On Heterogeneous Logic

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Intro to Formal Logic (With AI)
4/2/2026

HyperSlate *first* …

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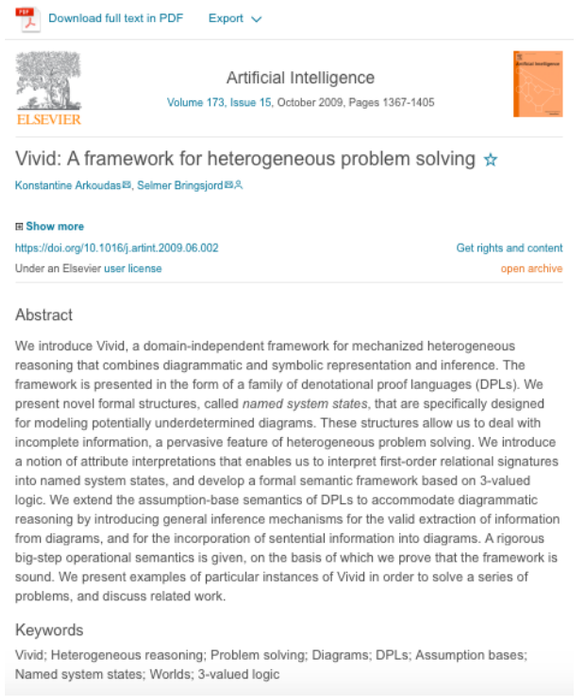

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Vivid: A framework for heterogeneous problem solving ☆

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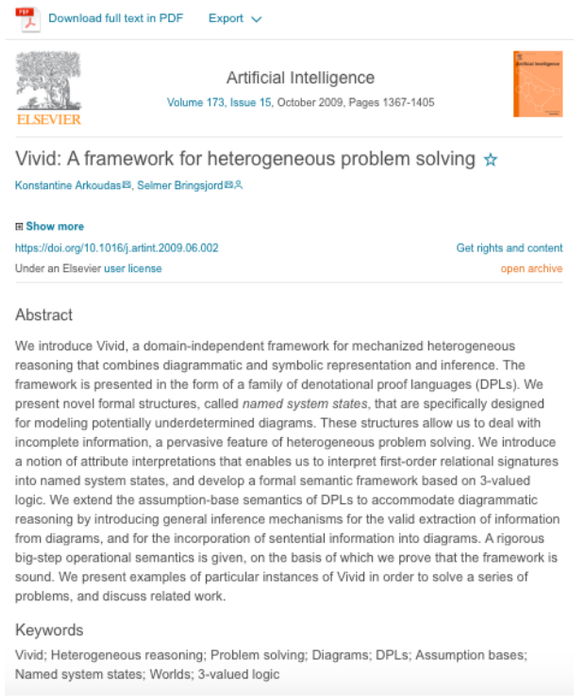

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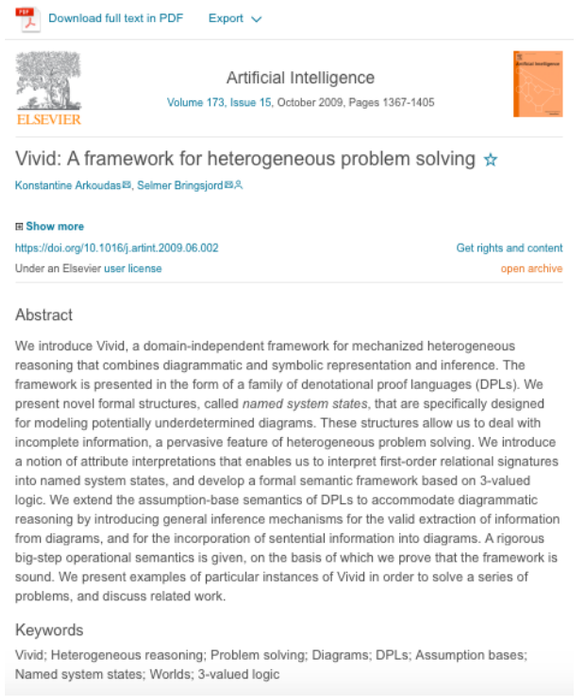

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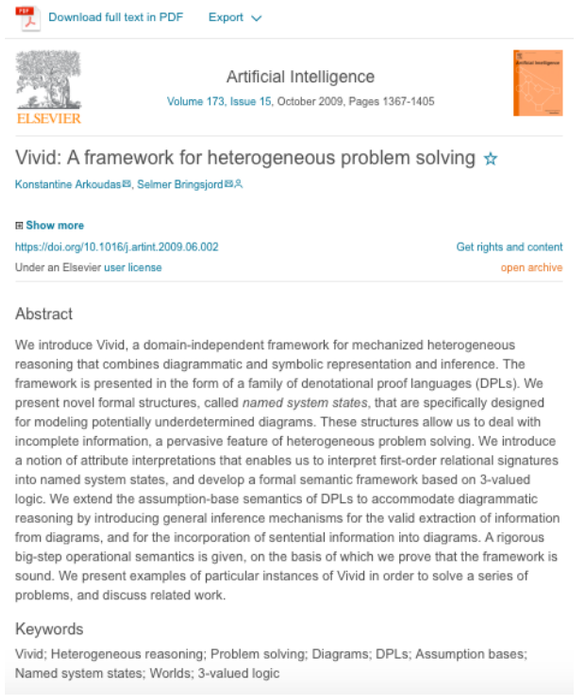

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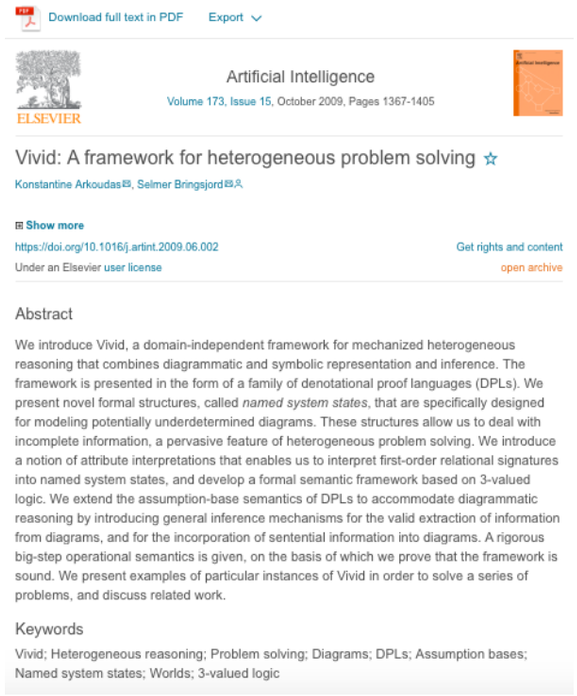

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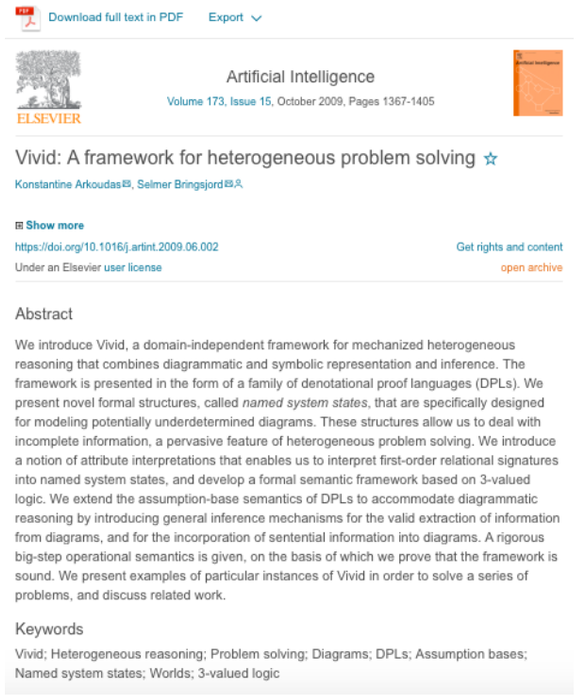

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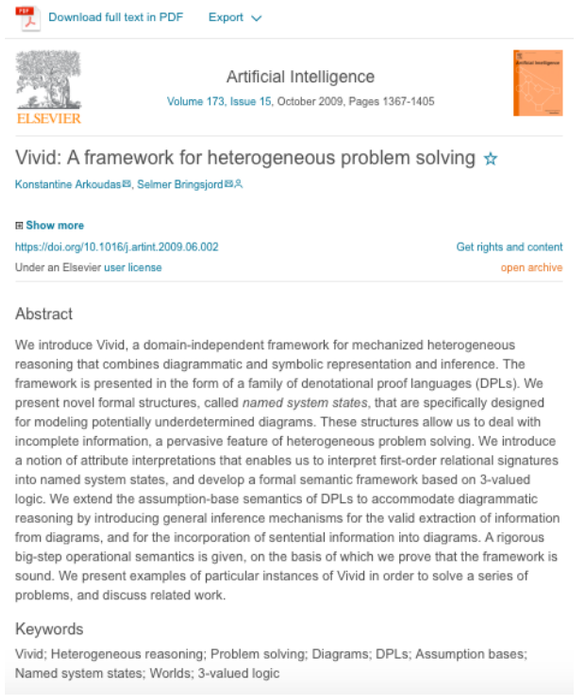

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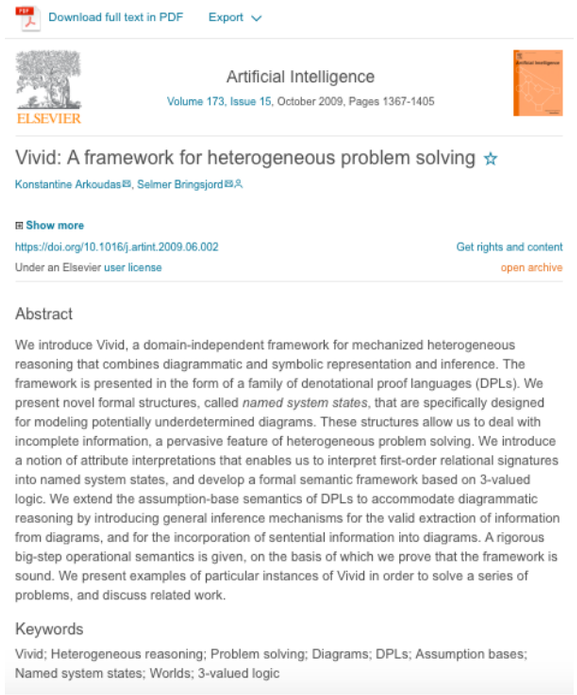

