline Script Notes Characters FeelFactor Reference Library PowerView Step Cards Story Tasks

1. TWIRL - DAY

68-year-old Harriet Smith sits with two wrinkled hands firmly on the wheel of her rust-eaten Subaru wagon, staring straight ahead through the top level of bifocals as she waits serenely at a red light.

Harriet is alone in the car. To her right is another vehicle, also waiting, in this case to make a right turn; it's a sleek, low-slung, black Camaro.

We are inside the cabin with Harriet. The Subaru's sound system softly plays choral music. Harriet's lips move slightly as she internally sings along, mouthing a slow aria. Her head weaves slightly side to side, in the rhythm with the music.

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Harriet's eyes swivel up to the light: still red. We wait with her.

Suddenly there is a piercing SCREECH outside. Harriet jerks her head to the right and we follow her line of sight.

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1. TWIRL - DAY

Step 1 of 3

1. TWIRL - DAY
2. YES, THAT'S HIM - LATER
3. SECOND HOME - LATER

[illegible]

Double-Minded Man

The Contemporary Craft of Creating Characters
Meets Today's Cognitive Architecture:
A Case Study in Expressivity*

Selmer Bringsjord • John Linn • Alexander Bringsjord
version of 022101000V

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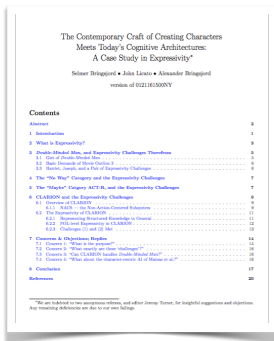
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Double-Minded Man

Double-Minded Man
by
S Bringsjord & A Bringsjord

DRAFT #5
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Selmer.Bringsjord@gmail.com



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The Contemporary Craft of Creating Characters

Memo Today's Cognitive Architecture:

A Case Study in Expressivity*

Silver Stripling • John Linn • Alexander Stripling

version of 02/16/2020

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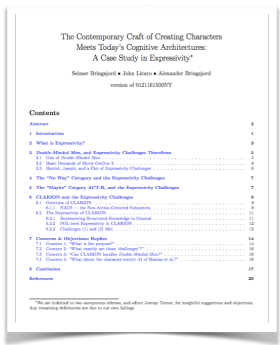
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$$\exists X[X(joseph) \wedge \neg X(m(harriet, joseph)) \wedge Sleazy(X)]$$

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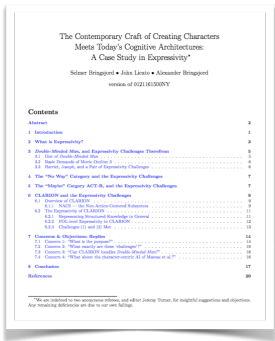
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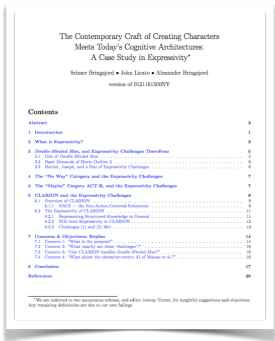
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Climbing the k -order Ladder

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a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

$$\exists x [Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$$

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Climbing the k -order Ladder

FOL $\exists x [Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

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Climbing the k -order Ladder

Things x and y , along with the father of x ,
share a certain property (and x likes y).

FOL $\exists x [Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes b (which
is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

$$\exists x \exists y \exists R [R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$$

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Climbing the k -order Ladder

Things x and y , along with the father of x , share a certain property; and, $x R^2s y$, where R^2 is a positive property.

SOL $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$

Things x and y , along with the father of x , share a certain property (and x likes y).

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Climbing the k -order Ladder

$$\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \textit{Positive}(R^2) \wedge R(\textit{fatherOf}(x))]$$

Things x and y , along with the father of x , share a certain property; and, x R^2 s y , where R^2 is a positive property.

SOL $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \textit{Likes}(x, y) \wedge R(\textit{fatherOf}(x))]$

Things x and y , along with the father of x , share a certain property (and x likes y).

FOL $\exists x [Llama(x) \wedge Llama(b) \wedge \textit{Likes}(x, b) \wedge Llama(\textit{fatherOf}(x))]$

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Climbing the k -order Ladder

TOL $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \textit{Positive}(R^2) \wedge R(\textit{fatherOf}(x))]$

Things x and y , along with the father of x , share a certain property; and, x R^2 s y , where R^2 is a positive property.

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Things x and y , along with the father of x , share a certain property (and x likes y).

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Climbing the k -order Ladder

TOL $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \textit{Positive}(R^2) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_3$ Things x and y , along with the father of x , share a certain property; and, x R^2 s y , where R^2 is a positive property.

SOL $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \textit{Likes}(x, y) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_2$ Things x and y , along with the father of x , share a certain property (and x likes y).

FOL $\exists x [Llama(x) \wedge Llama(b) \wedge \textit{Likes}(x, b) \wedge Llama(\textit{fatherOf}(x))]$

$\mathcal{L}_1$ There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge \textit{Likes}(a, b) \wedge Llama(\textit{fatherOf}(a))$

$\mathcal{L}_0$ a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

⋮

TOL $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \textit{Positive}(R^2) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_3$ Things x and y , along with the father of x , share a certain property; and, x R^2 s y , where R^2 is a positive property.

SOL $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \textit{Likes}(x, y) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_2$ Things x and y , along with the father of x , share a certain property (and x likes y).

