

# On to *Intensional* Logics

**Selmer Bringsjord**

Rensselaer AI & Reasoning (RAIR) Lab  
Department of Cognitive Science  
Department of Computer Science  
Lally School of Management & Technology  
**RPI** (Rensselaer Polytechnic Institute)  
Troy, New York 12180 USA

IFLAI  
3/26/2026



**In The Logic-and-AI News**

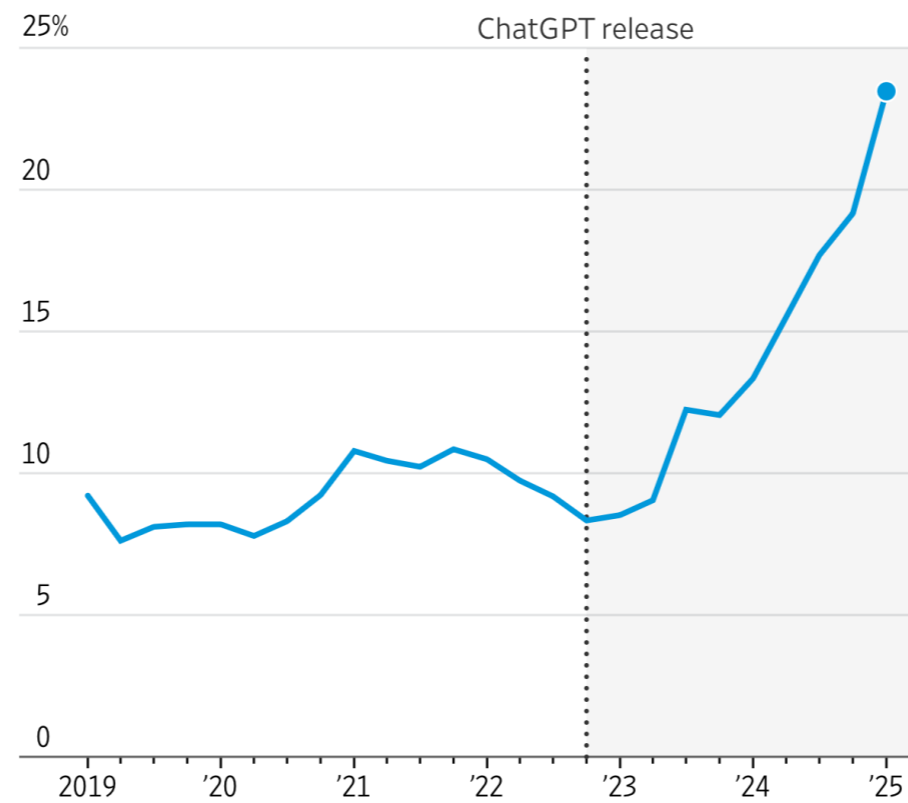
...

TECH

# How the AI Talent Race Is Reshaping the Tech Job Market

In industries from finance to retail, companies are seeking artificial-intelligence skills when looking for technology staff

Share of newly listed technology\* jobs that are AI-related, quarterly



\*Technology jobs are a broad group of computer- and math-related occupations

Note: U.S. only. 1Q 2025 figures are as of January.

Source: UMD-LinkUp AI Maps

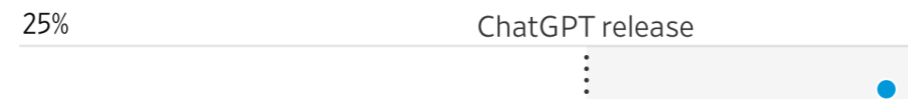
By Nate Rattner

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Consider therefore taking ILBAI & I(ML)AI!



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On the esemplastic  
extensional-logic ladder ...  
questions?

# Climbing the $k$ -order Ladder

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$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

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⋮

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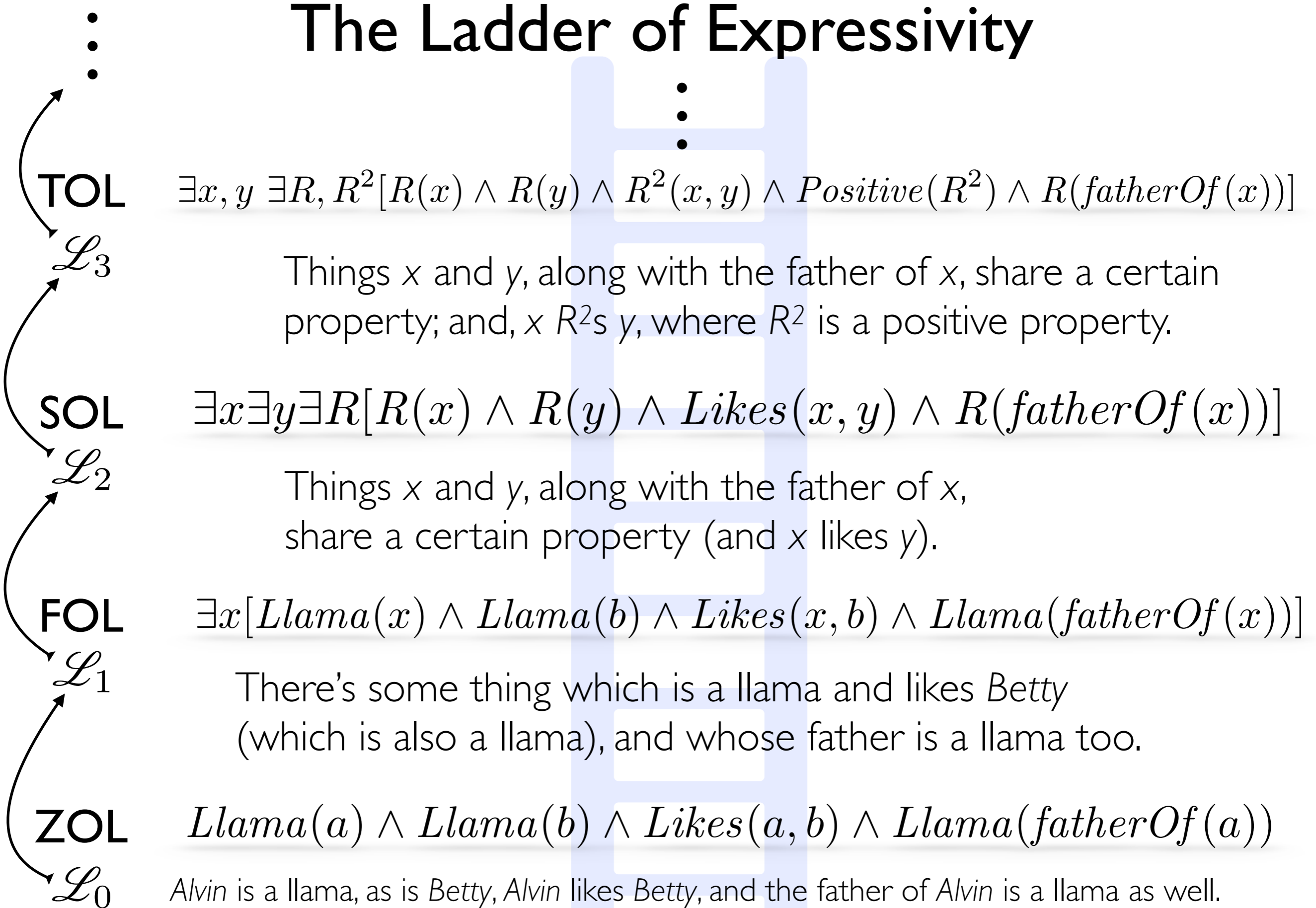
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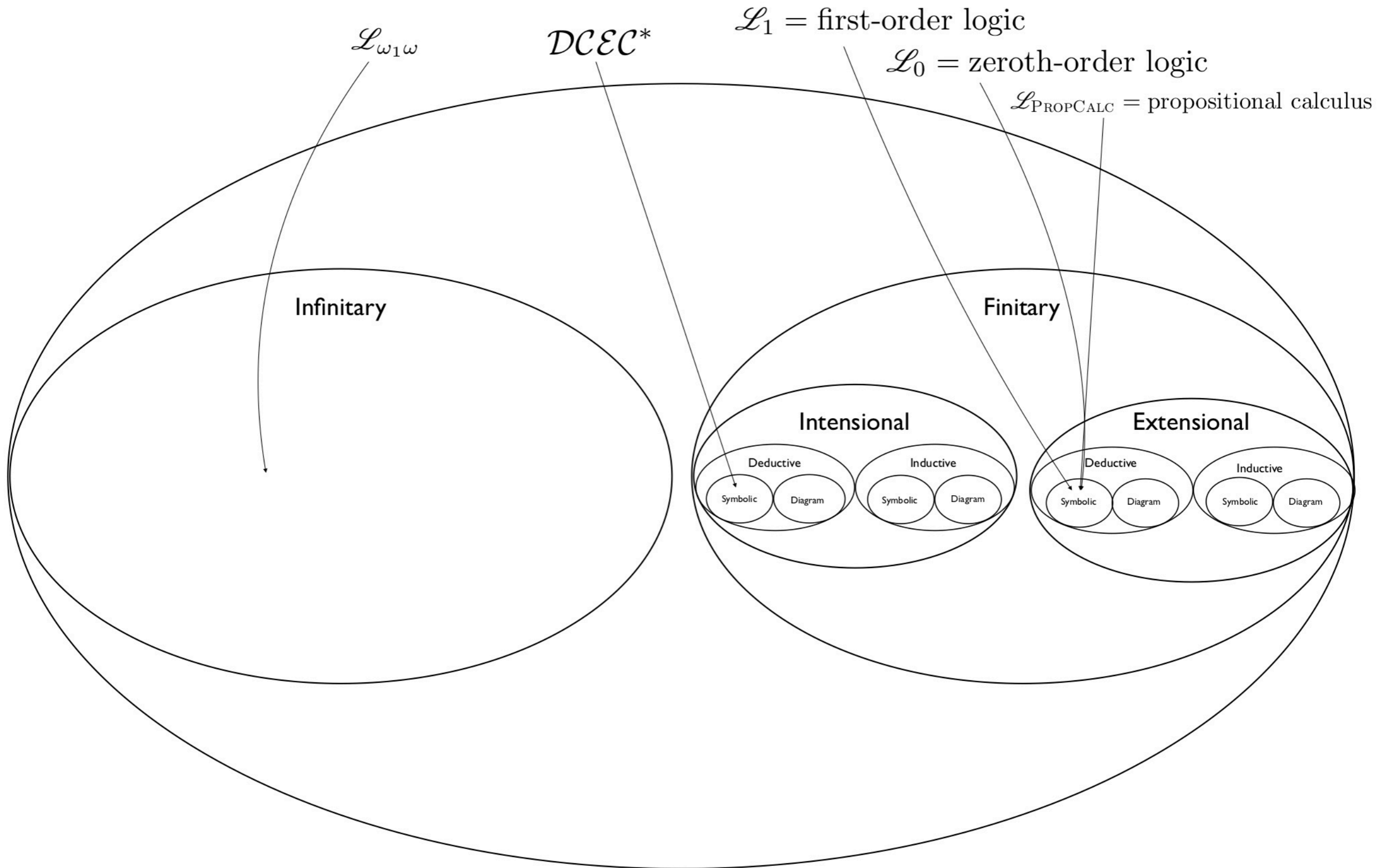
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# The Ladder of Expressivity

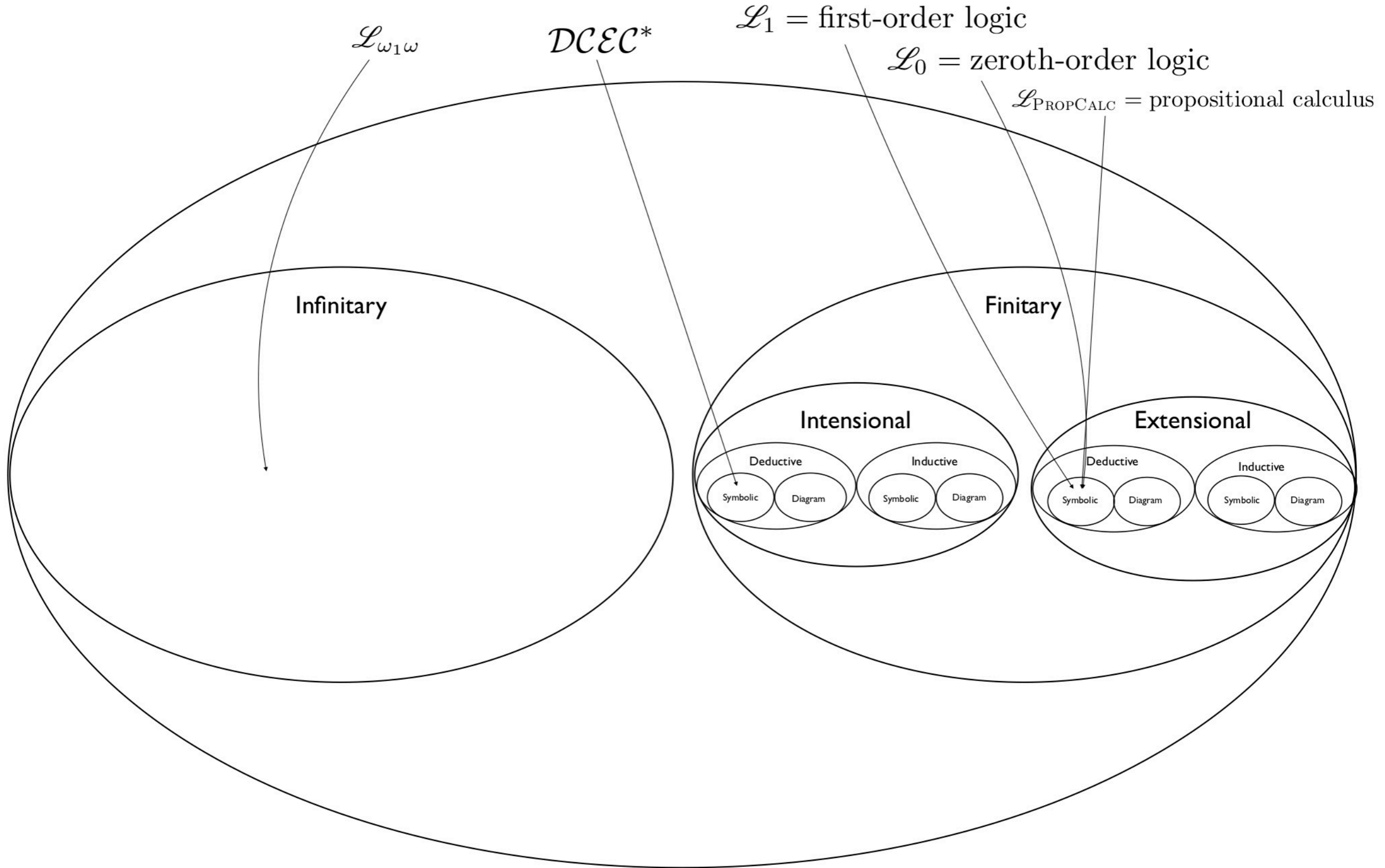


# The Universe of Logics



$\mathcal{L}_3$   
 $\mathcal{L}_2$

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$\mathcal{L}_3$

$\mathcal{L}_2$

$\mathcal{L}_{\omega_1\omega}$

$\mathcal{DCEC}^*$

$\mathcal{L}_1 =$  first-order logic

$\mathcal{L}_0 =$  zeroth-order logic

$\mathcal{L}_{\text{PROP-CALC}} =$  propositional calculus

Infinitary

Finitary

Intensional

Extensional

Deductive

Inductive

Deductive

Inductive

Symbolic

Diagram

Symbolic

Diagram

Symbolic

Diagram

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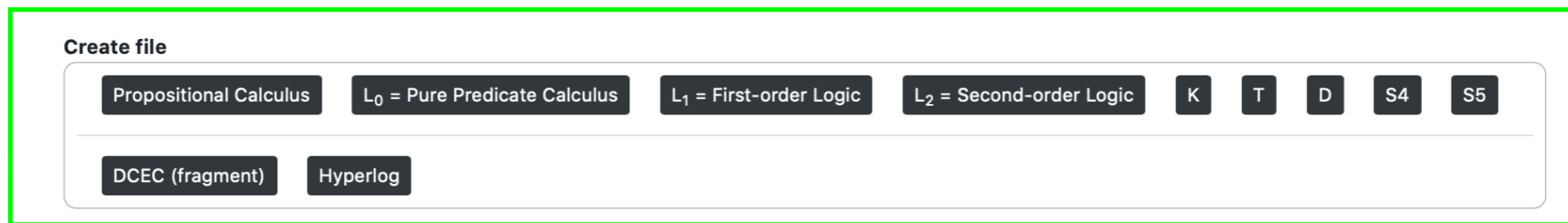
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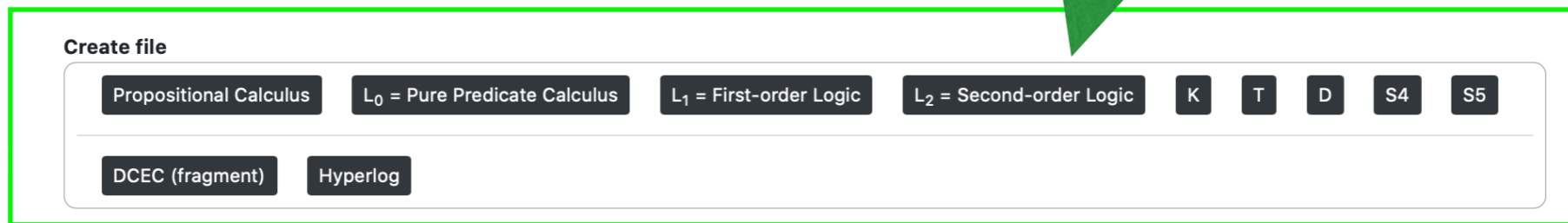
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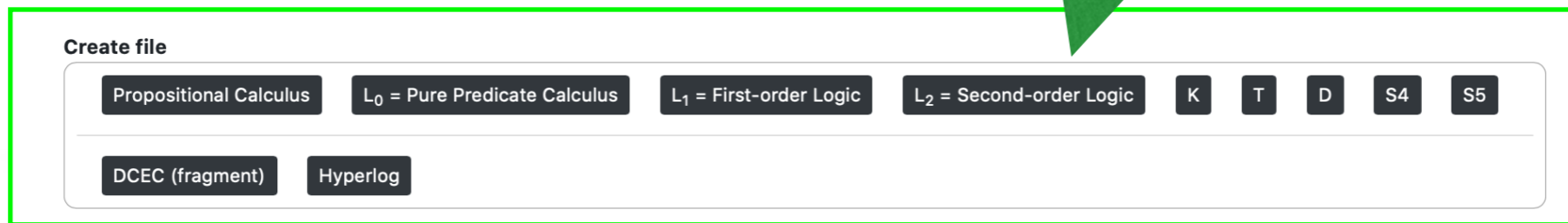
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**SOL**

$\mathcal{L}_2$

Create file

Propositional Calculus

$L_0$  = Pure Predicate Calculus

$L_1$  = First-order Logic

$L_2$  = Second-order Logic

K

T

D

S4

S5

DCEC (fragment)

Hyperlog

$\text{fatherOf}(x)]$

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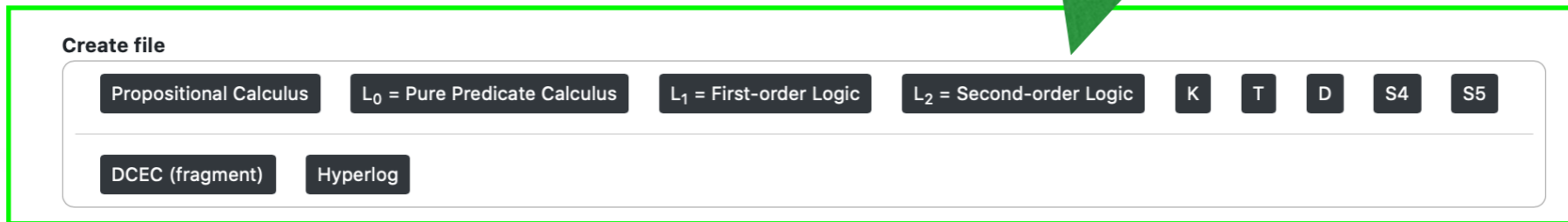
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⋮

Incomplete!

TOL

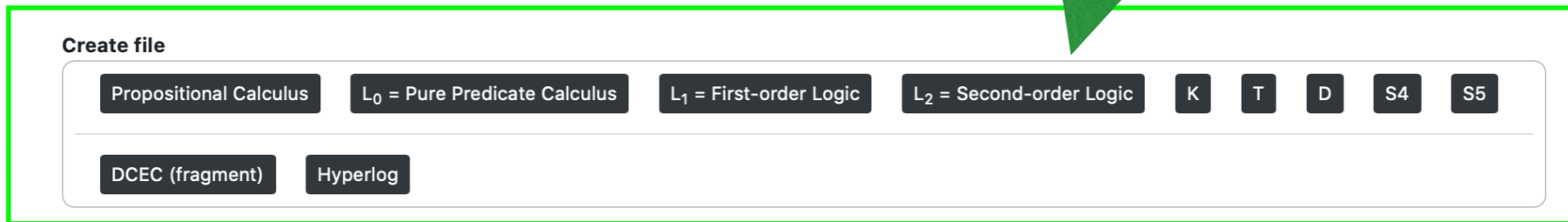
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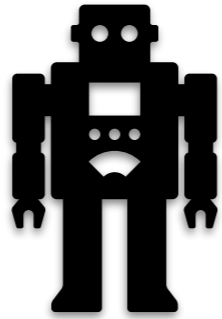
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**Blinky as portal to  
intensional logics ...**

Blinky



1

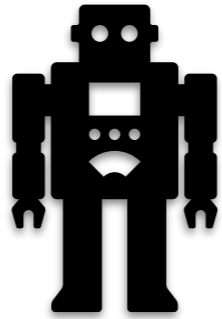


2



3

Blinky



1



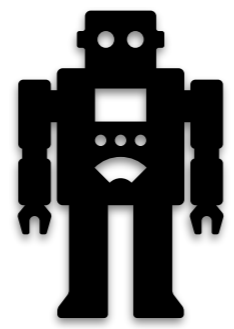
2



3



Blinky



1

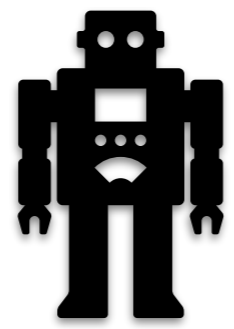


2



3

Blinky



1

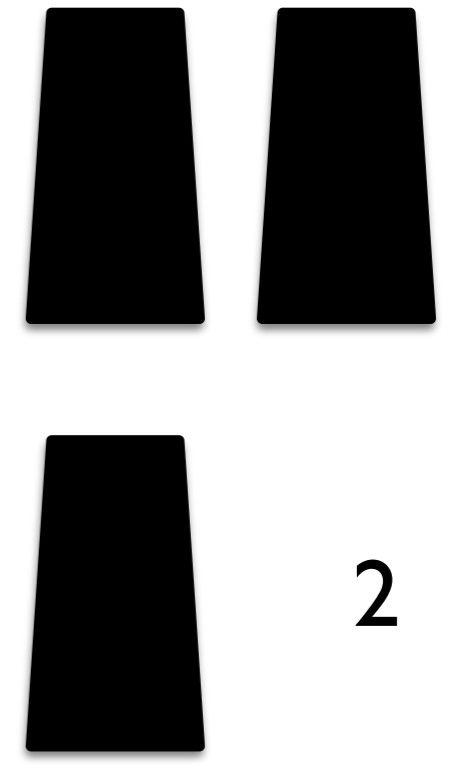
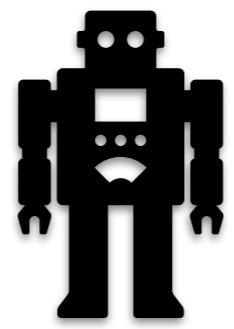


2



3

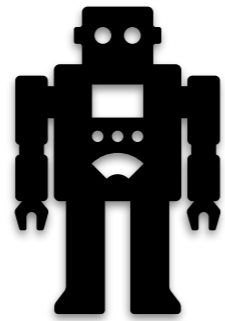
Blinky



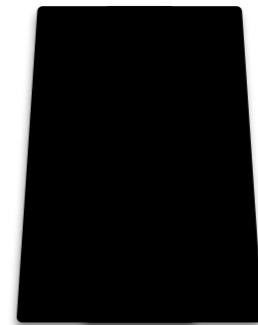
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3

Blinky



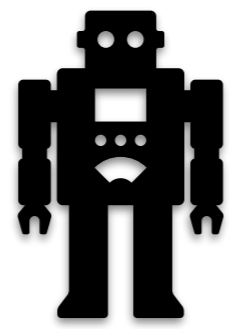
1



2

3

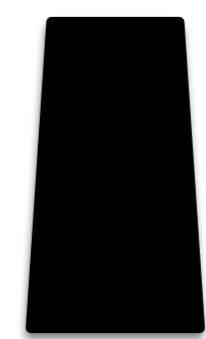
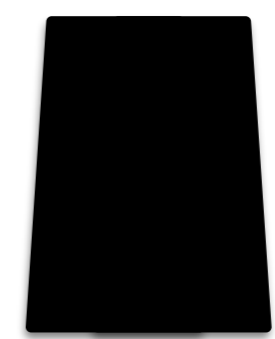
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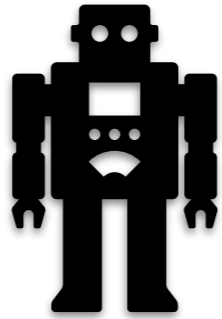
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2

3



Blinky



1

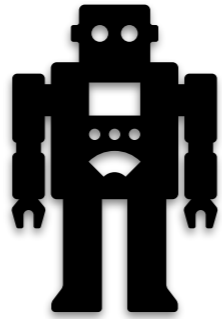


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3

Blinky



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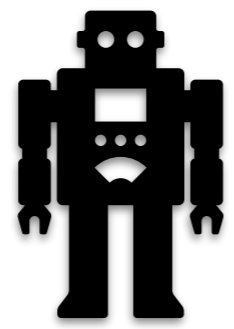


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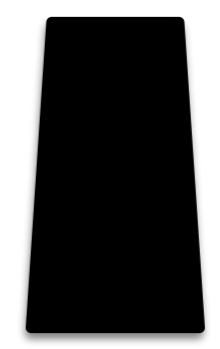
Blinky



1



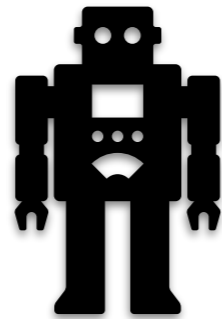
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3

Selmers believes Alex believes that the ball is in the cup at location #1.

Blinky



1



2

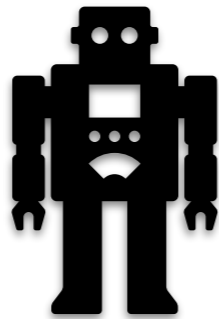


3

Selmers believes Alex believes that the ball is in the cup at location #1.

**B**(blinky, loc-ball-1)

Blinky



1



2



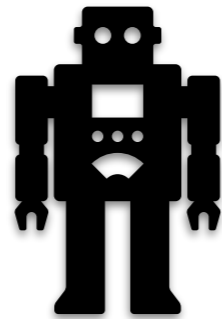
3

Selmers believes Alex believes that the ball is in the cup at location #1.

**B**(blinky, loc-ball-1)

(Believes! blinky loc-ball-1)

Blinky



1



2



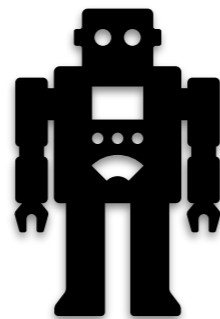
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Selmers believes Alex believes that the ball is in the cup at location #1.

**B**(blinky, loc-ball-1)

(Believes! blinky loc-ball-1)

Blinky



1



2



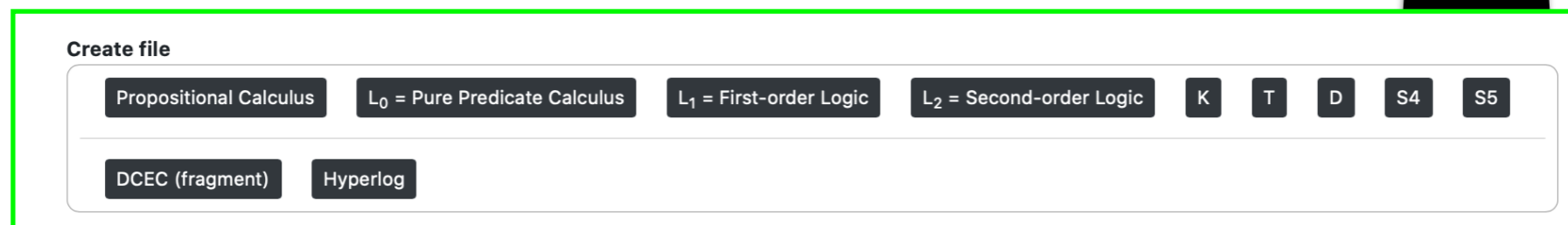
3

In intensional logics, meaning and designation are separated, and compositionality is abandoned.

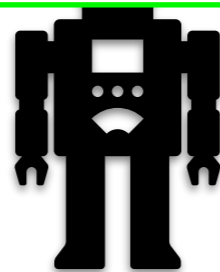
Selmers believes Alex believes that the ball is in the cup at location #1.

**B**(blinky, loc-ball-1)

(Believes! blinky loc-ball-1)



Blinky



1



2



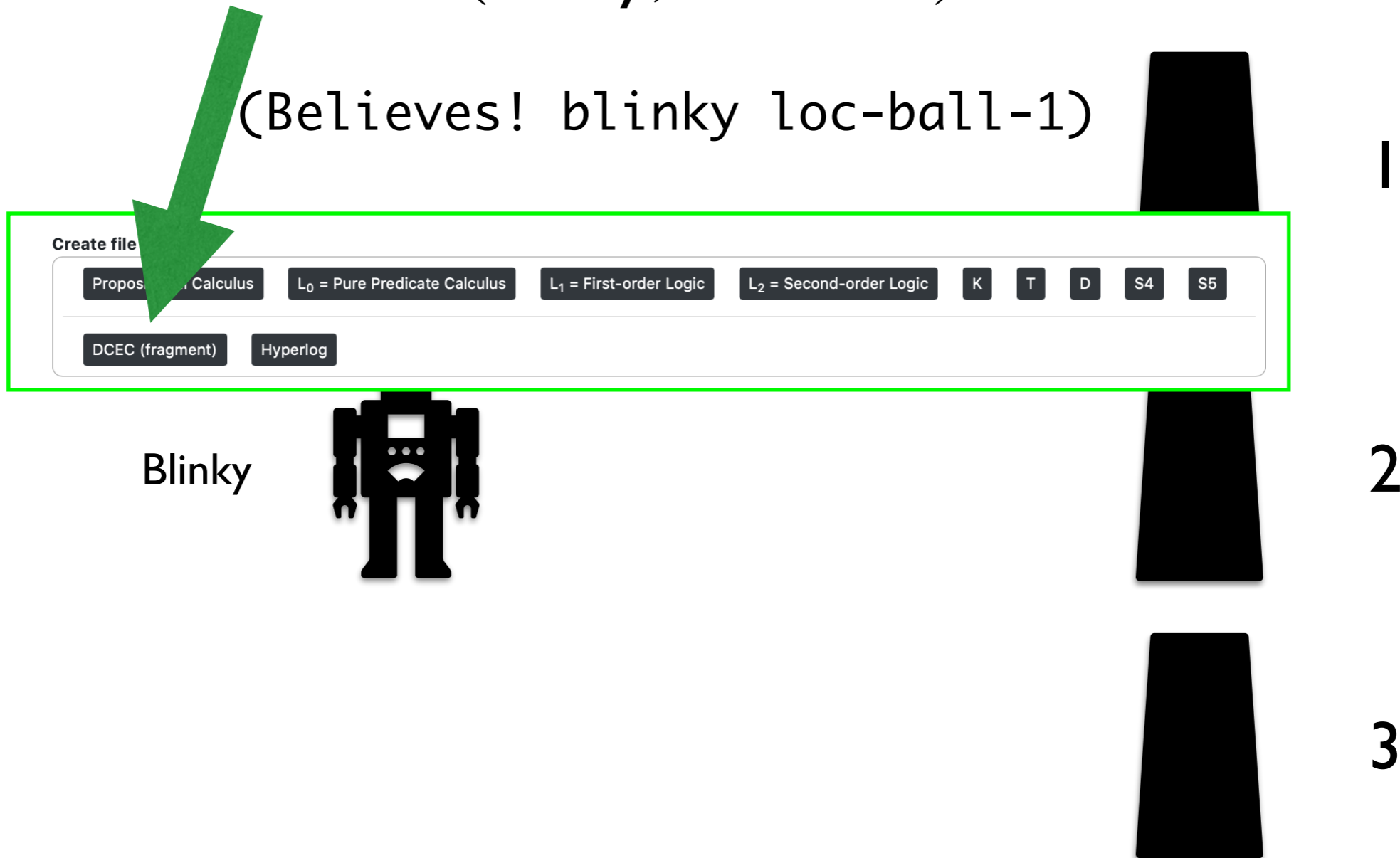
3

In intensional logics, meaning and designation are separated, and compositionality is abandoned.

Selmers believes Alex believes that the ball is in the cup at location #1.

**B**(blinky, loc-ball-1)

(Believes! blinky loc-ball-1)



In intensional logics, meaning and designation are separated, and compositionality is abandoned.

Blinky believes that the ball is in the cup at location #1.

Selmers believes Alex believes that the ball is in the cup at location #1.

**B**(blinky, loc-ball-1)

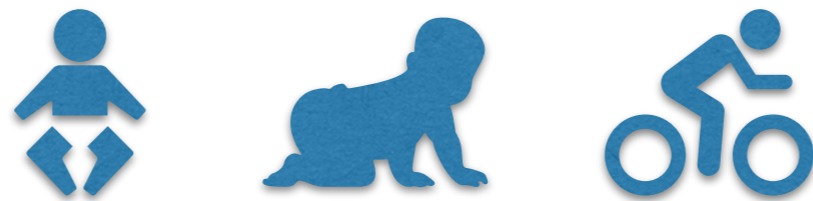
(Believes! blinky loc-ball-1)



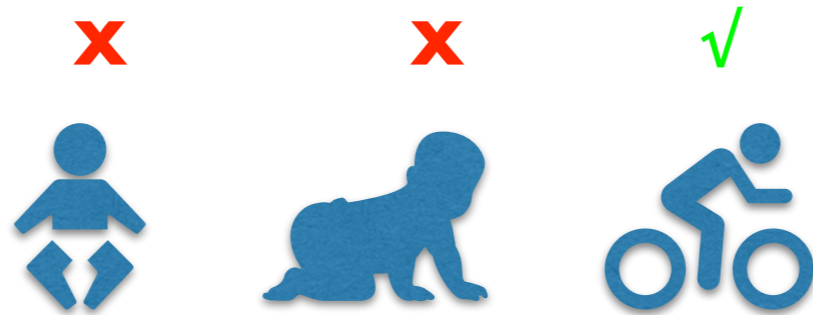
In intensional logics, meaning and designation are separated, and compositionality is abandoned.

**False Belief Task Demands  
Intensional Logic ...**

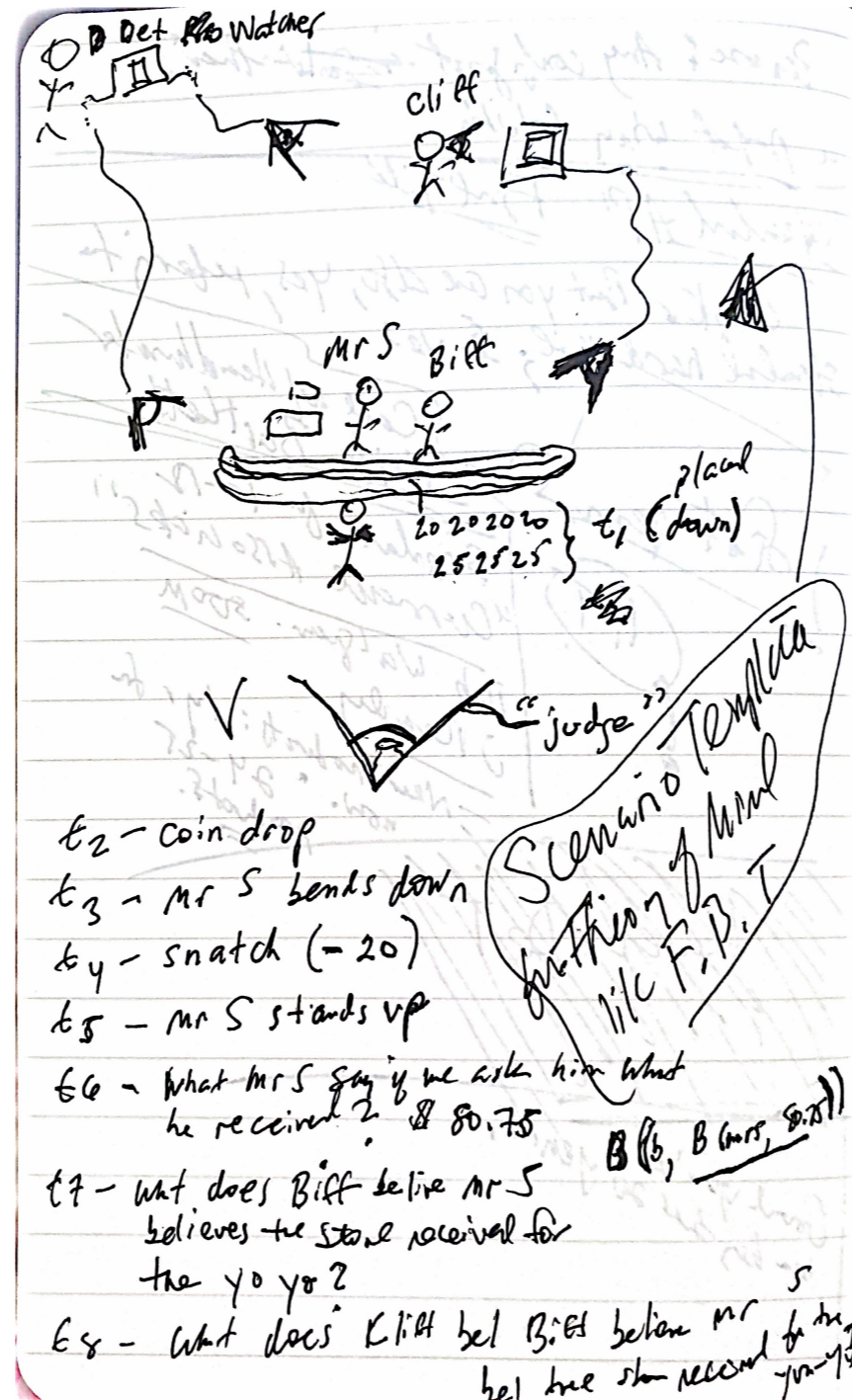
# False Belief Task Demands Intensional Logic ...