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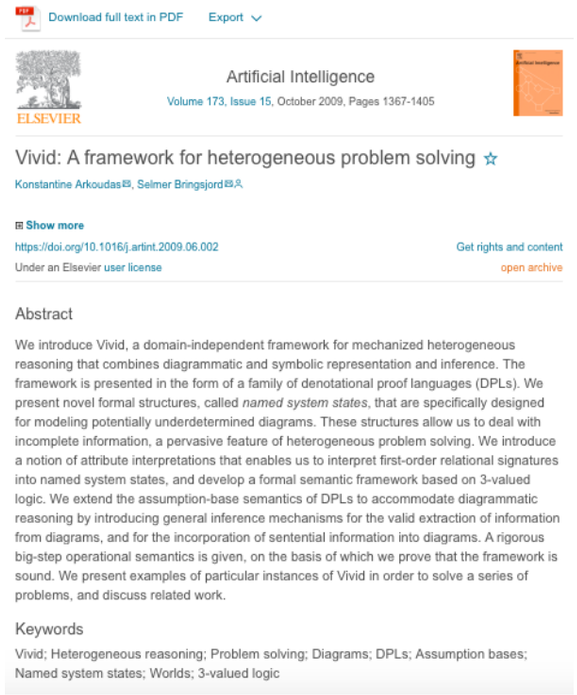

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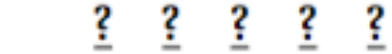
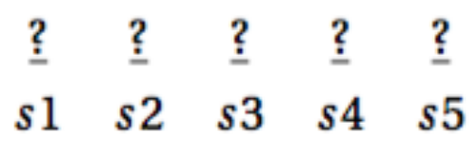
ANIMATION

8.2 Jumping In With Seating Puzzles

Specifically, we now get started in earnest with a class of diagrams that denote seating arrangements. Here's a simple diagram intended to denote five chairs in a row, each of which is either empty or filled with one of four possible people:



To make it easier on ourselves, we label the seats from left to right, starting with 1:



In addition, assume that the only people who can occupy seats are Alvin (a), Billy (b), Cindy (c), and Dora (d). A seat can also be empty; we denote this condition by the constant e. (Here, obviously, these lower-case Roman letters are symbolic

Our four people are to be seated in a row of five seats. This seating arrangement must satisfy the following three conditions:

C1 a and c should flank the empty seat.

C2 c should be closer to the middle seat than b.

C3 b and d should be seated next to each other.

Given this information, reach the following three goals:

G1 prove that the empty seat can't be in the middle and can't be on either end;

G2 settle whether it can be determined who must be seated in the middle seat;

G3 settle whether it can be determined who is to be seated on the two ends.

Proof: From all the permutations in which our four characters are seated (with a remaining empty seat), which totals 5! = 120), we can eliminate as inconsistent with condition C1 all but these six possibilities:

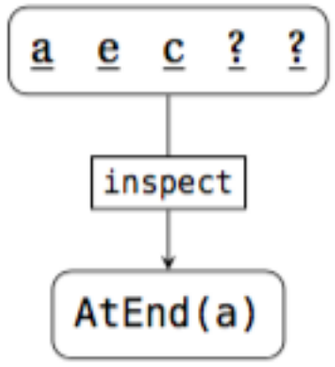
P1	a	e	c	?	?
P2	?	a	e	c	?
P3	?	?	a	e	c
P4	c	e	a	?	?
P5	?	c	e	a	?
P6	?	?	c	e	a

But possibilities P2–P5 each lead to absurdity when combined with the conjunction of C2 and C3. Hence by `disjunctive syllogism` over the disjunction of all six possibilities P1–P6 we deduce P1 ∨ P6, from which we in turn can deduce (i) G1, (ii) that *c* is in the middle seat and hence an affirmative answer settles G2, and (iii) that any of *a*, *b*, *d* can occupy an end seat, and from this a negative answer to the query presented in G3. **QED**

proof given above is a diagram. We set Pi to $\mathfrak{d}_i$. Now consider specifically $\mathfrak{d}_1$, that is:

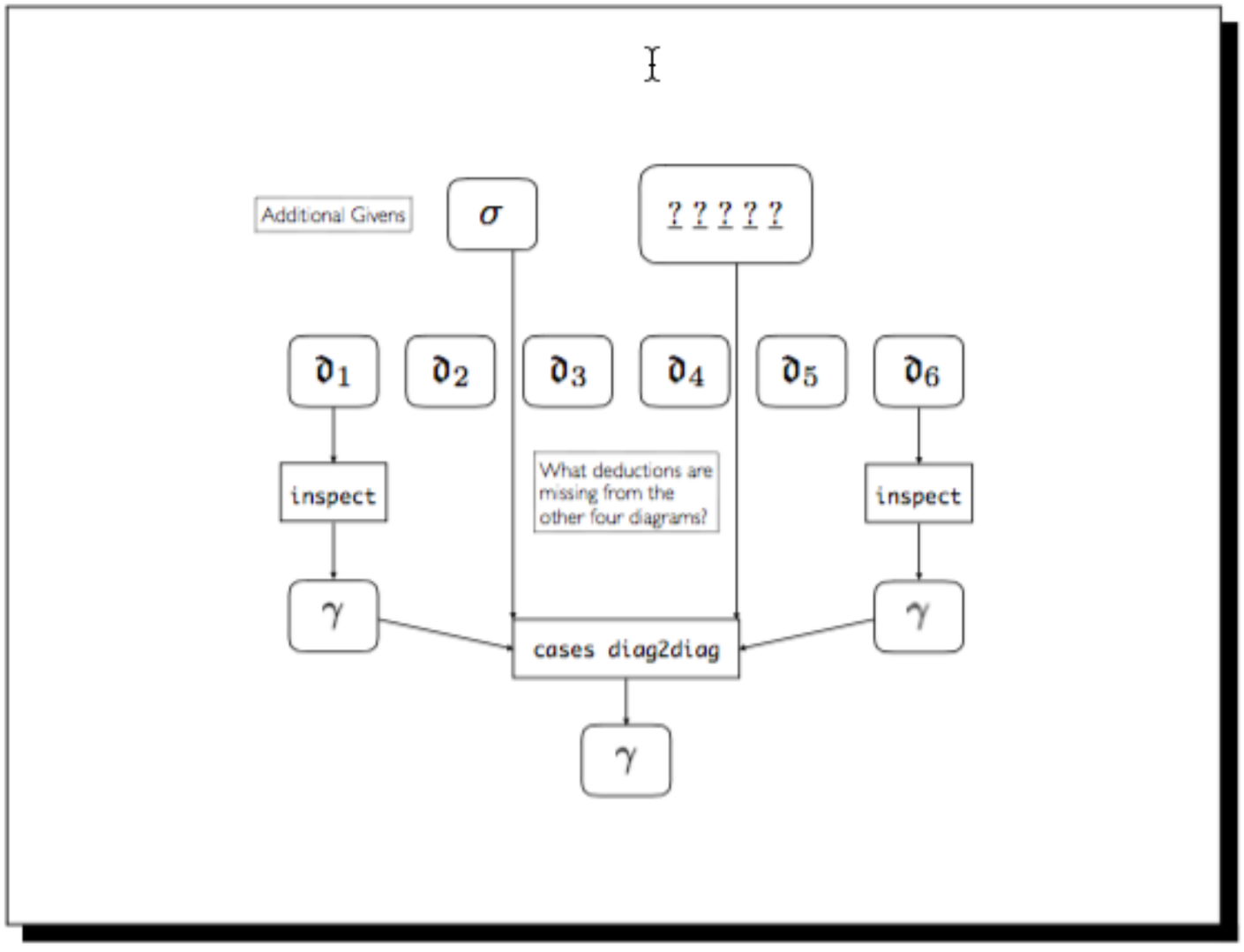
a e c ? ?

We can deduce by `inspect`[5] that `AtEnd(a)`. We can also deduce by `inspect` that `At(a,s1)`. Here's the first of these inferences in Slate style:



(8.1)

Figure 8.3: Hetergeneous Slate Proof that Solves the Seating Puzzle

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Is your person wearing a hat?
Does your person have blue eyes?
WARNING:


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Figure 8.4: Portrait of an "Emotionally Mysterious" Larry (diagram $\mathfrak{d}_?$; by KB Foushée)

Now, suppose that the following conjunction, ϕ, holds:

$$\neg\mathsf{Happy}(\mathsf{larry},t) \wedge \neg\mathsf{Angry}(\mathsf{larry},t) \wedge \neg\mathsf{Distrustful}(\mathsf{larry},t) \wedge \neg\mathsf{Fearful}(\mathsf{larry},t)$$

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Figure 8.5: Result of Deduction by `thinning` Applied to Diagram $\mathfrak{d}_?$ and ϕ (by KB Foushée)

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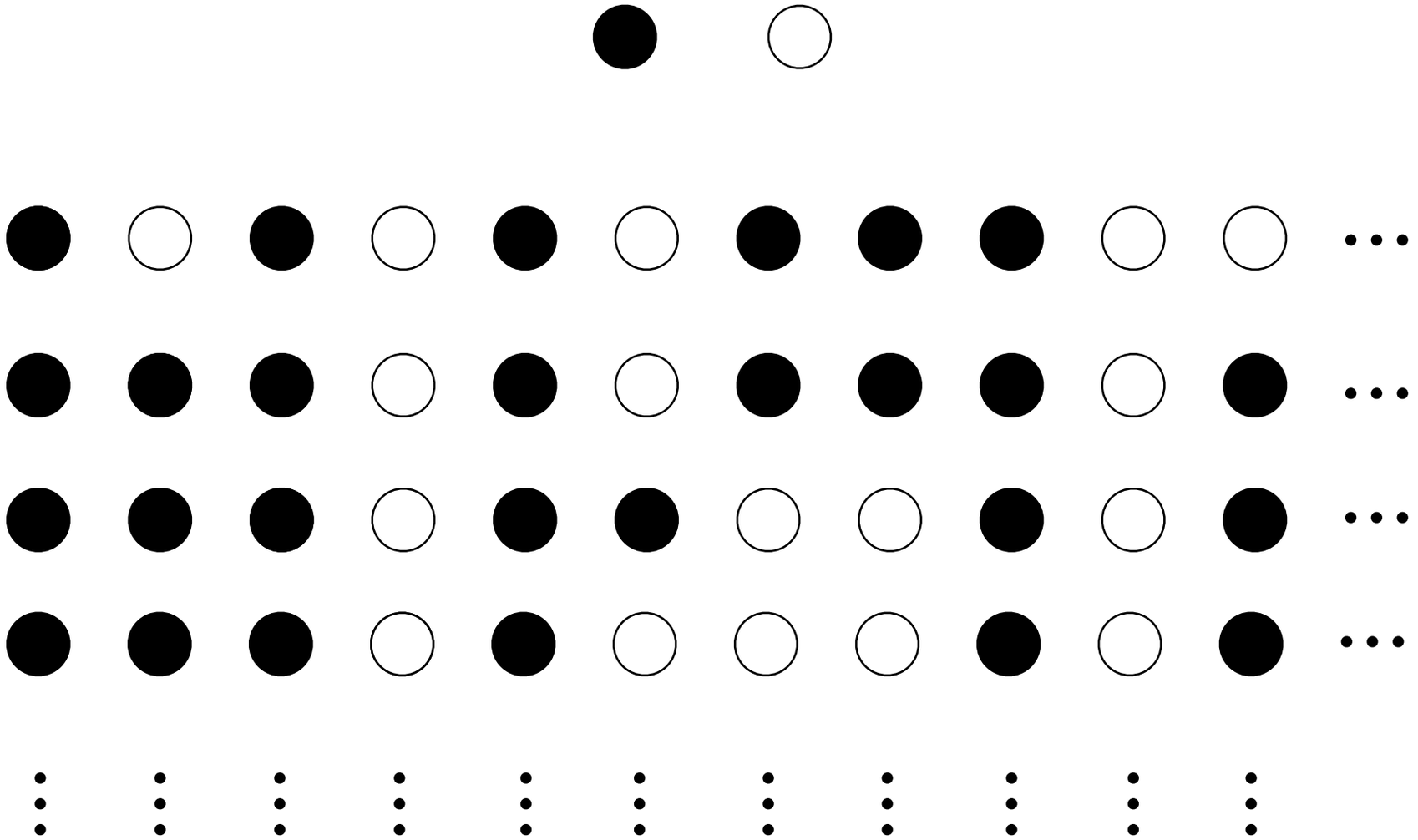
8.3 Exercises