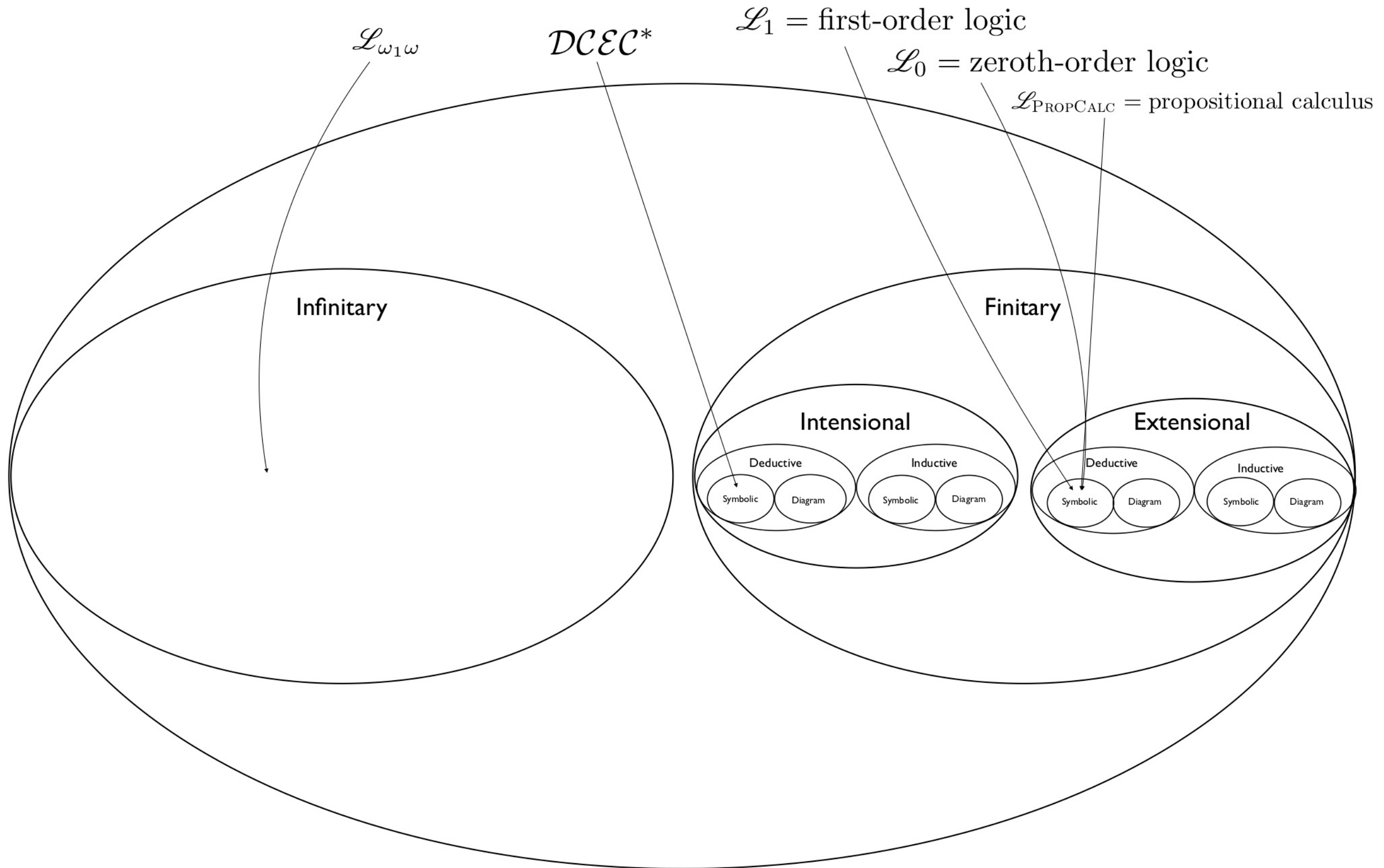
FOL $\exists x [Llama(x) \wedge Llama(b) \wedge \textit{Likes}(x, b) \wedge Llama(\textit{fatherOf}(x))]$

$\mathcal{L}_1$ There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge \textit{Likes}(a, b) \wedge Llama(\textit{fatherOf}(a))$

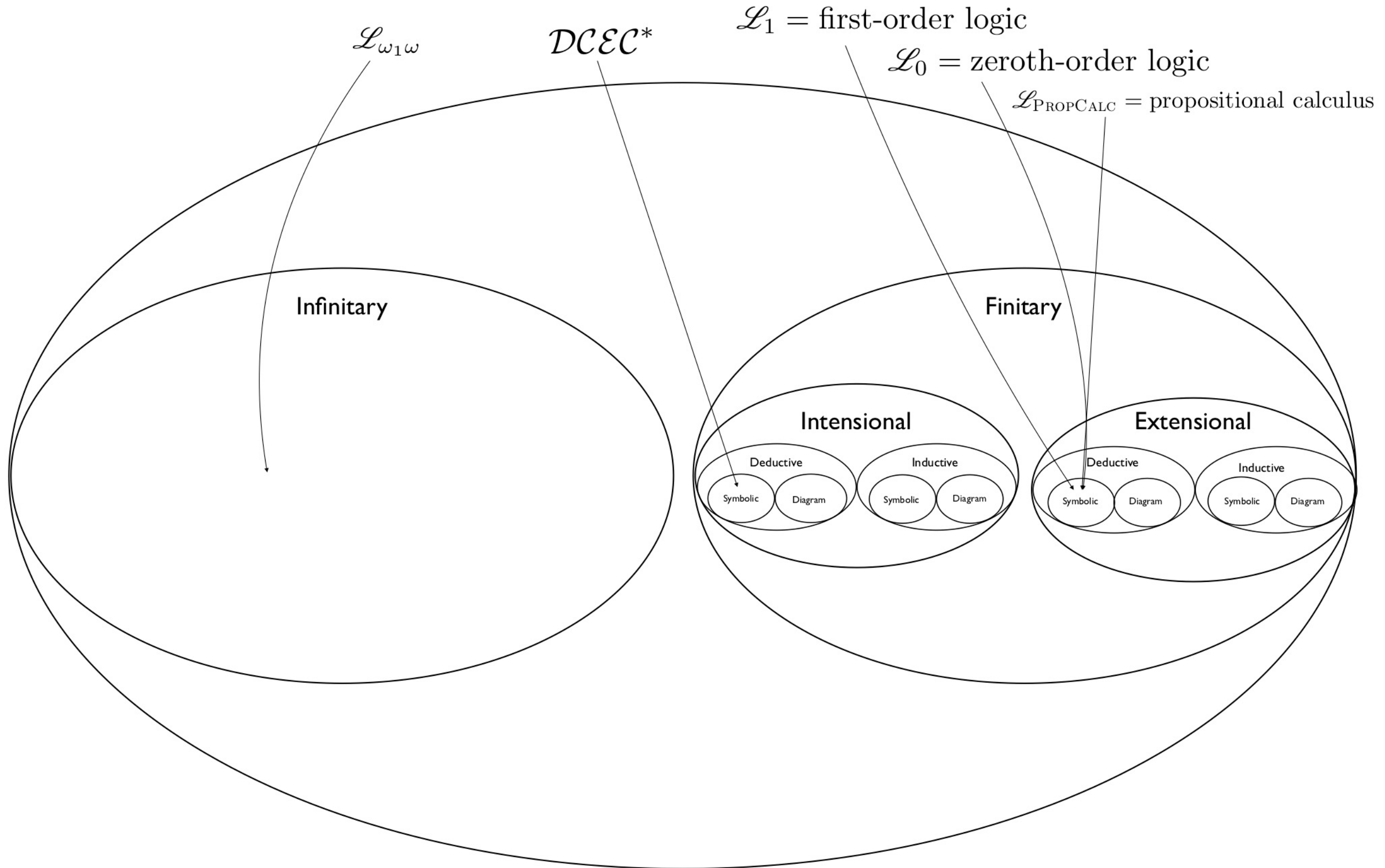
$\mathcal{L}_0$ a is a llama, as is b , a likes b , and the father of a is a llama as well.