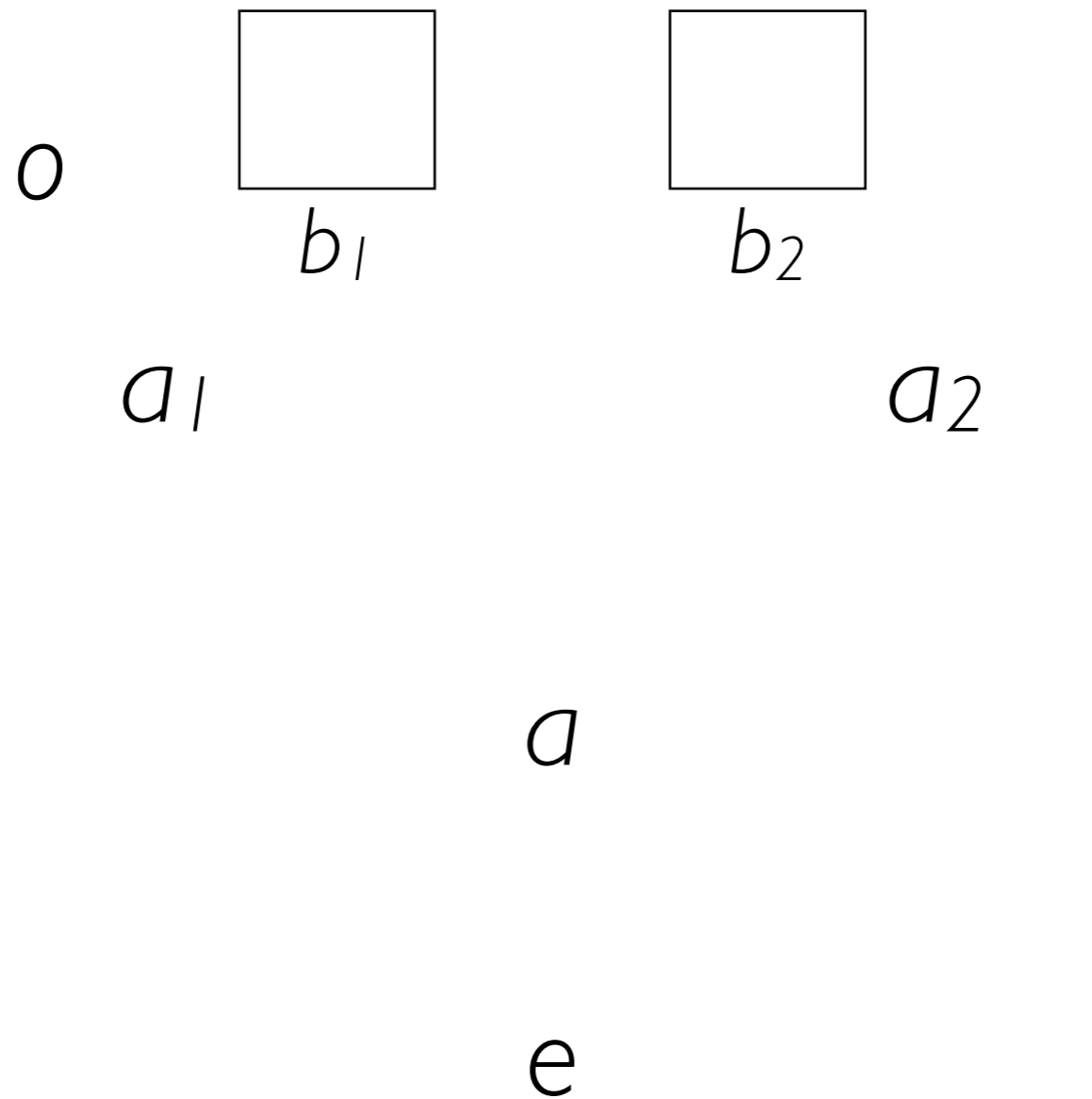
# False Belief Task Demands Intensional Logic ...



# Better, But Embryonic: The ToM Pawn Shop

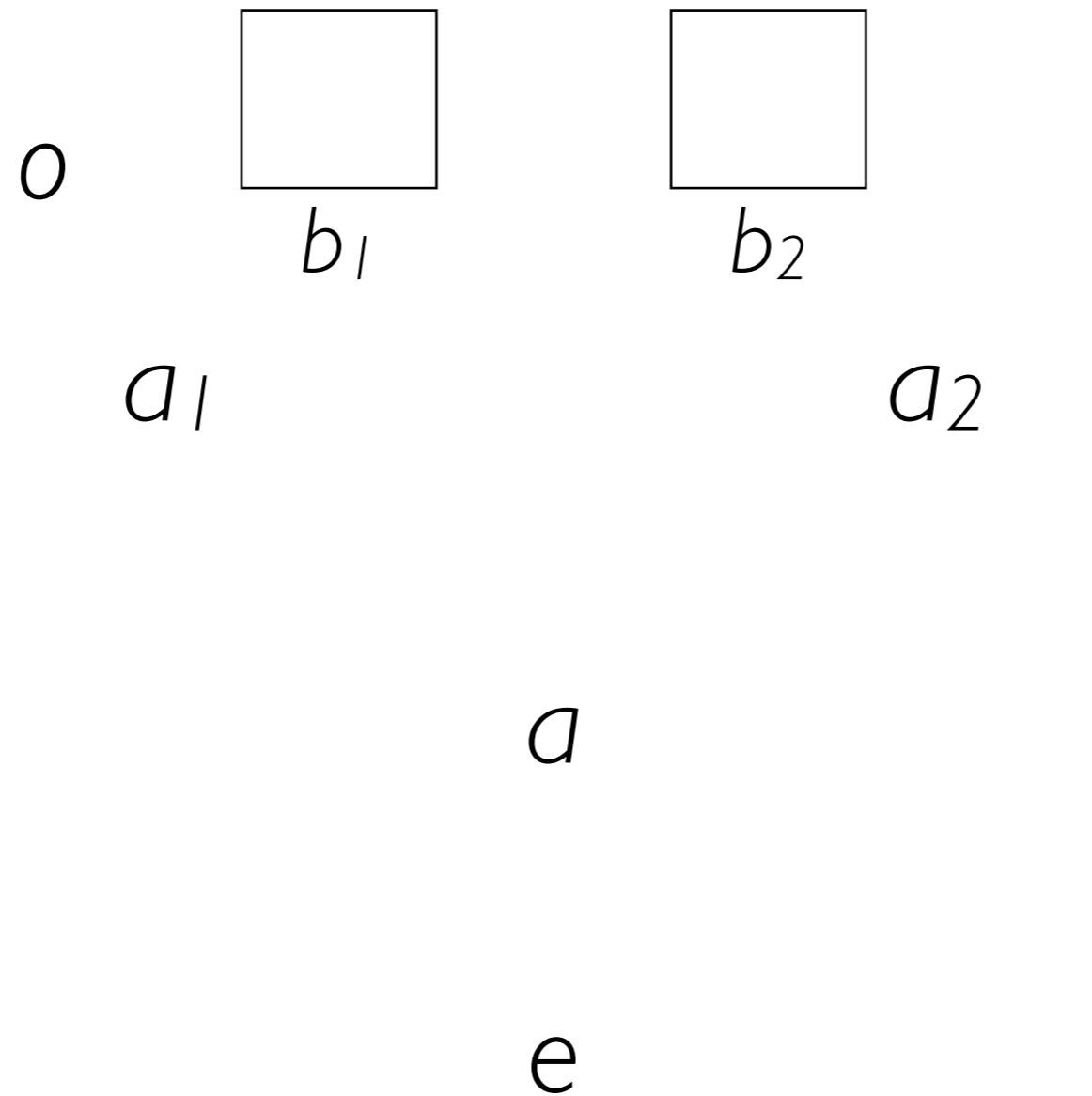


# Framework for $\text{FBT}^0_1$



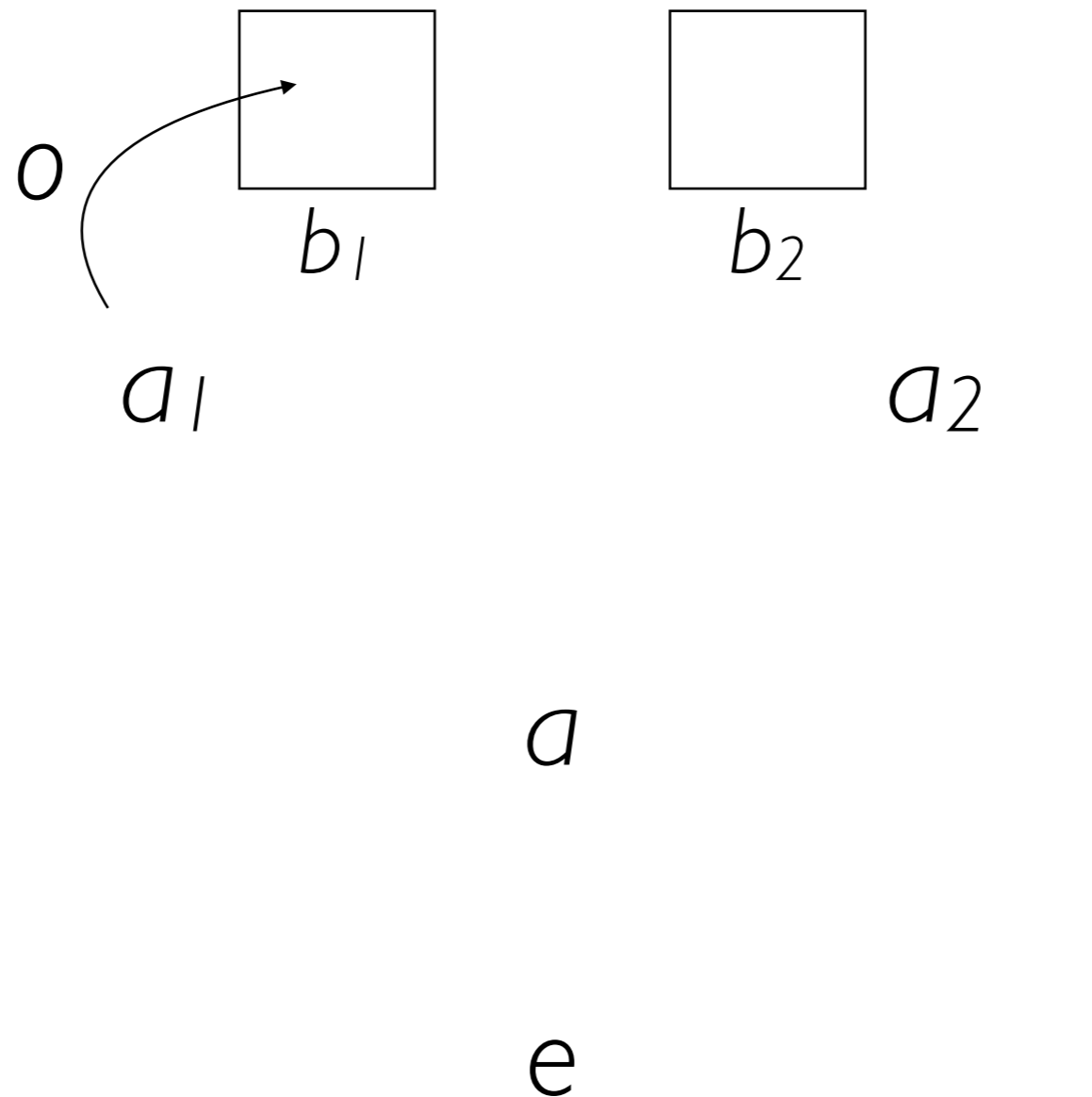
# Framework for $FBT^0_1$

(five timepoints)



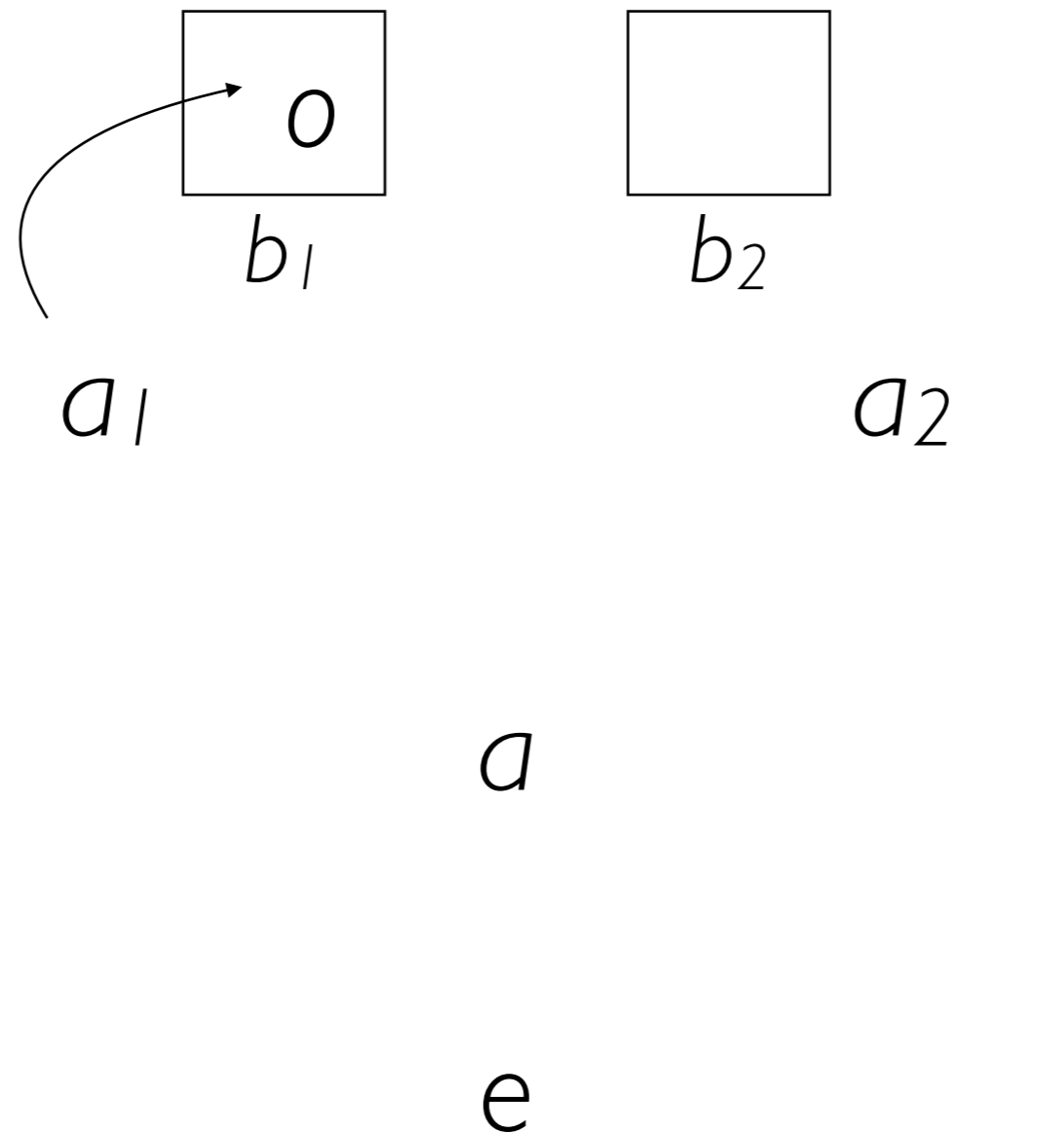
# Framework for $\text{FBT}^0_1$

(five timepoints)



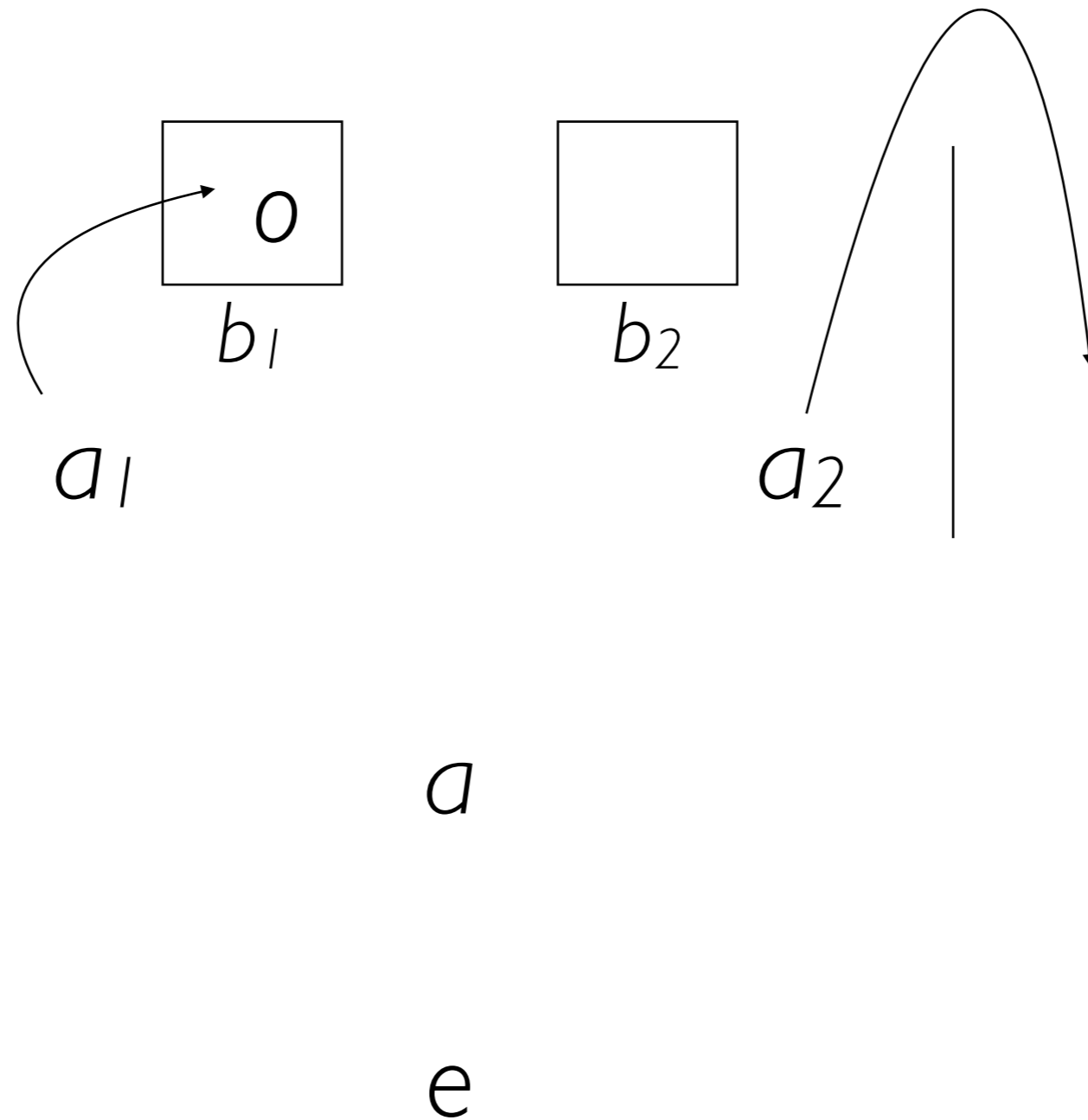
# Framework for $\text{FBT}^0_1$

(five timepoints)



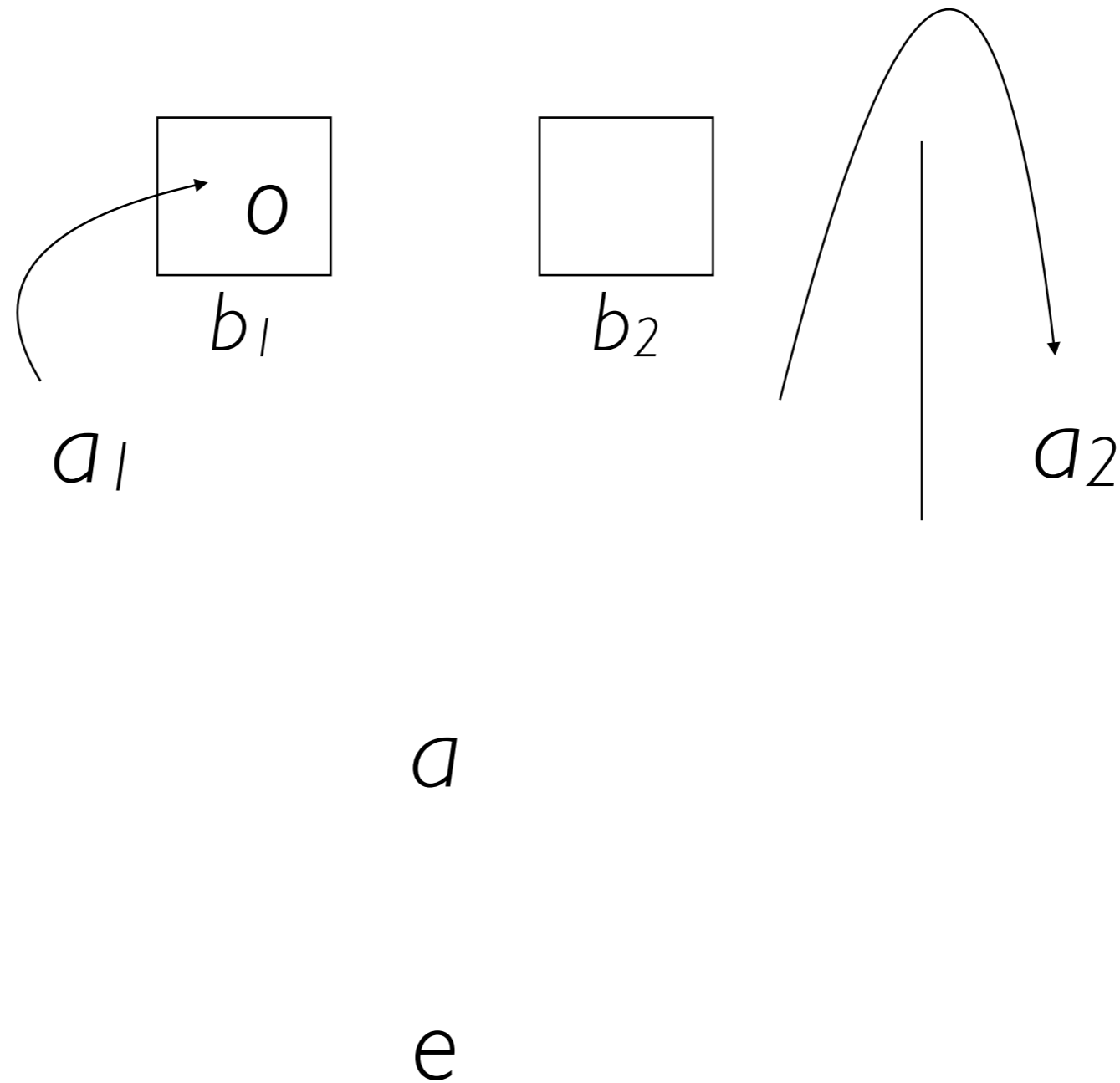
# Framework for $\text{FBT}^0_1$

(five timepoints)



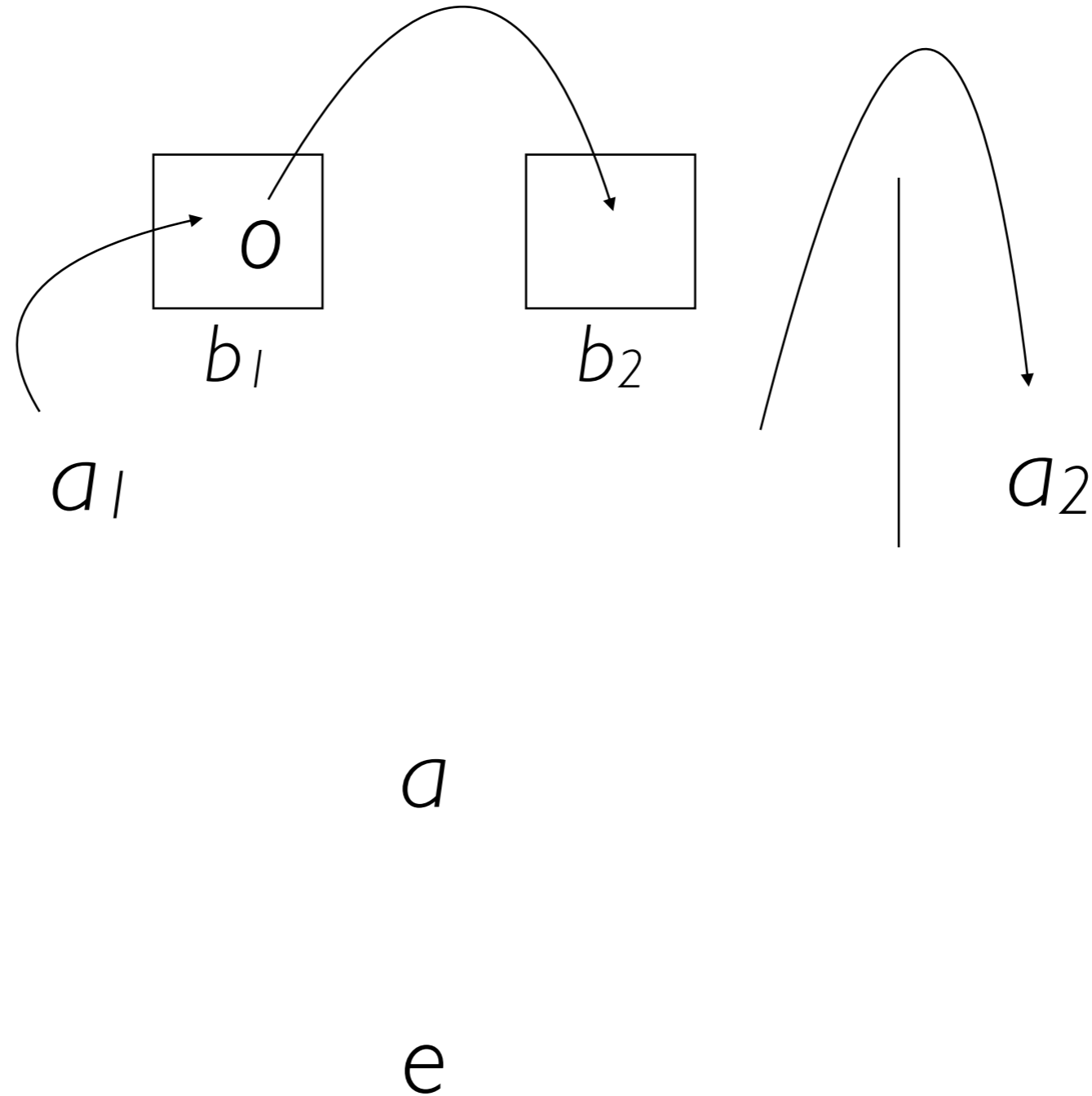
# Framework for $\text{FBT}^0_1$

(five timepoints)



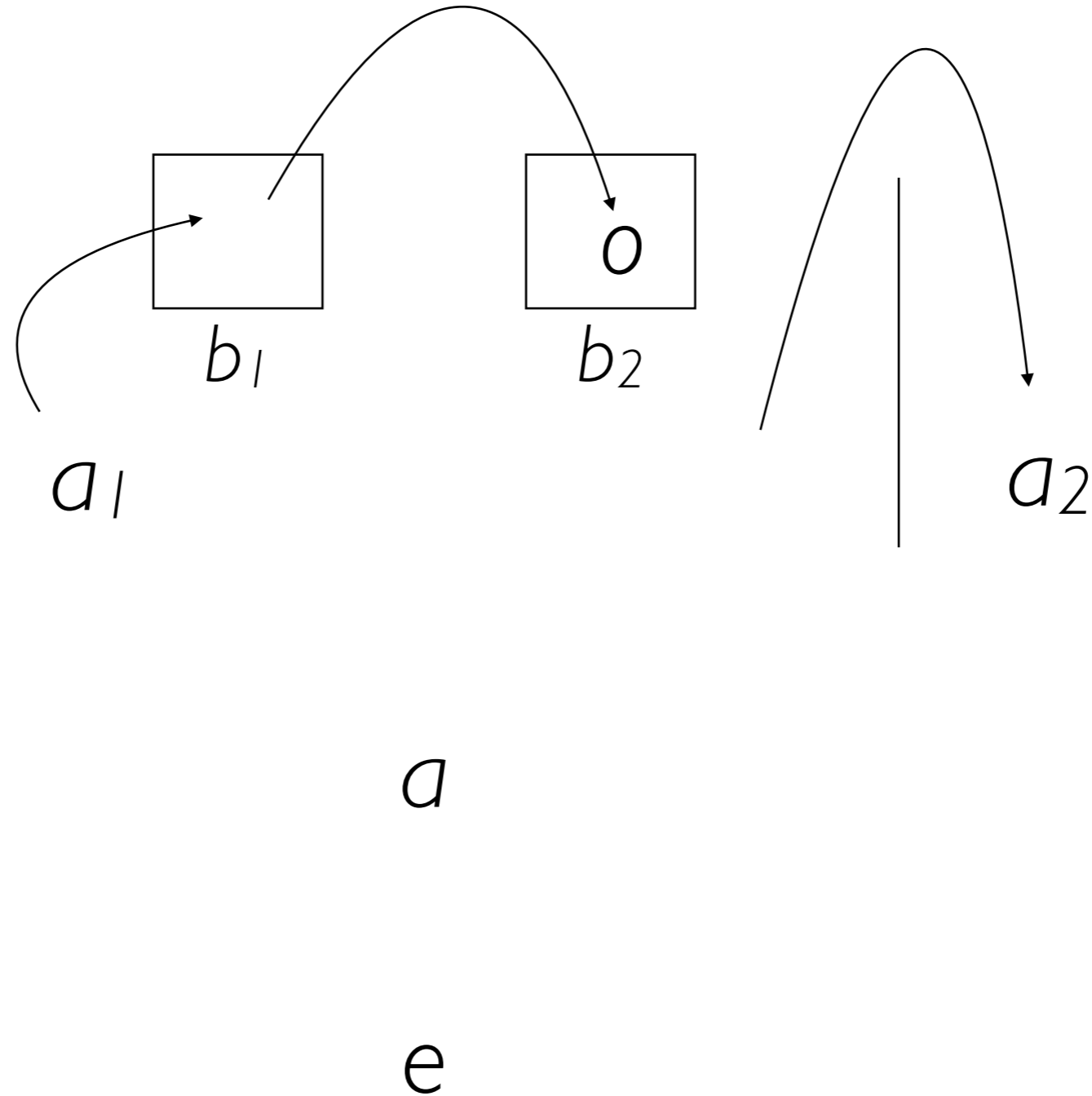
# Framework for $\text{FBT}^0_1$

(five timepoints)



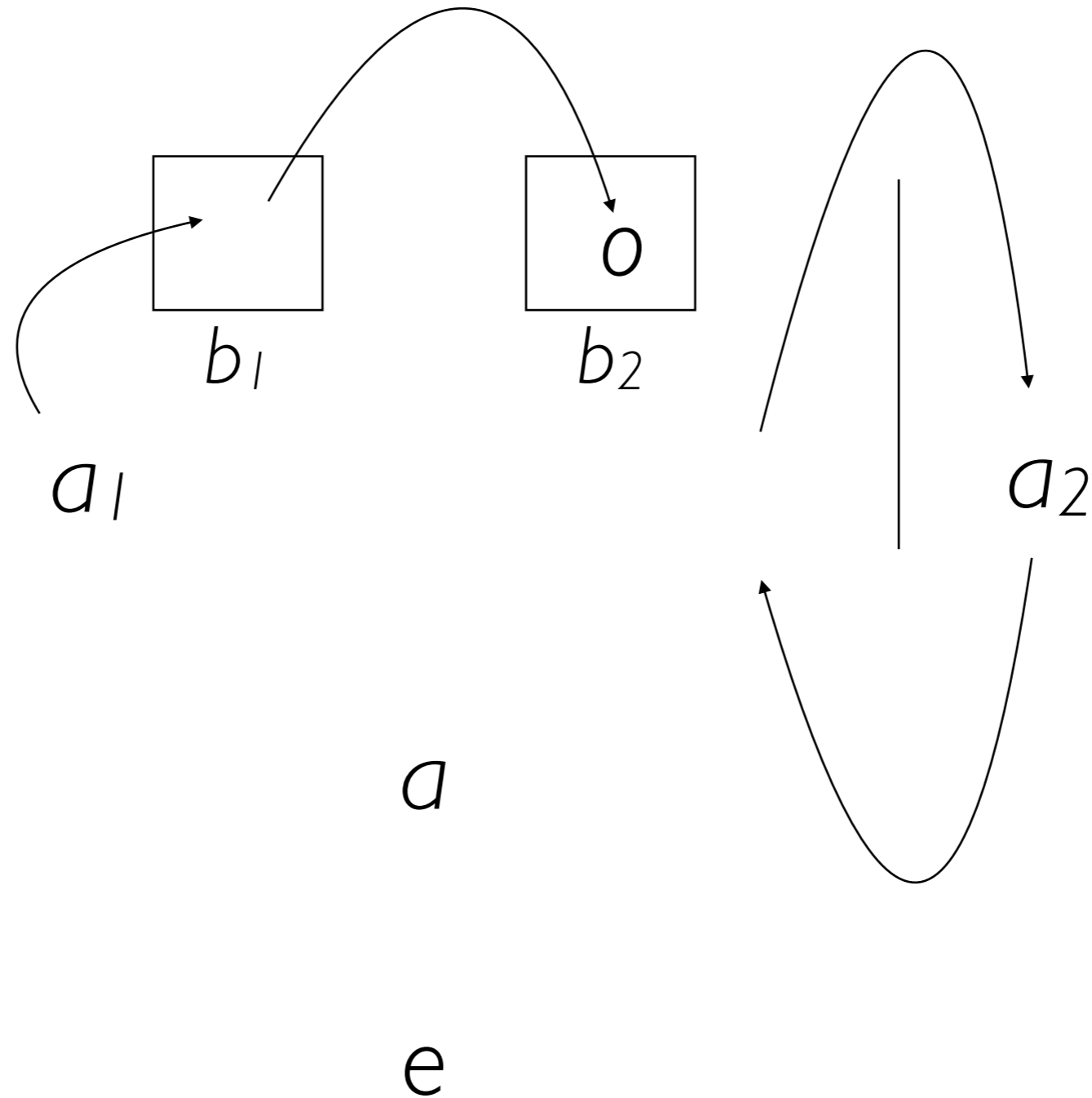
# Framework for $\text{FBT}^0_1$

(five timepoints)



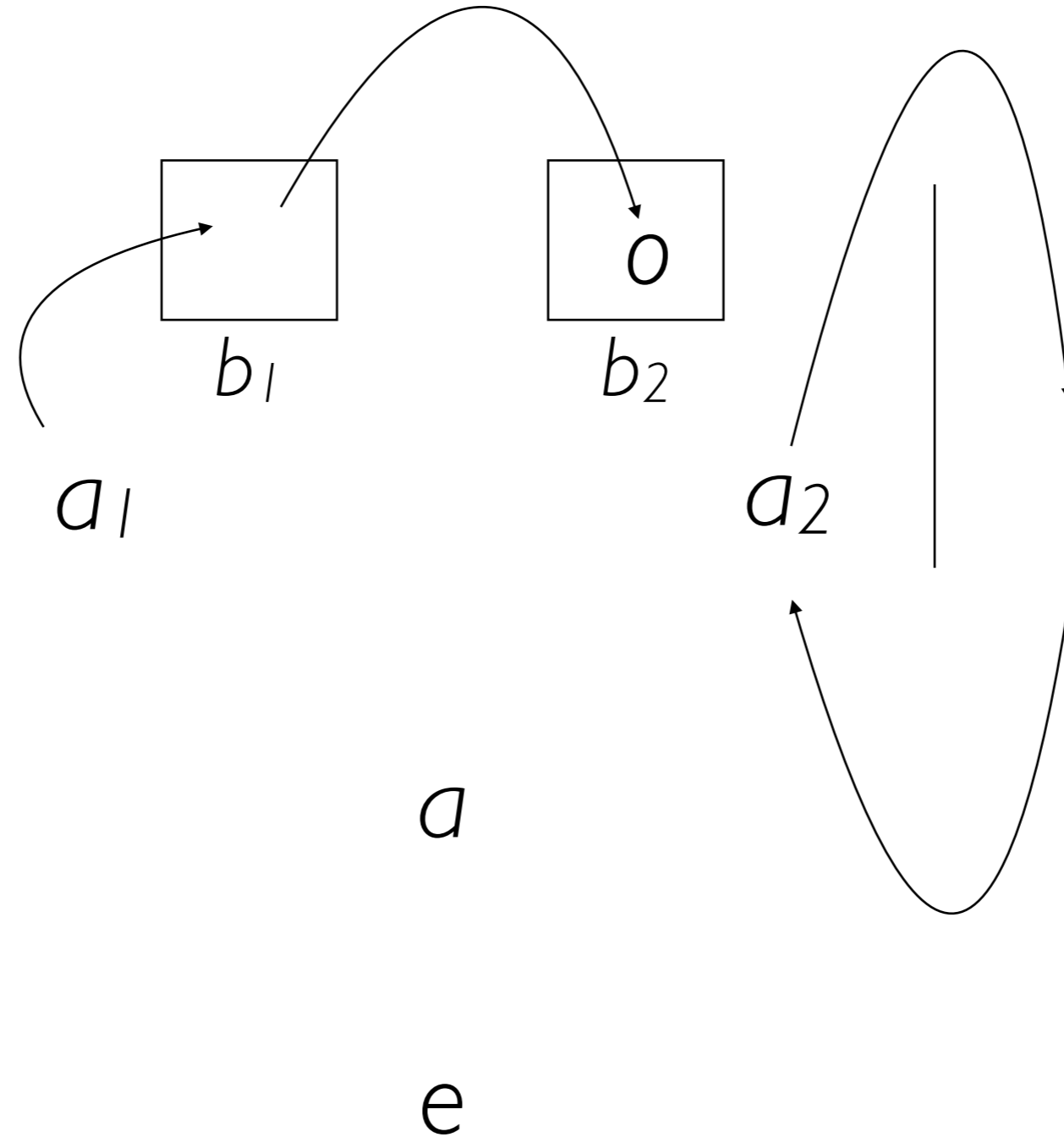
# Framework for $FBT^0_1$

(five timepoints)

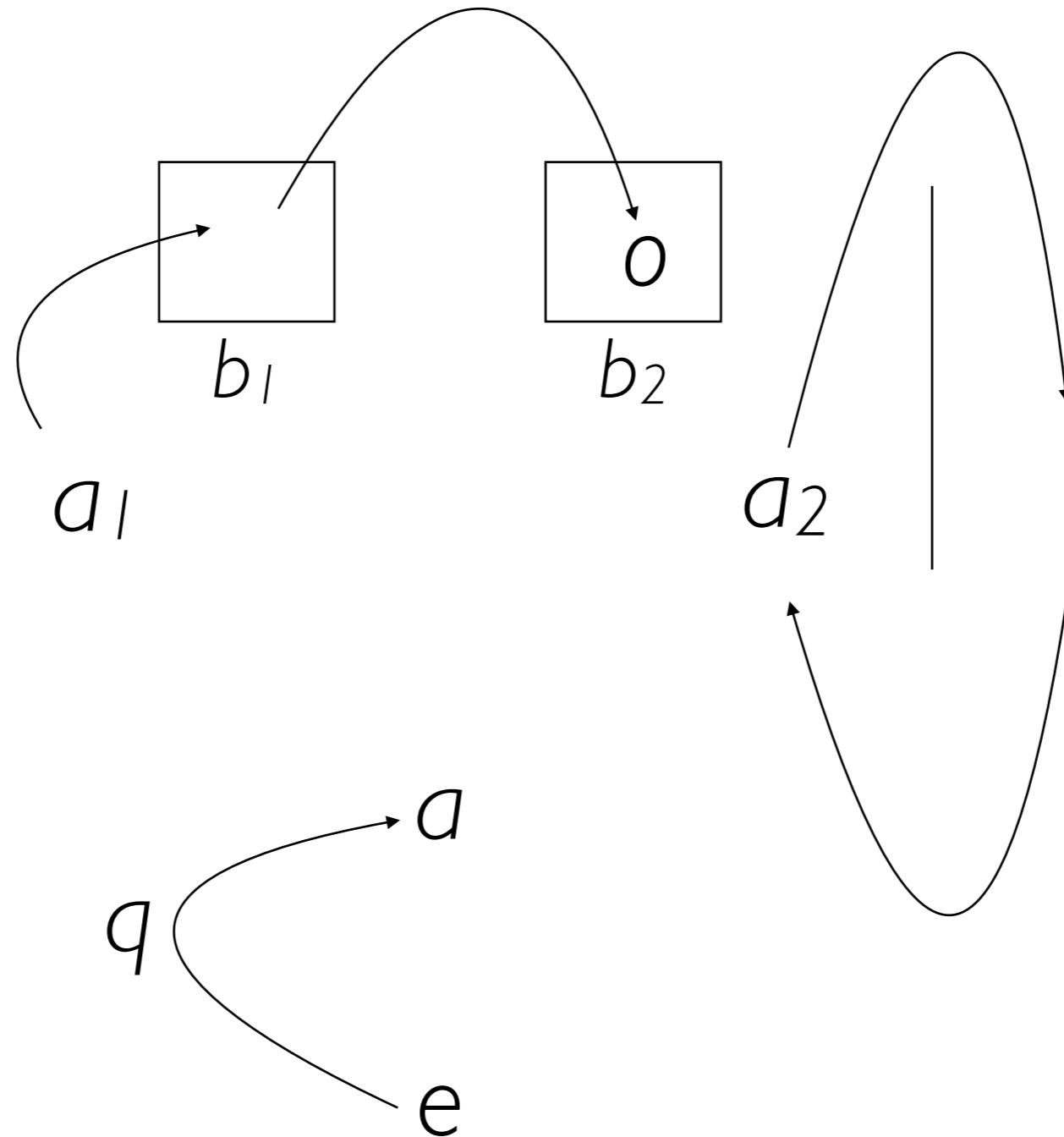


# Framework for $FBT^0_1$

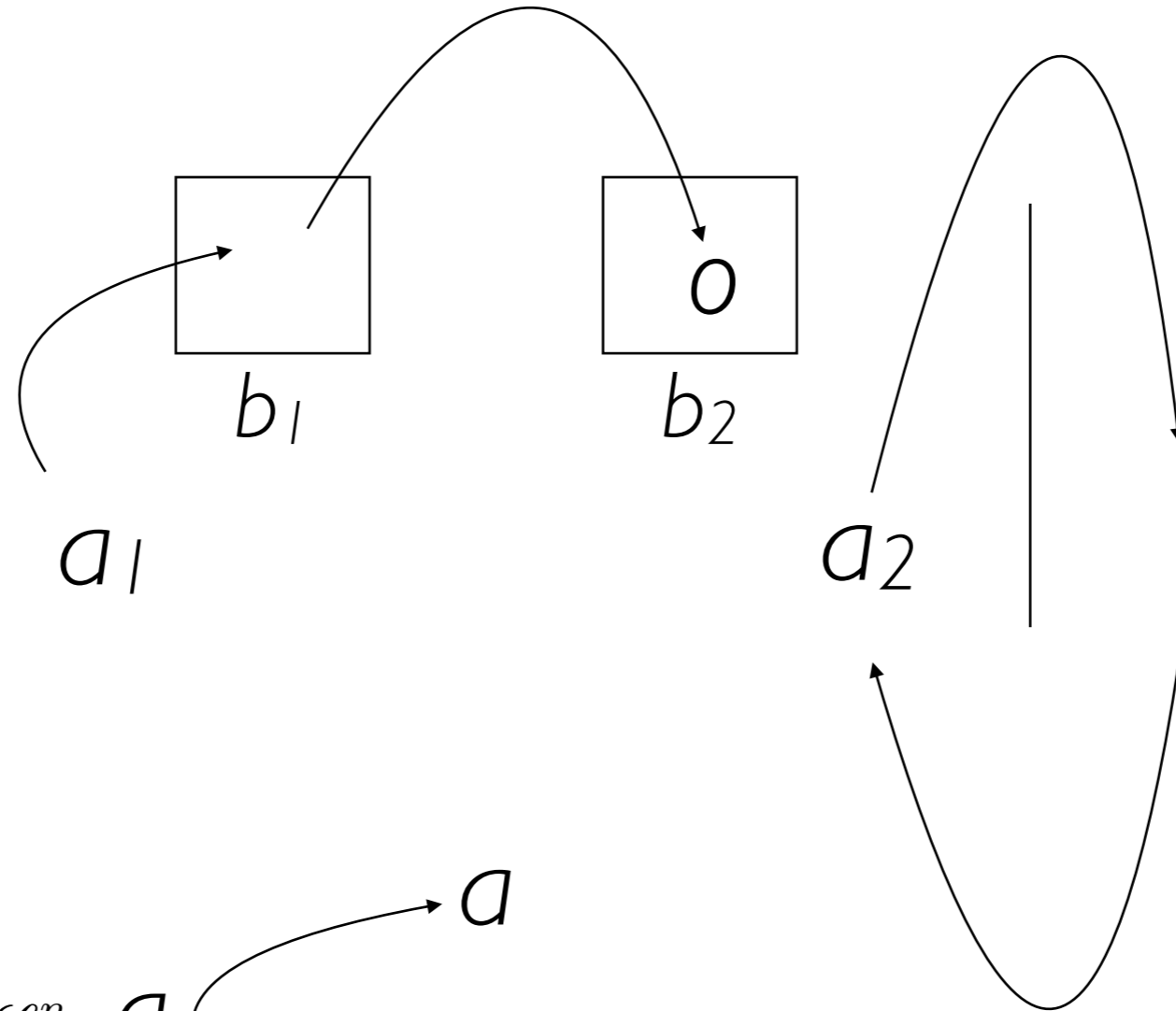
(five timepoints)