1. Examine Figure 8.3 carefully, if you haven't done so already. Complete the proof, by specifying and deploying the missing givens, and by adding the deductions that are missing below each of the diagrams $\mathfrak{d}_2$–$\mathfrak{d}_4$.
2. What, exactly, is the general form of `cases diag2diag`? Write down the inference schema in graphical form. (We suggest that you generalize from the proof produced by an answer to the previous exercise.)
3. What, exactly, is the general form of `thinning`? Write down the inference schema in graphical form.
4. As you know, we have (informally) introduced the inference schema `thinning`. What do you think the inference schema called `widening` is, given that it is accurately said to be the inverse of `thinning`? Write down a specification of it, as your best hypothesis.

Yes, but animations! …

Animation-Based Proof of Cantor's $\mathscr{P}(\mathbb{Z}^+) > \mathbb{Z}^+$

Ideal Observer



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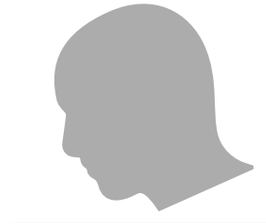

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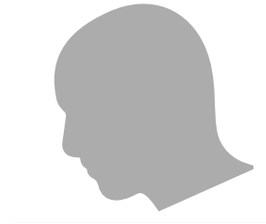

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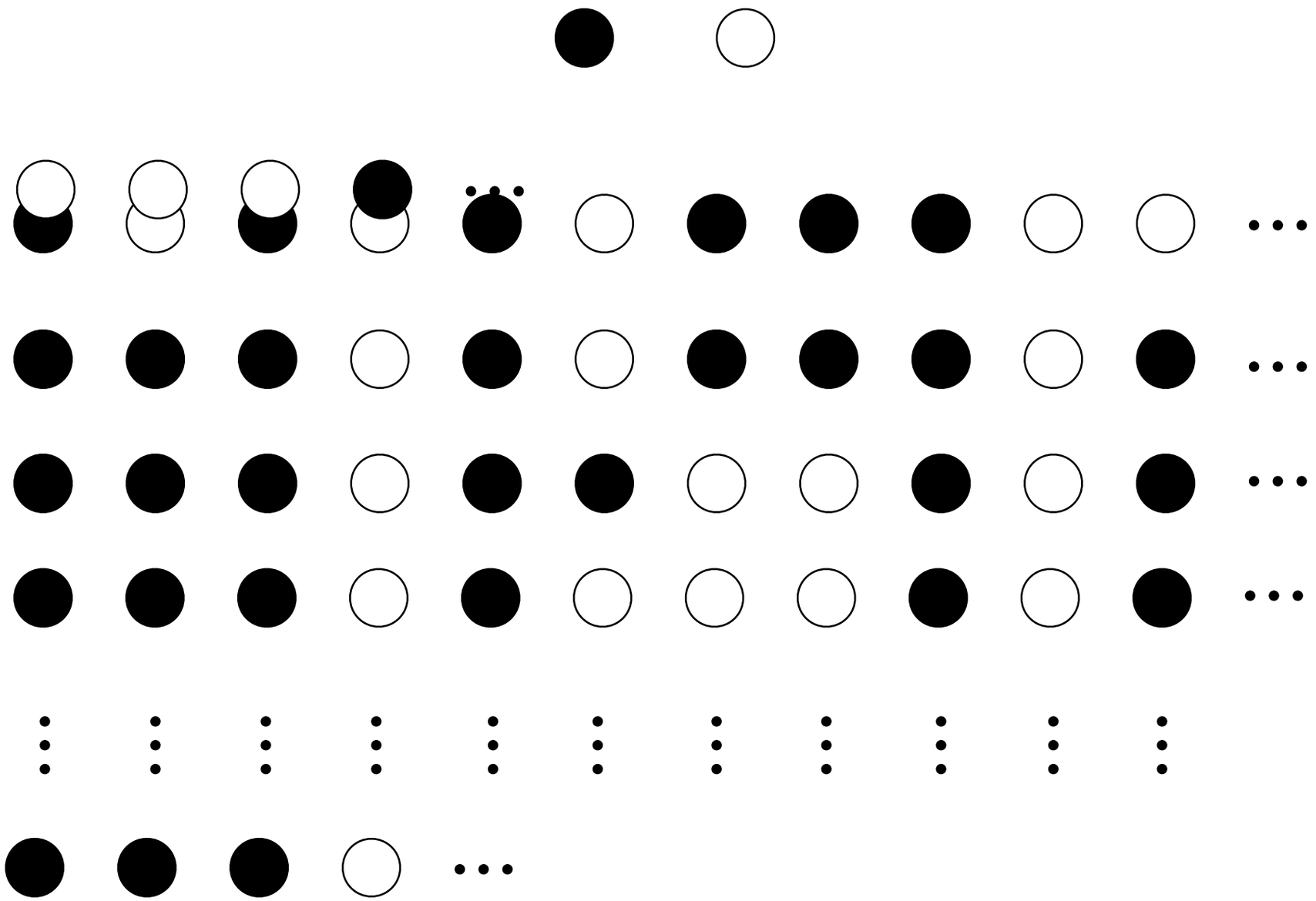

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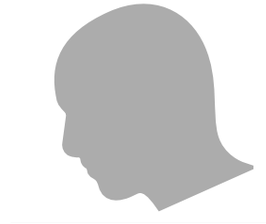

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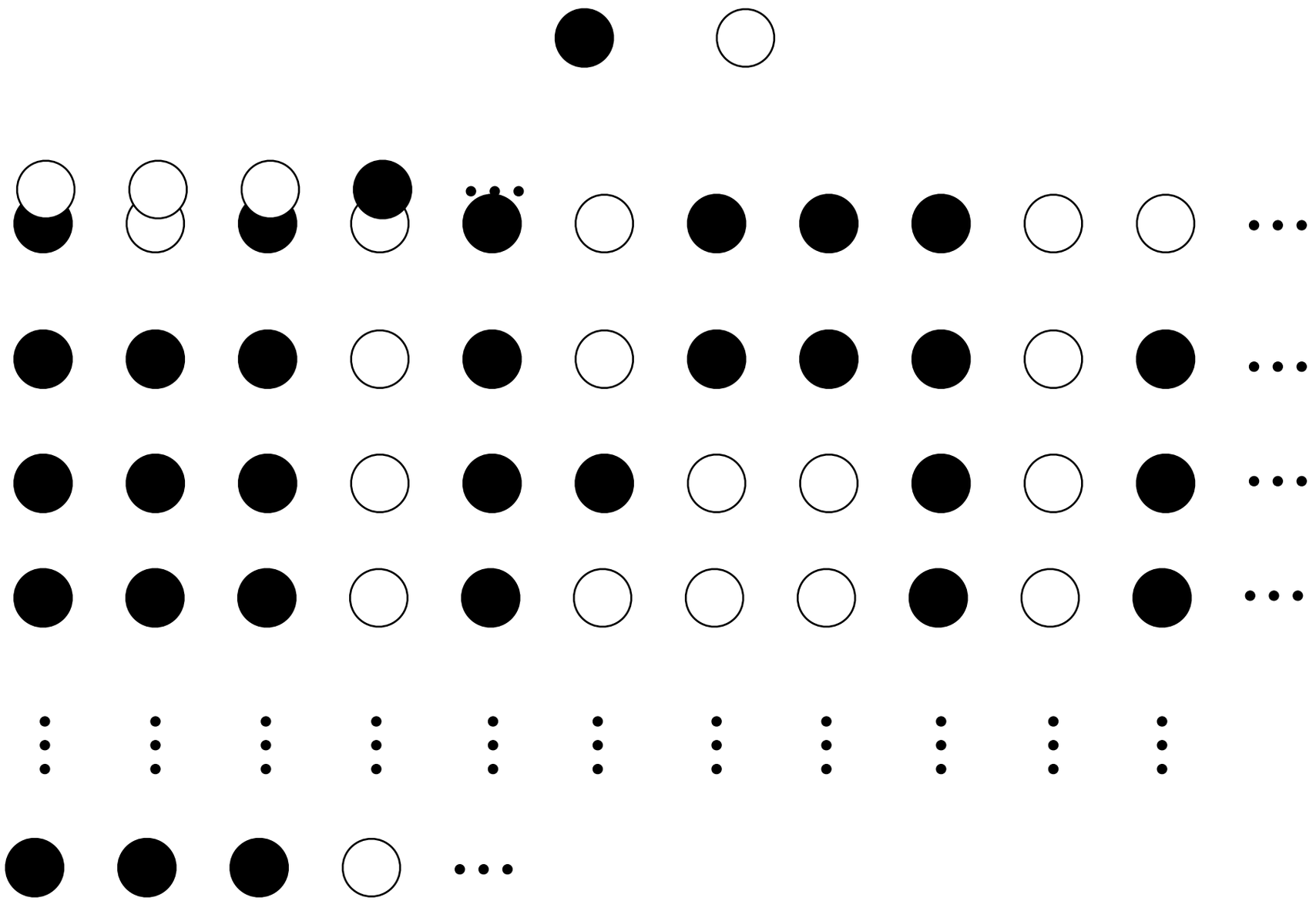