The Universe of Logics

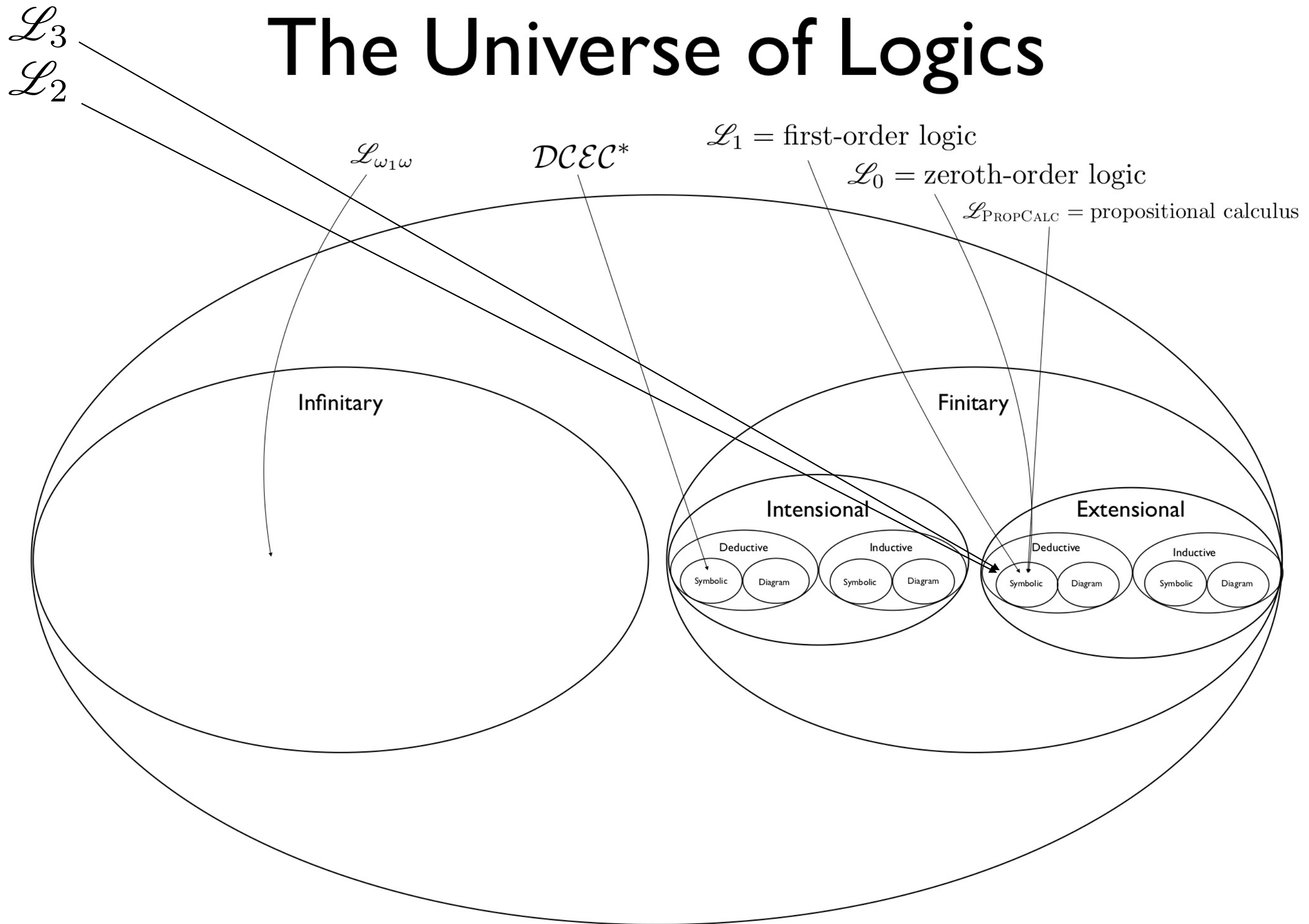


$\mathcal{L}_3$
 $\mathcal{L}_2$

The Universe of Logics



The Universe of Logics

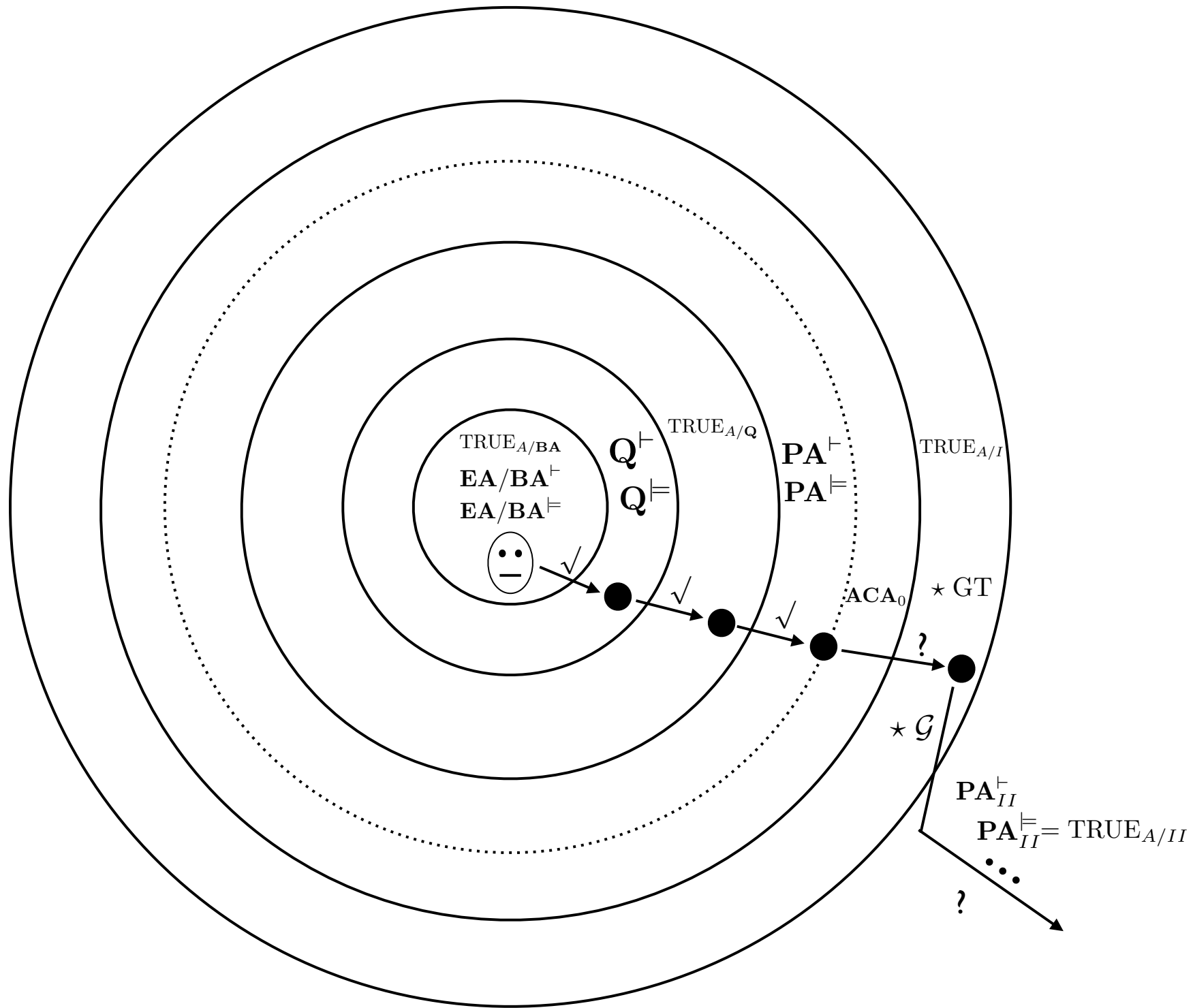


“Higher-Order” Remarks,
Glimpse ahead to Gödel

...