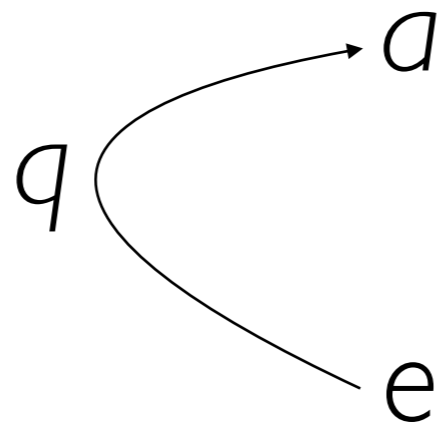
# Framework for $FBT^0_1$ (five timepoints)



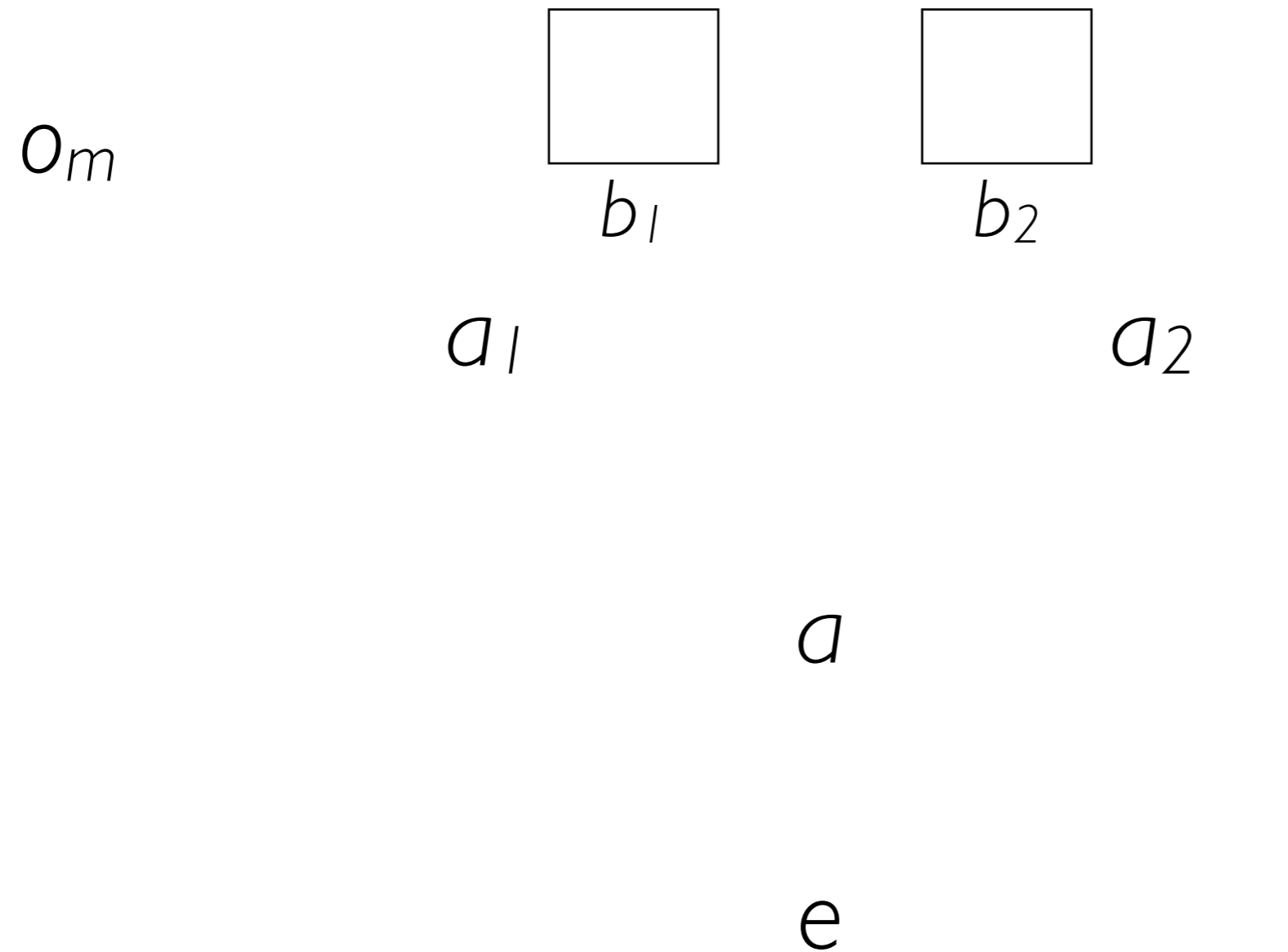
# Framework for $\text{FBT}^0_1$ (five timepoints)



$q$  a formula in modal  $\mathcal{L}^n$

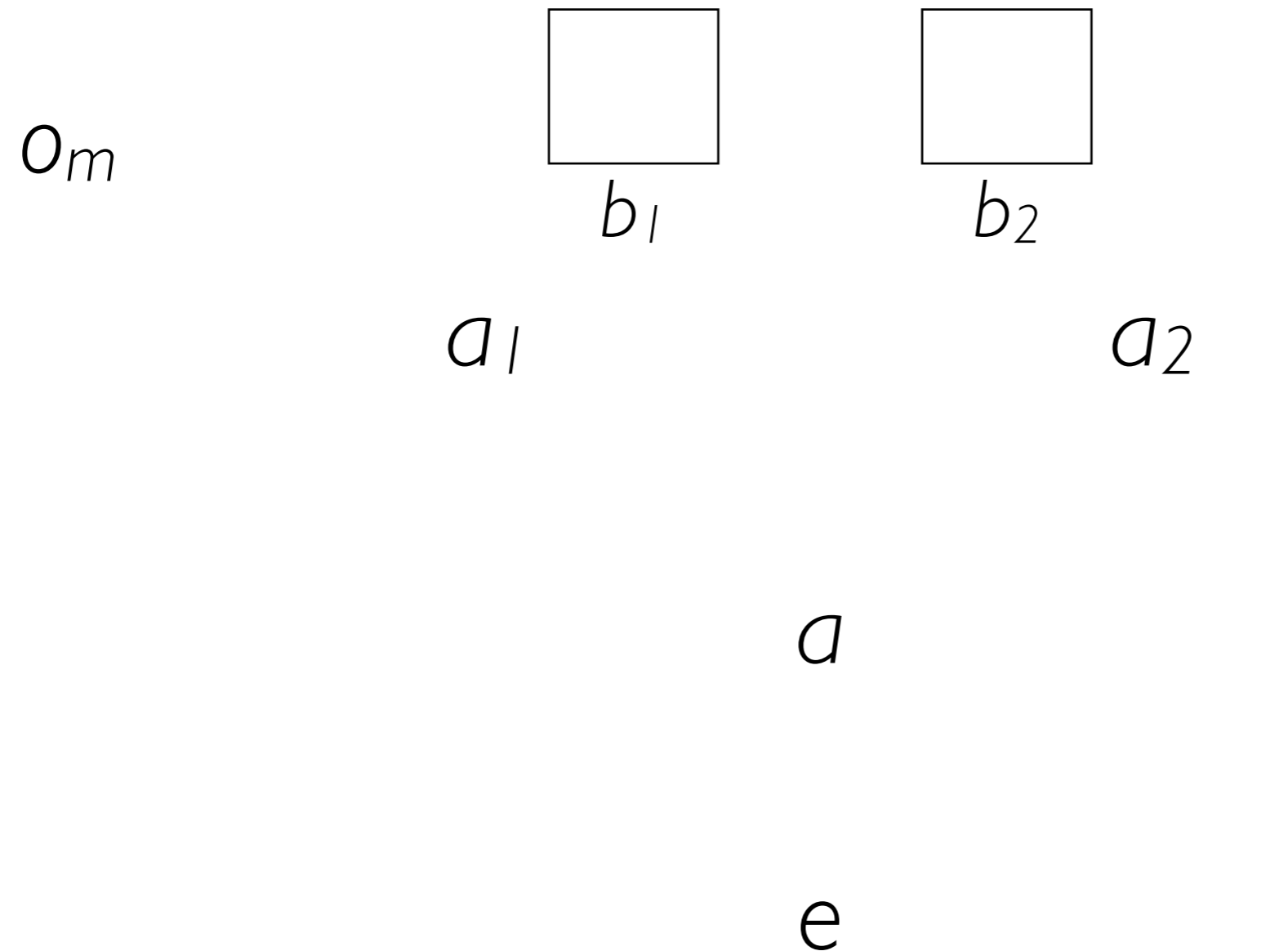


# Framework for FBT<sub>1</sub>



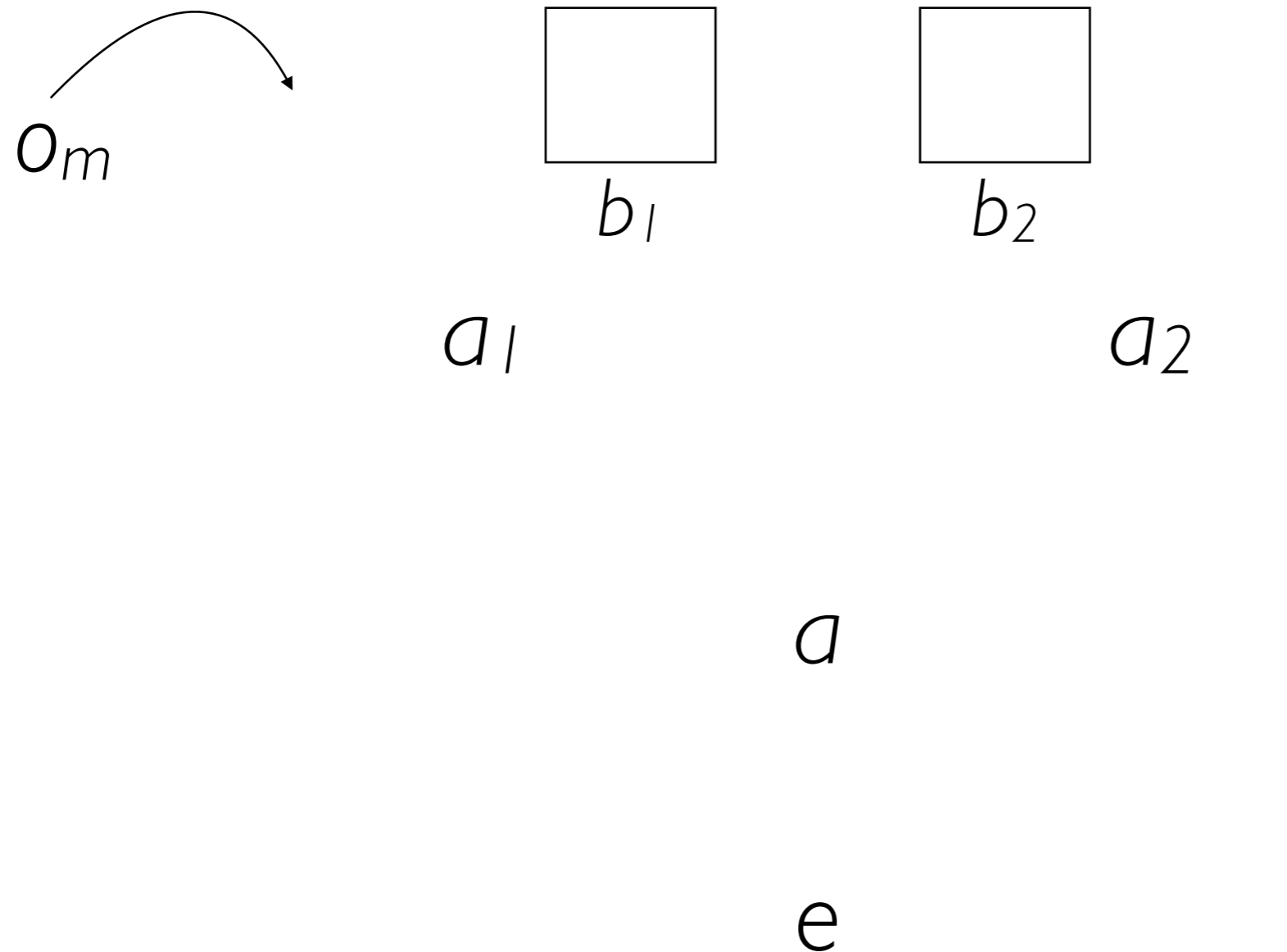
# Framework for FBT<sub>1</sub>

(six timepoints)



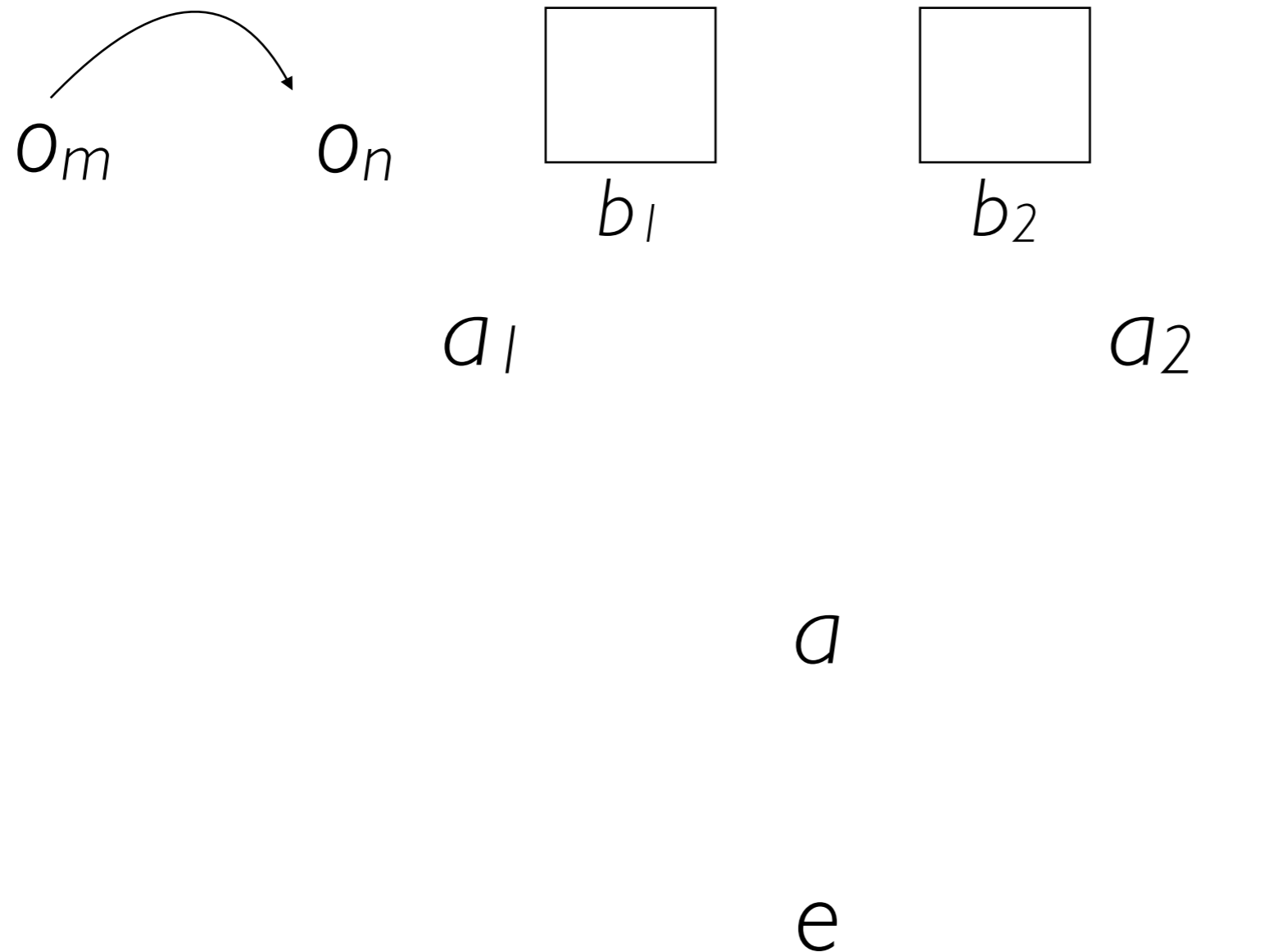
# Framework for FBT<sub>1</sub>

(six timepoints)



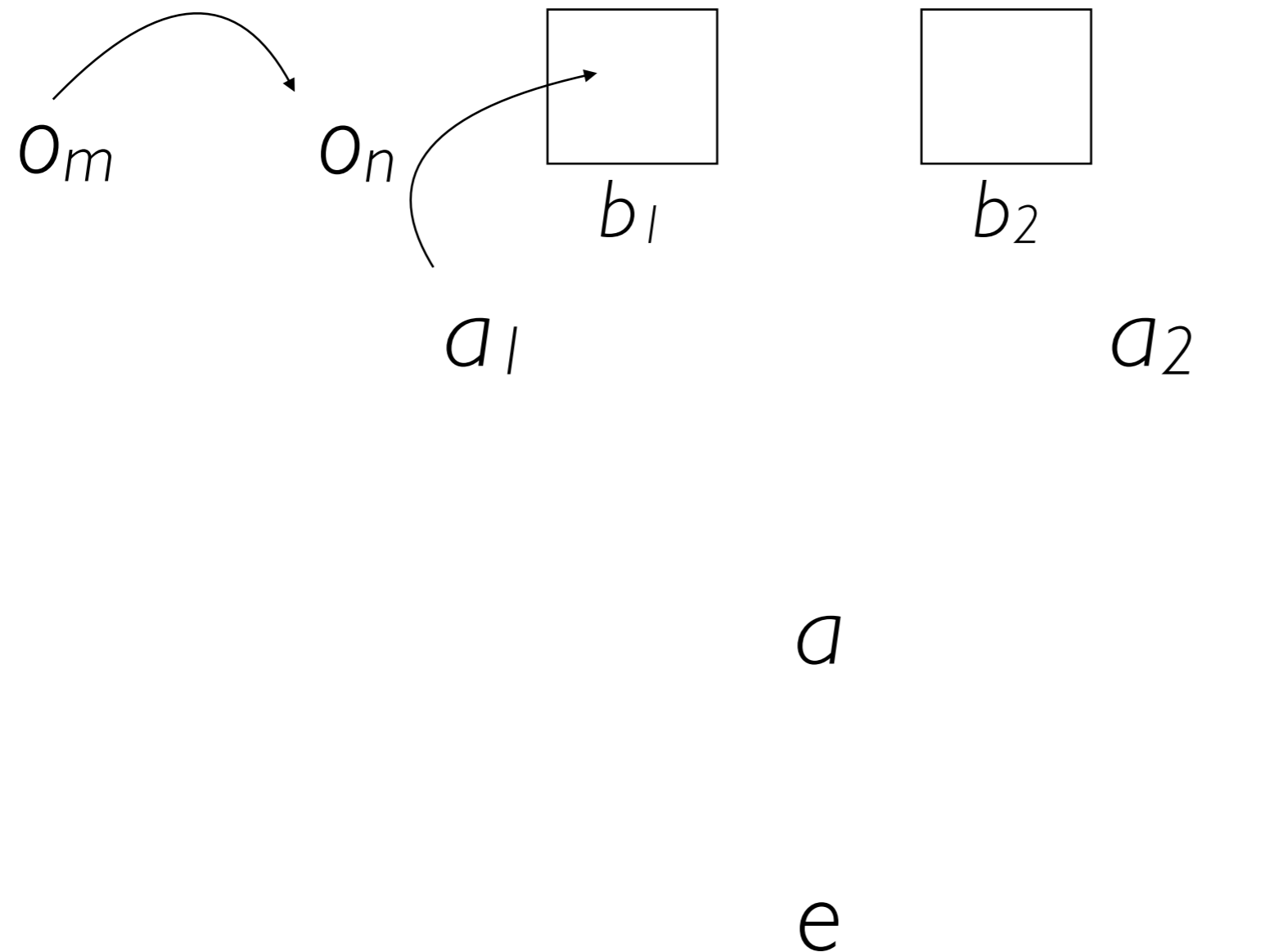
# Framework for FBT<sub>1</sub>

(six timepoints)



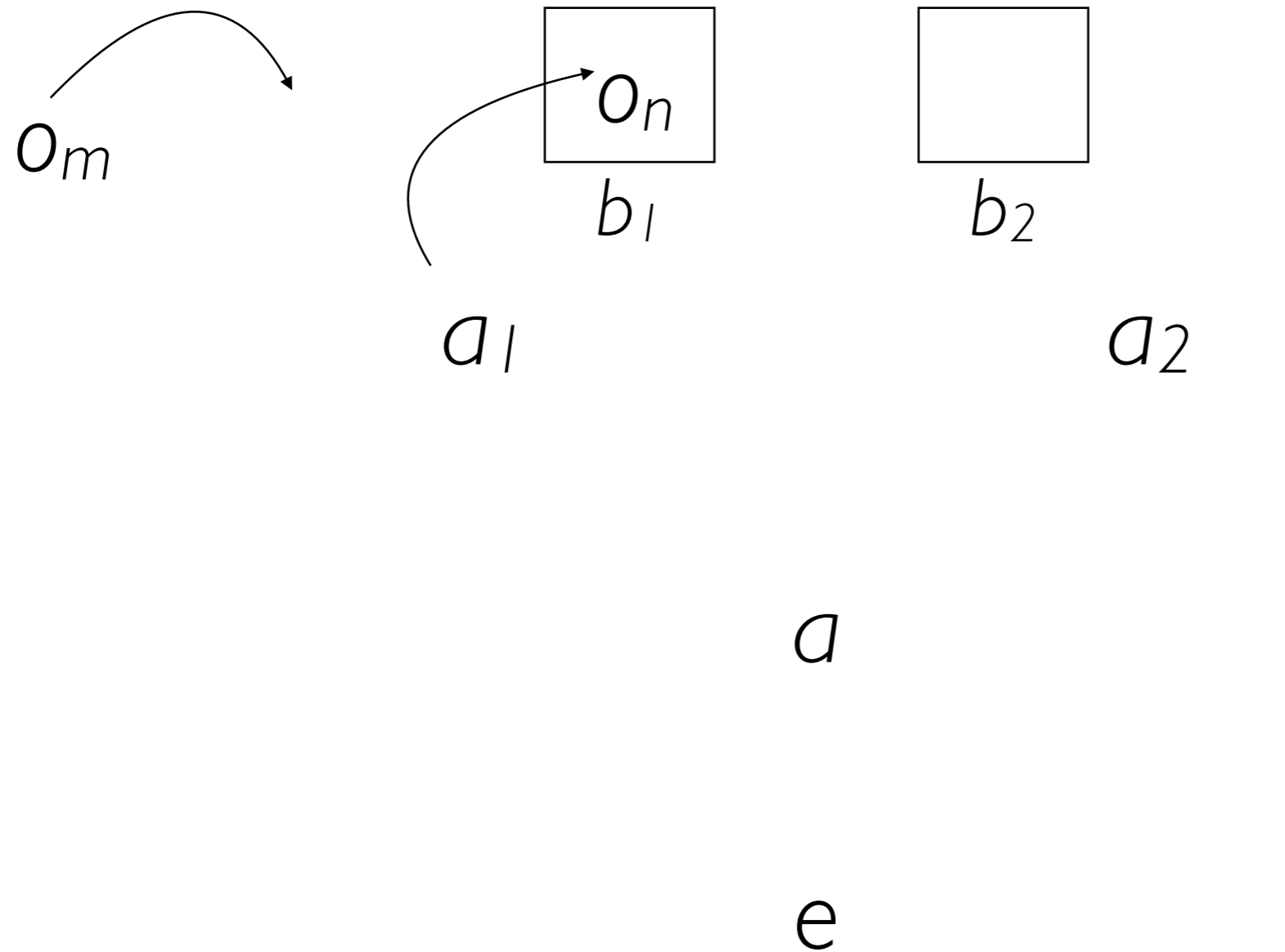
# Framework for FBT<sub>1</sub>

(six timepoints)



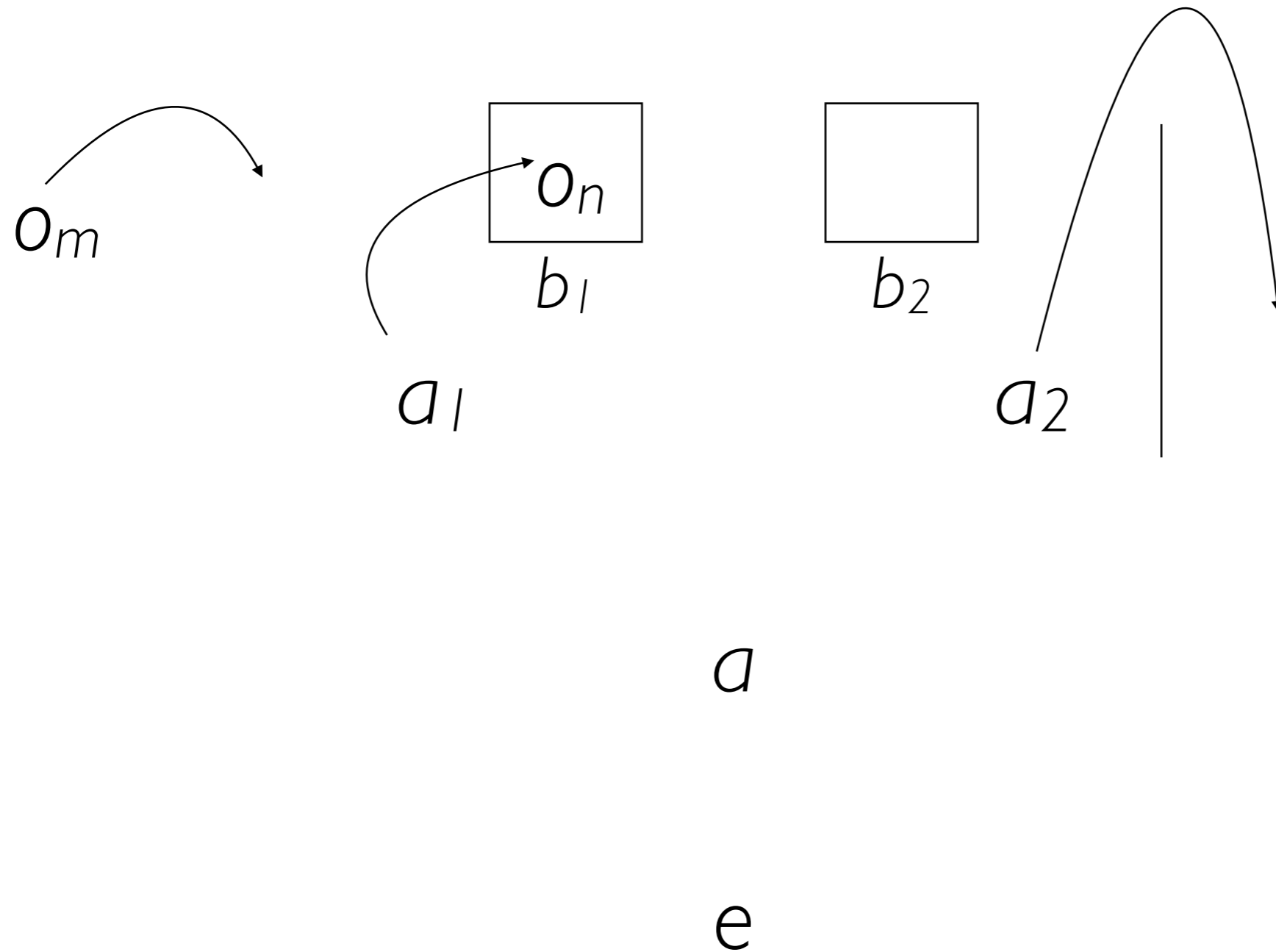
# Framework for FBT<sub>1</sub>

(six timepoints)



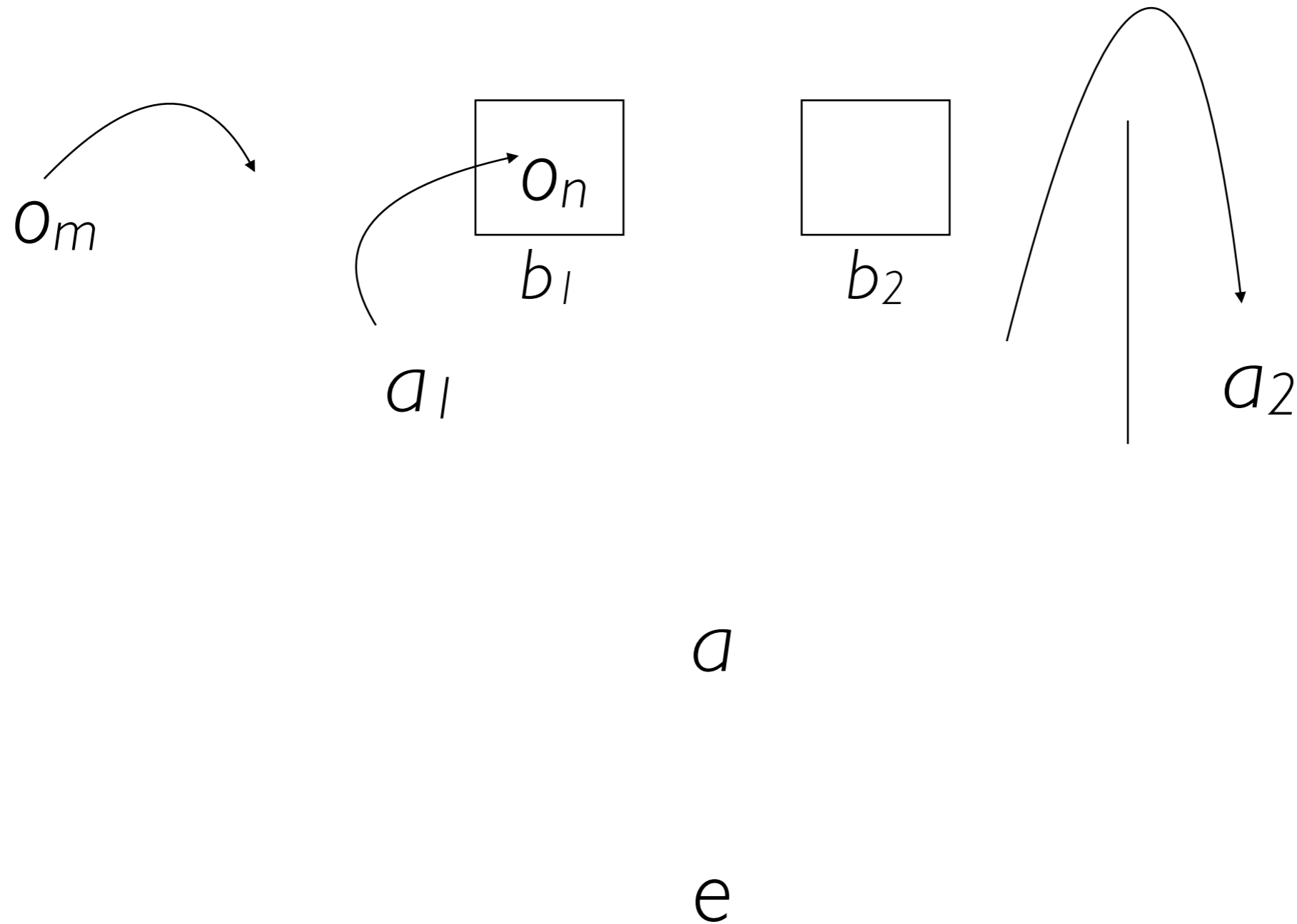
# Framework for FBT<sub>1</sub>

(six timepoints)



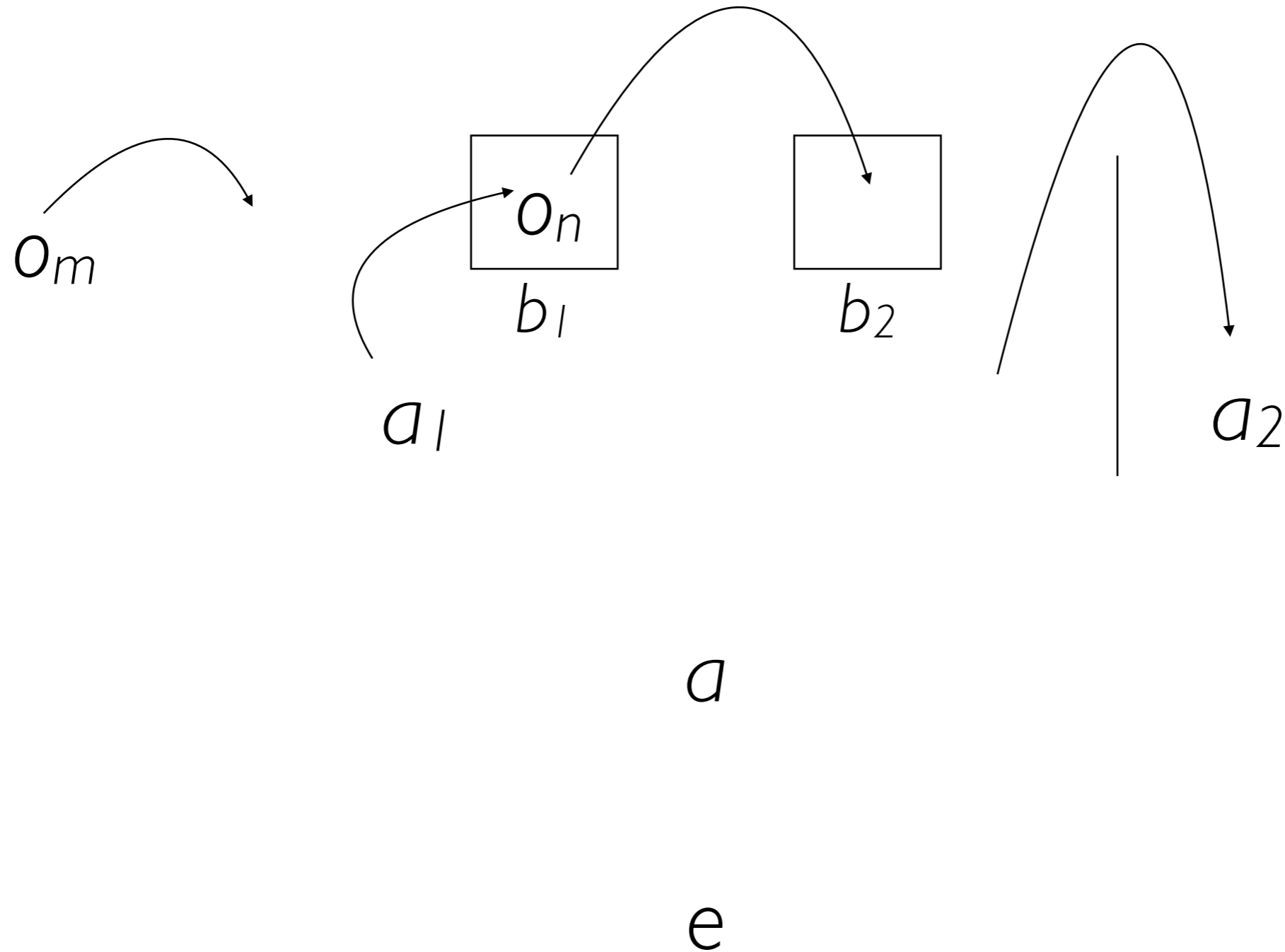
# Framework for FBT<sub>1</sub>

(six timepoints)