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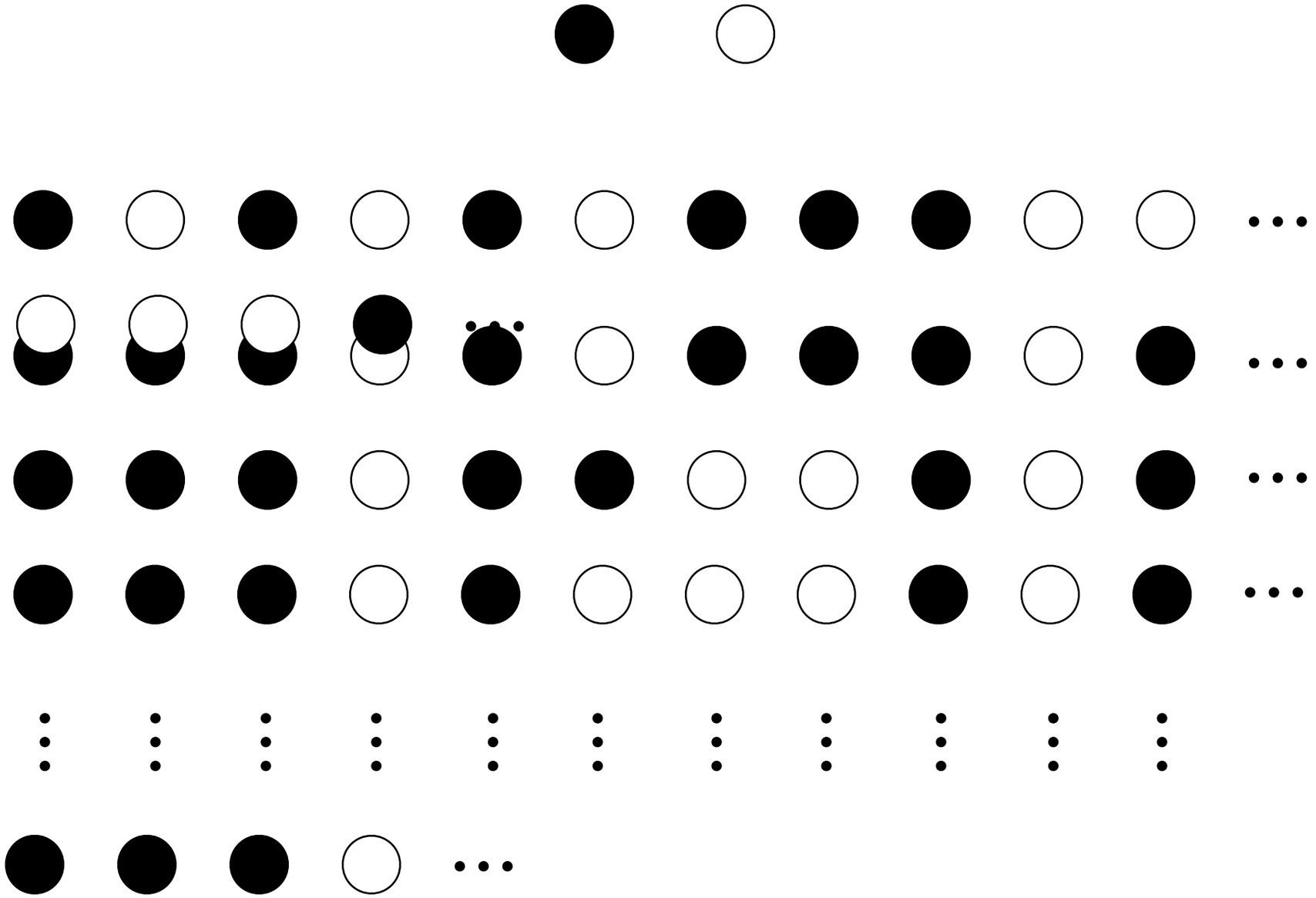

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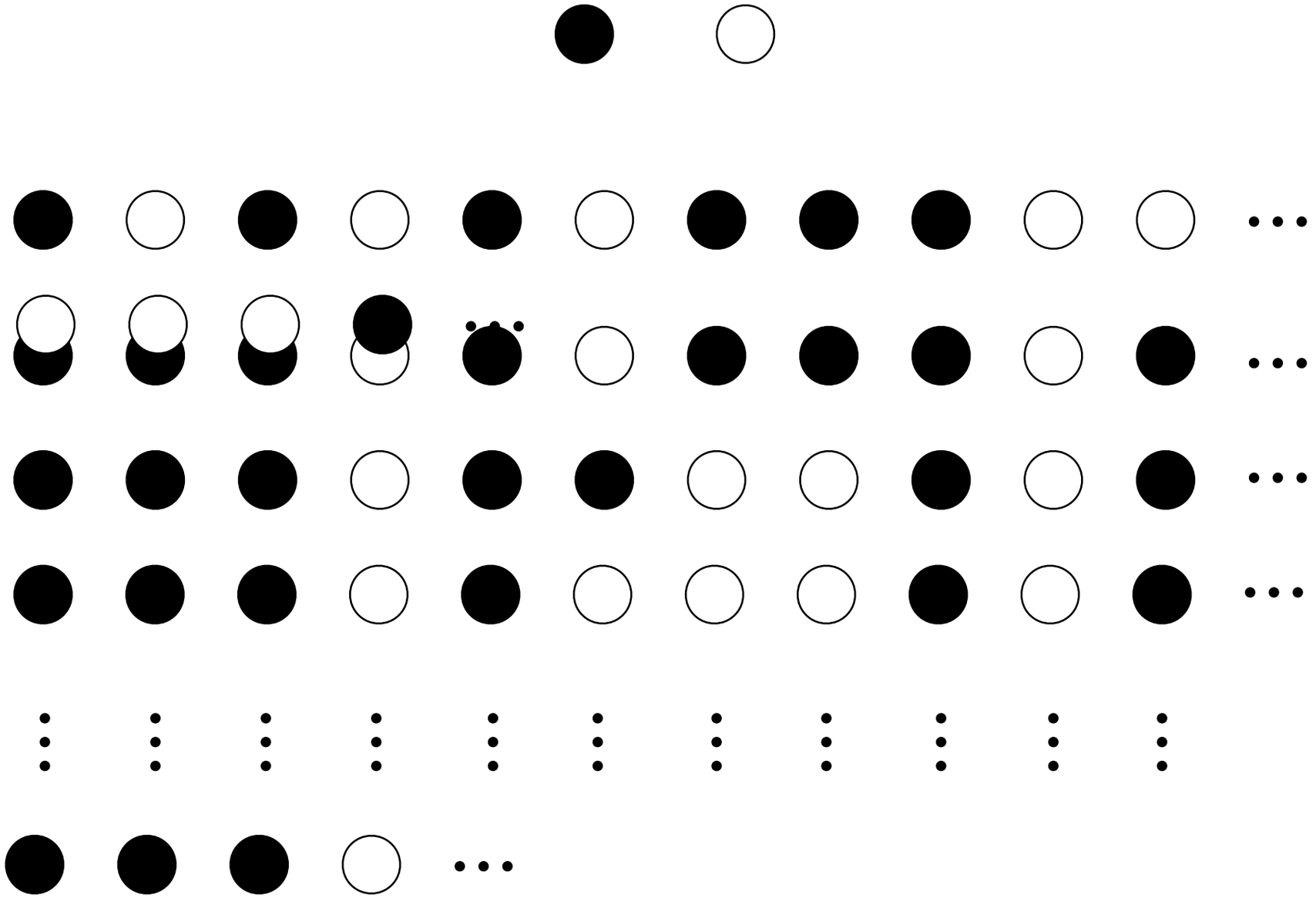
Ideal Observer



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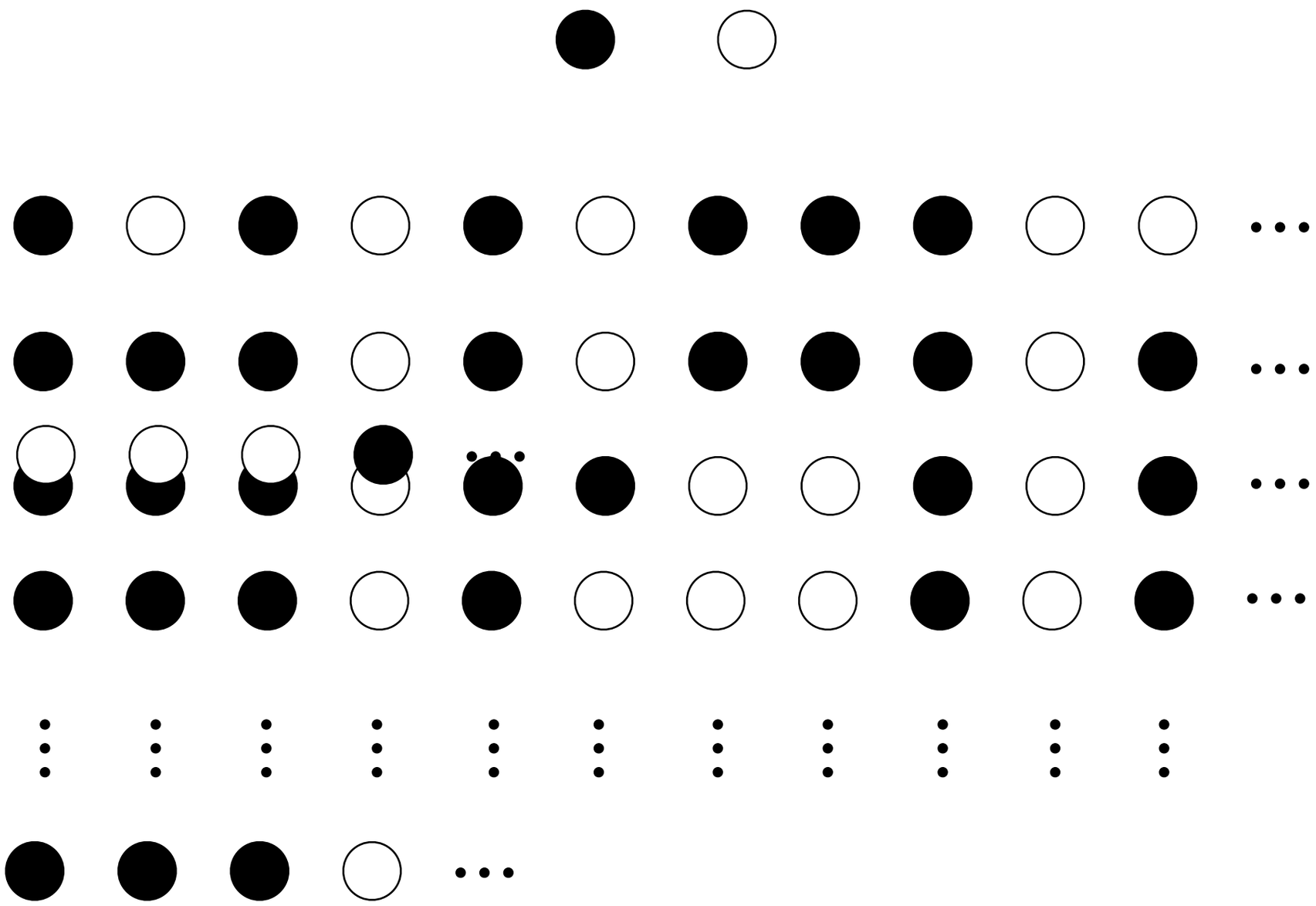