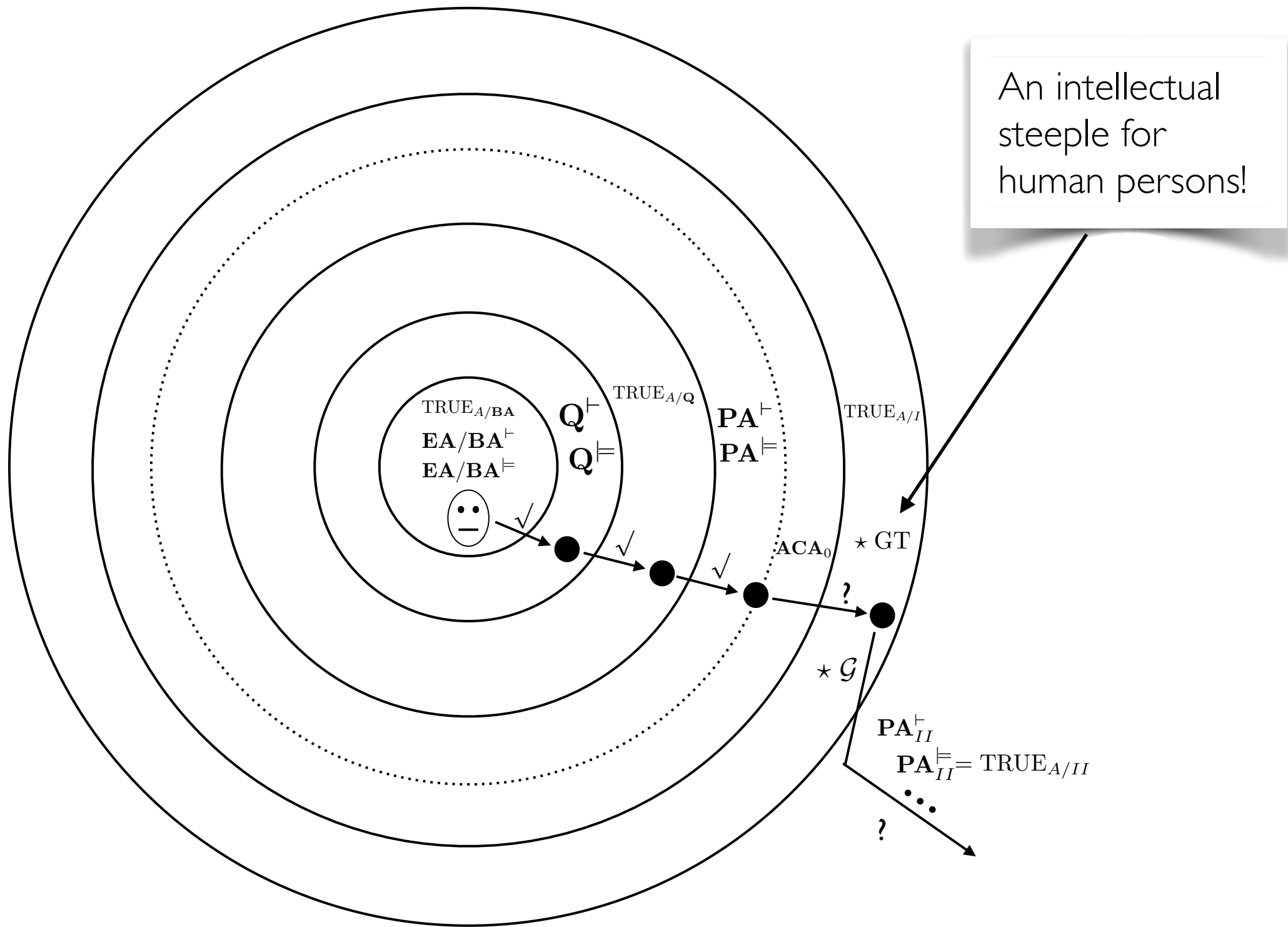
Astrologic

(Aliens & Angels on the Same “Race Track”)