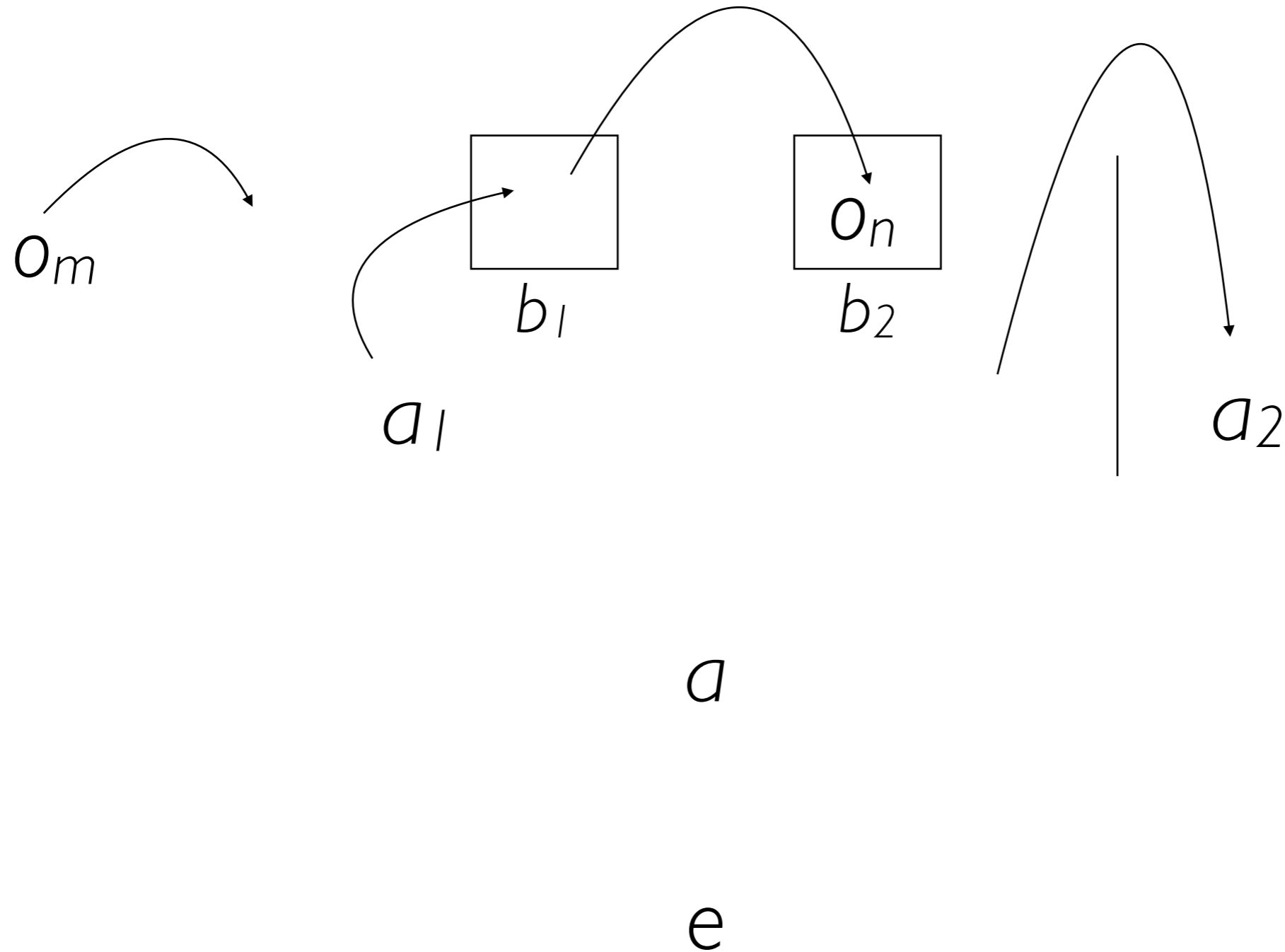
# Framework for FBT<sub>1</sub>

(six timepoints)



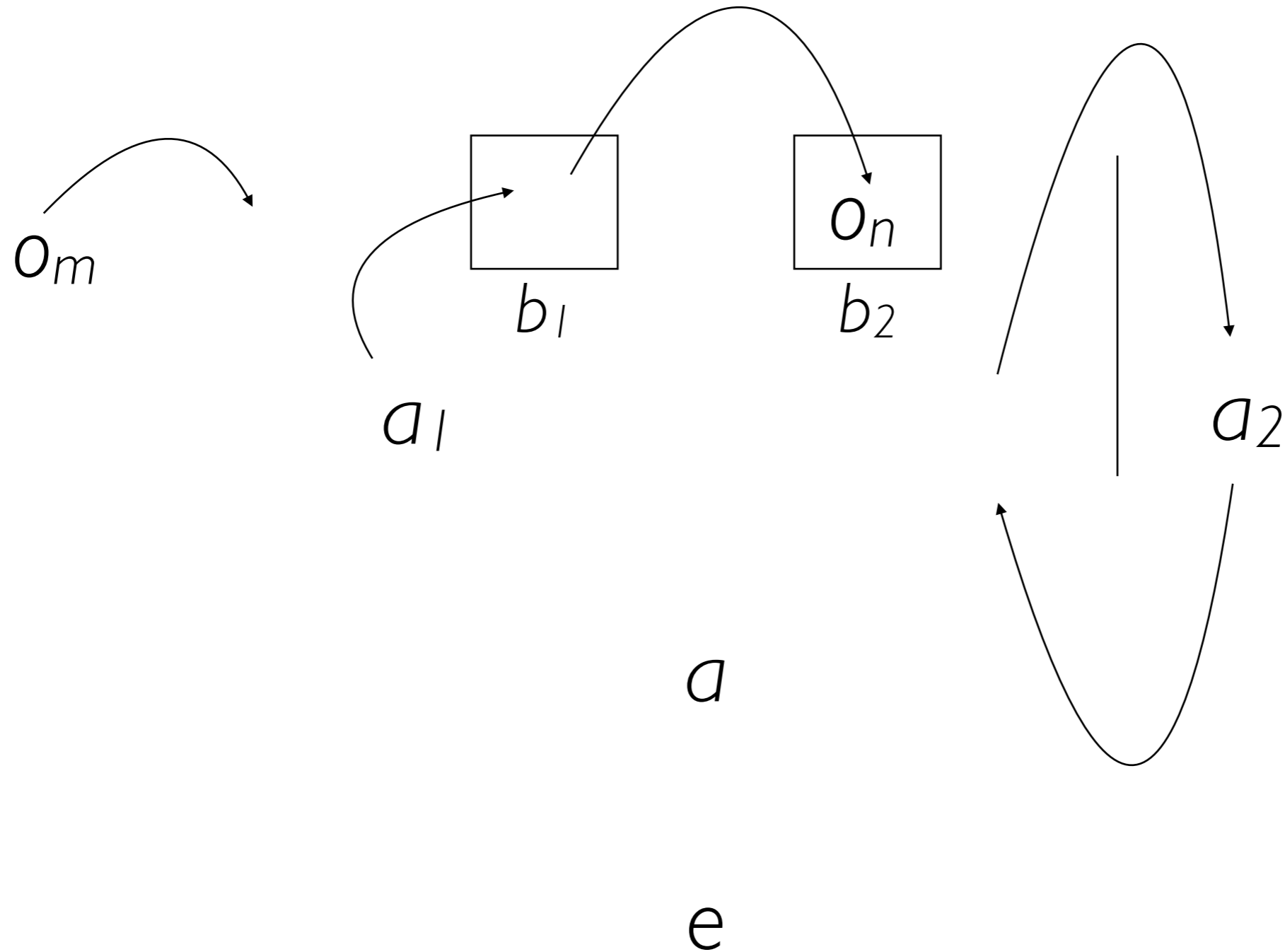
# Framework for FBT<sub>1</sub>

(six timepoints)



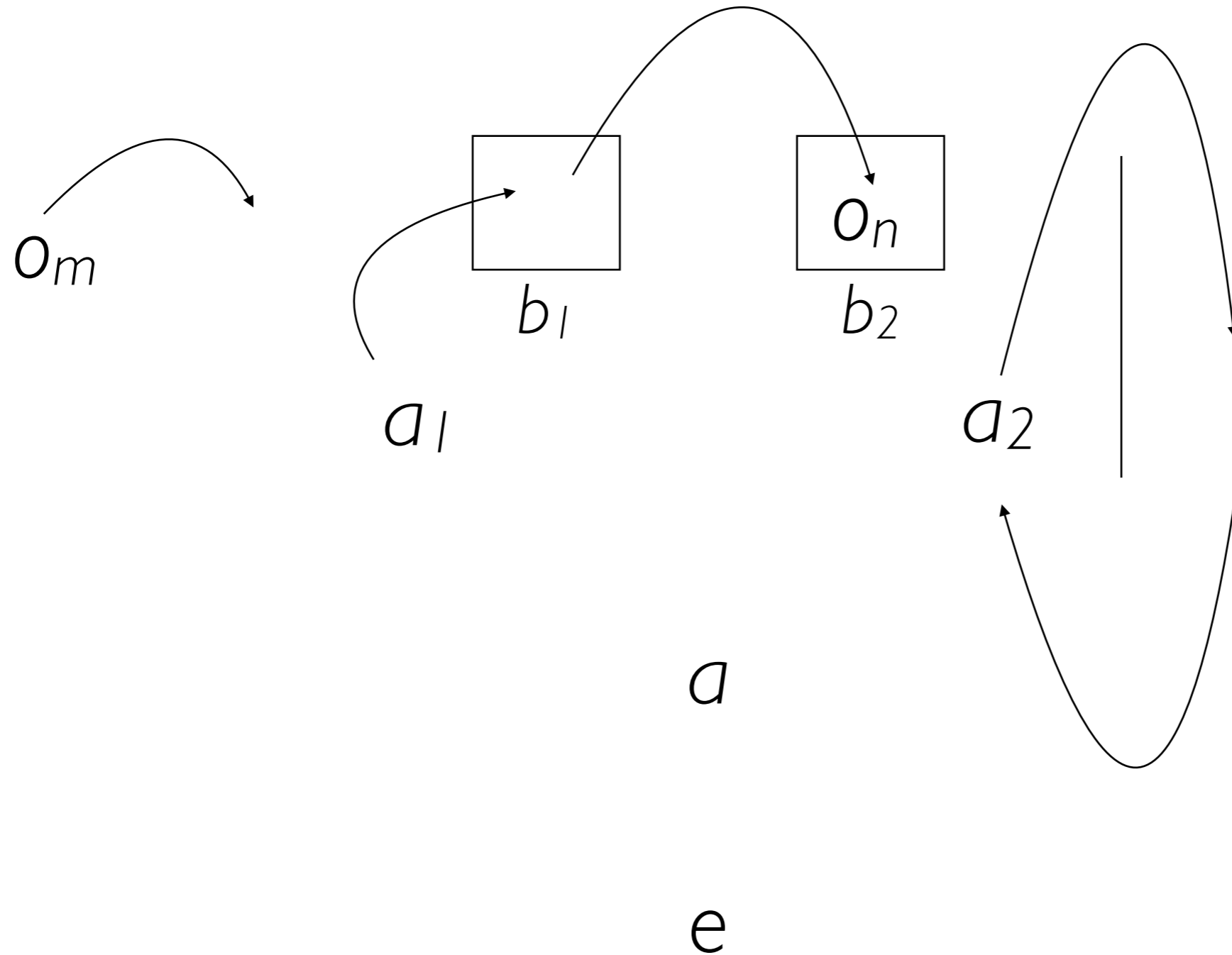
# Framework for FBT<sub>1</sub>

(six timepoints)



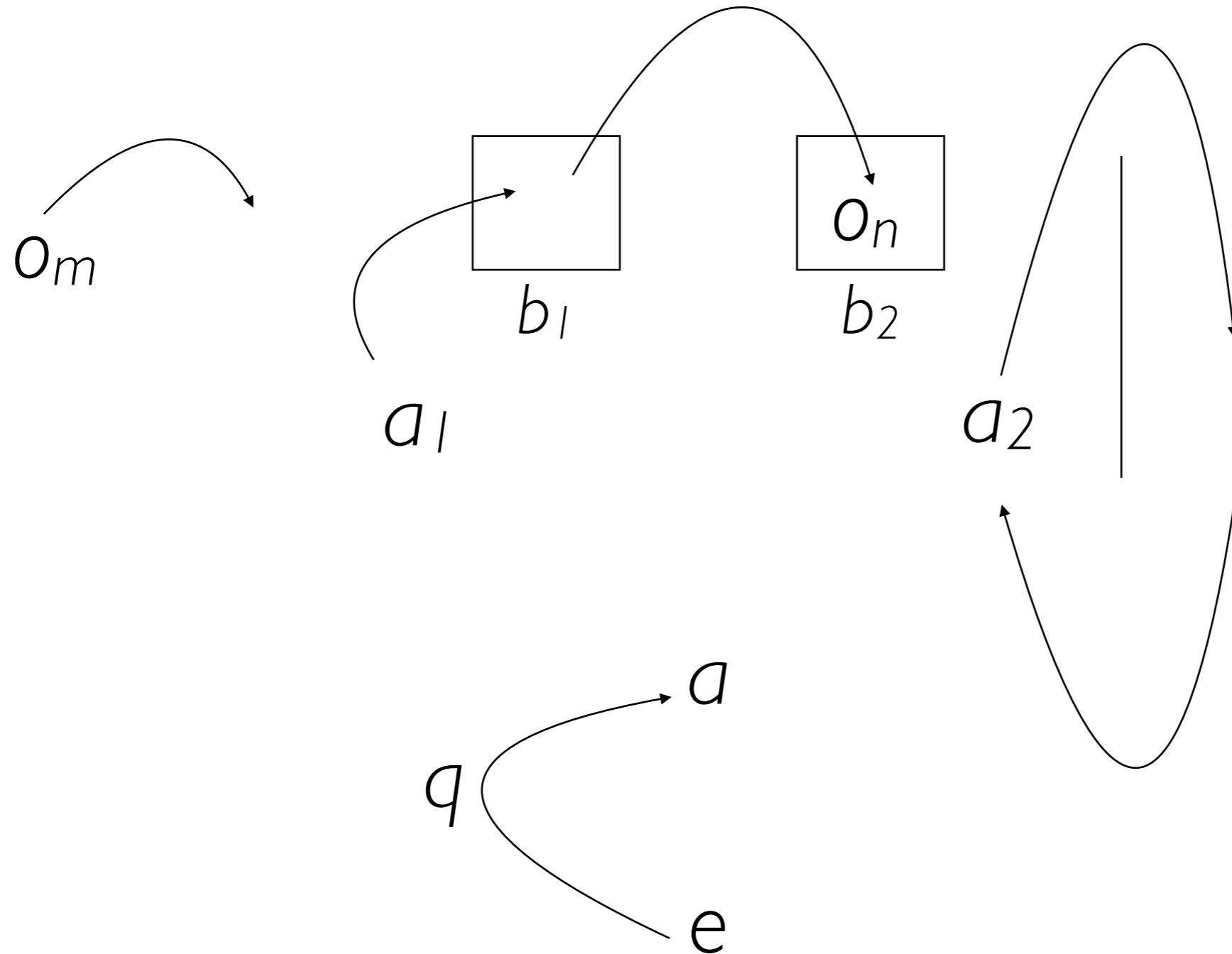
# Framework for FBT<sub>1</sub>

(six timepoints)



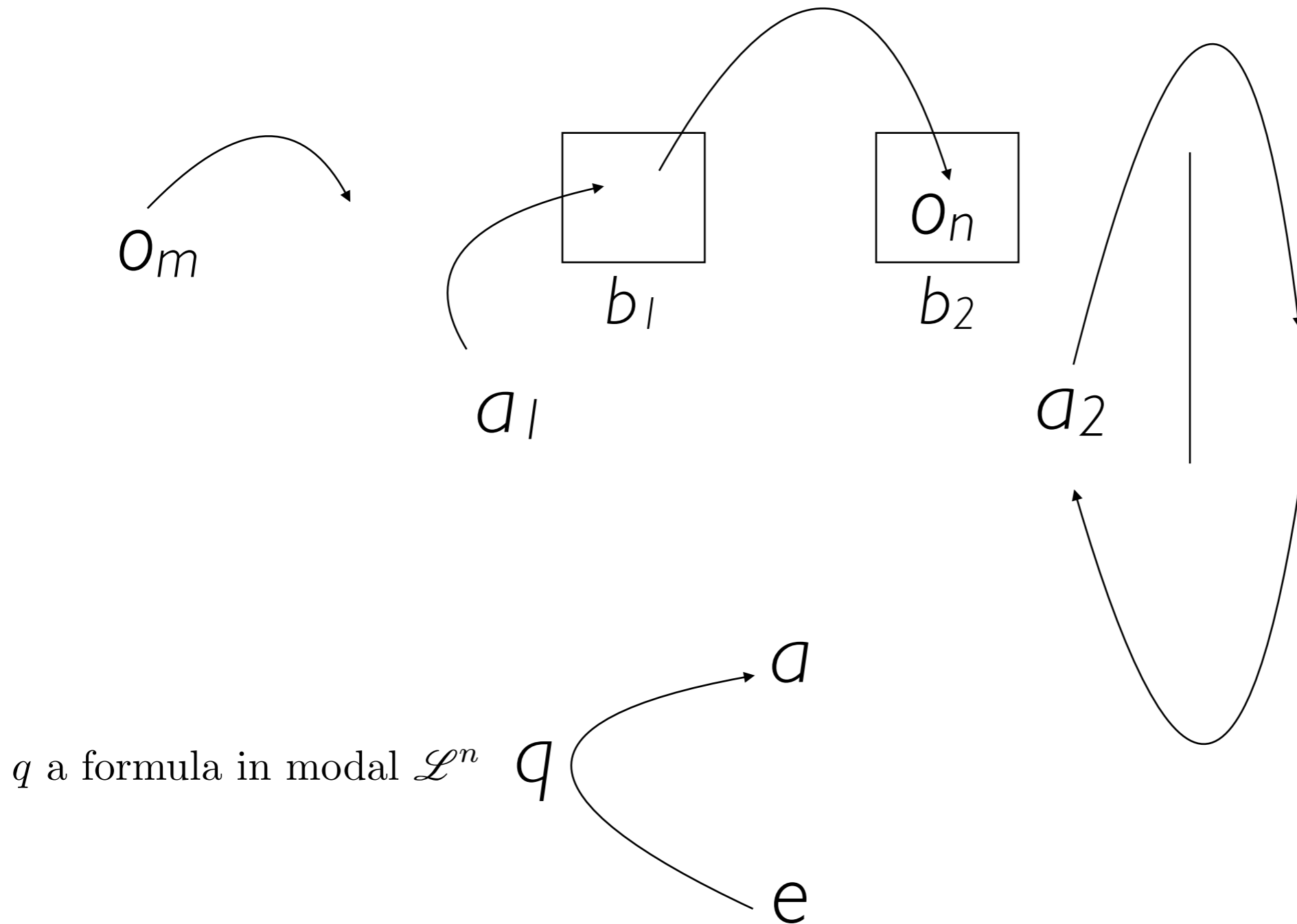
# Framework for FBT<sub>1</sub>

(six timepoints)



# Framework for $\text{FBT}^1_1$

(six timepoints)



# Done, a Decade Ago, Formally & Implementation/Simulation

Arkoudas, K. & Bringsjord, S.  
(2009) “Propositional  
Attitudes and Causation”  
*International Journal of Software  
and Informatics* **3.1**: 47–65.

[http://kryten.mm.rpi.edu/PRICAI\\_w\\_sequentcalc\\_041709.pdf](http://kryten.mm.rpi.edu/PRICAI_w_sequentcalc_041709.pdf)

## Propositional attitudes and causation

Konstantine Arkoudas and Selmer Bringsjord

Cognitive Science and Computer Science Departments, RPI  
arkouk@rpi.edu, brings@rpi.edu

**Abstract.** Predicting and explaining the behavior of others in terms of mental states is indispensable for everyday life. It will be equally important for artificial agents. We present an inference system for representing and reasoning about mental states, and use it to provide a formal analysis of the false-belief task. The system allows for the representation of information about events, causation, and perceptual, doxastic, and epistemic states (vision, belief, and knowledge), incorporating ideas from the event calculus and multi-agent epistemic logic. Unlike previous AI formalisms, our focus here is on mechanized proofs and proof programmability, not on metamathematical results. Reasoning is performed via relatively cognitively plausible inference rules, and a degree of automation is achieved by general-purpose inference methods and by a syntactic embedding of the system in first-order logic.

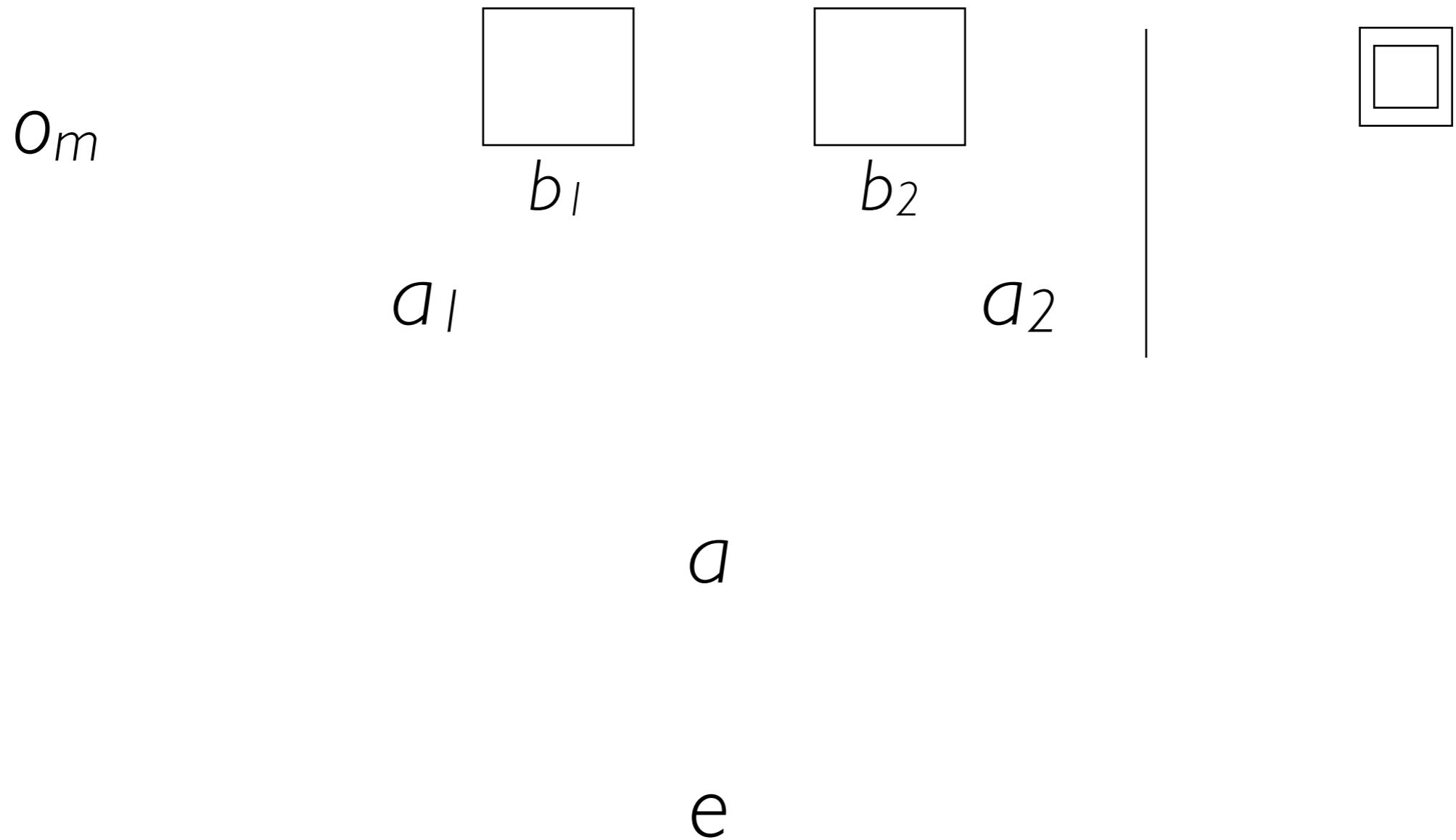
### 1 Introduction

Interpreting the behavior of other people is indispensable for everyday life. It is something that we do constantly, on a daily basis, and it helps us not only to make sense of human behavior, but also to predict it and—to a certain extent—to control it. How exactly do we manage that? That is not currently known, but many have argued that the ability to ascribe mental states to others and to reason about such mental states is a key component of our capacity to understand human behavior. In particular, all social transactions, from engaging in commerce and negotiating to making jokes and empathizing with other people’s pain or joy, appear to require at least a rudimentary grasp of common-sense psychology (CSP), i.e., a large body of truisms such as the following: When an agent  $a$  (1) wants to achieve a certain state of affairs  $p$ , and (2) believes that some action  $c$  can bring about  $p$ , and (3)  $a$  knows how to carry out  $c$ ; then, *ceteris paribus*,<sup>1</sup>  $a$  will carry out  $c$ ; when  $a$  sees that  $p$ ,  $a$  knows that  $p$ ; when  $a$  fears that  $p$  and  $a$  discovers that  $p$  is the case,  $a$  is disappointed; and so on.

Artificial agents without a mastery of CSP would be severely handicapped in their interactions with humans. This could present problems not only for artificial agents trying to interpret human behavior, but also for artificial agents trying to interpret the behavior of one another. When a system exhibits a complex but rational behavior, and detailed knowledge of its internal structure is not

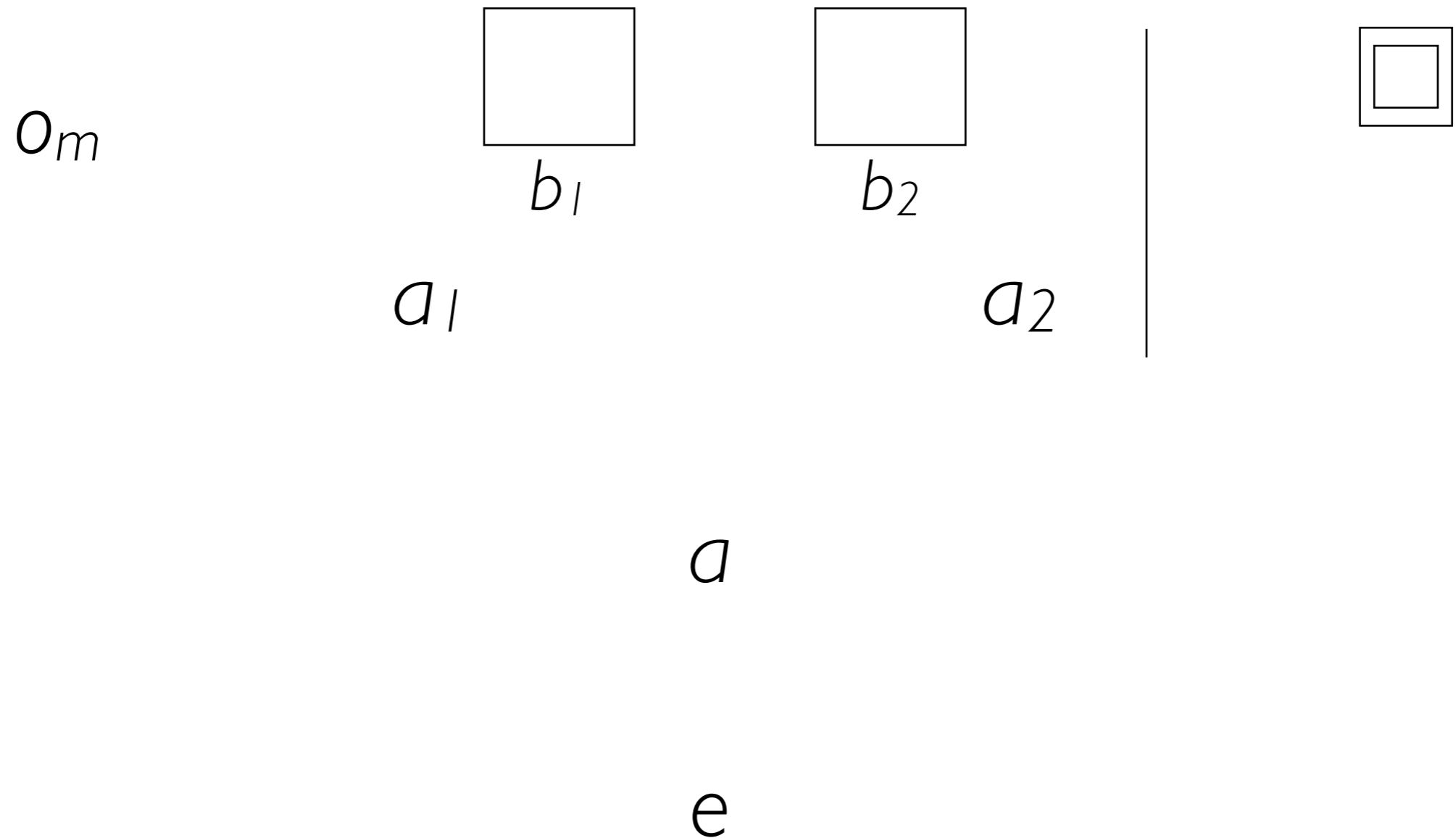
<sup>1</sup> Assuming that  $a$  is able to carry out  $c$ , that  $a$  has no conflicting desires that override his goal that  $p$ ; and so on.

# Framework for FBT<sub>2</sub>



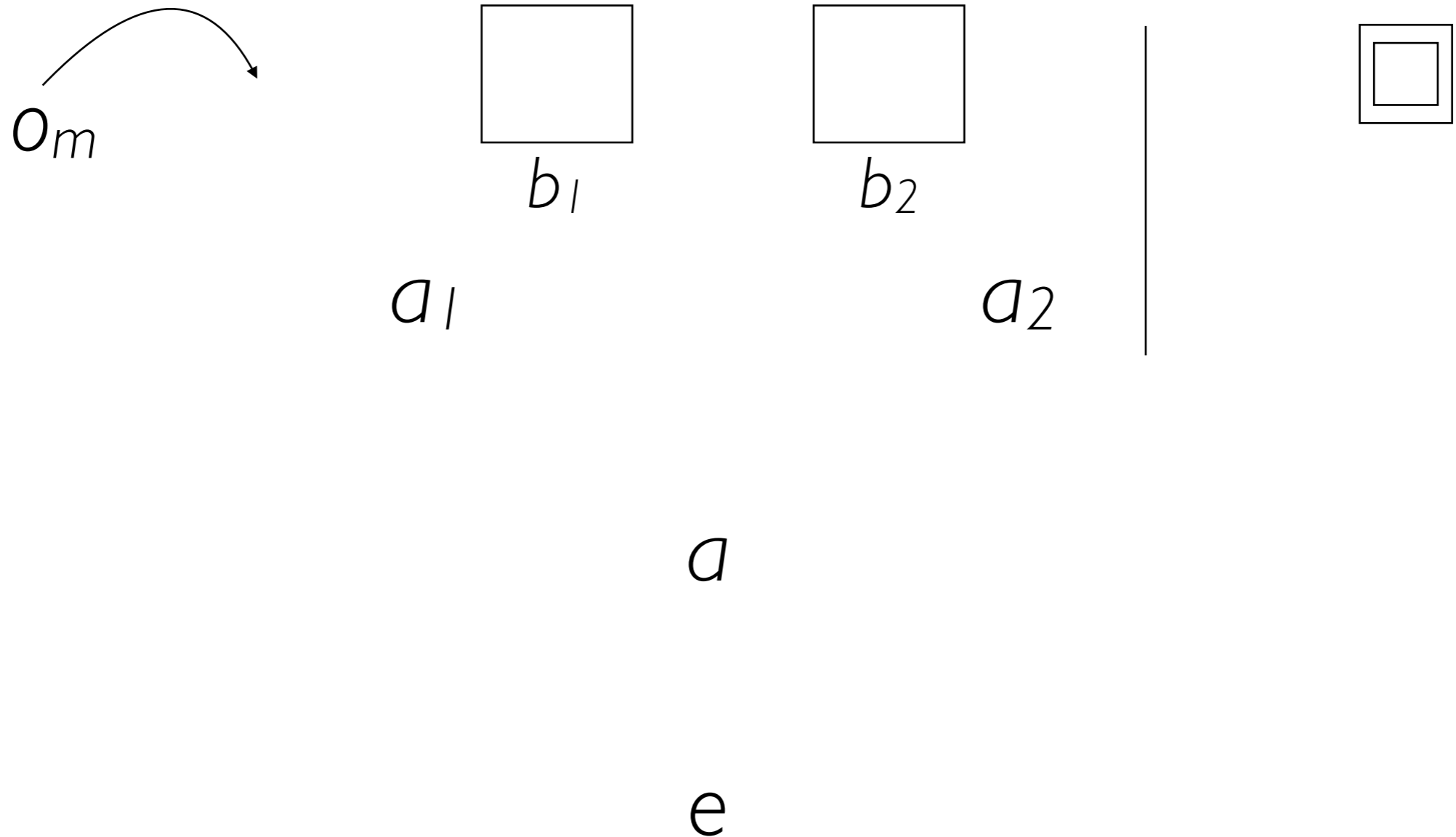
# Framework for FBT<sub>2</sub>

(seven timepoints)



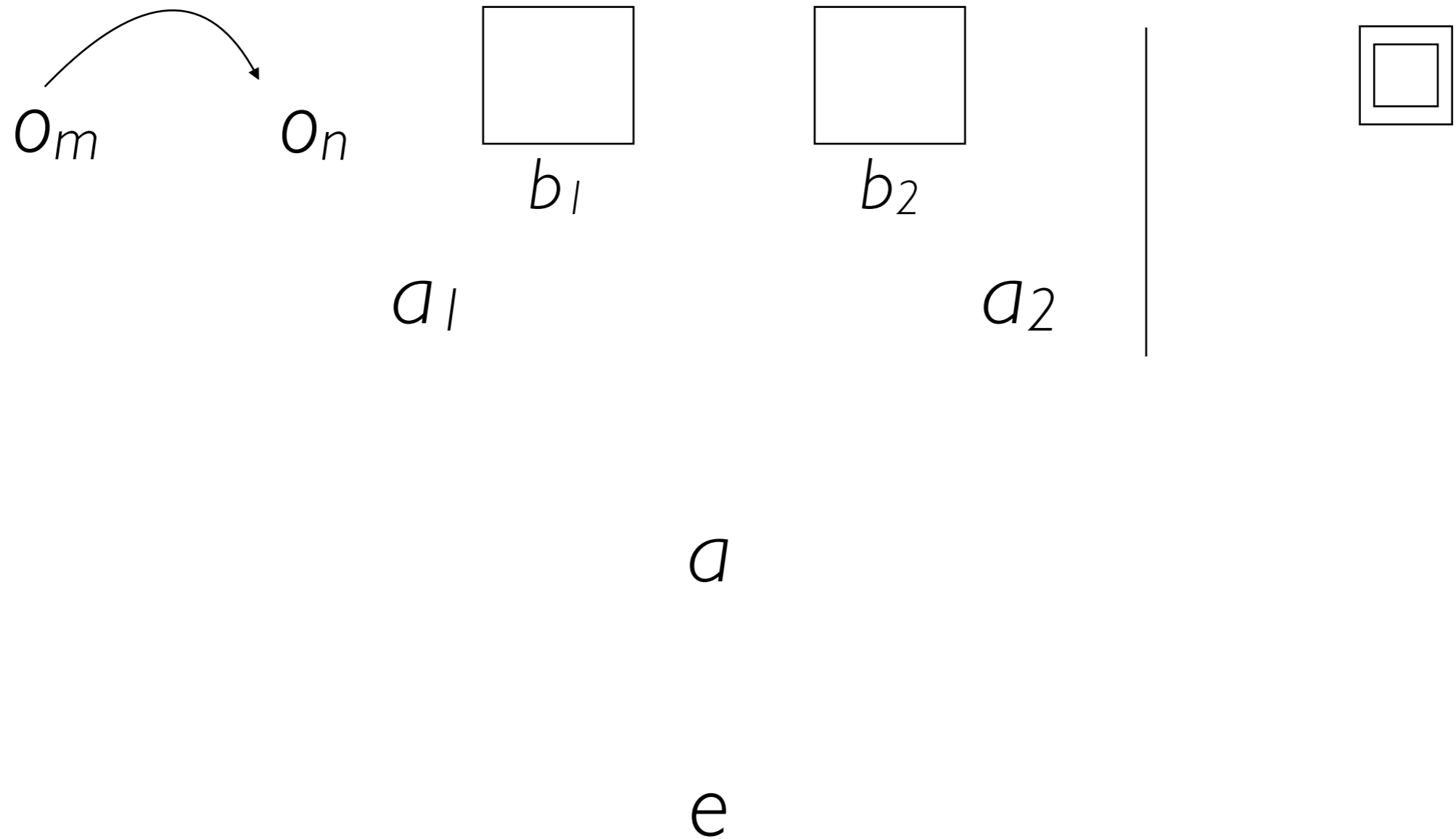
# Framework for $\text{FBT}^1_2$

(seven timepoints)



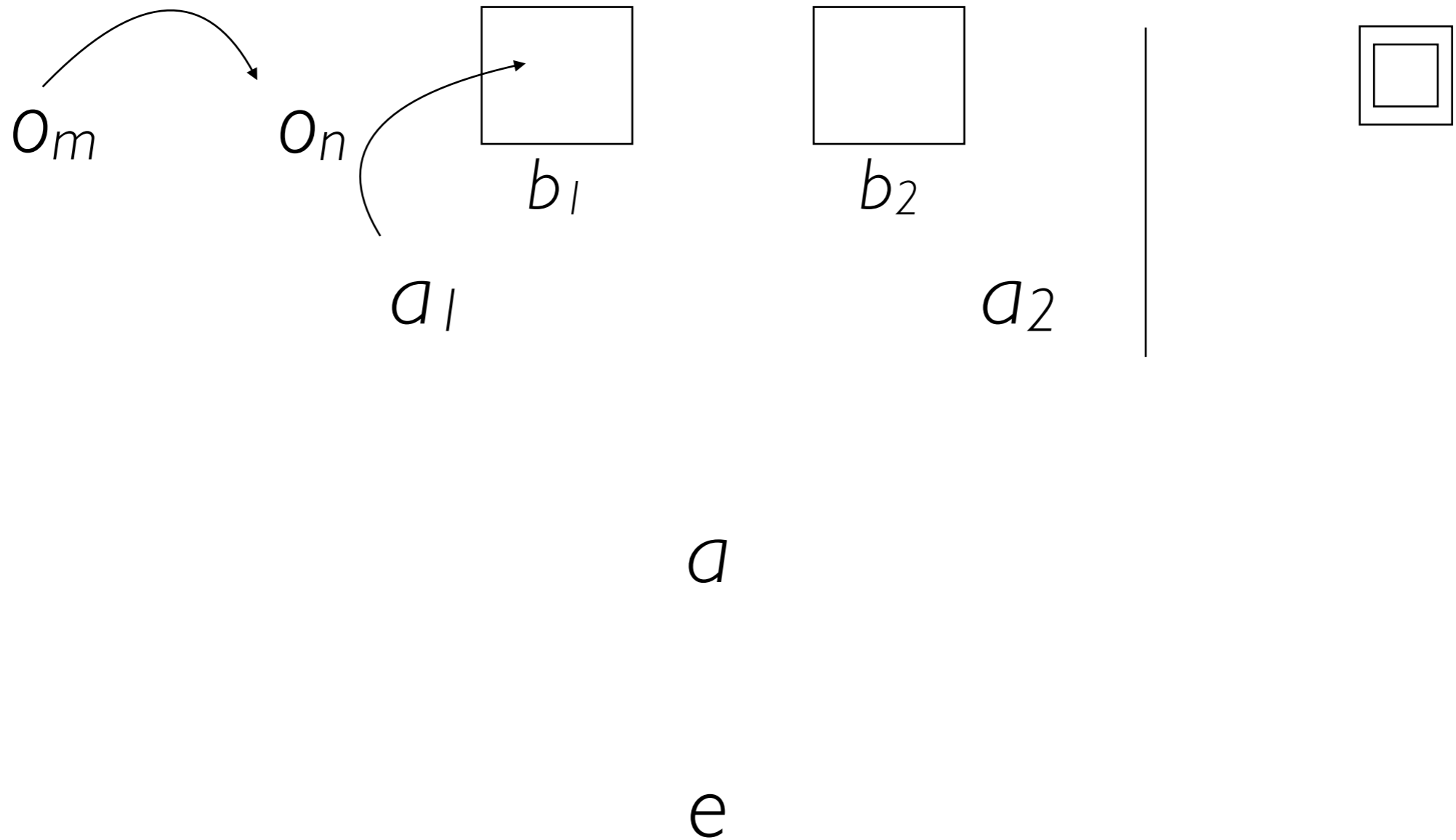
# Framework for FBT<sub>2</sub>

(seven timepoints)