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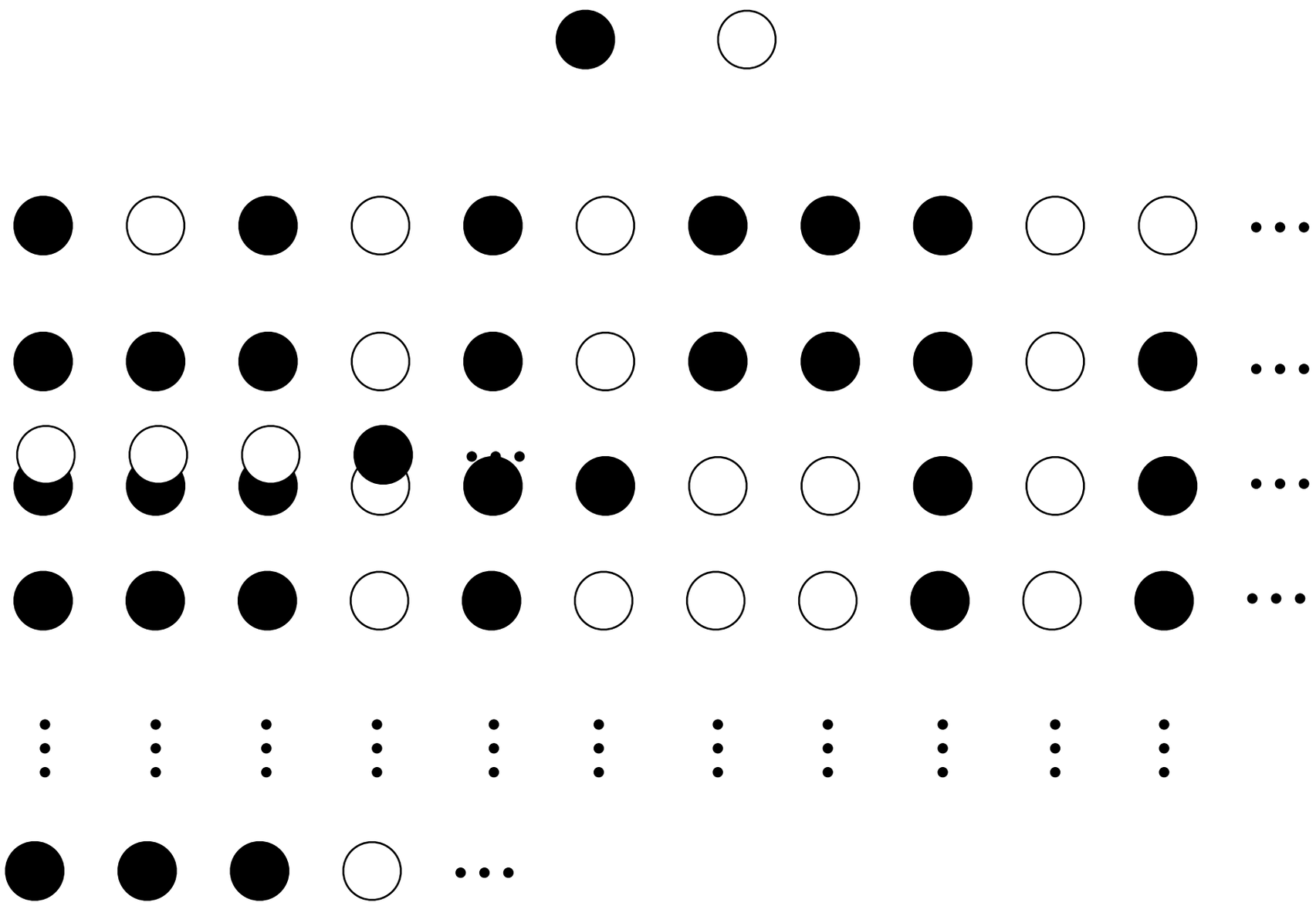

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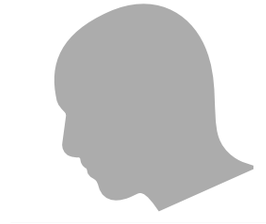

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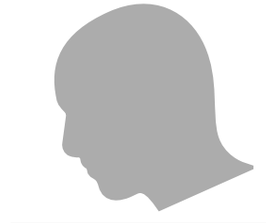

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Animation-Based Proof of Cantor's $\mathscr{P}(\mathbb{Z}^+) > \mathbb{Z}^+$

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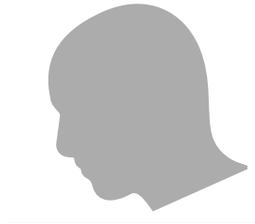

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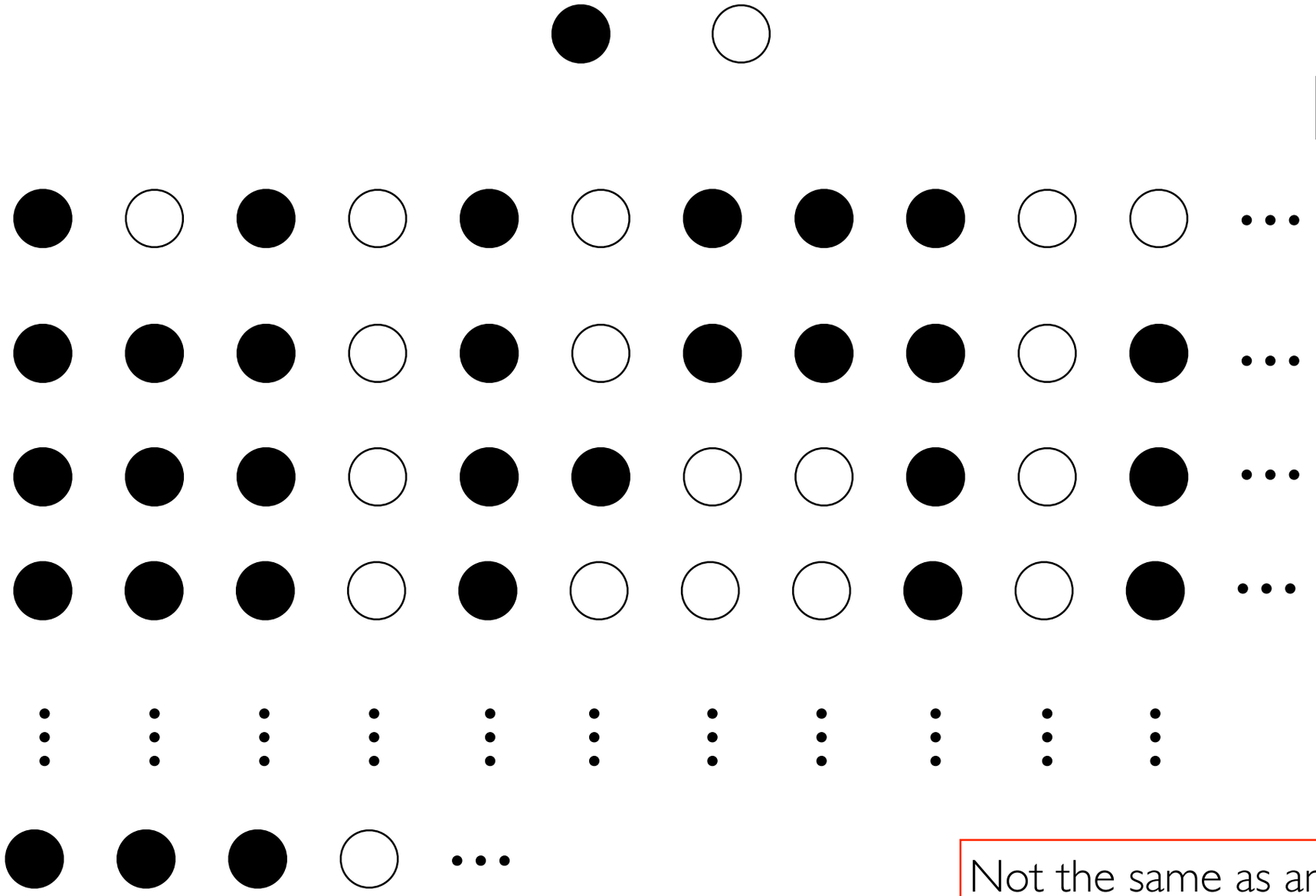
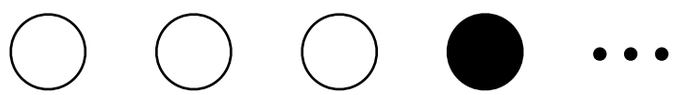
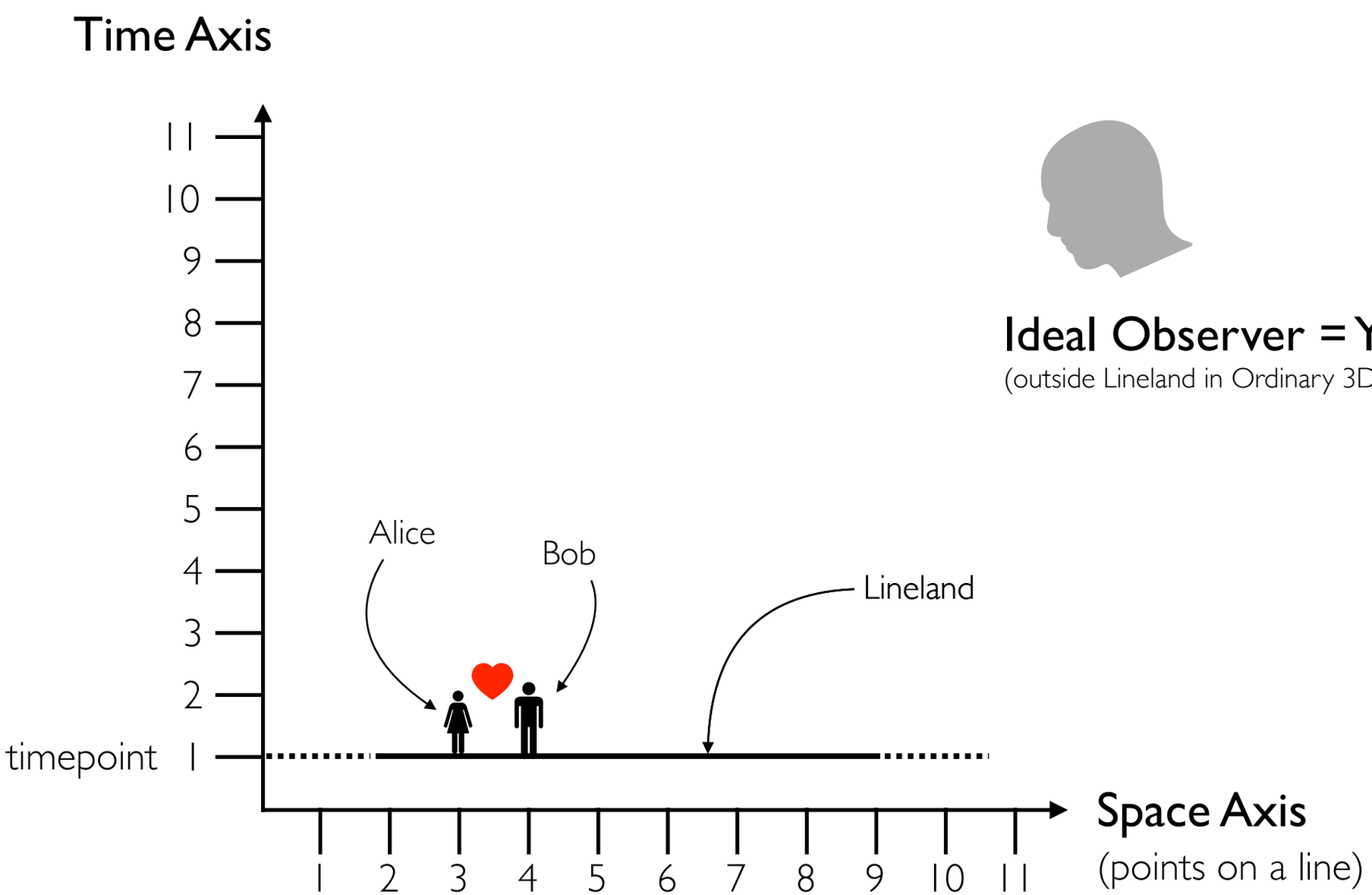