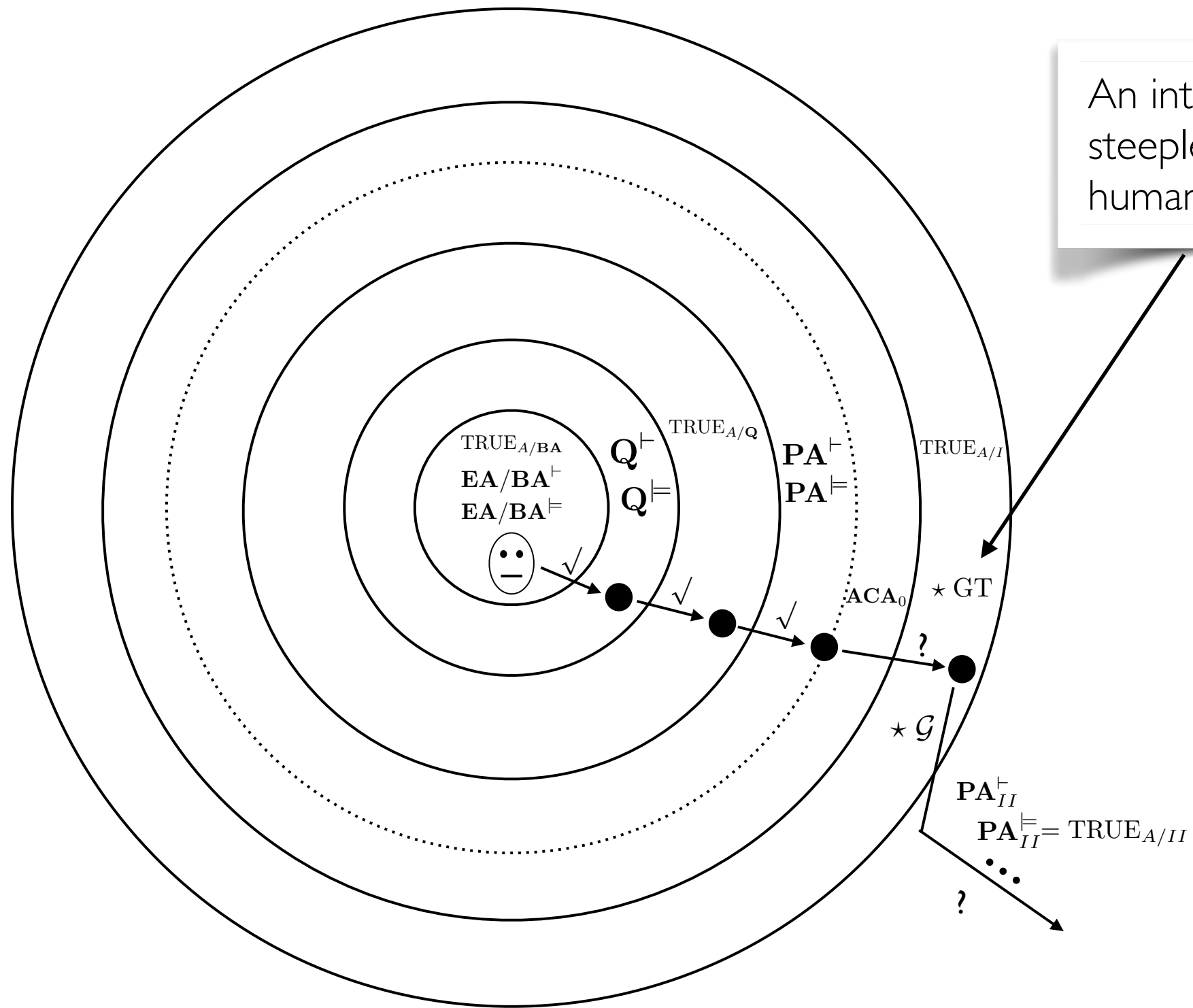
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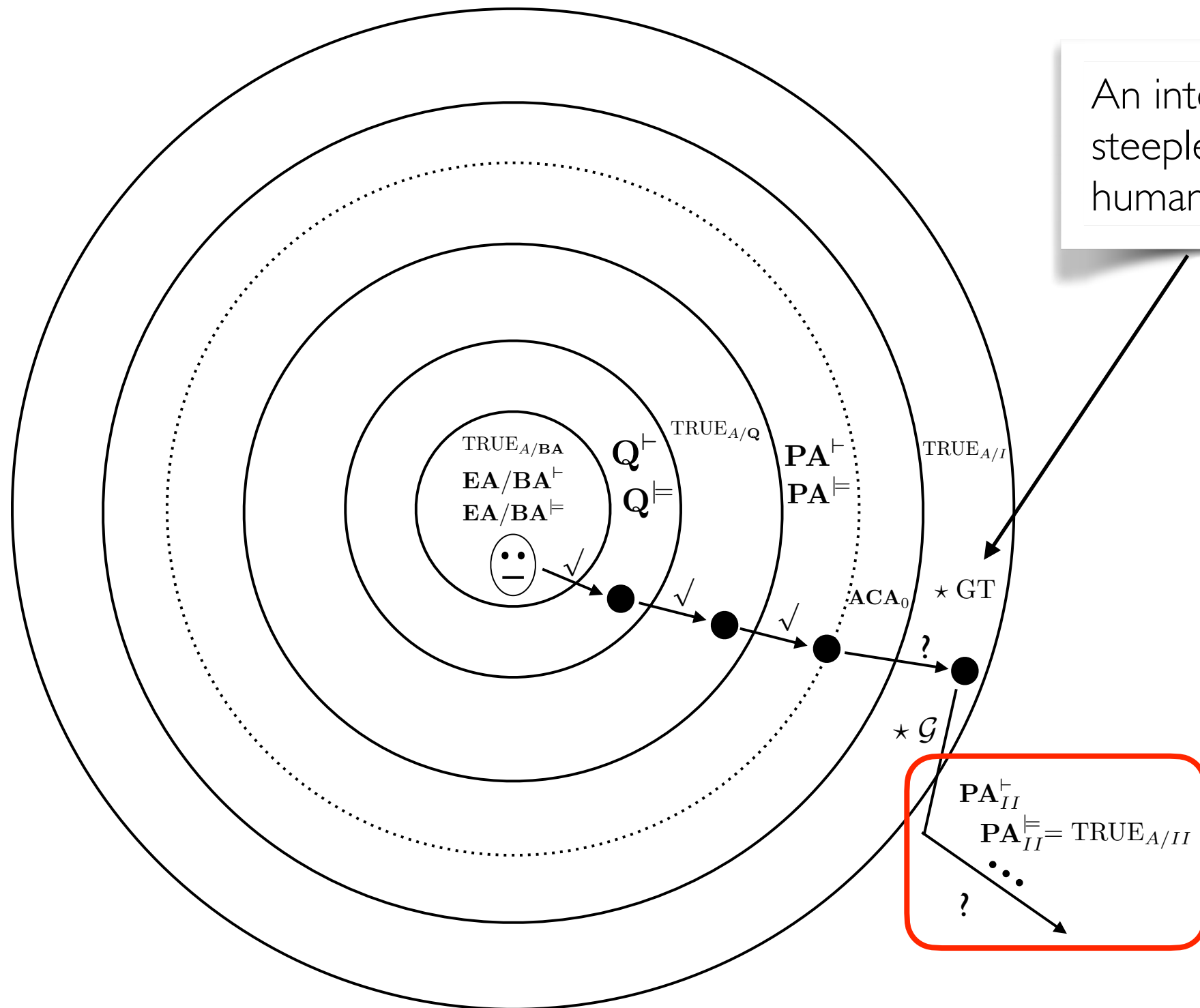