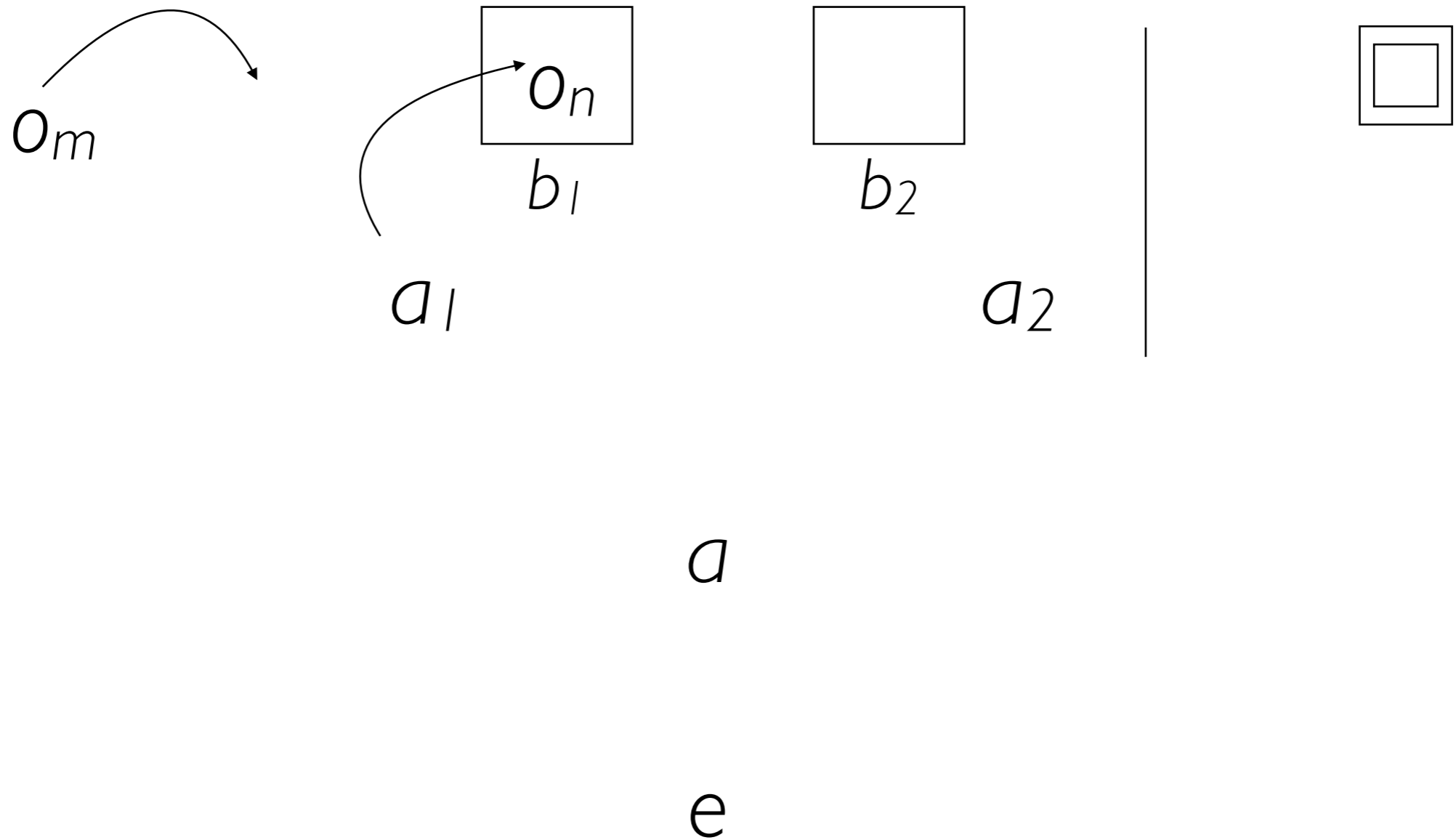
# Framework for $\text{FBT}^1_2$

(seven timepoints)



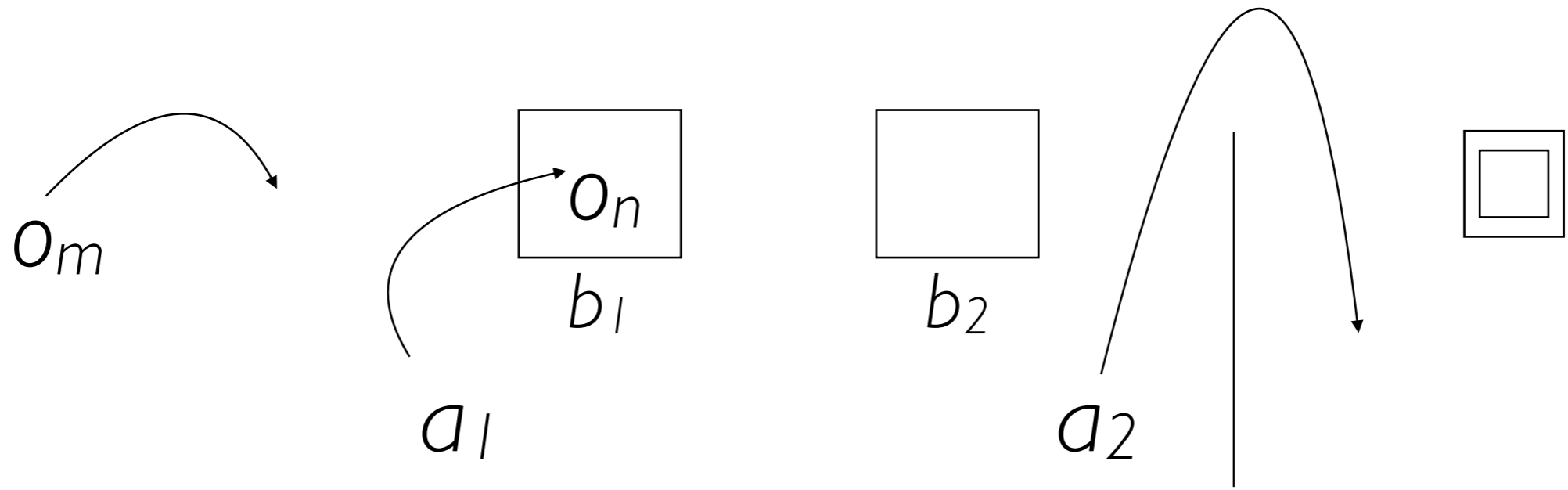
# Framework for $\text{FBT}^1_2$

(seven timepoints)



# Framework for $\text{FBT}^1_2$

(seven timepoints)

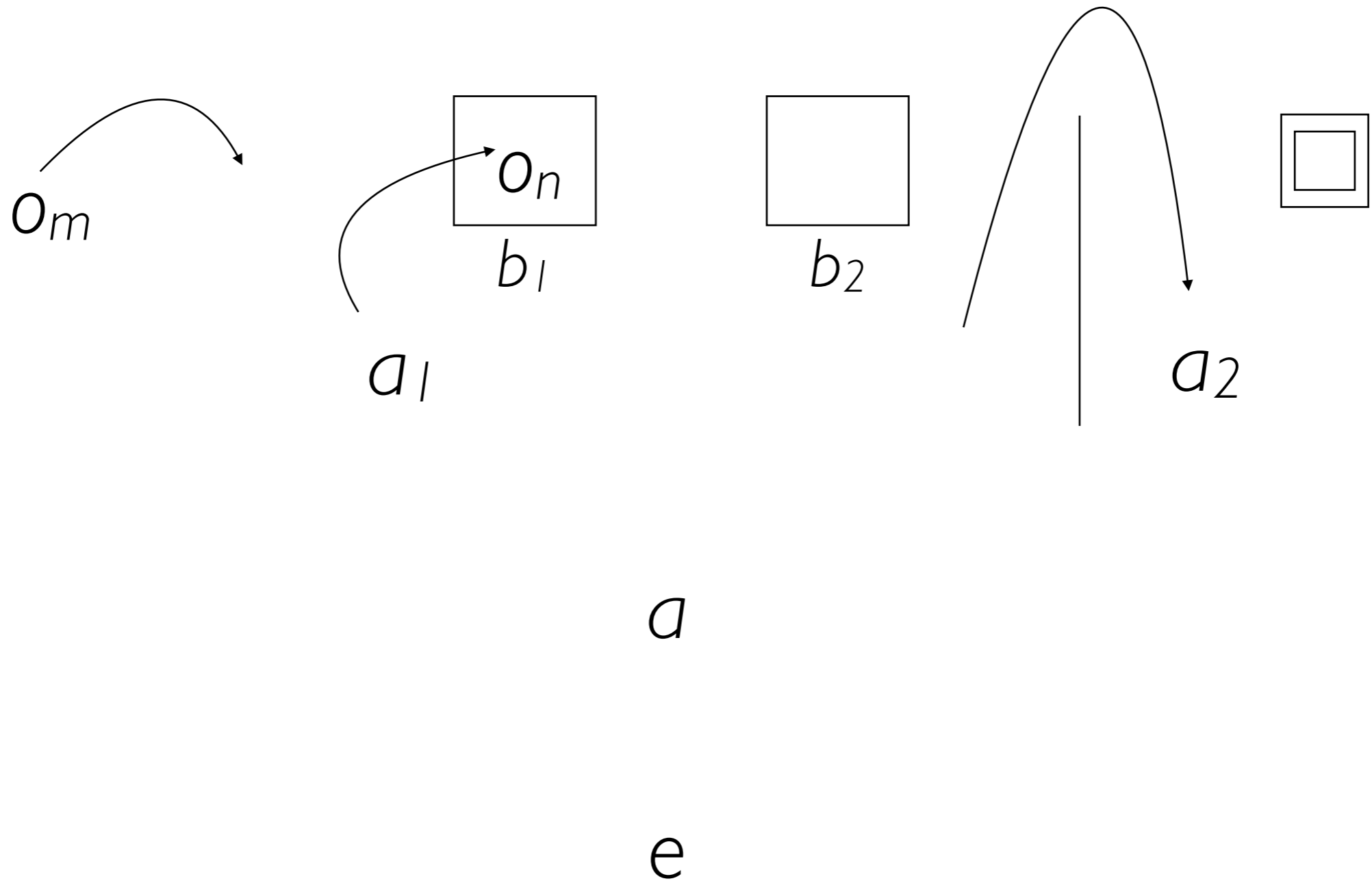


$a$

$e$

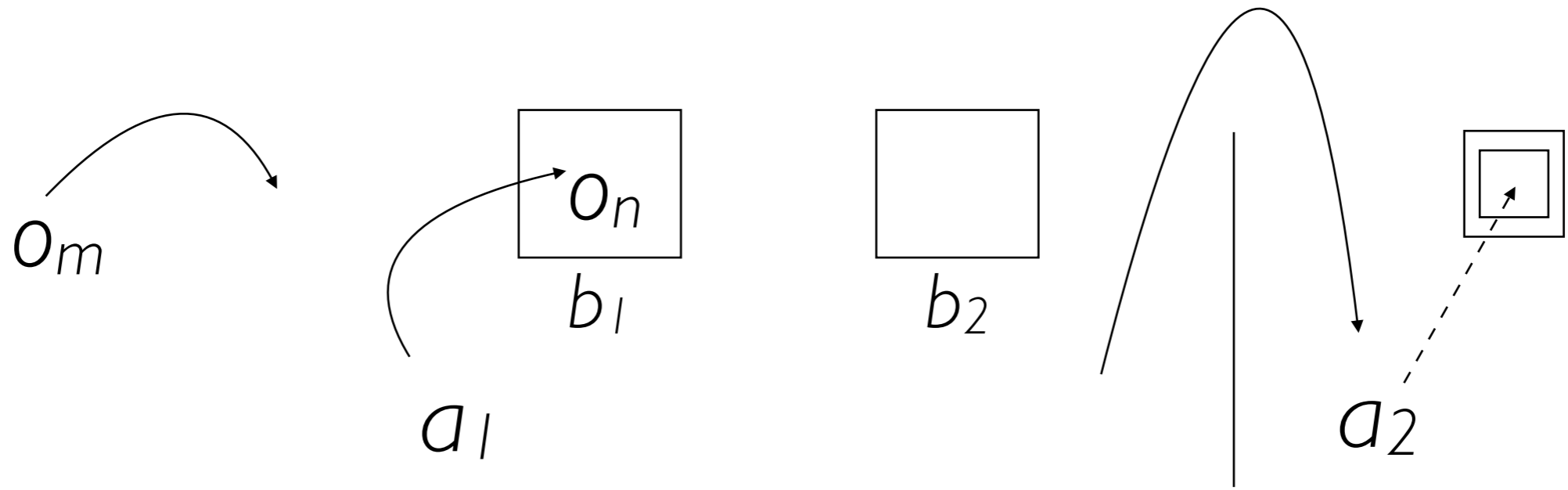
# Framework for $\text{FBT}^1_2$

(seven timepoints)



# Framework for $\text{FBT}^1_2$

(seven timepoints)

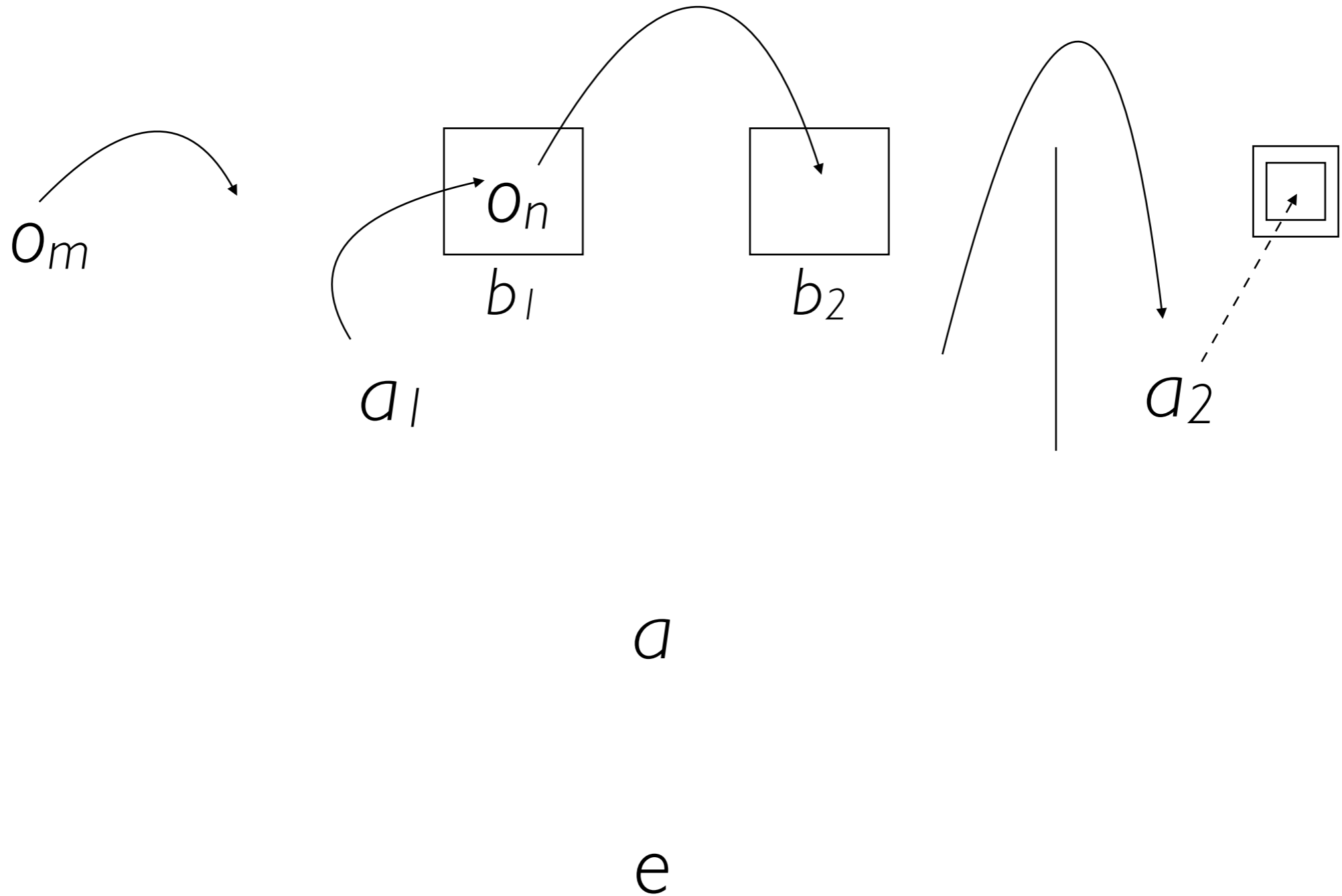


$a$

$e$

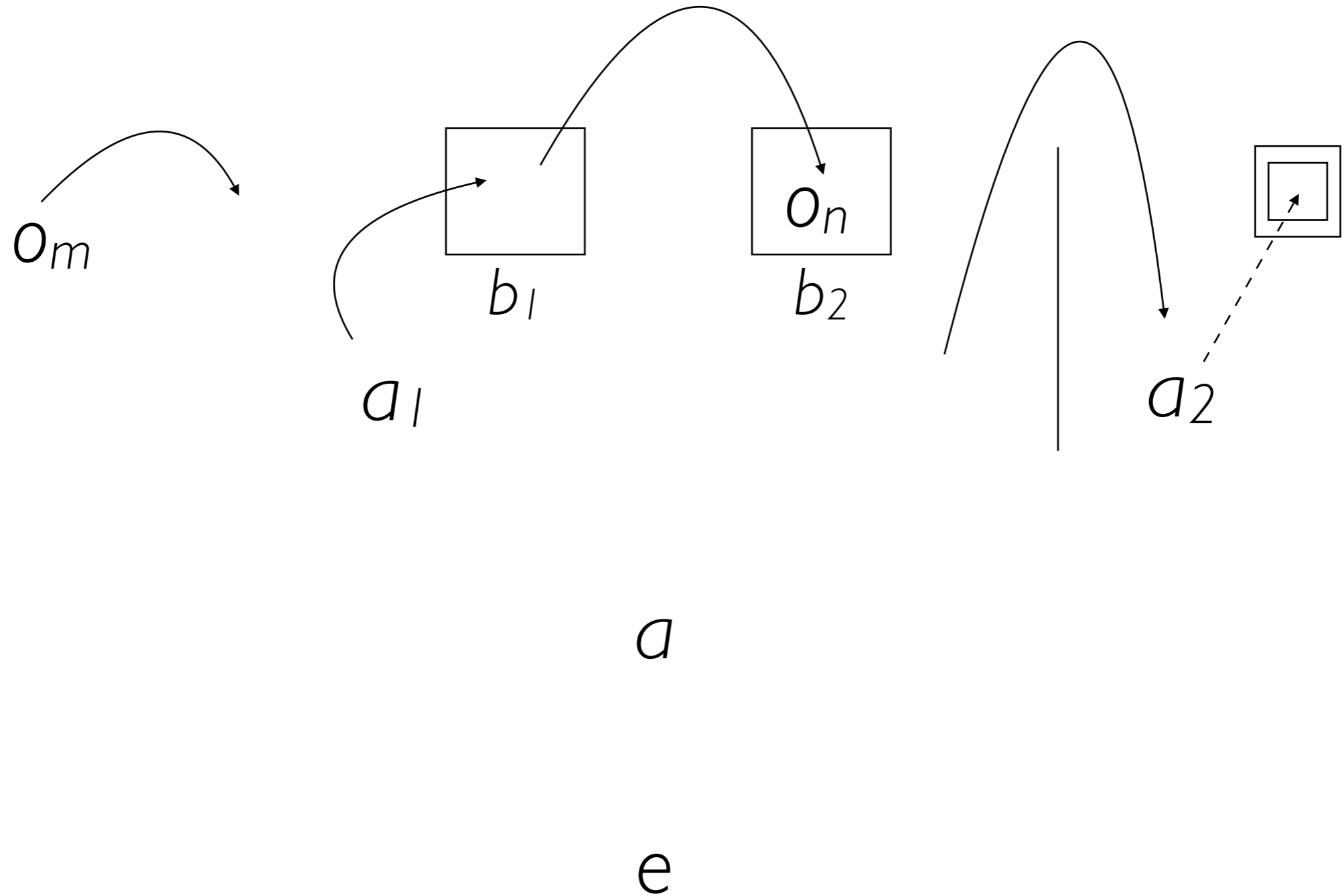
# Framework for $\text{FBT}^1_2$

(seven timepoints)



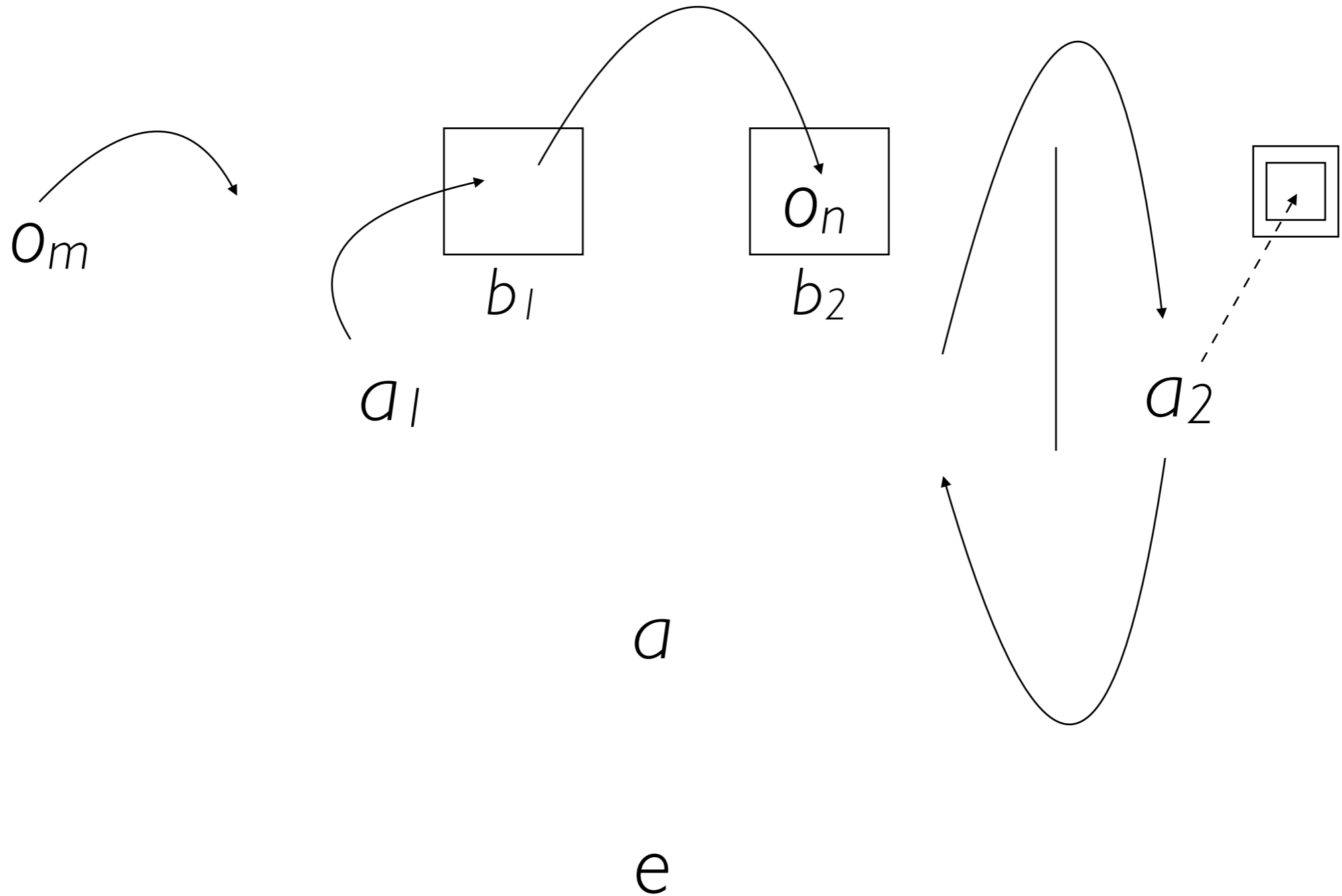
# Framework for $\text{FBT}^1_2$

(seven timepoints)



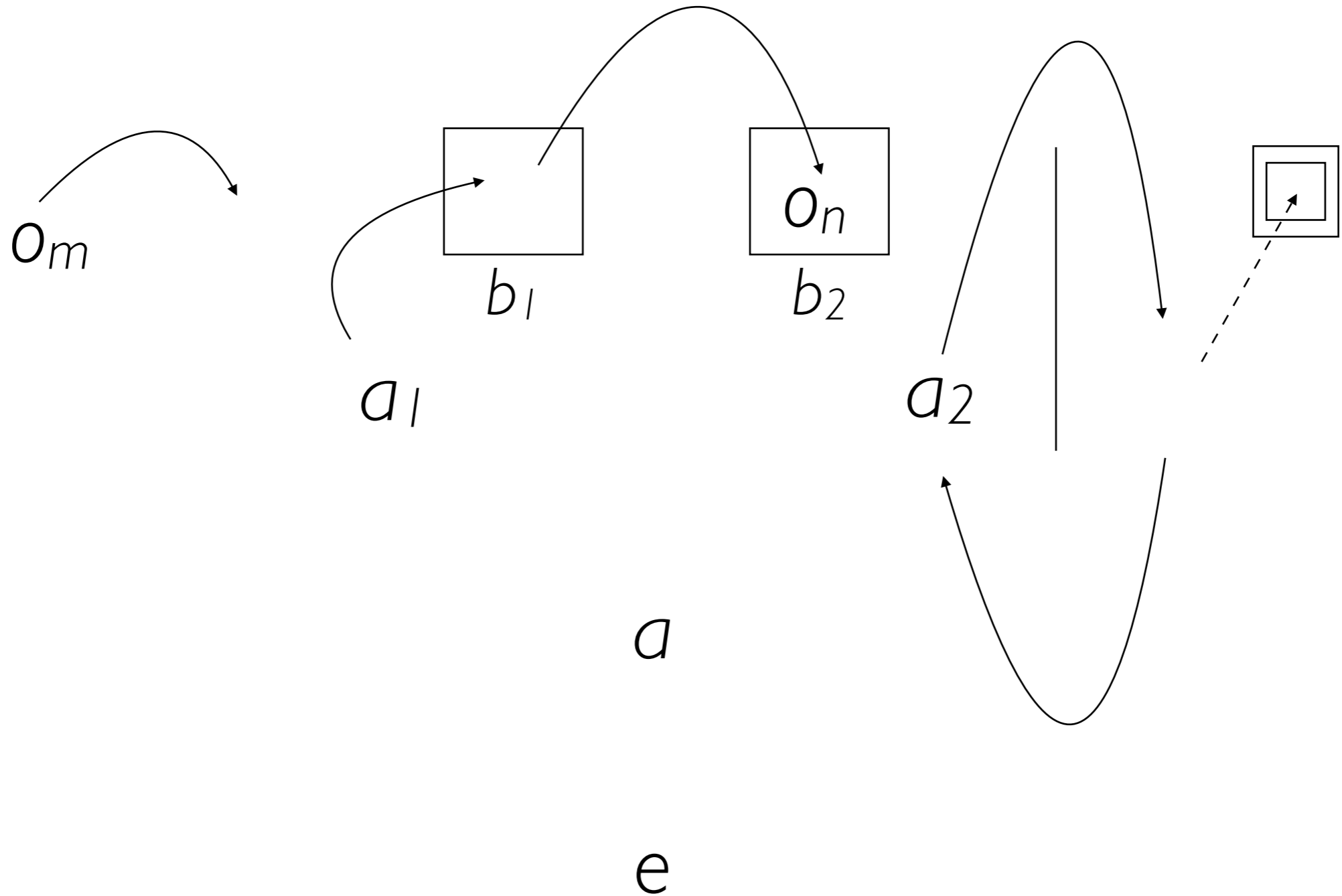
# Framework for $\text{FBT}^1_2$

(seven timepoints)



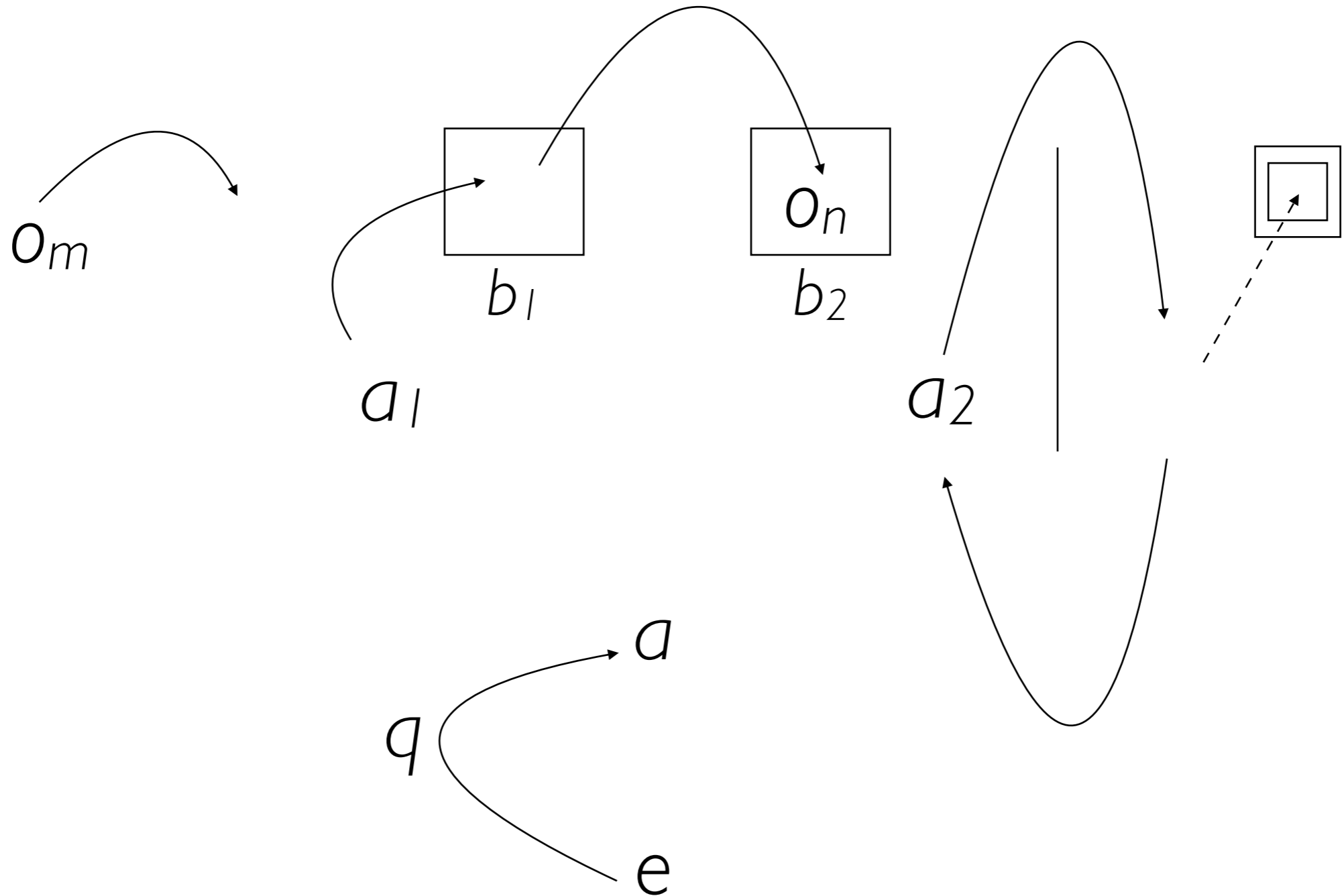
# Framework for $\text{FBT}^1_2$

(seven timepoints)



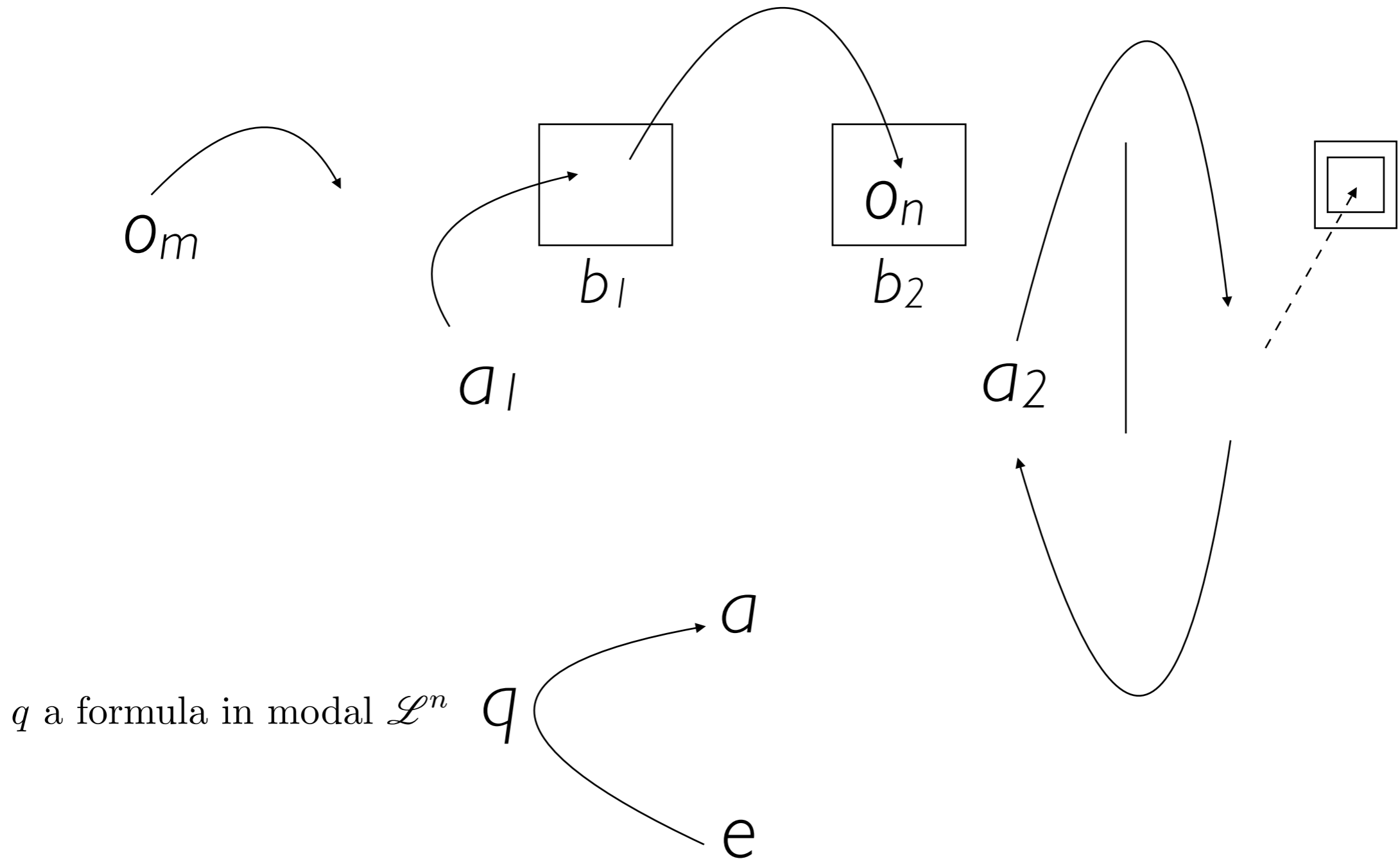
# Framework for $\text{FBT}^1_2$

(seven timepoints)

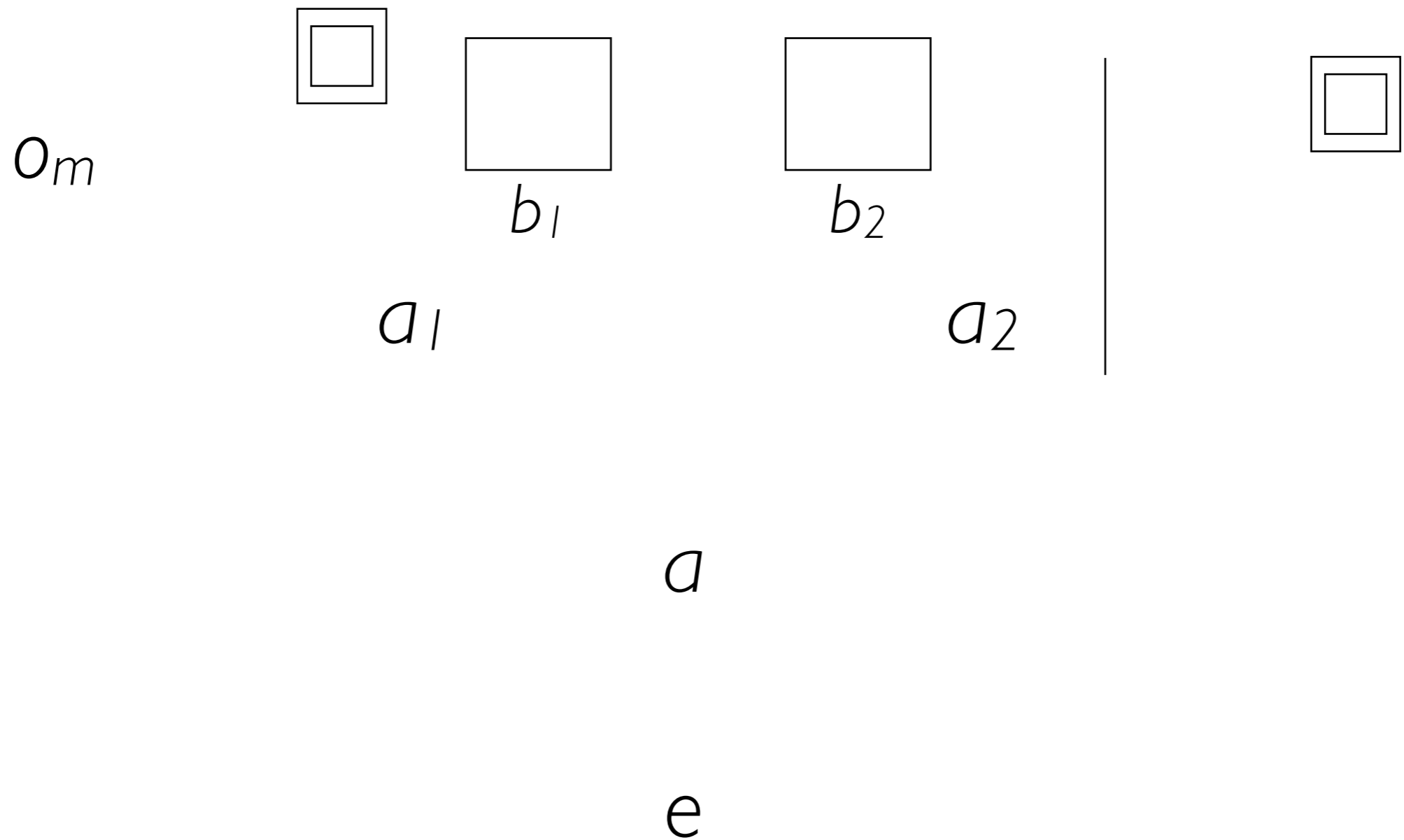


# Framework for $\text{FBT}^1_2$

(seven timepoints)