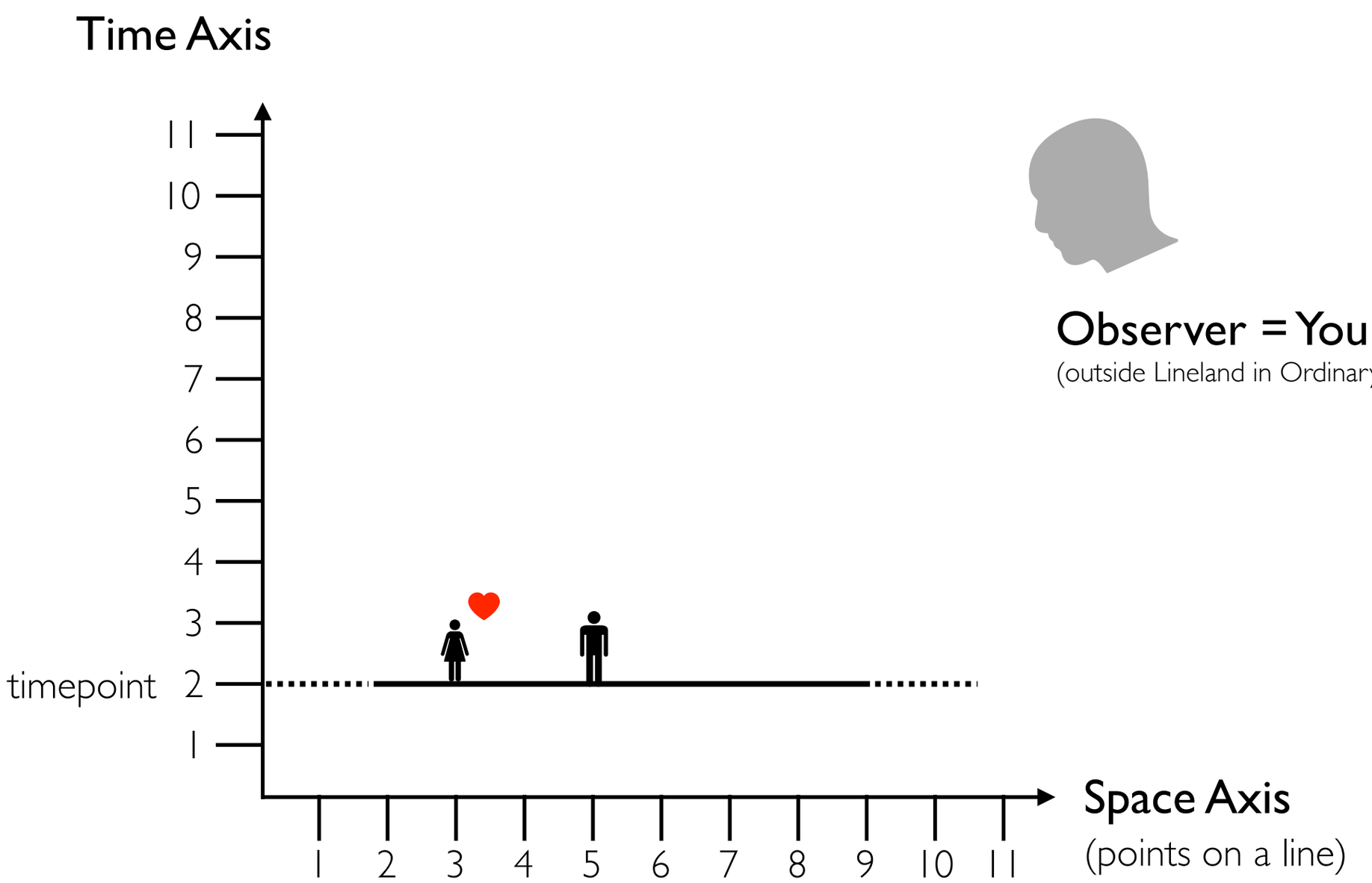


Not the same as any row! Ergo, the set of all subsets of $\mathbb{Z}^+$, i.e. $\mathscr{P}(\mathbb{Z}^+)$, is greater than $\mathbb{Z}^+$ itself!

TimeTravel



Ideal Observer = You
(outside Lineland in Ordinary 3D)



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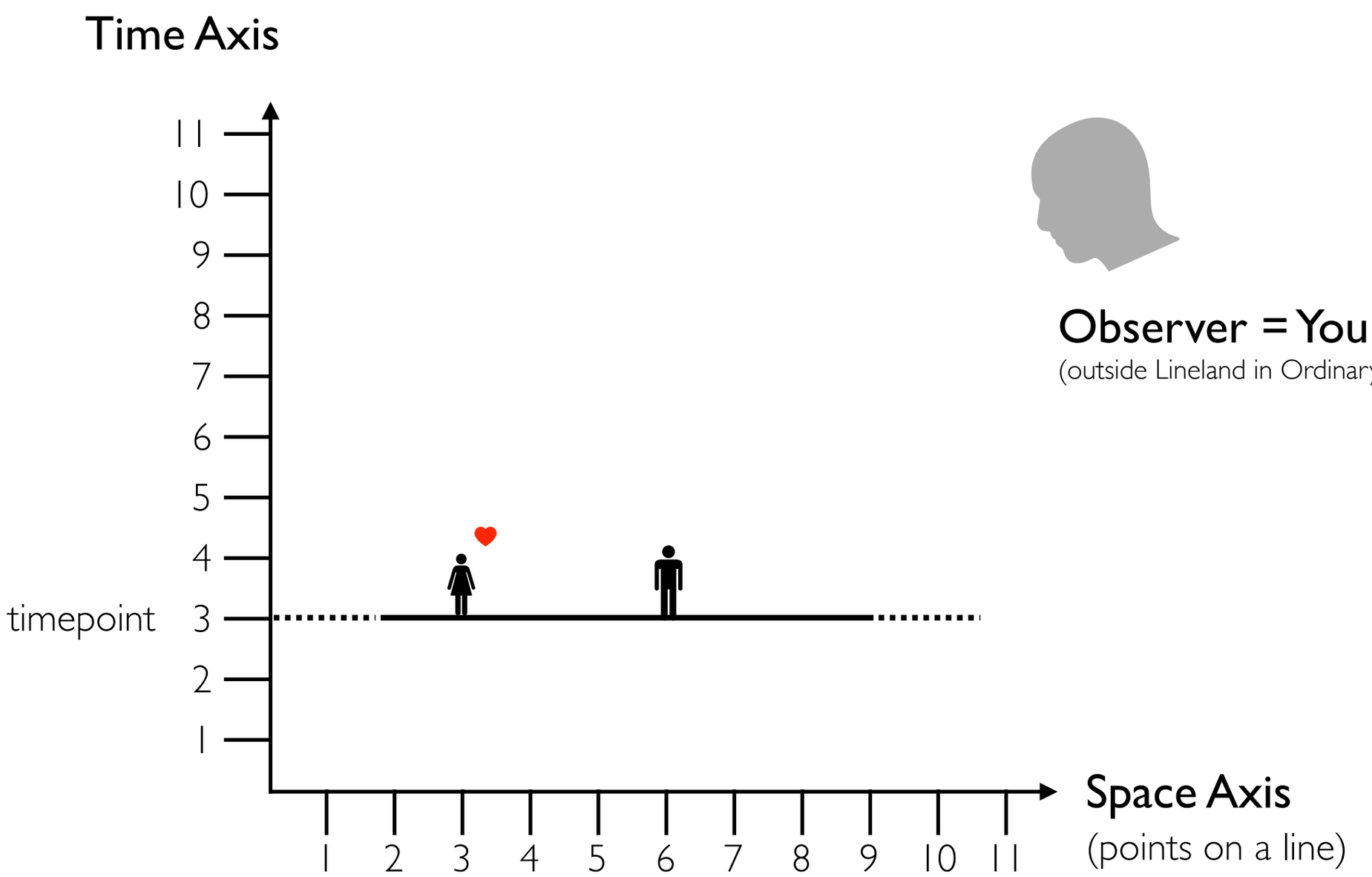