An intellectual
steeple for
human persons!



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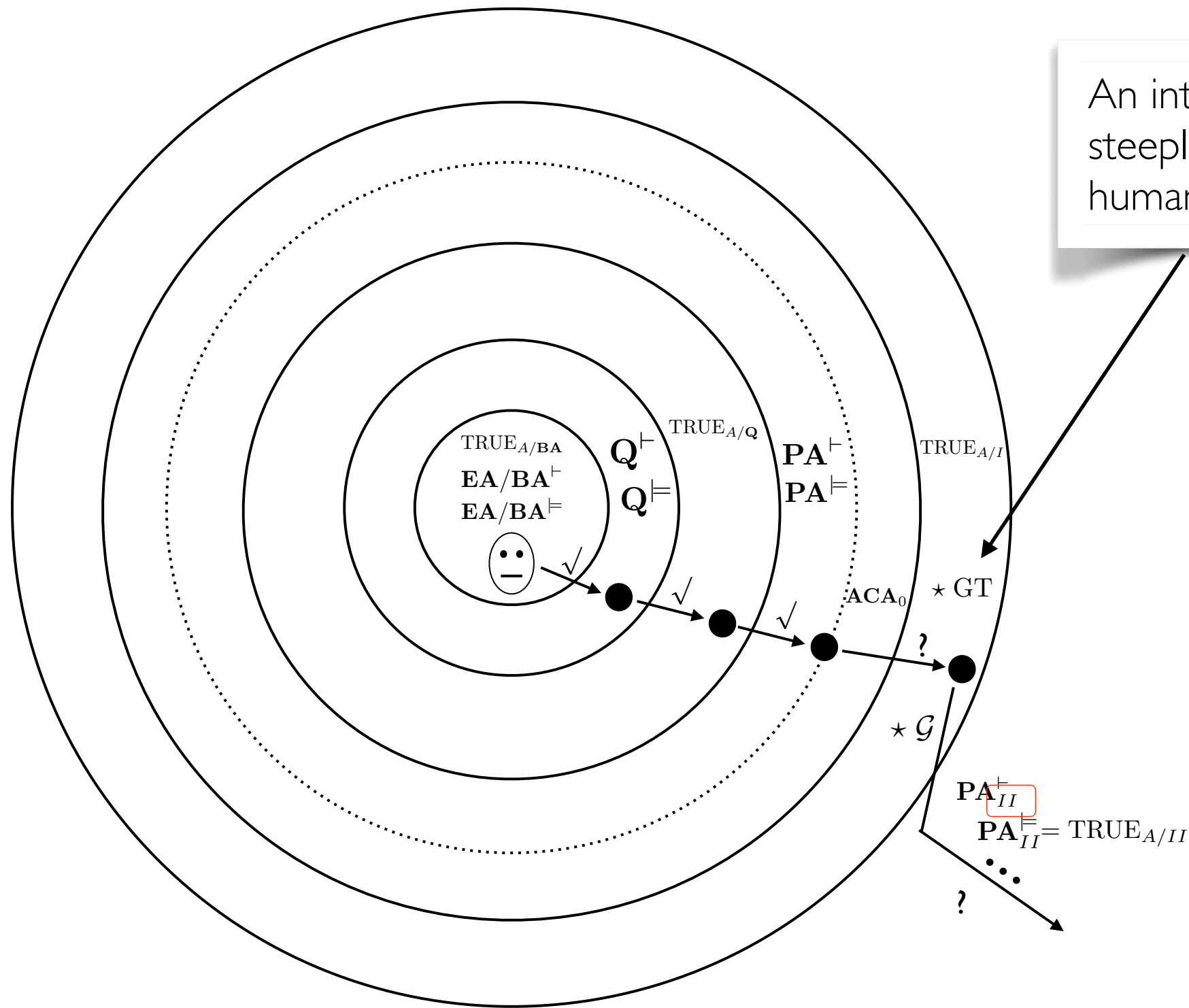