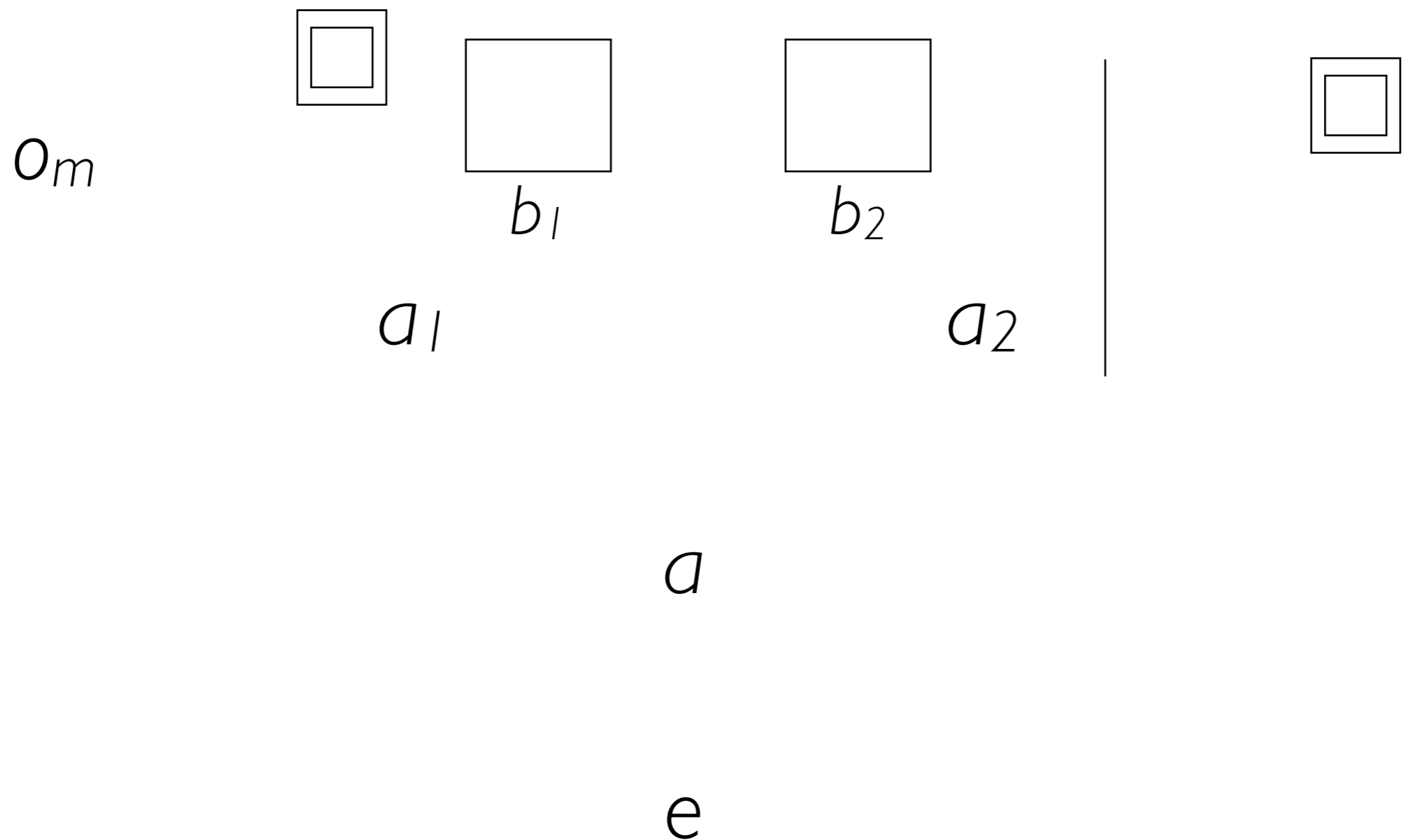


# Framework for $\text{FBT}^1_3$



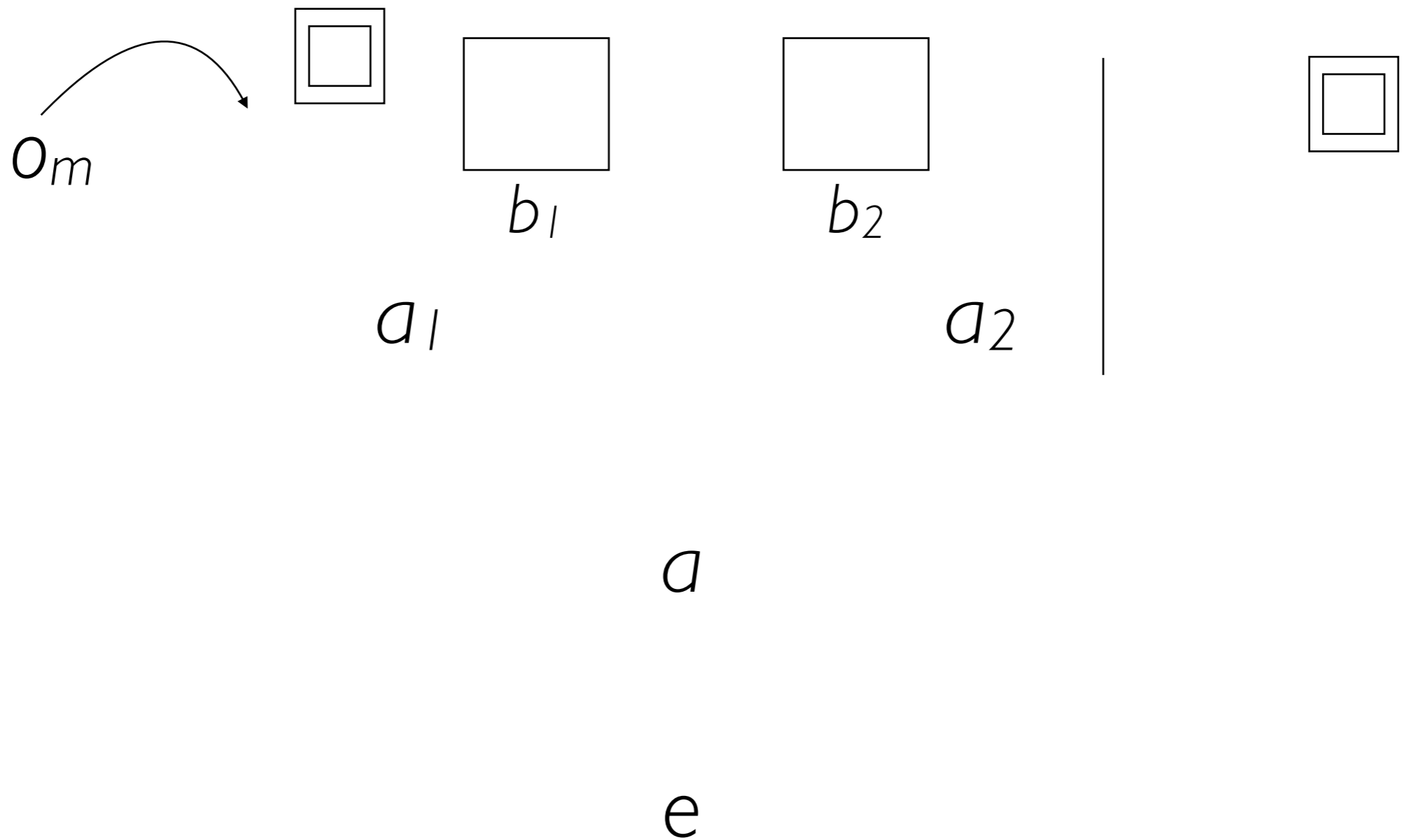
# Framework for $\text{FBT}^1_3$

(eight timepoints)



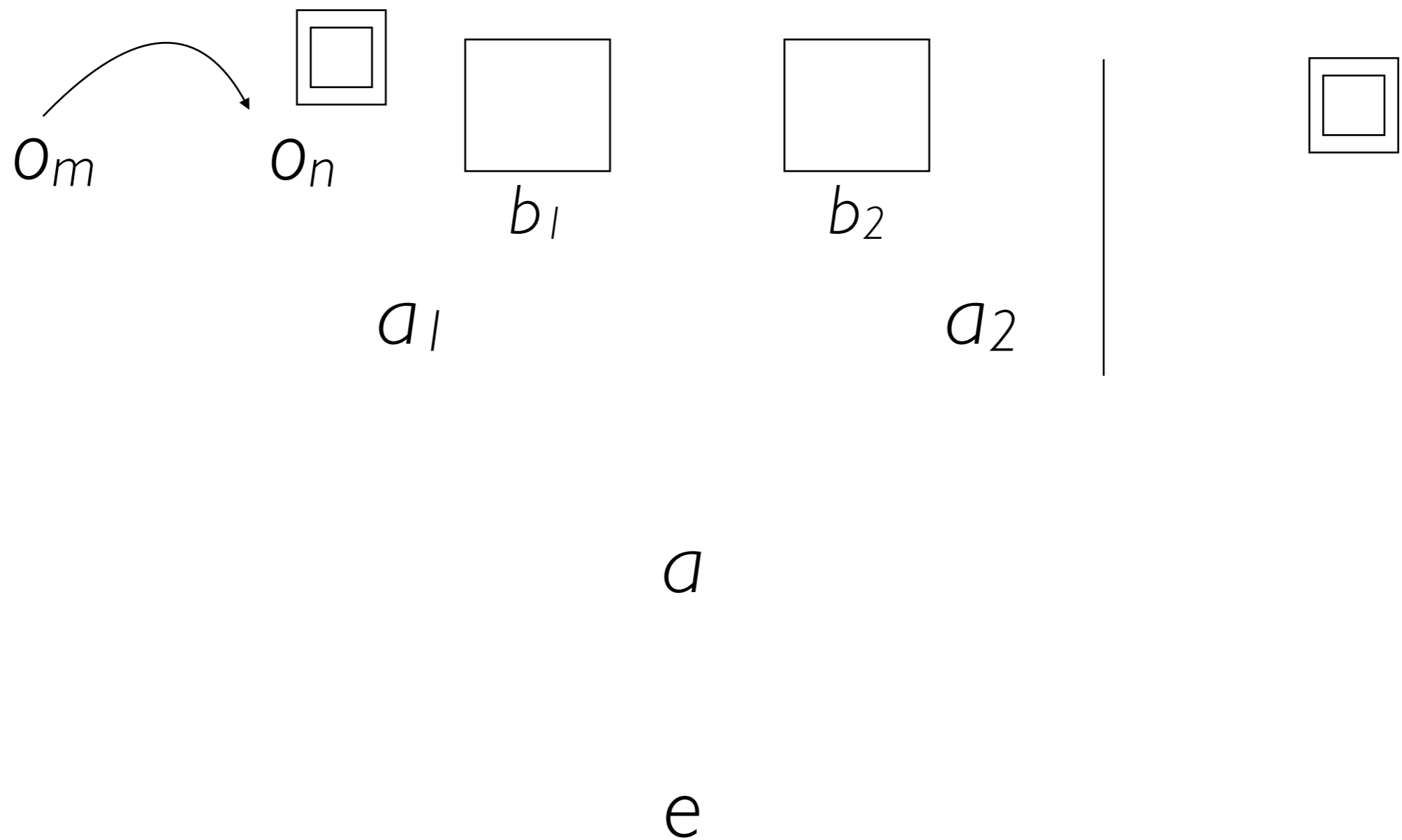
# Framework for $\text{FBT}^1_3$

(eight timepoints)



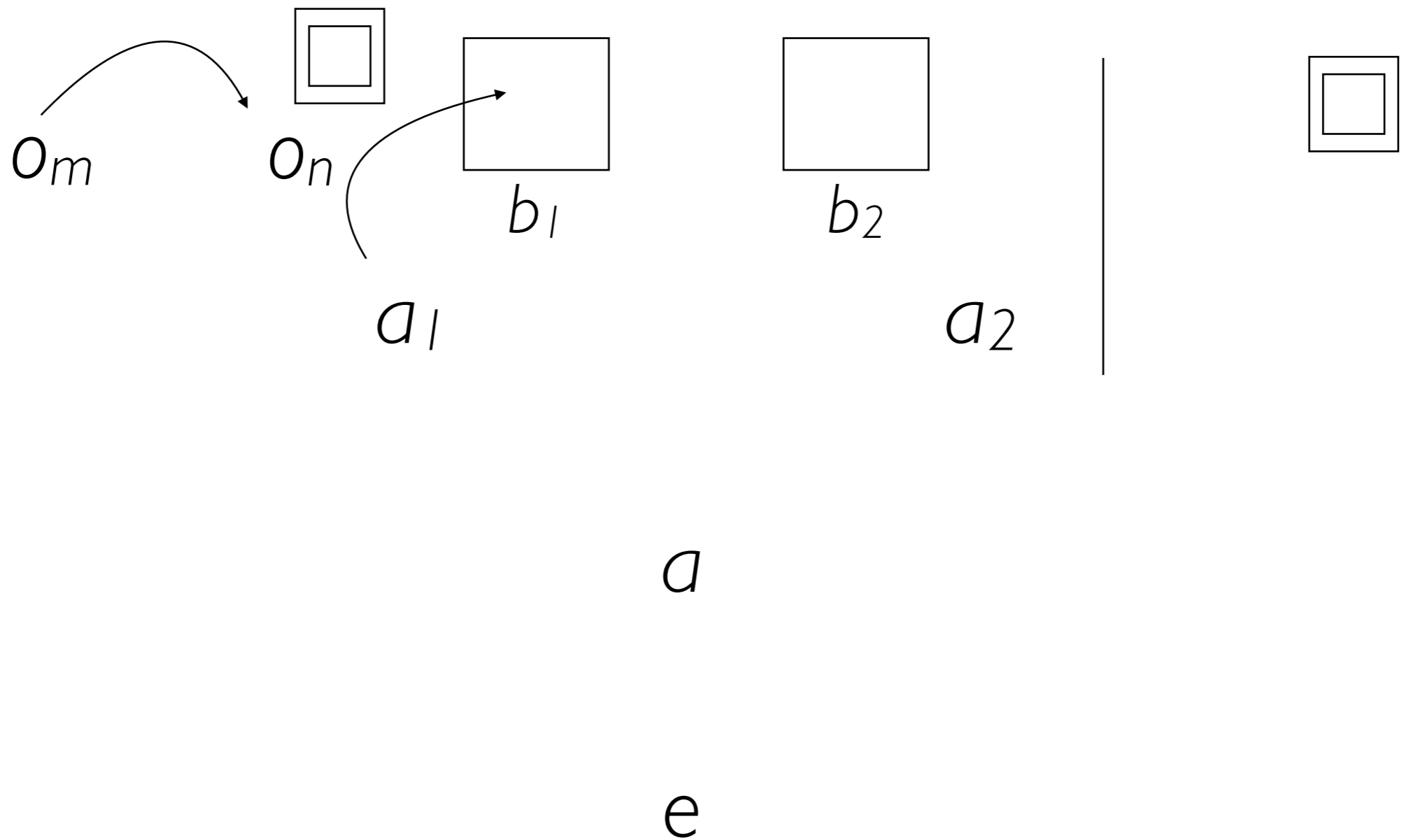
# Framework for $\text{FBT}^1_3$

(eight timepoints)



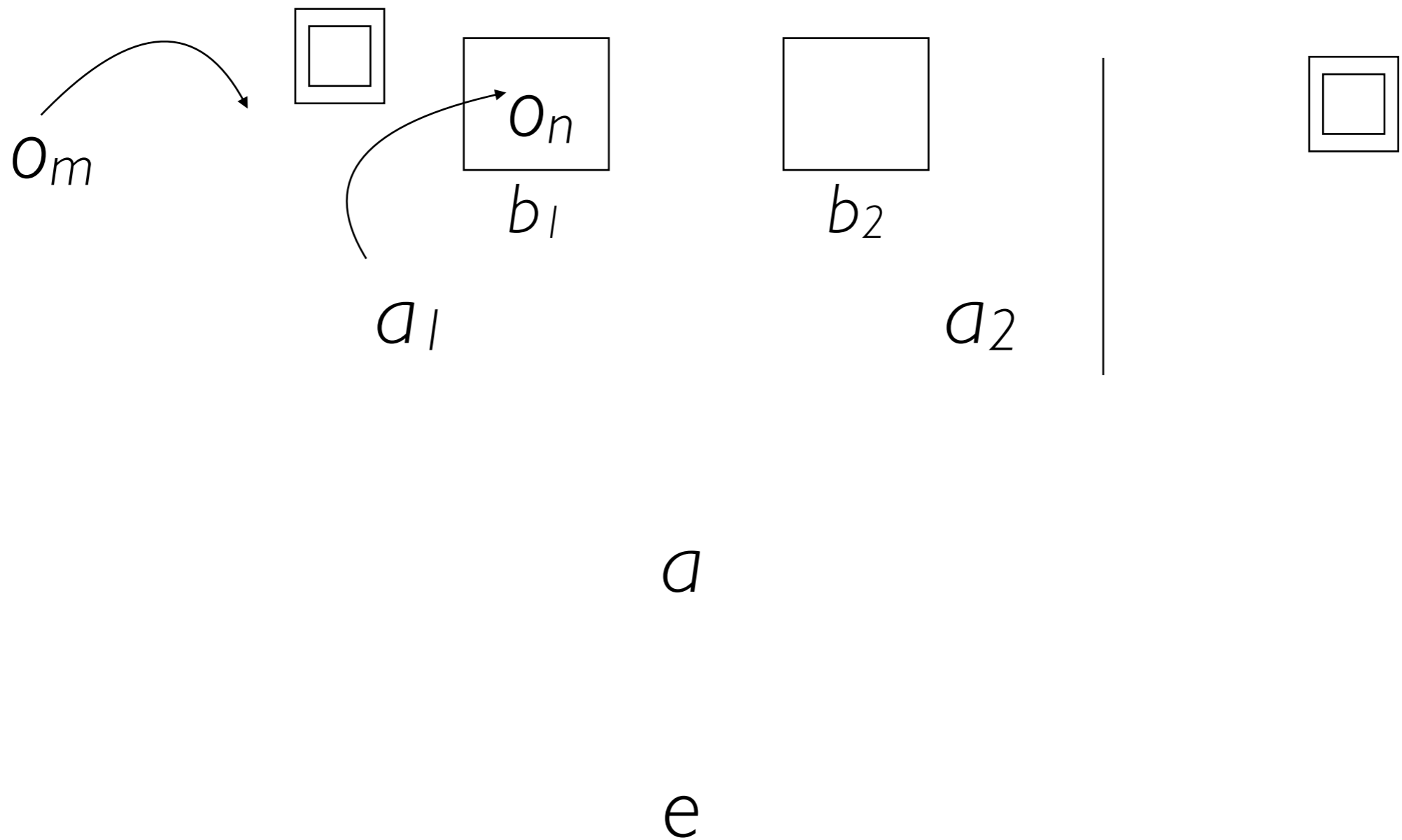
# Framework for $\text{FBT}^1_3$

(eight timepoints)



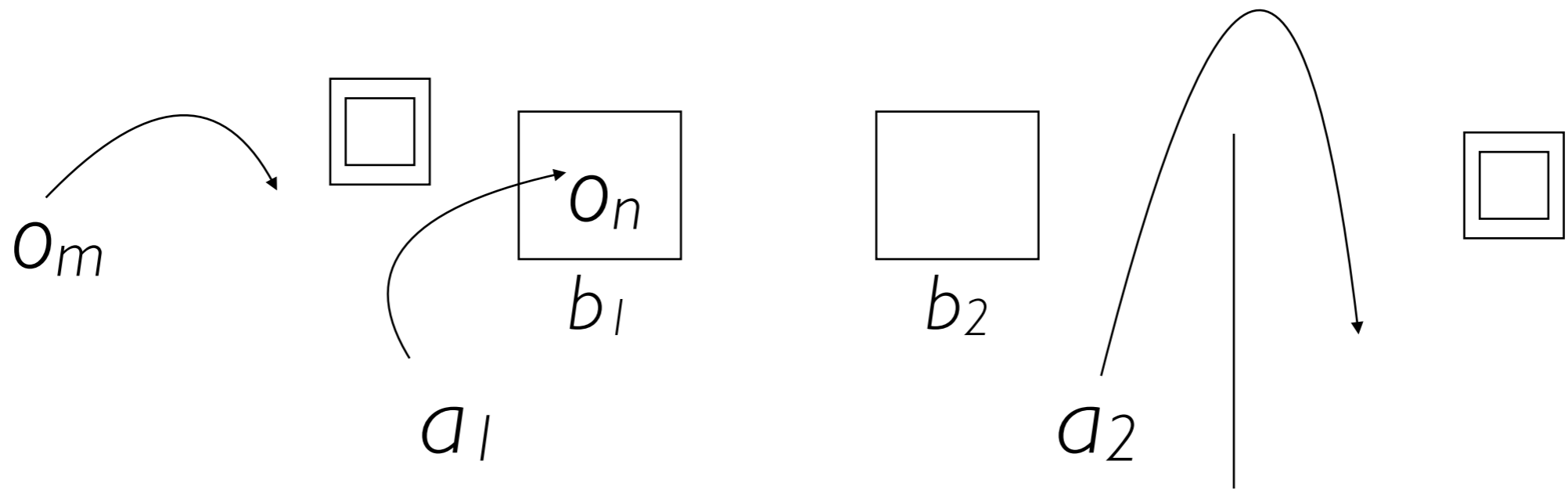
# Framework for $\text{FBT}^1_3$

(eight timepoints)



# Framework for $\text{FBT}^1_3$

(eight timepoints)

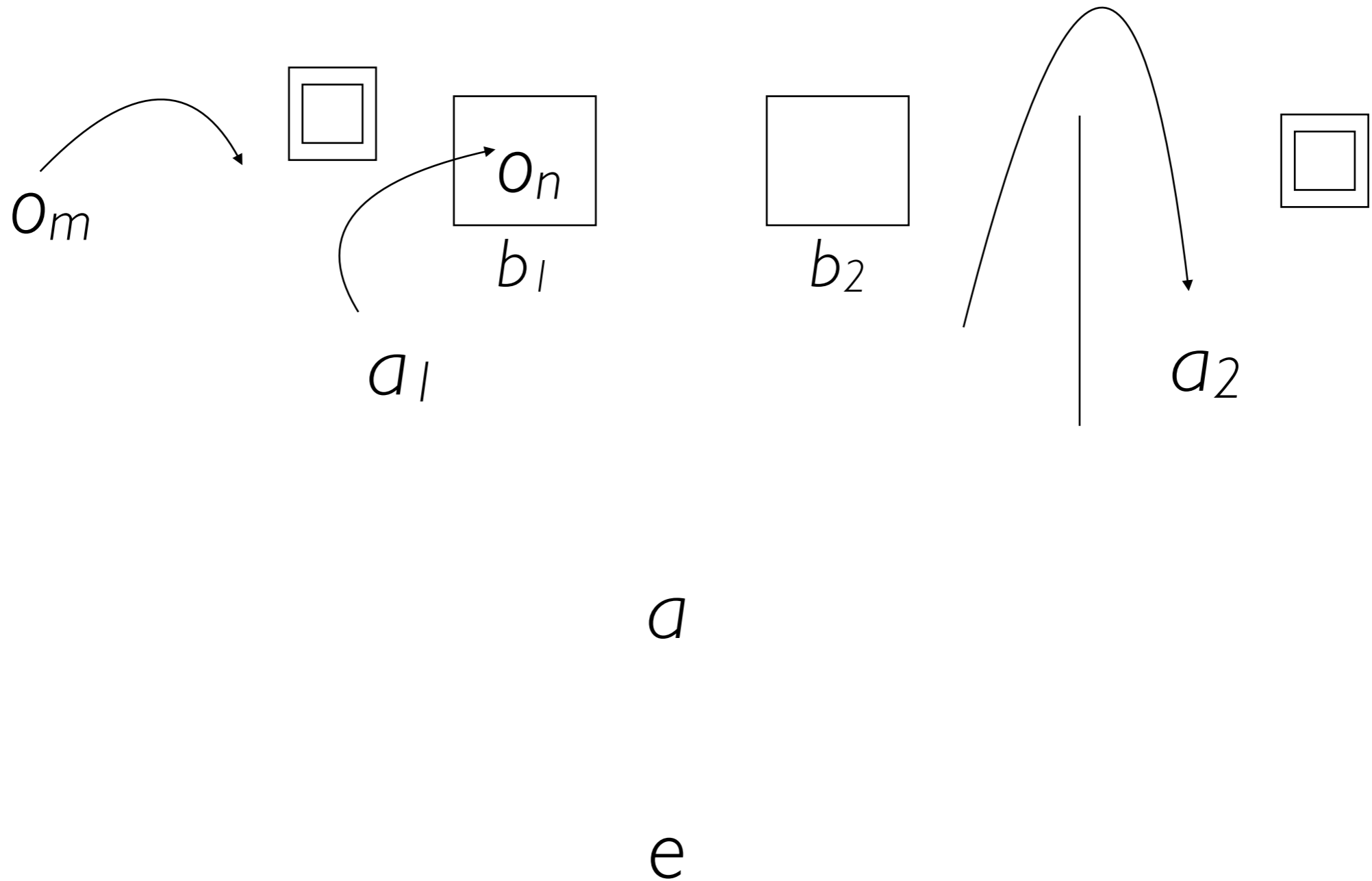


$a$

$e$

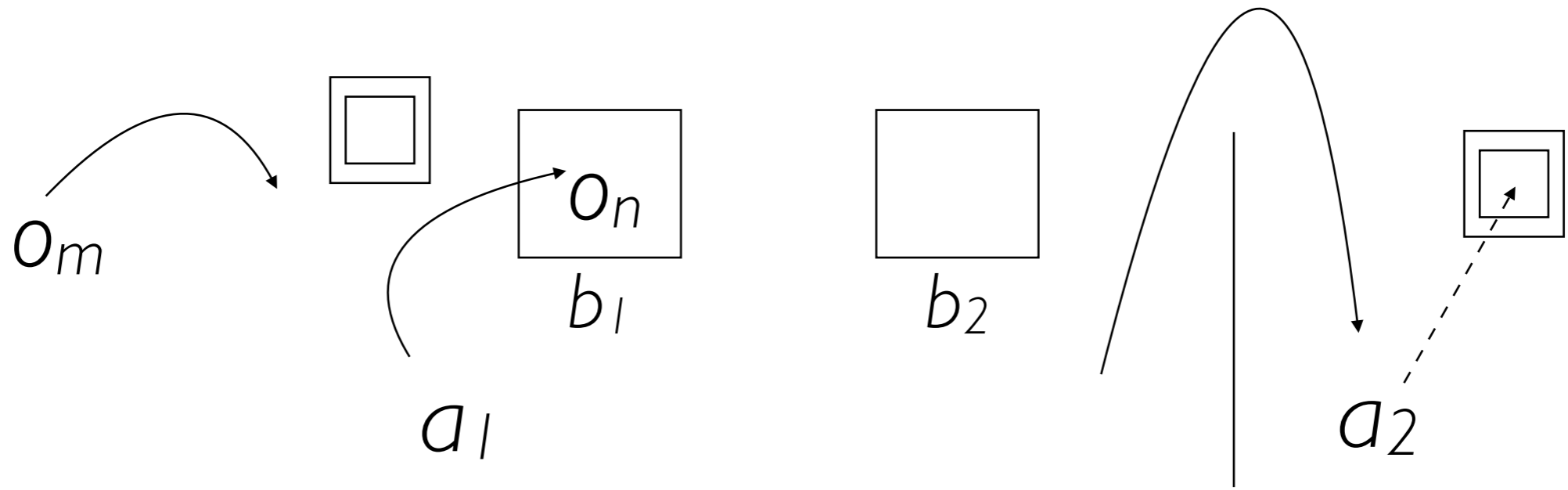
# Framework for $\text{FBT}^1_3$

(eight timepoints)



# Framework for $\text{FBT}^1_3$

(eight timepoints)

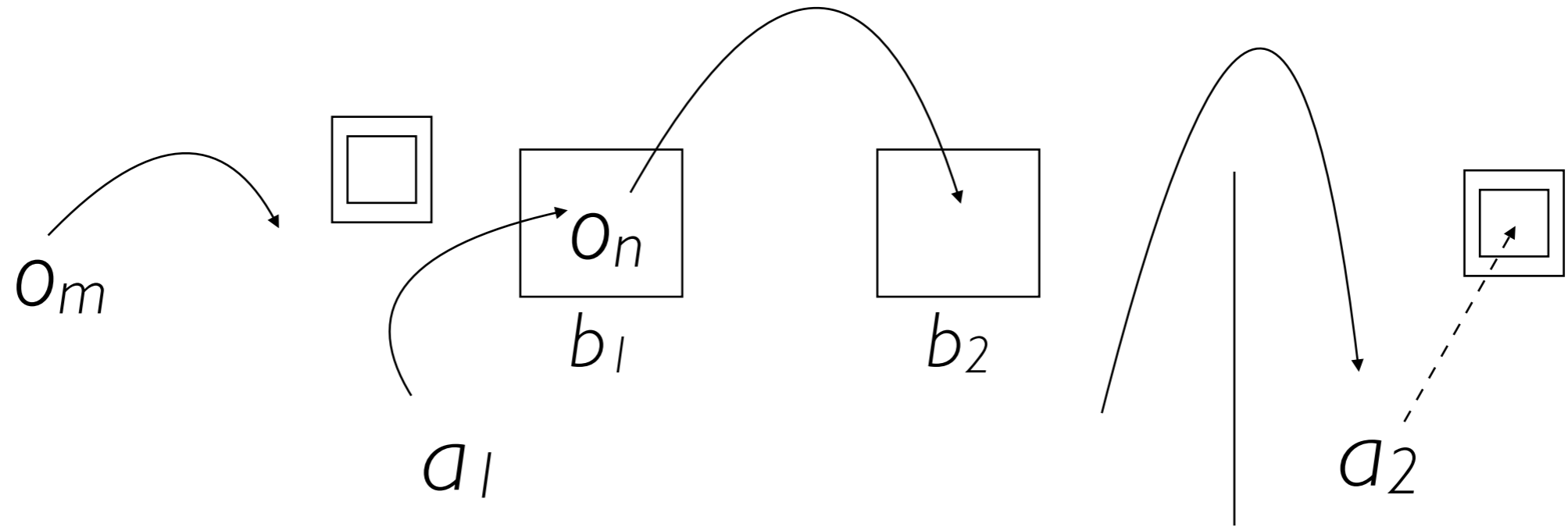


$a$

$e$

# Framework for $\text{FBT}^1_3$

(eight timepoints)

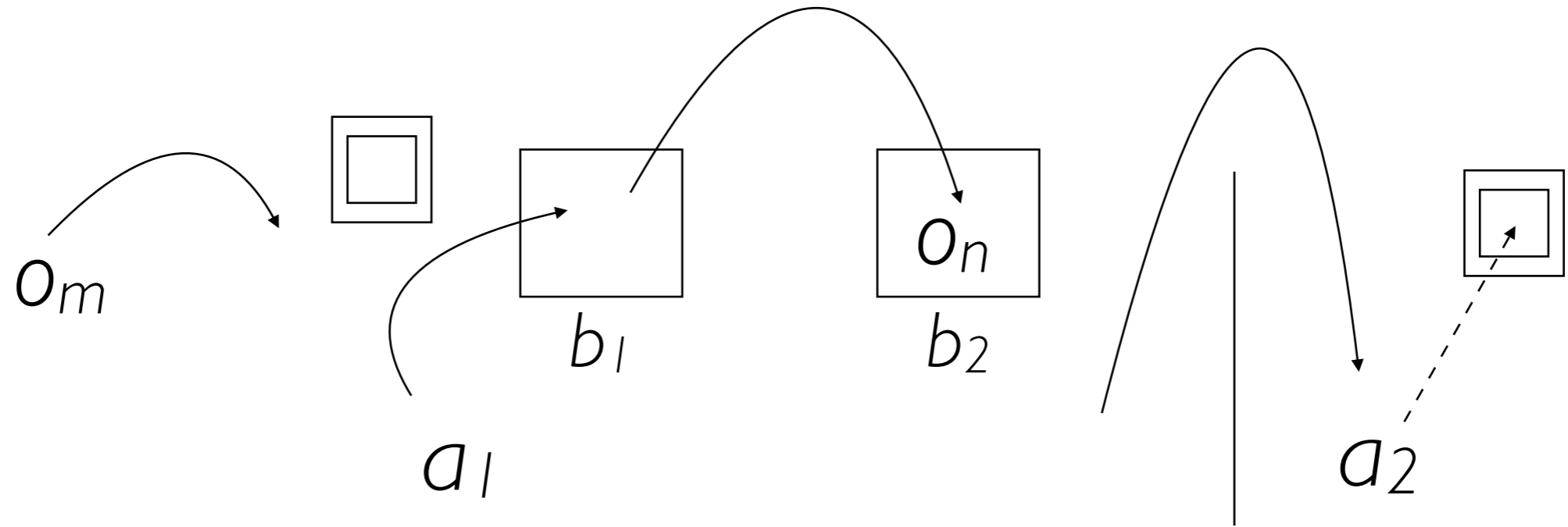


$a$

$e$

# Framework for $\text{FBT}^1_3$

(eight timepoints)

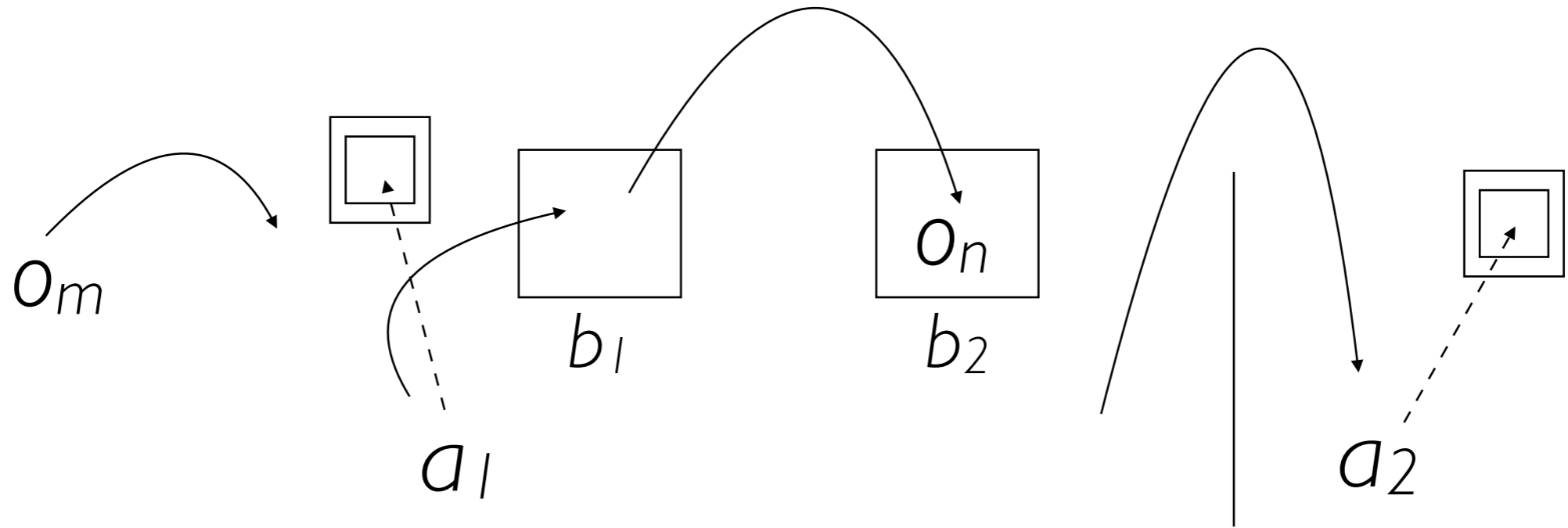


$a$

$e$

# Framework for $\text{FBT}^1_3$

(eight timepoints)

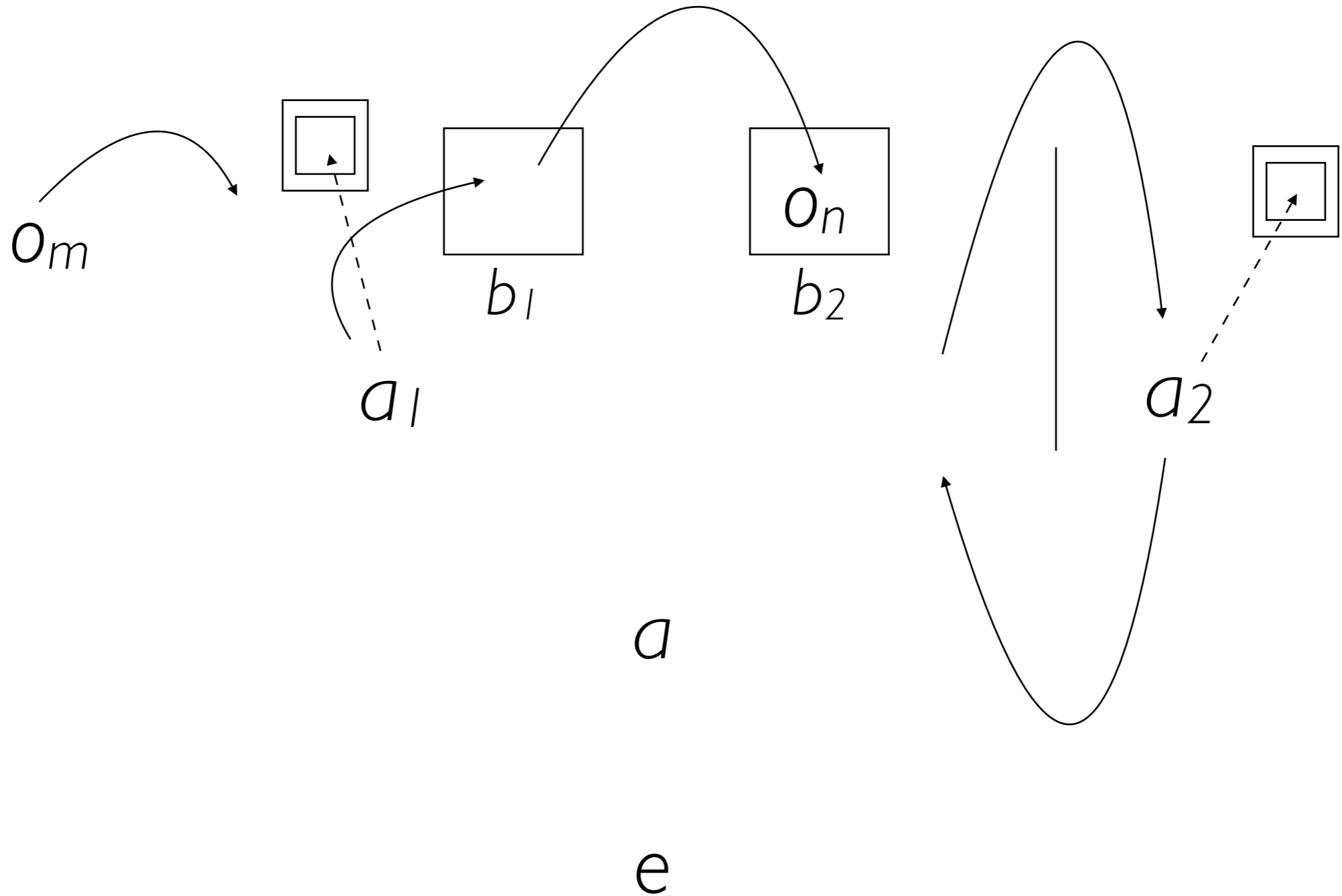


$a$

$e$

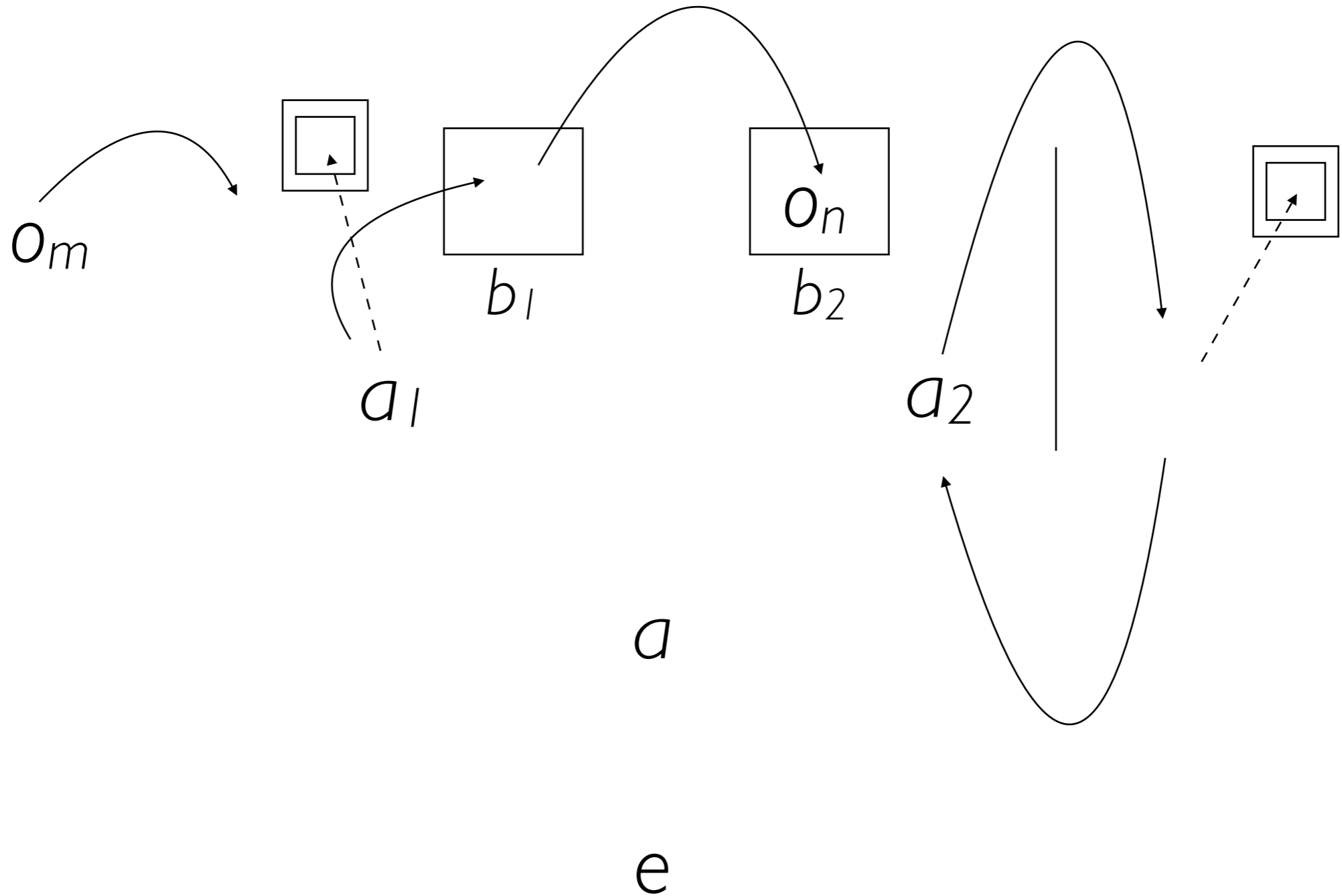
# Framework for $\text{FBT}^1_3$

(eight timepoints)