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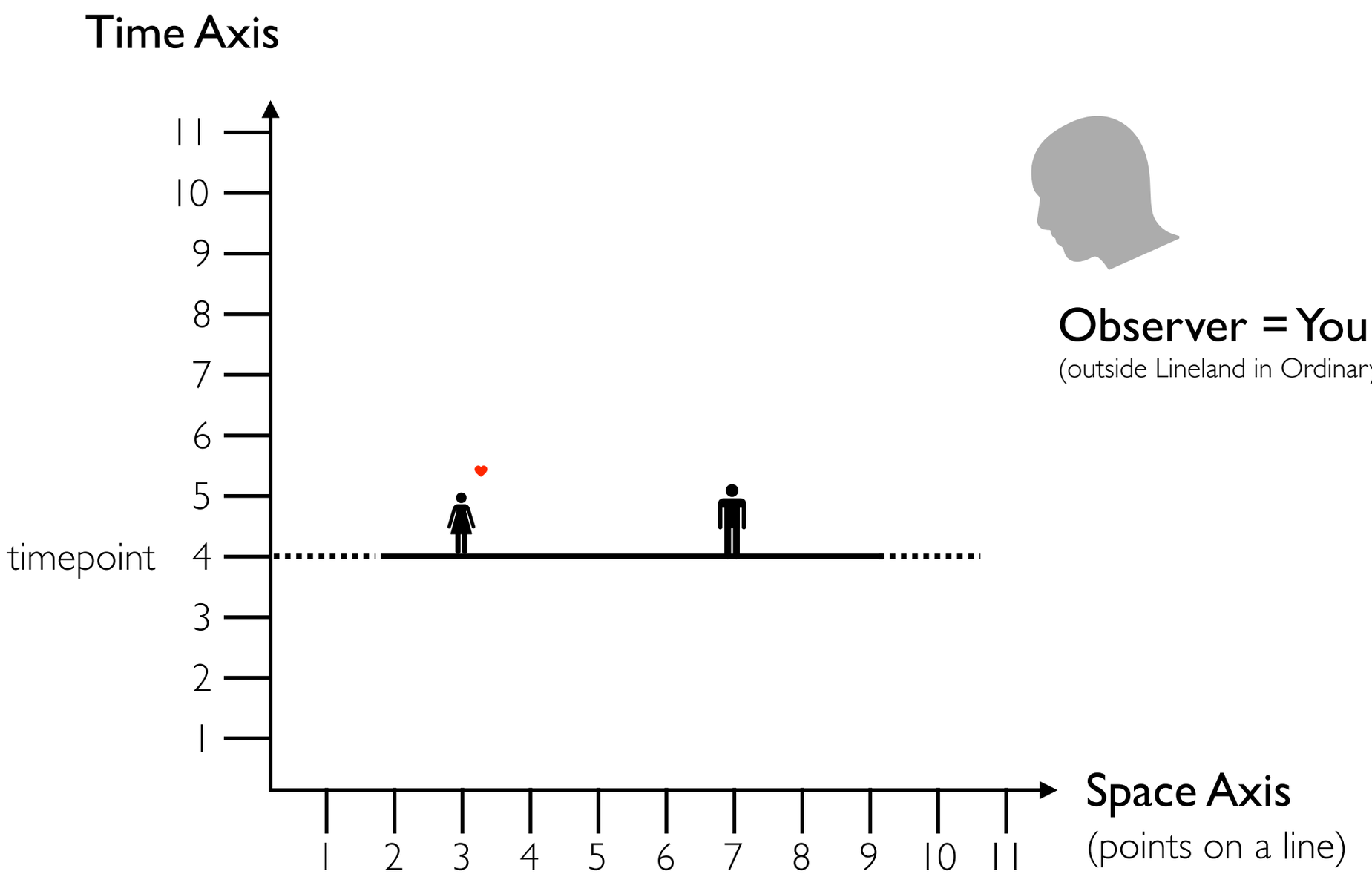

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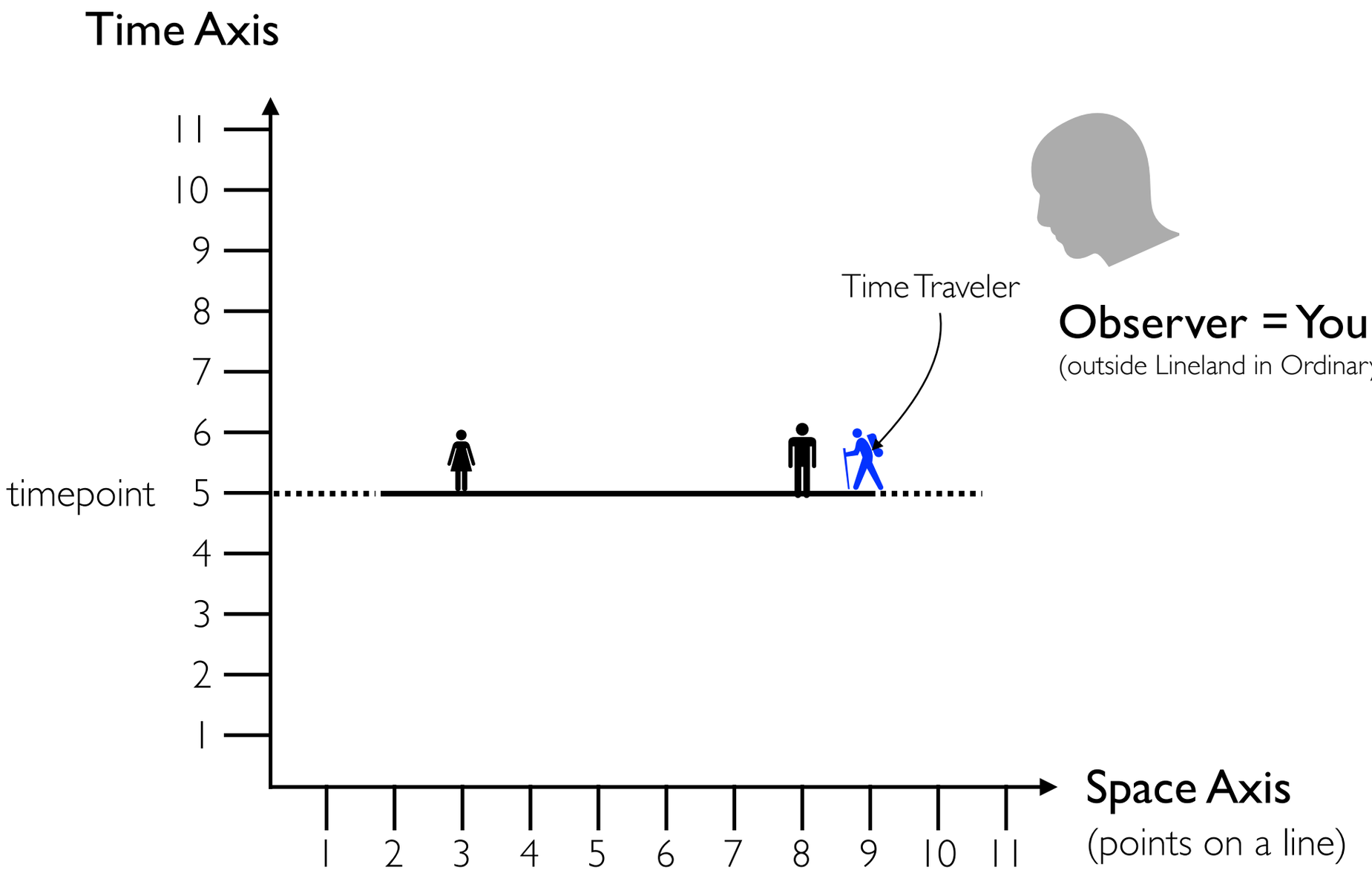
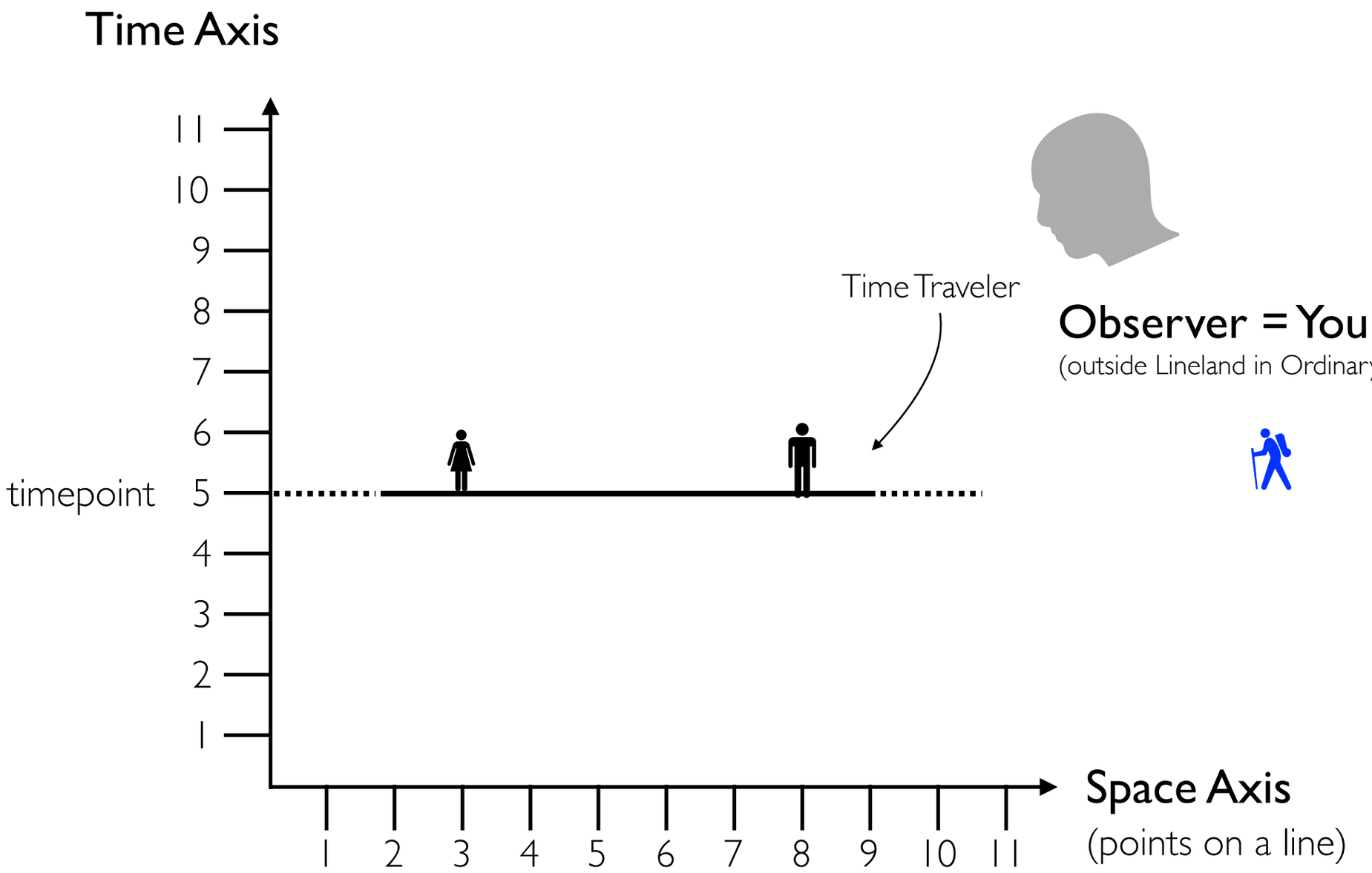