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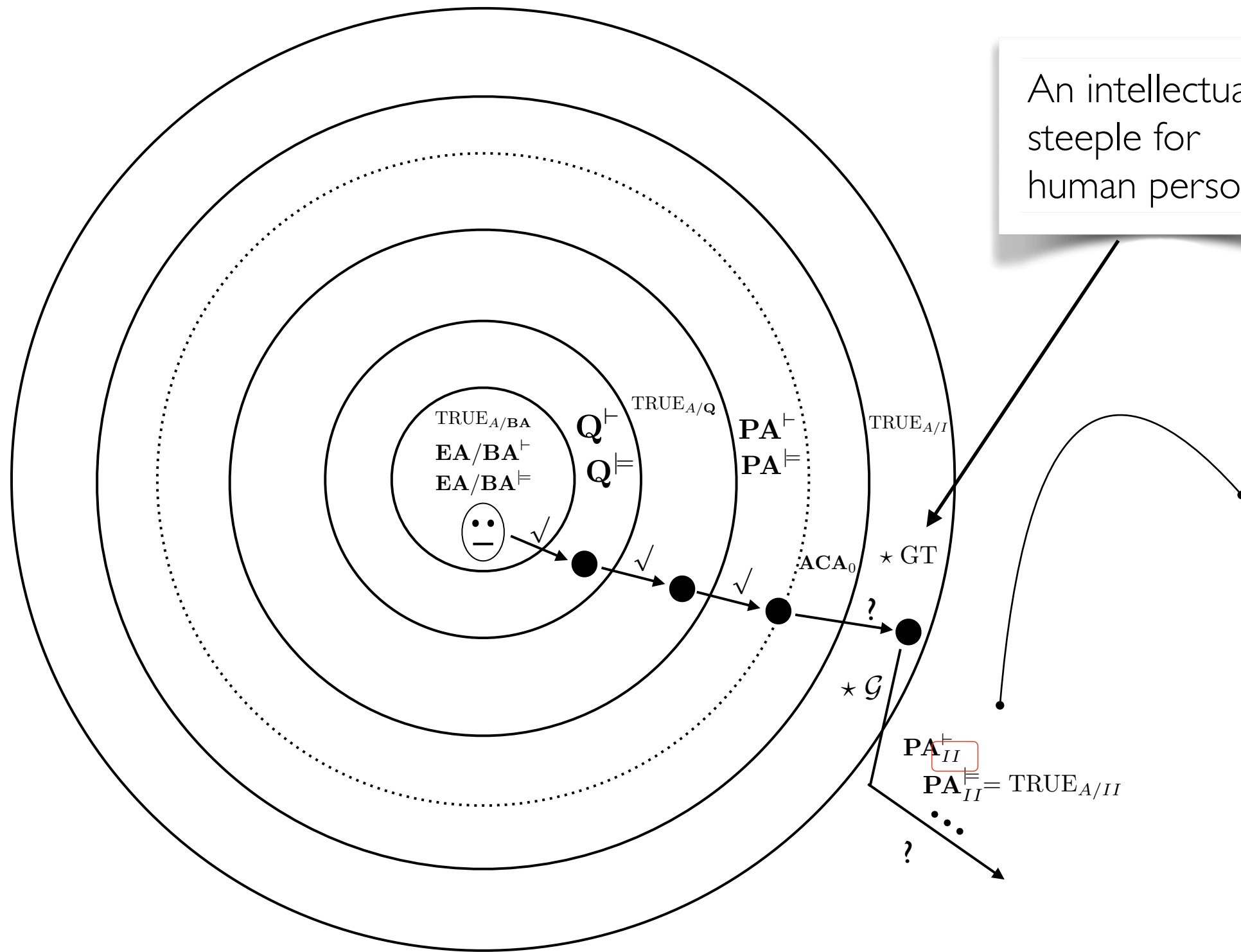