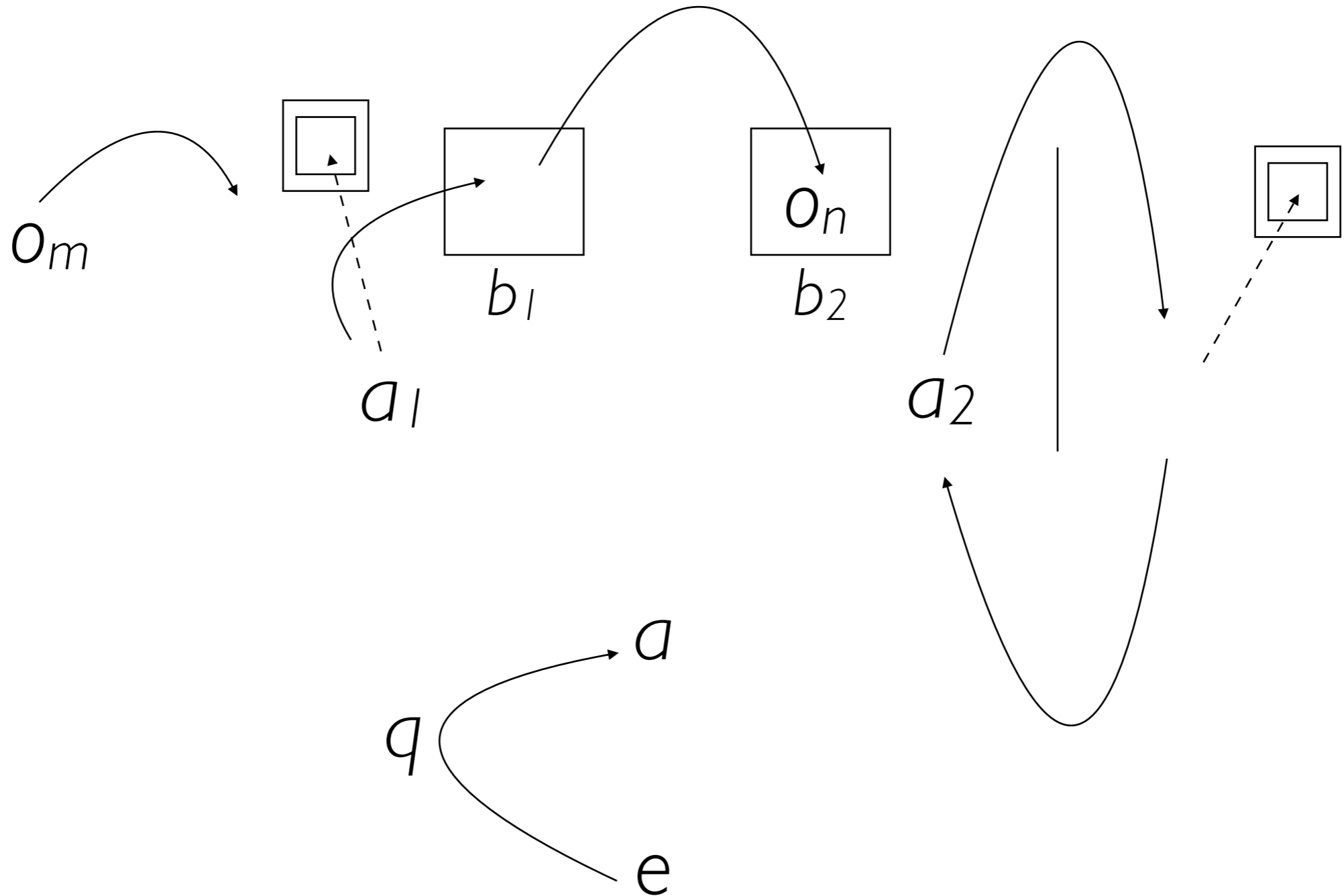
# Framework for $\text{FBT}^1_3$

(eight timepoints)



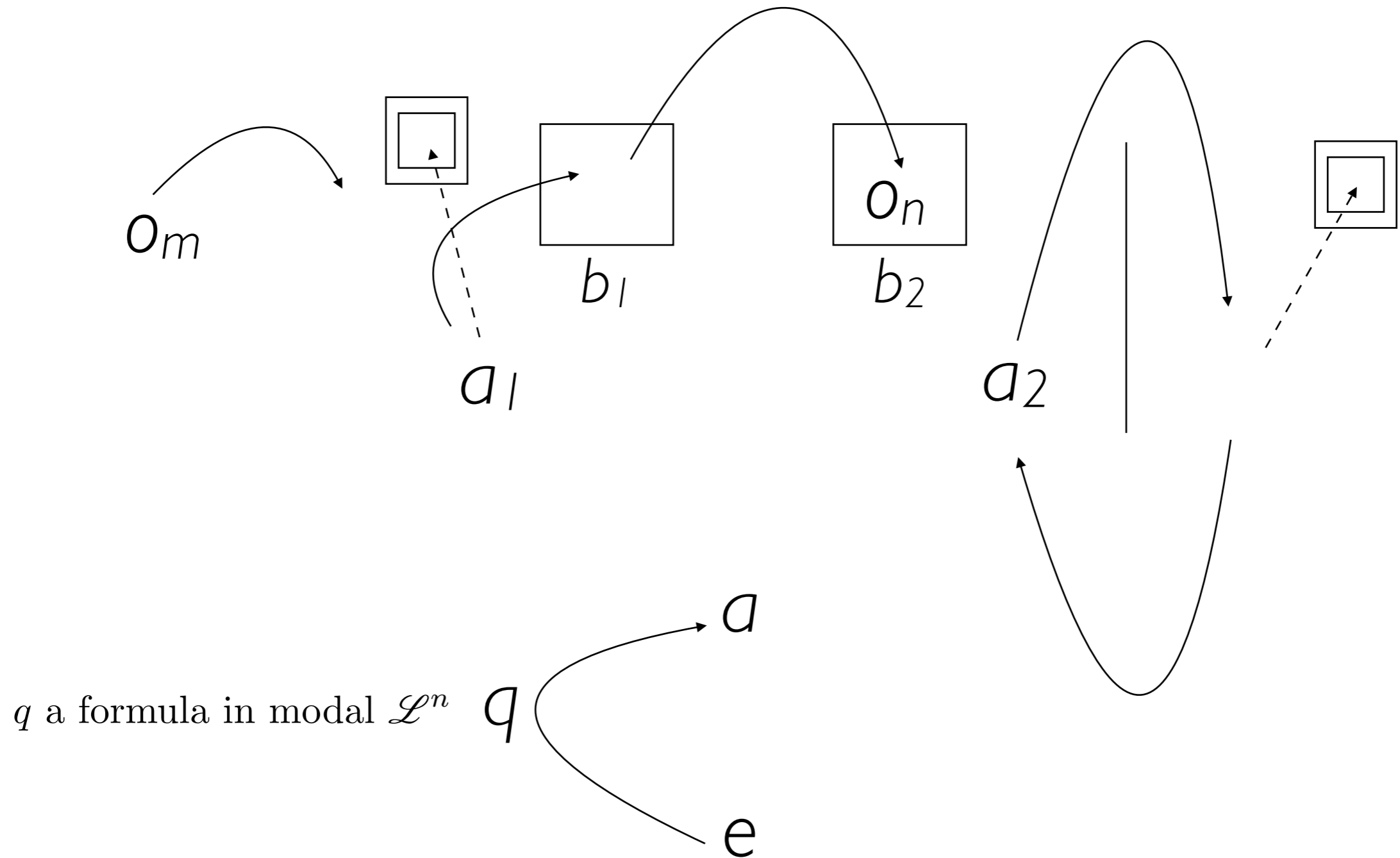
# Framework for $\text{FBT}^1_3$

(eight timepoints)

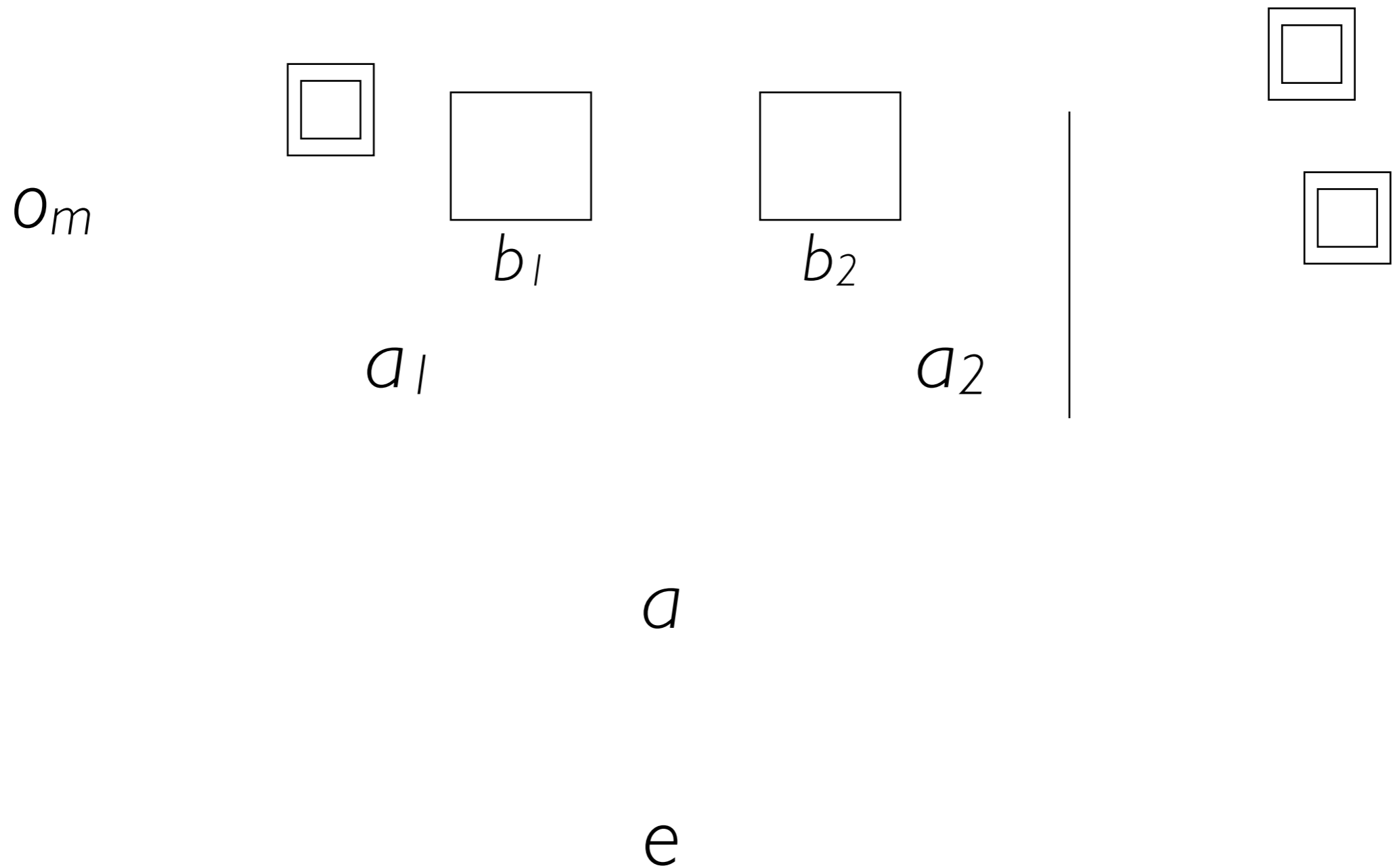


# Framework for $\text{FBT}^1_3$

(eight timepoints)

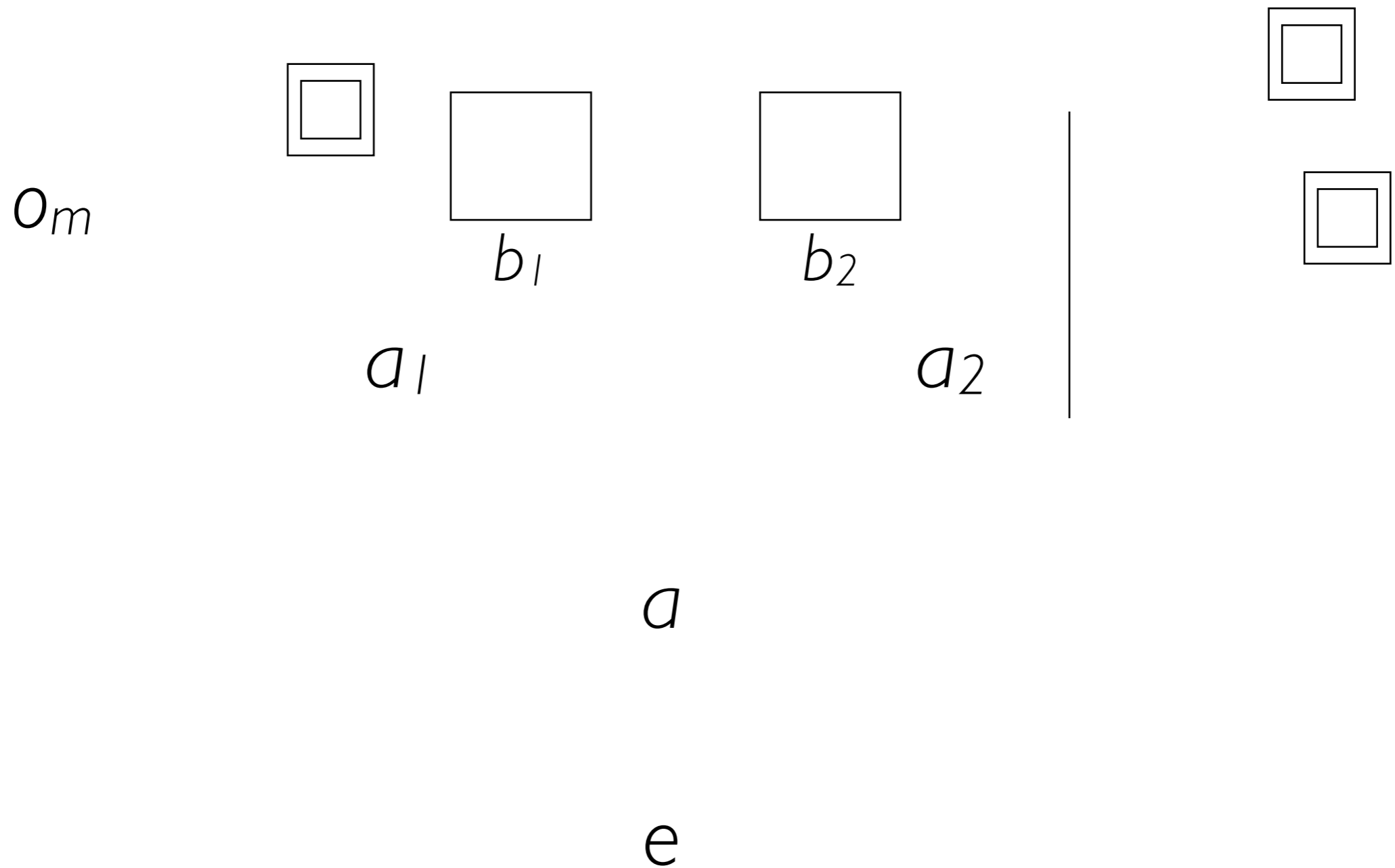


# Framework for $\text{FBT}^1_4$



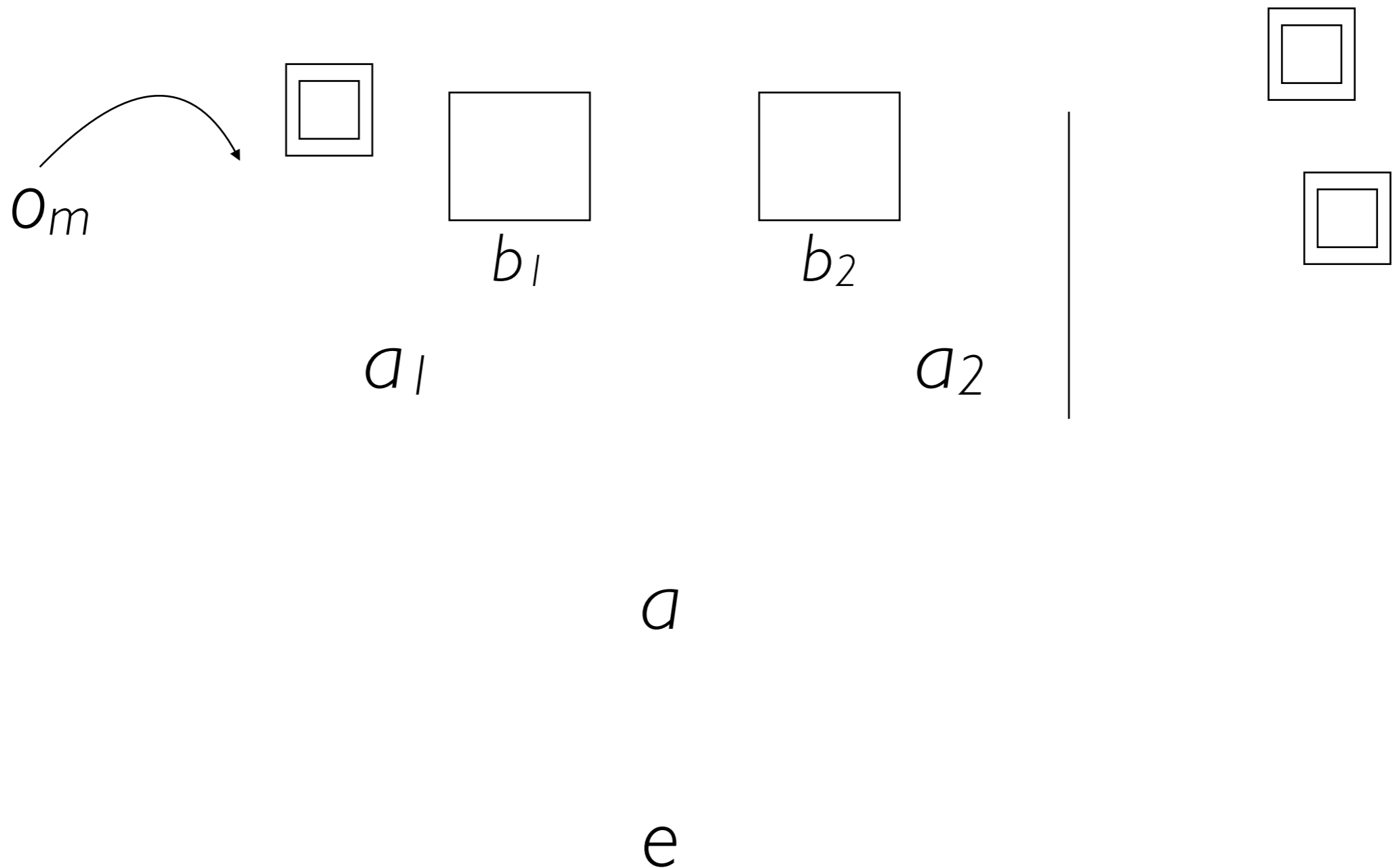
# Framework for $\text{FBT}^1_4$

(nine timepoints)



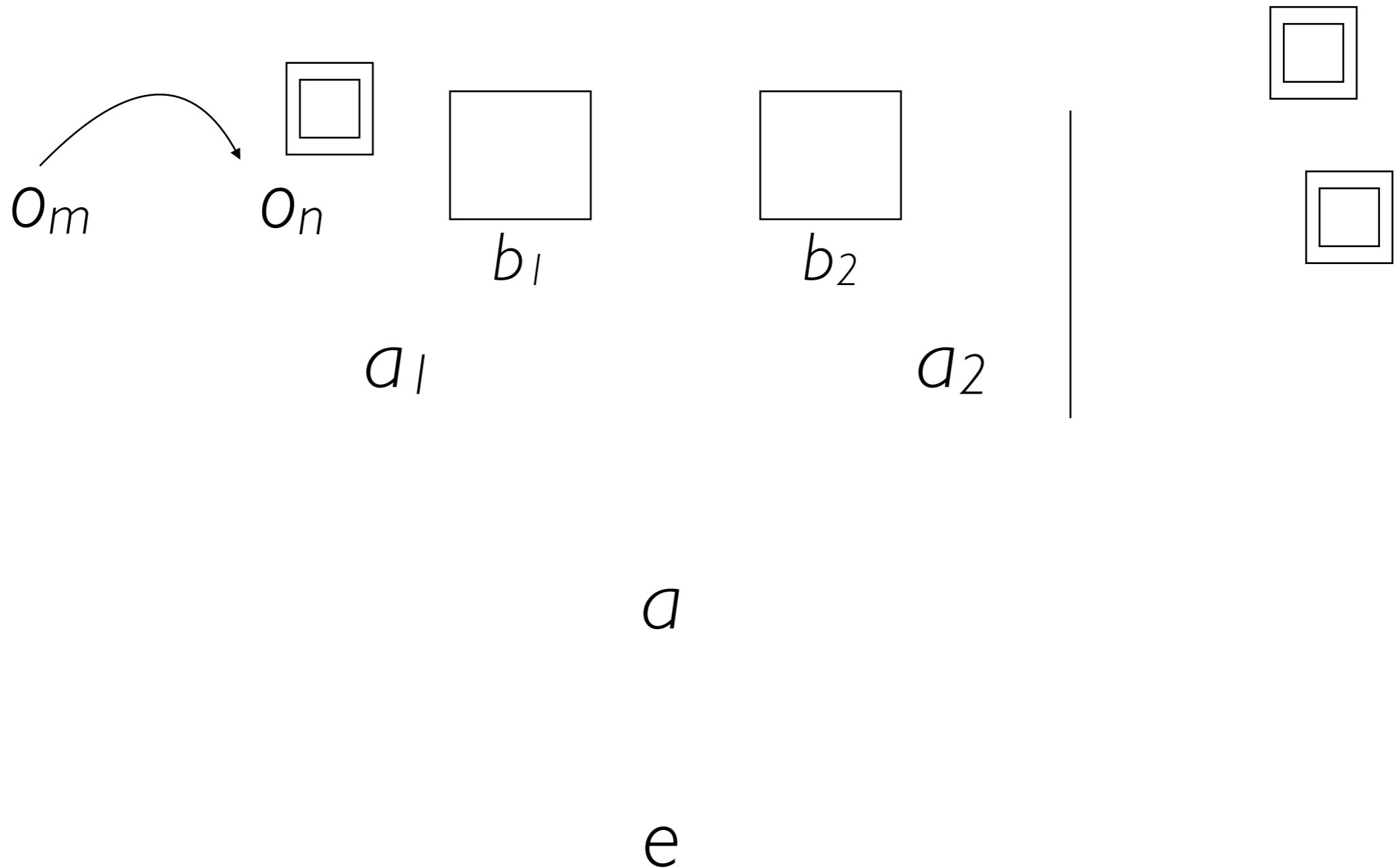
# Framework for $\text{FBT}^1_4$

(nine timepoints)



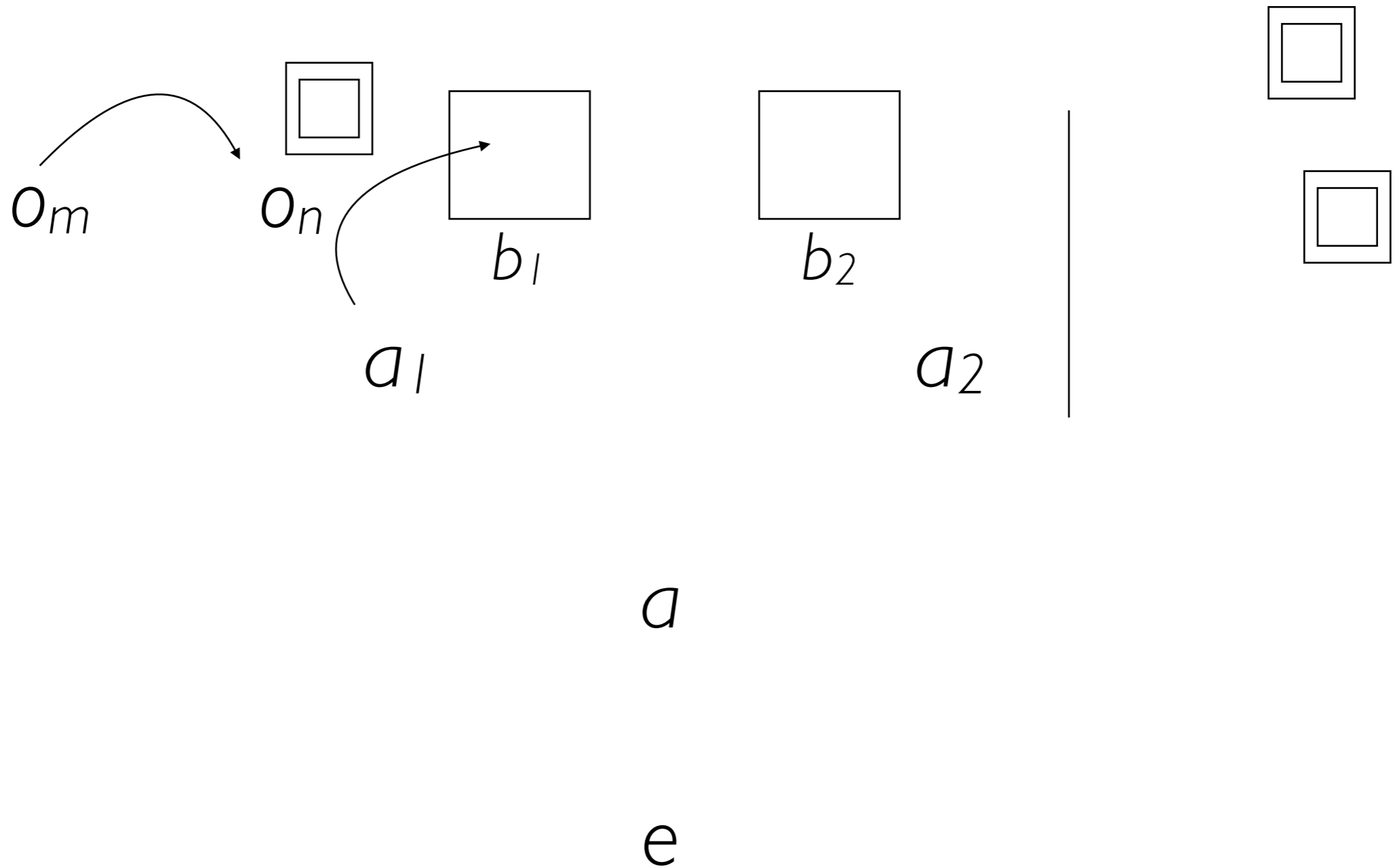
# Framework for $\text{FBT}^1_4$

(nine timepoints)



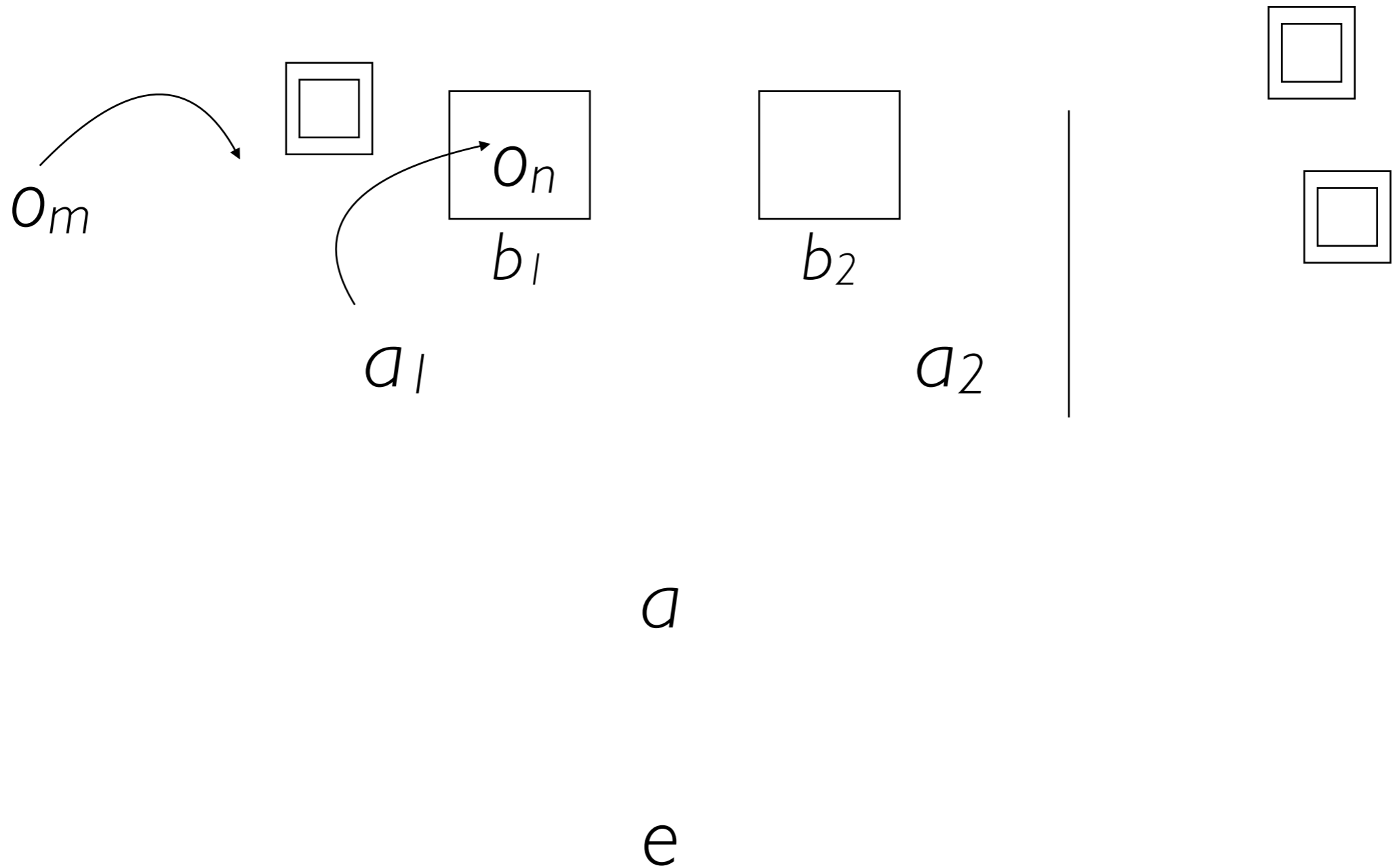
# Framework for $\text{FBT}^1_4$

(nine timepoints)

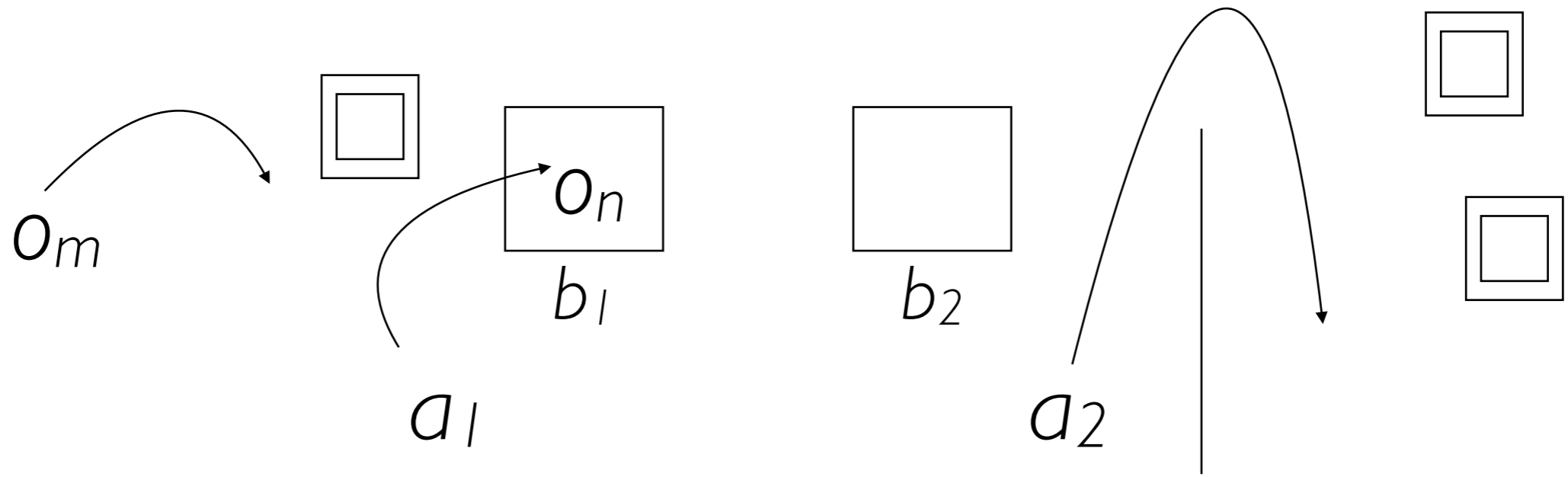


# Framework for $\text{FBT}^1_4$

(nine timepoints)



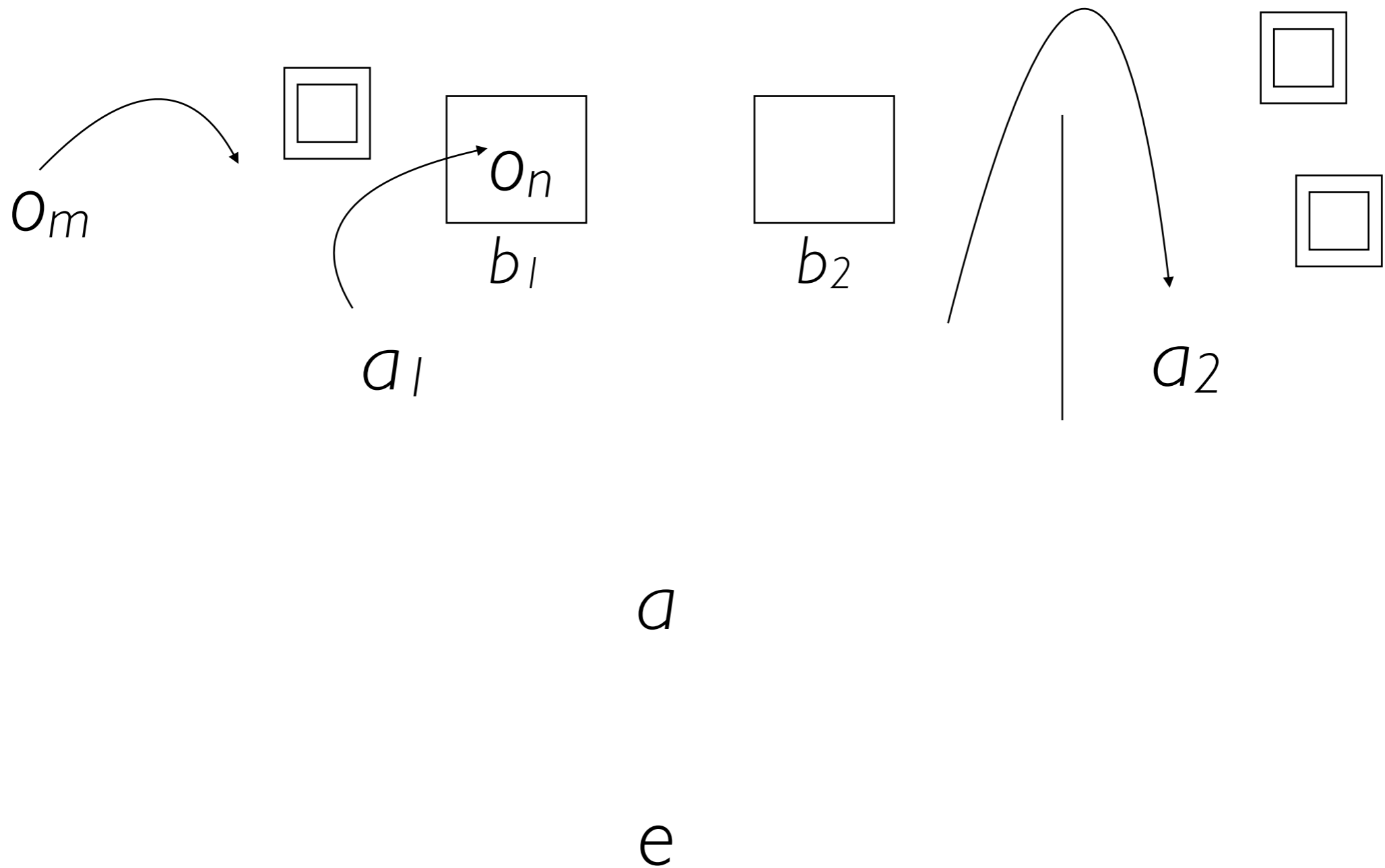
# Framework for $\text{FBT}^4$ (nine timepoints)



$a$

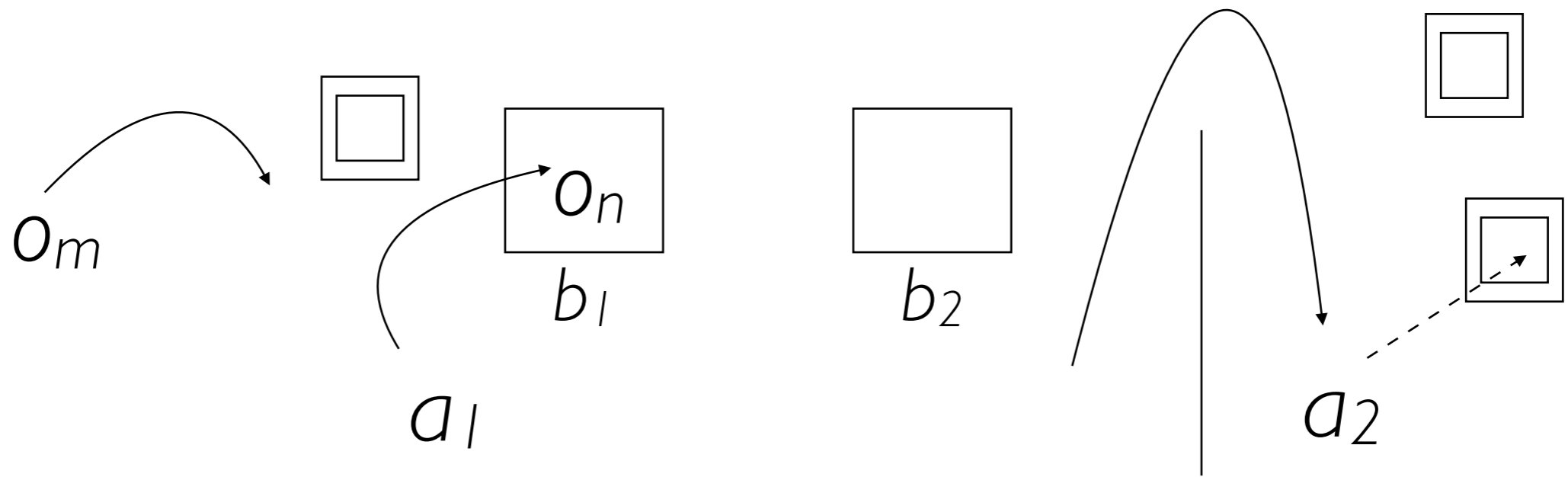
$e$

# Framework for $\text{FBT}^1_4$ (nine timepoints)



# Framework for $\text{FBT}^1_4$

(nine timepoints)