Time Axis
11
10
9
8
7
6
timepoint 5
4
3
2
1
Time Traveler
Observer = You
(outside Lineland in Ordinary 3D)
1 2 3 4 5 6 7 8 9 10 11
Space Axis
(points on a line)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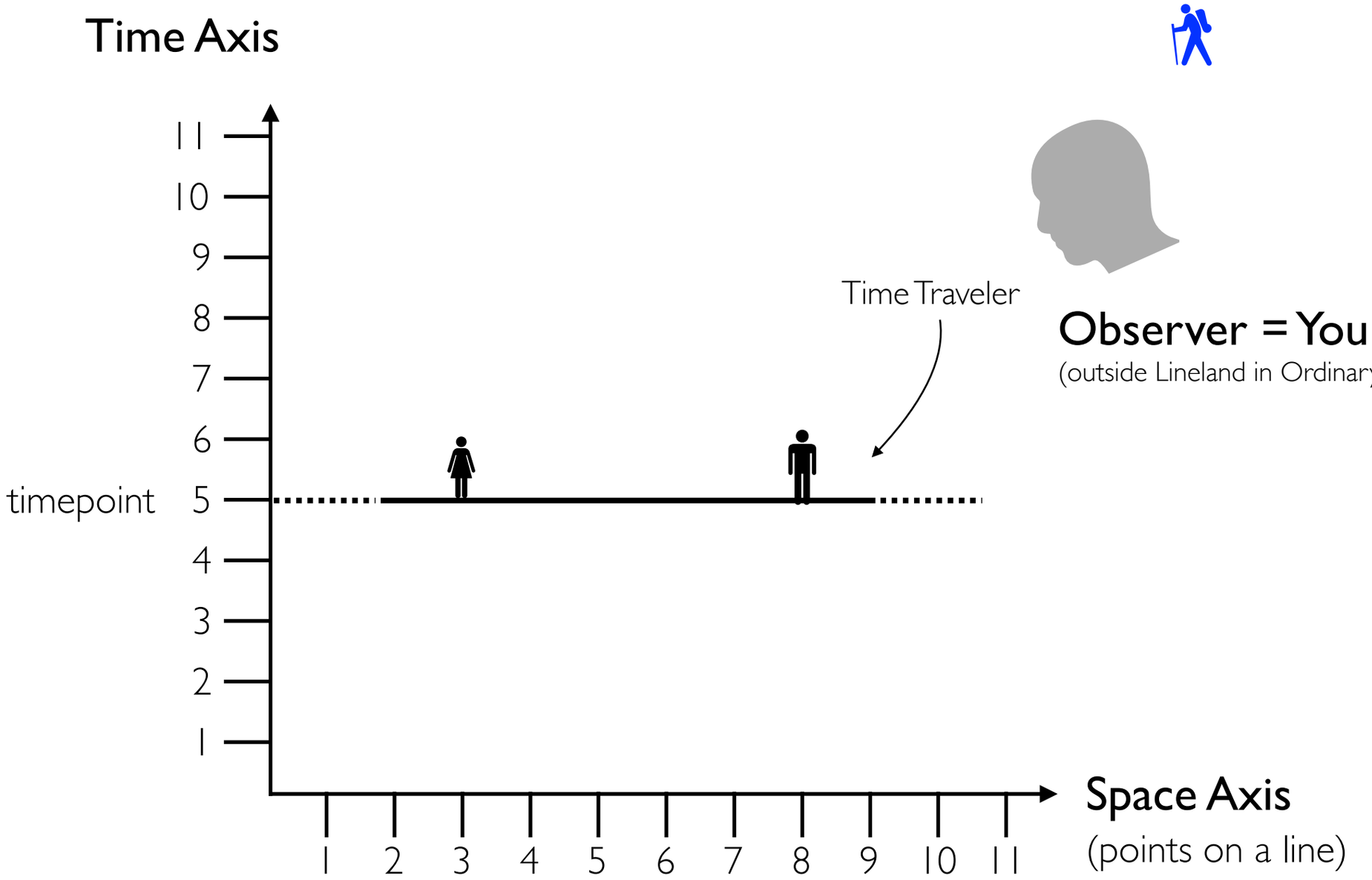
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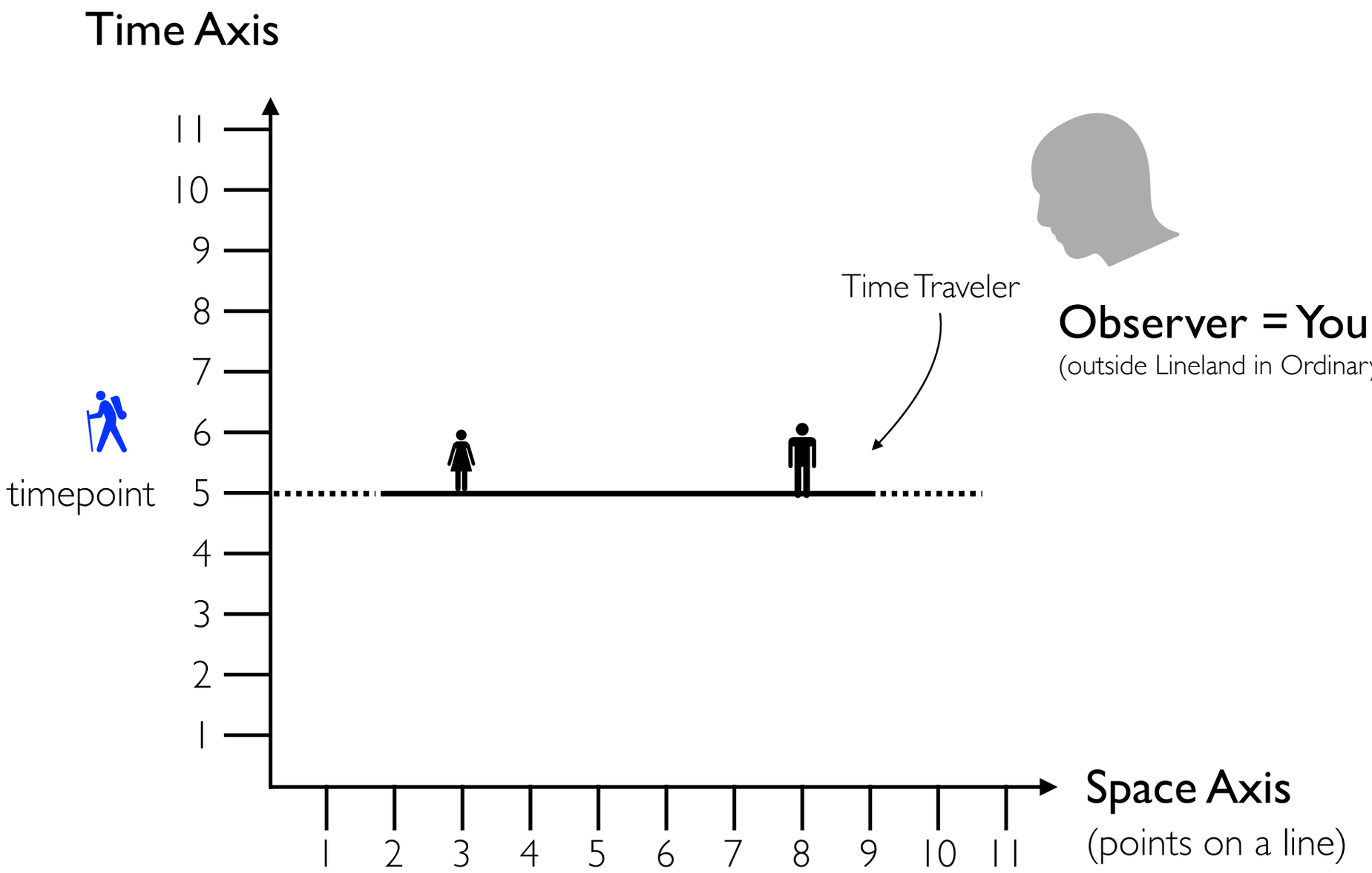
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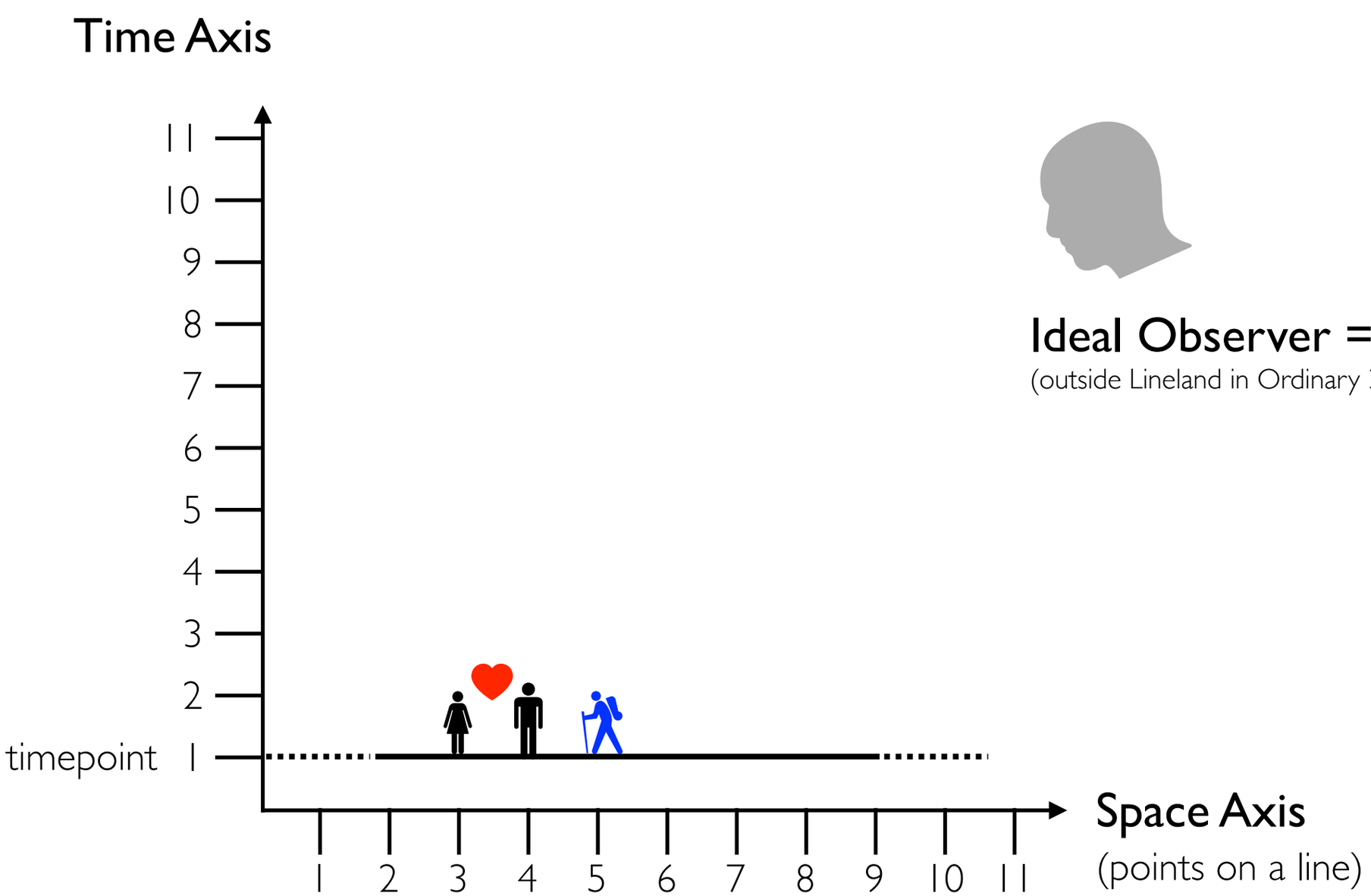

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(outside Lineland in Ordinary 3D)

Med nok penger er Strong Weak AI mulig.