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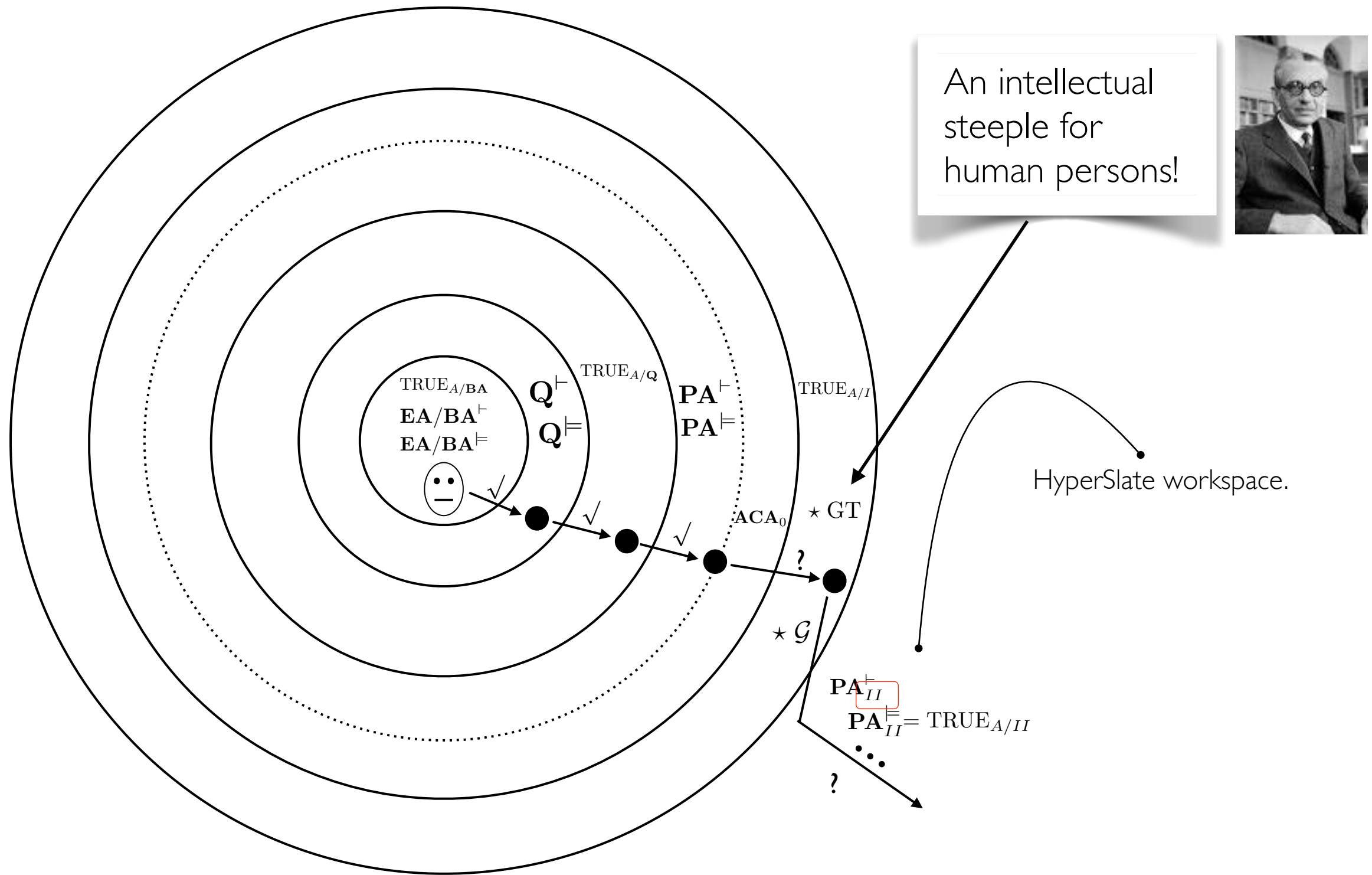


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Astrologic

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(First-Order) Peano Arithmetic = $\text{PA}_1 = \text{Z}_1$

$$\text{A1} \quad \forall x(0 \neq s(x))$$

$$\text{A2} \quad \forall x \forall y (s(x) = s(y) \rightarrow x = y)$$

$$\text{A3} \quad \forall x (x \neq 0 \rightarrow \exists y (x = s(y)))$$

$$\text{A4} \quad \forall x (x + 0 = x)$$

$$\text{A5} \quad \forall x \forall y (x + s(y) = s(x + y))$$

$$\text{A6} \quad \forall x (x \times 0 = 0)$$

$$\text{A7} \quad \forall x \forall y (x \times s(y) = (x \times y) + x)$$

} $\mathcal{Q}$

And, every sentence that is the universal closure of an instance of

$$([\phi(0) \wedge \forall x(\phi(x) \rightarrow \phi(s(x)))] \rightarrow \forall x \phi(x))$$

where $\phi(x)$ is open wff with variable x , and perhaps others, free.

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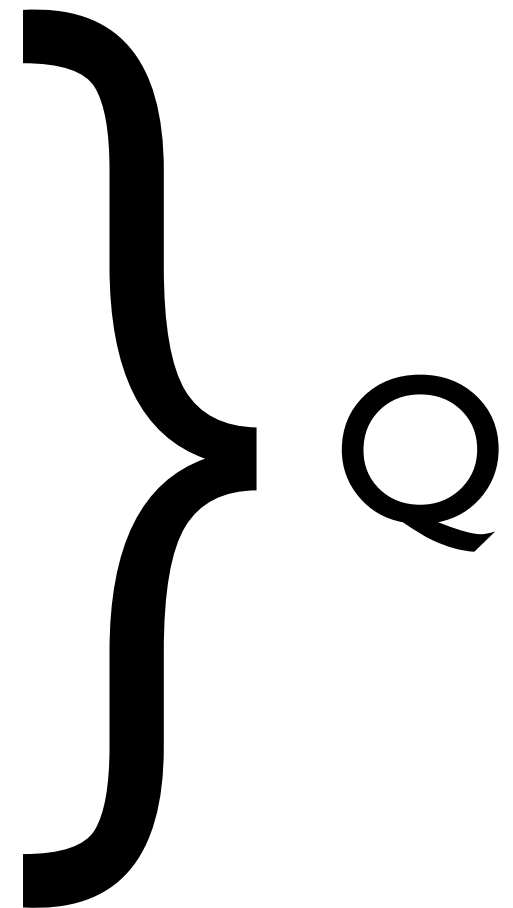
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$$\mathbf{PA}_2 = \mathbf{Z}_2$$

- A1 $\forall x(0 \neq s(x))$
- A2 $\forall x \forall y(s(x) = s(y) \rightarrow x = y)$
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(if we drop any restriction C, we have full second-order arithmetic.)

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$$\text{Induction Axiom} \quad \forall X ([X(0) \wedge \forall x (X(x) \rightarrow X(s(x)))] \rightarrow \forall x X(x))$$

(if we drop any restriction C, we have full second-order arithmetic.)

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$$\begin{array}{l} \text{Comprehension} \\ \text{Schema} \end{array} \quad \exists X (\forall x X(x) \leftrightarrow \phi(x)) \quad \text{where } \phi(x) \in \mathcal{C}$$

(if we drop any restriction $\mathcal{C}$, we have full second-order arithmetic.)

New!

ACA₀

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Comprehension

Schema $\exists X [\forall x X(x) \leftrightarrow \phi(x)]$ where $\phi(x)$ has no second-order quantifiers

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Schema $\exists X [\forall x X(x) \leftrightarrow \phi(x)]$ where $\phi(x)$ has no second-order quantifiers

Comprehension inference schema is
available in HS[®]! —
But order of biconditional matters!

Bonus “Explosion” Challenge: $\exists Y \forall X \forall z [Yz \leftrightarrow Xz] \vdash \zeta$

\$100 Challenge; Must sign up & get approved to tackle beforehand by Selmer:
Long reasonable proof in **PA**; short counterpart in **PA2**.

Slutten