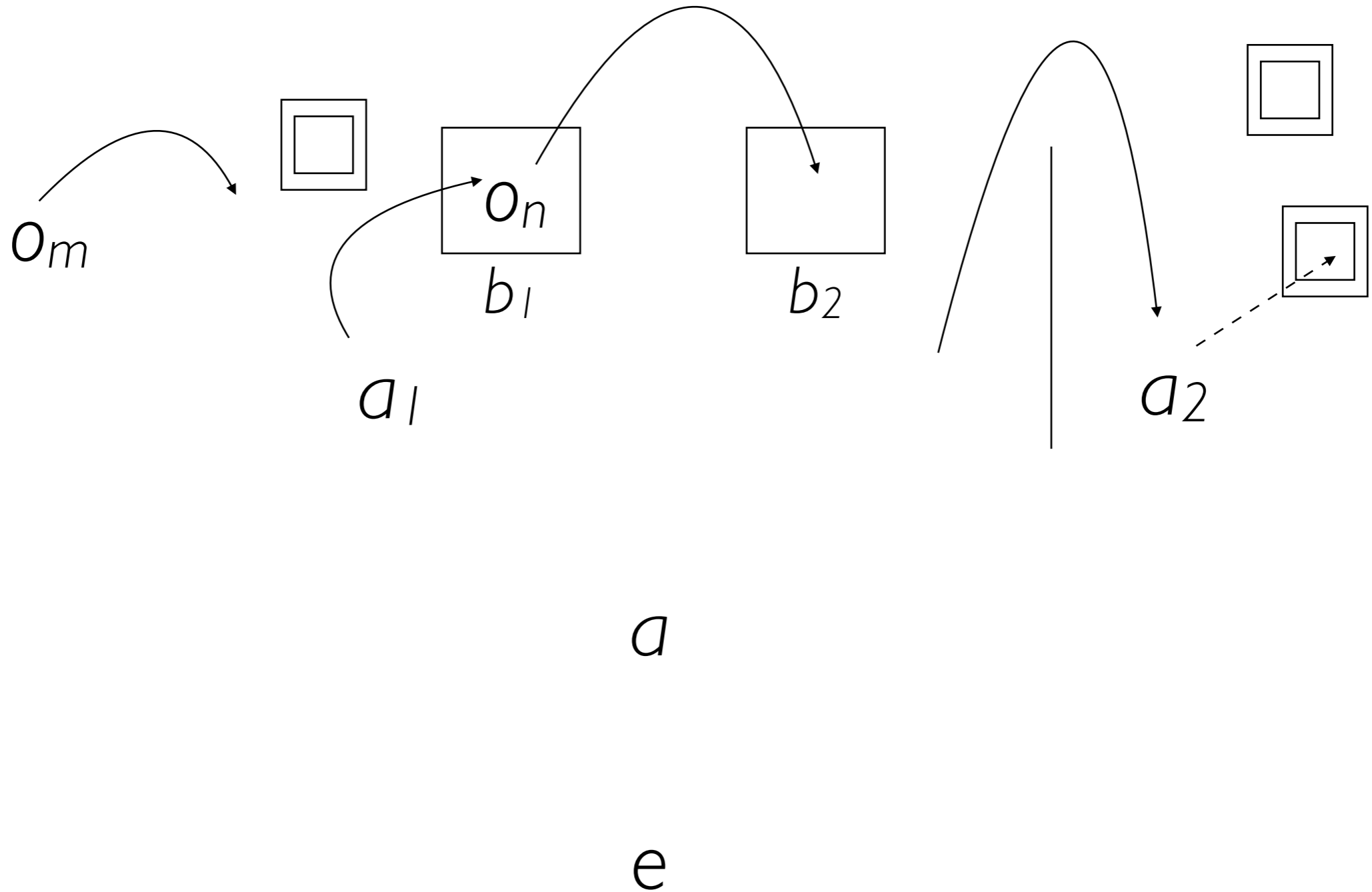


$a$

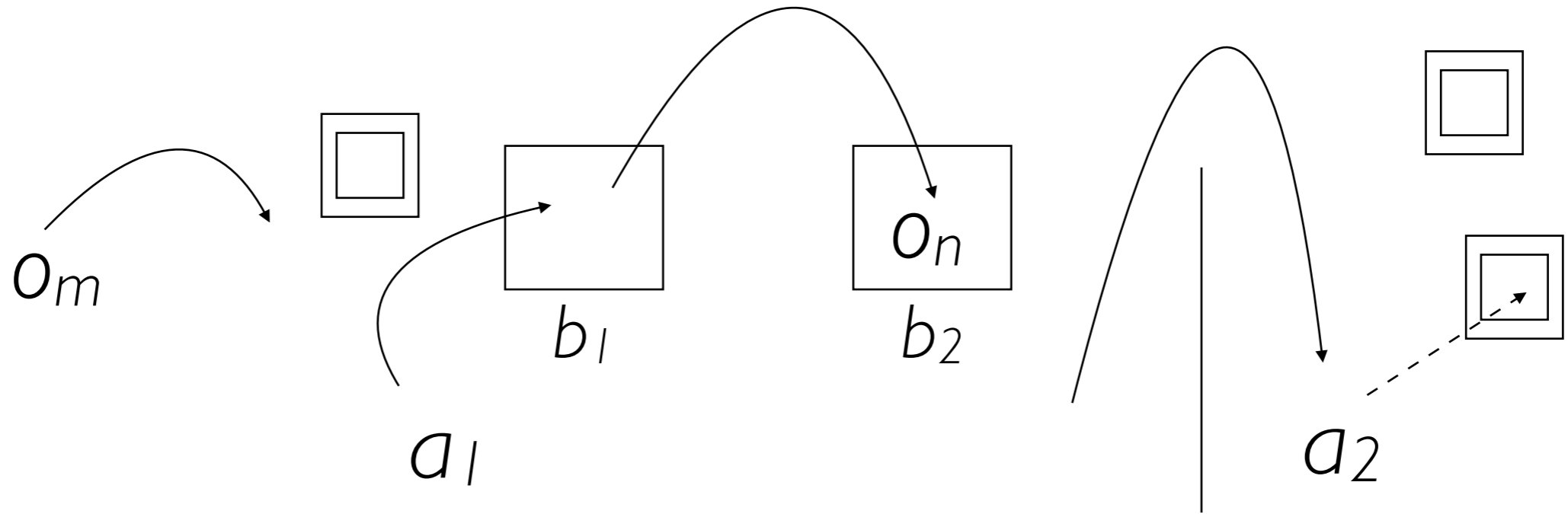
$e$

# Framework for $\text{FBT}^1_4$

(nine timepoints)



# Framework for $\text{FBT}^I_4$ (nine timepoints)

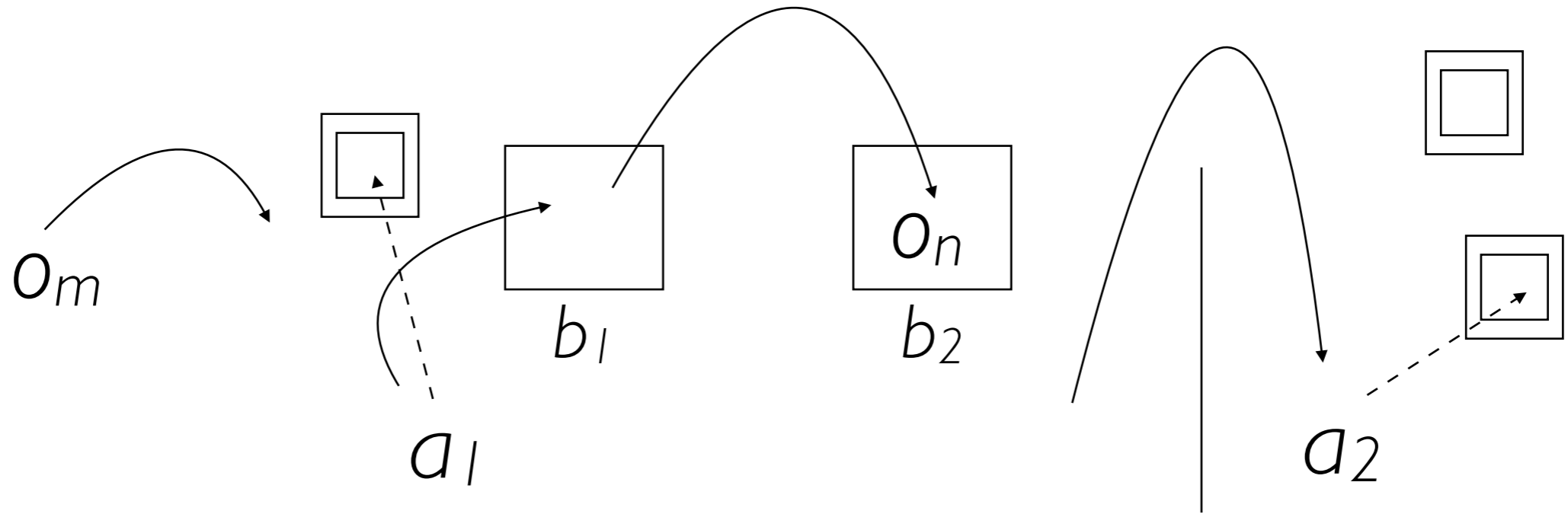


$a$

$e$

# Framework for $\text{FBT}^1_4$

(nine timepoints)

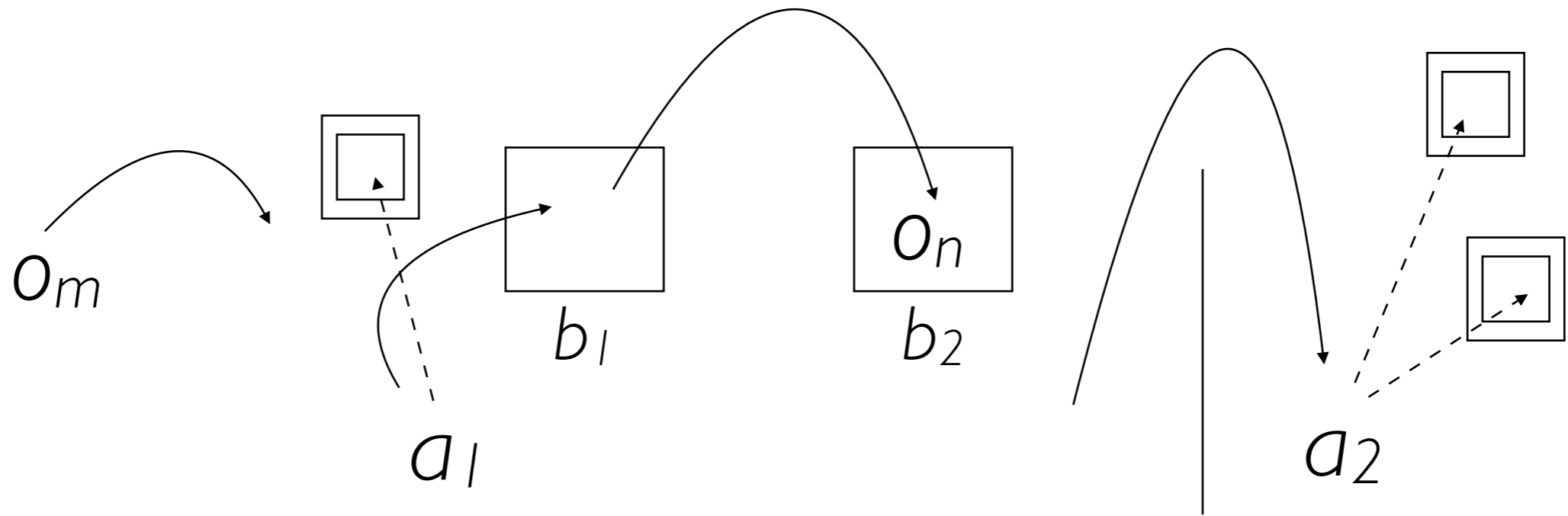


$a$

$e$

# Framework for $\text{FBT}^1_4$

(nine timepoints)

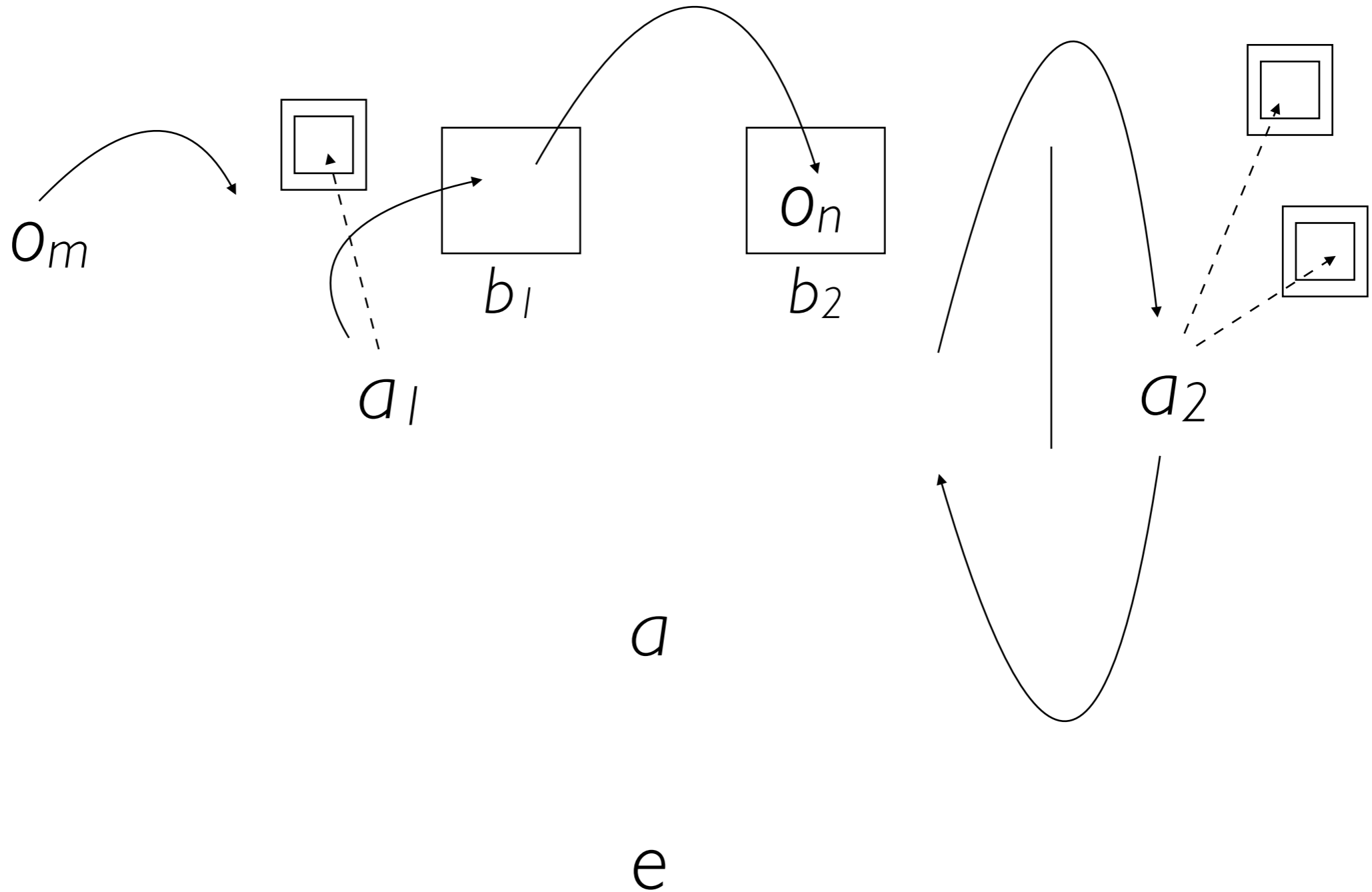


$a$

$e$

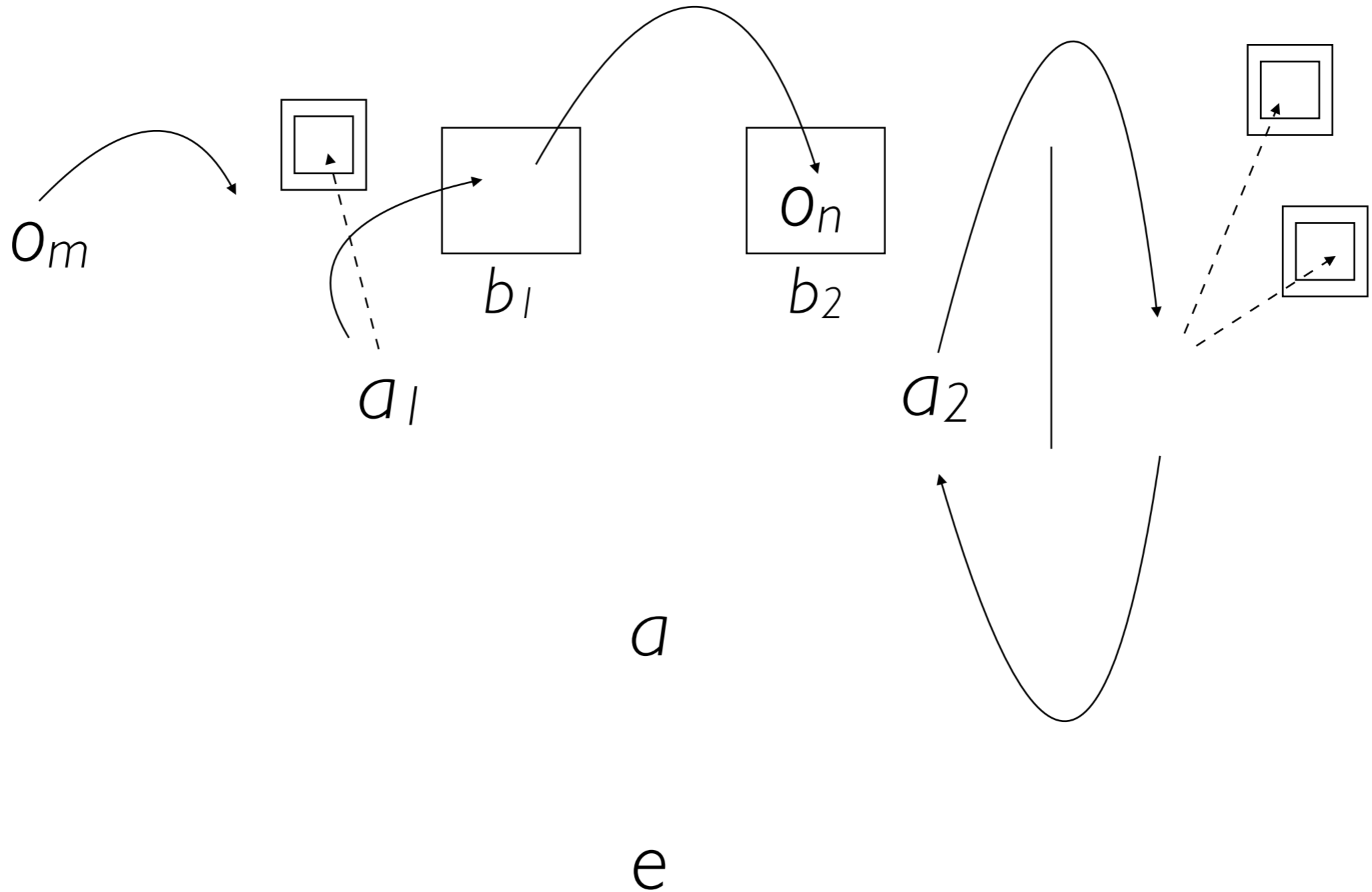
# Framework for $\text{FBT}^1_4$

(nine timepoints)

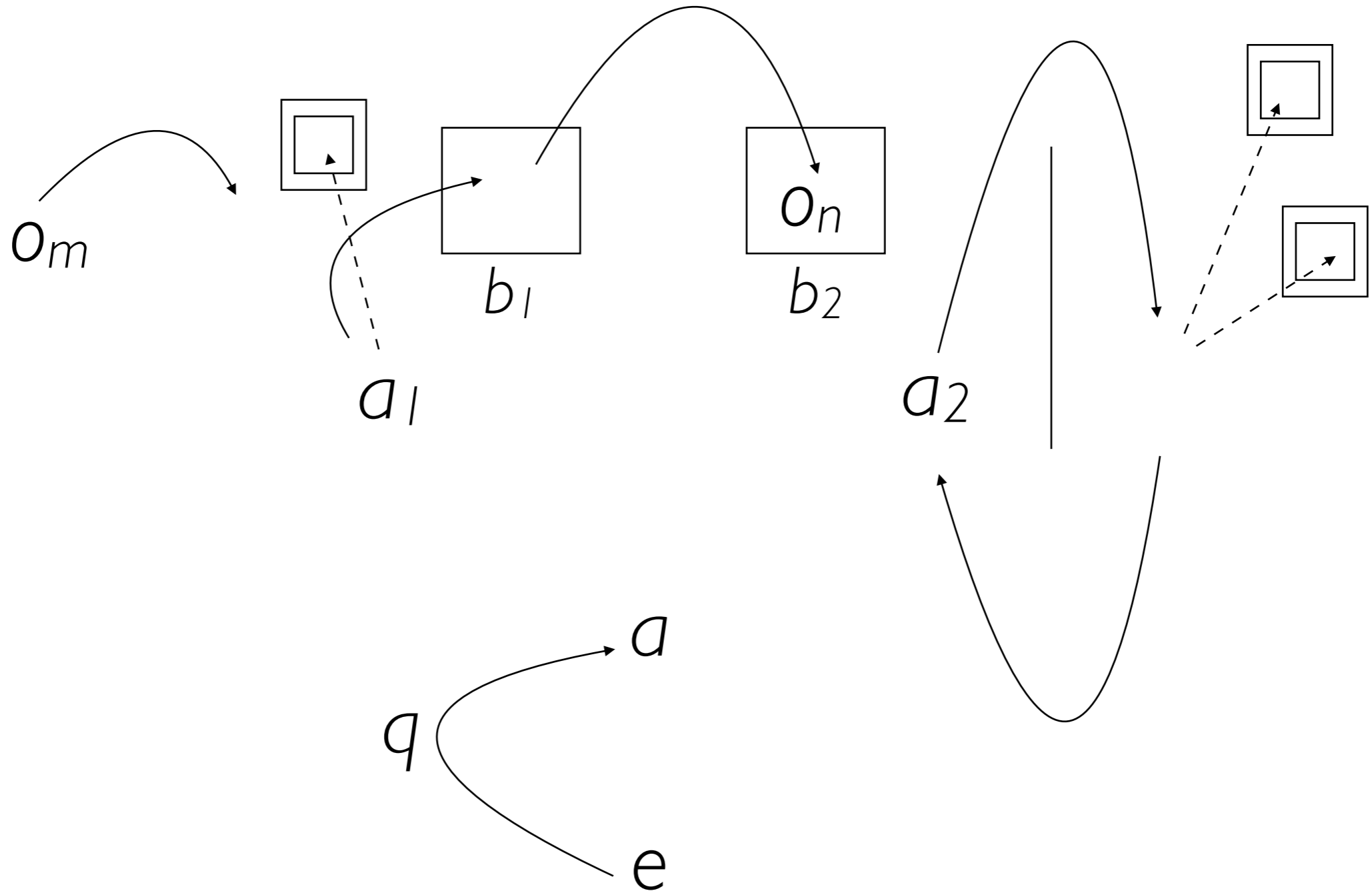


# Framework for $\text{FBT}^1_4$

(nine timepoints)

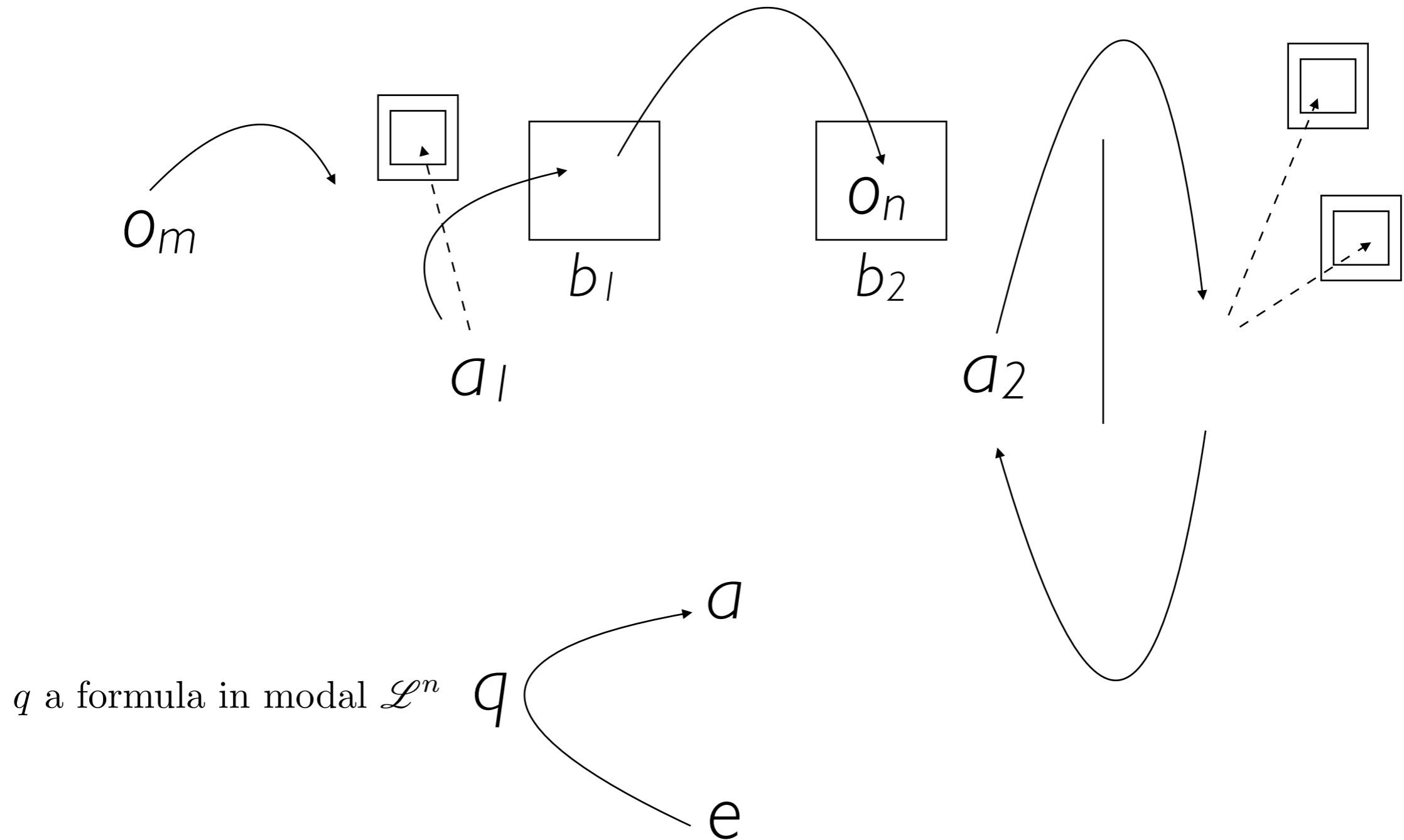


# Framework for $\text{FBT}^1_4$ (nine timepoints)

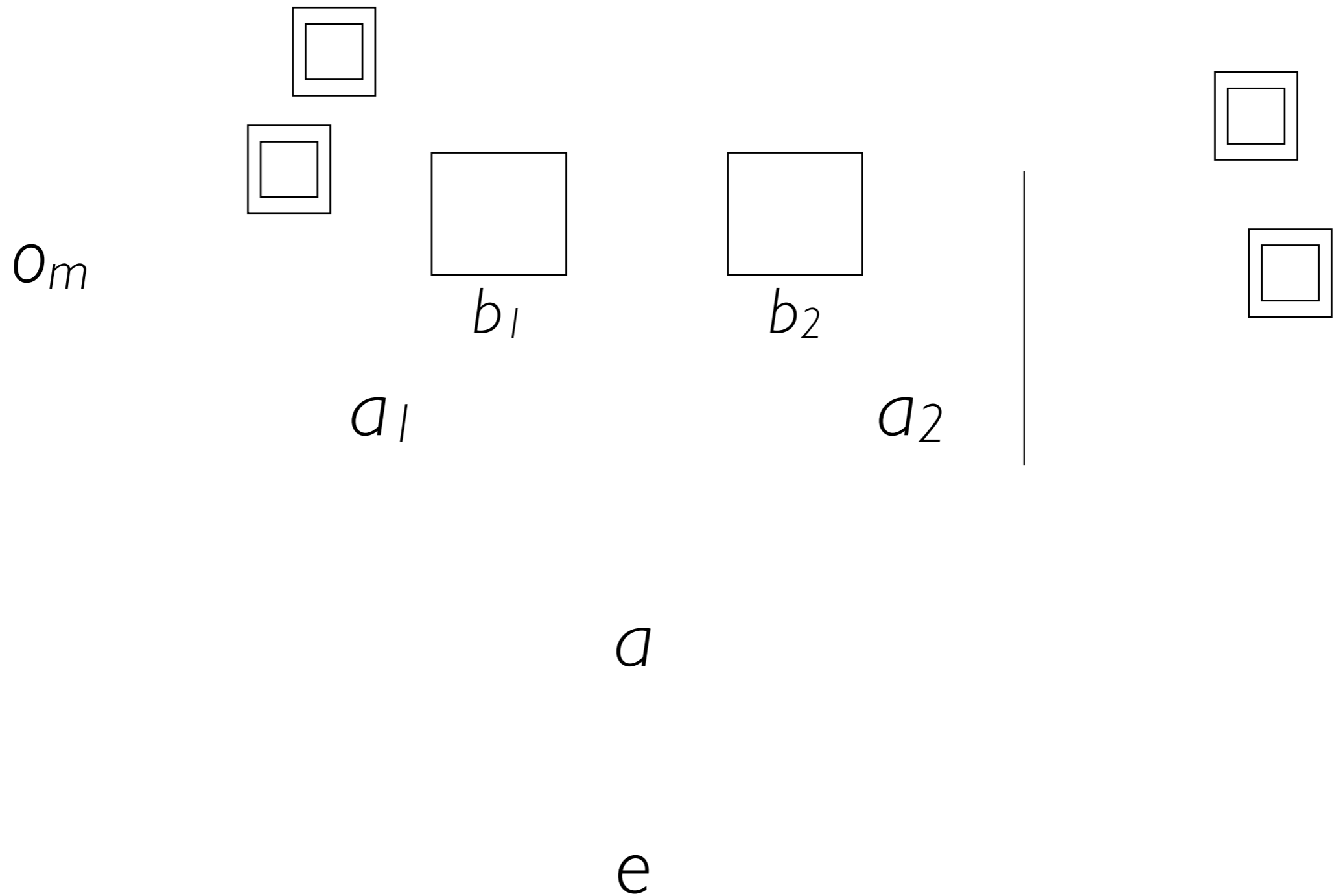


# Framework for $\text{FBT}^1_4$

(nine timepoints)

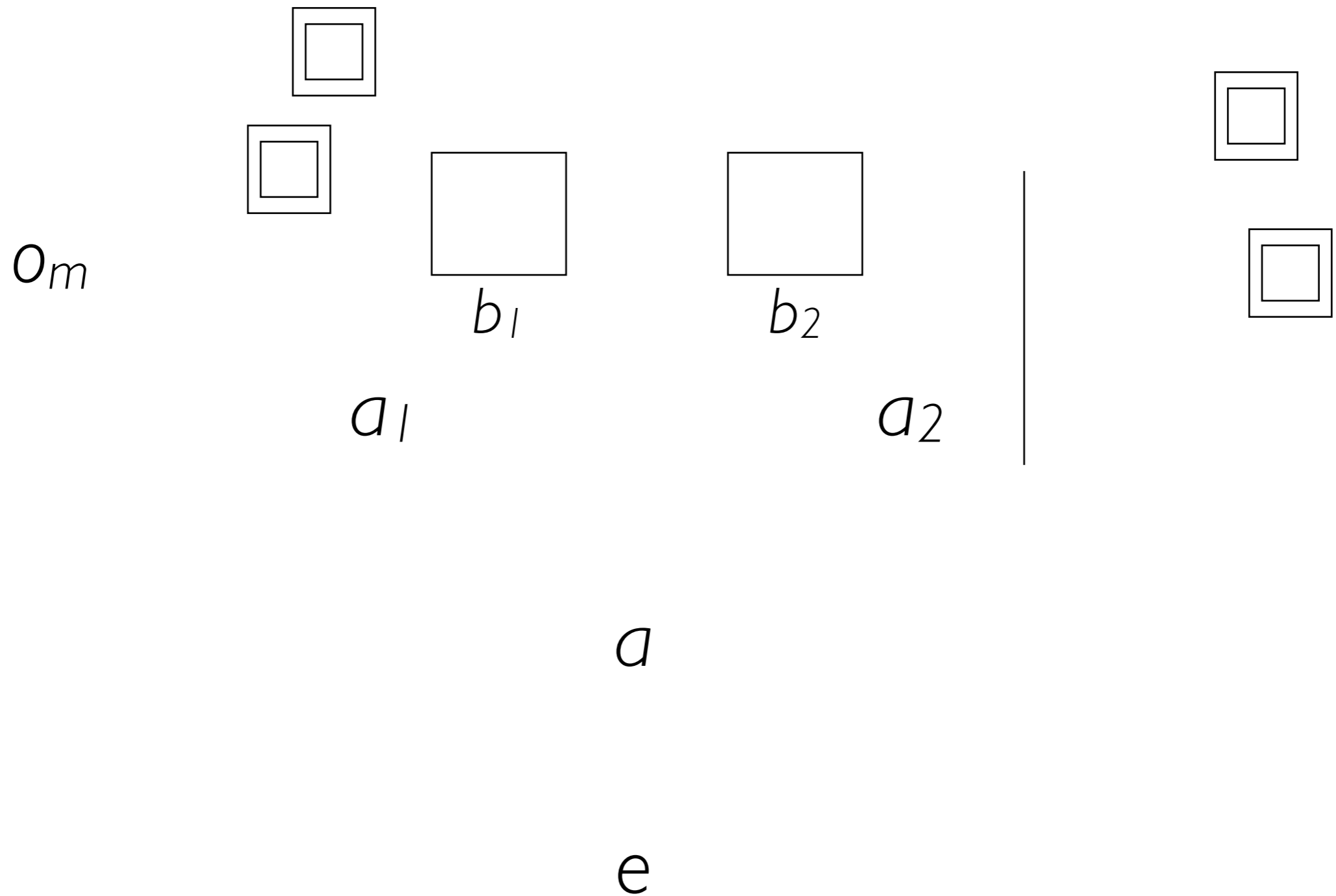


# Framework for $\text{FBT}^1_5$



# Framework for FBT<sup>15</sup>

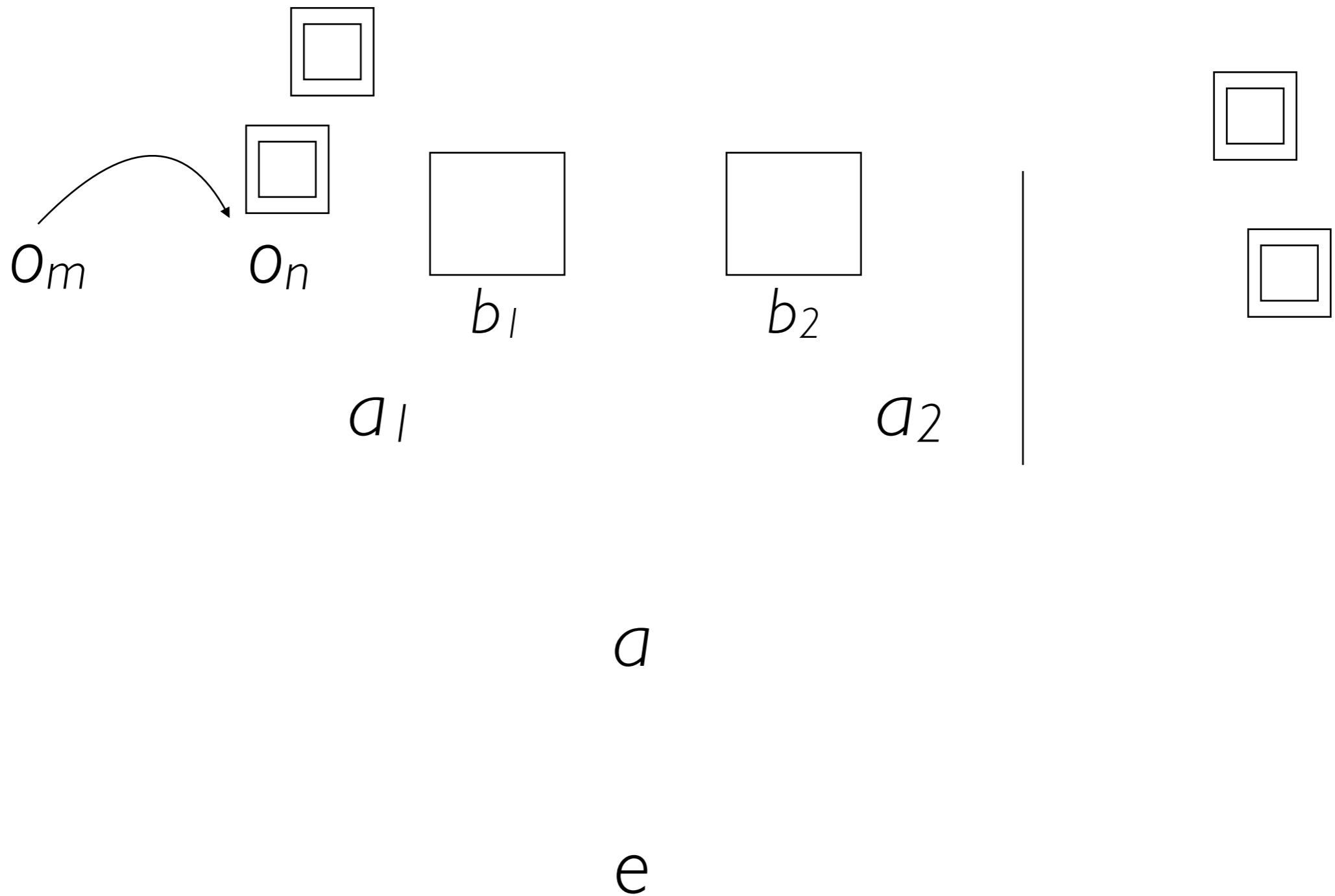
(ten timepoints)





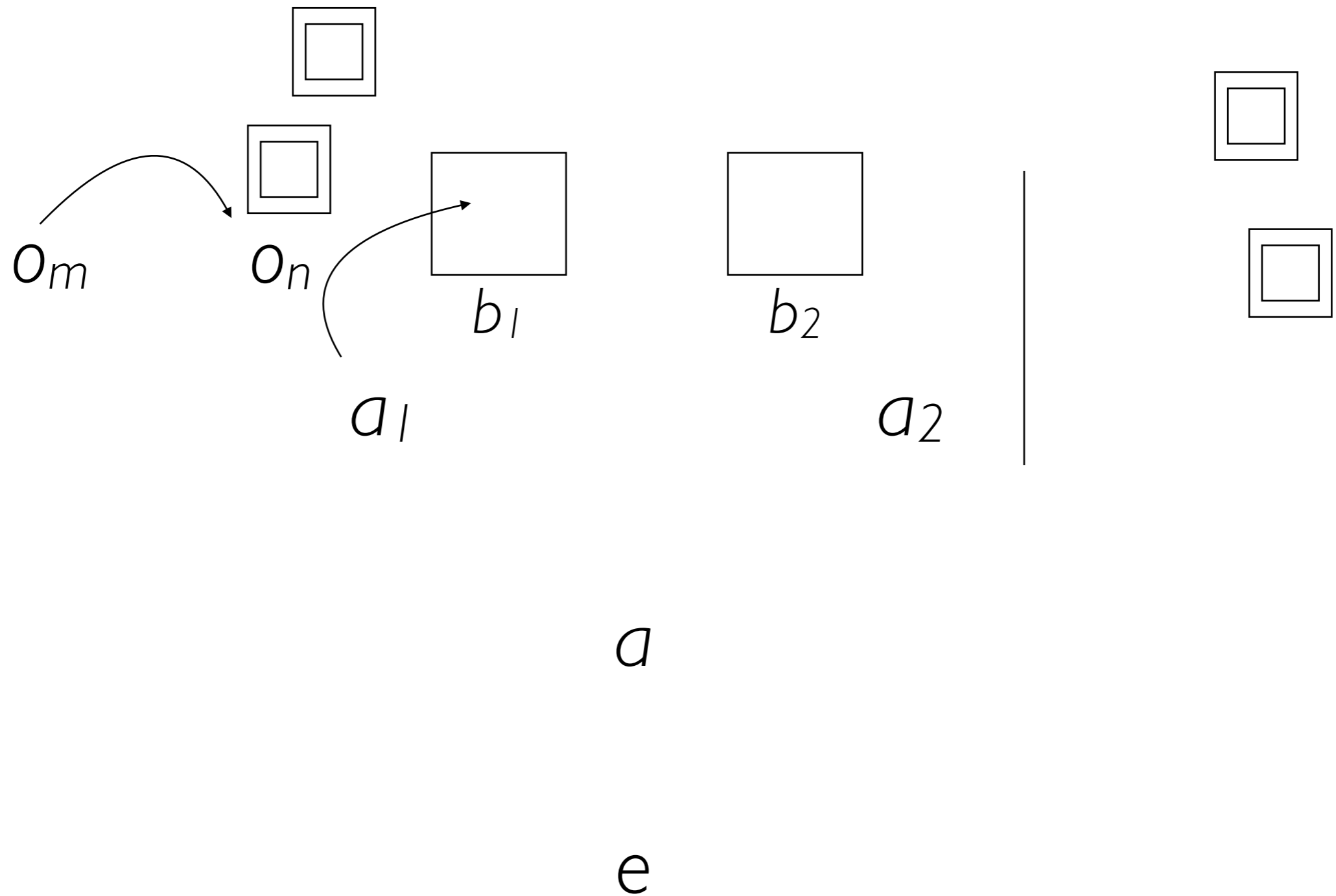
# Framework for FBT<sup>15</sup>

(ten timepoints)



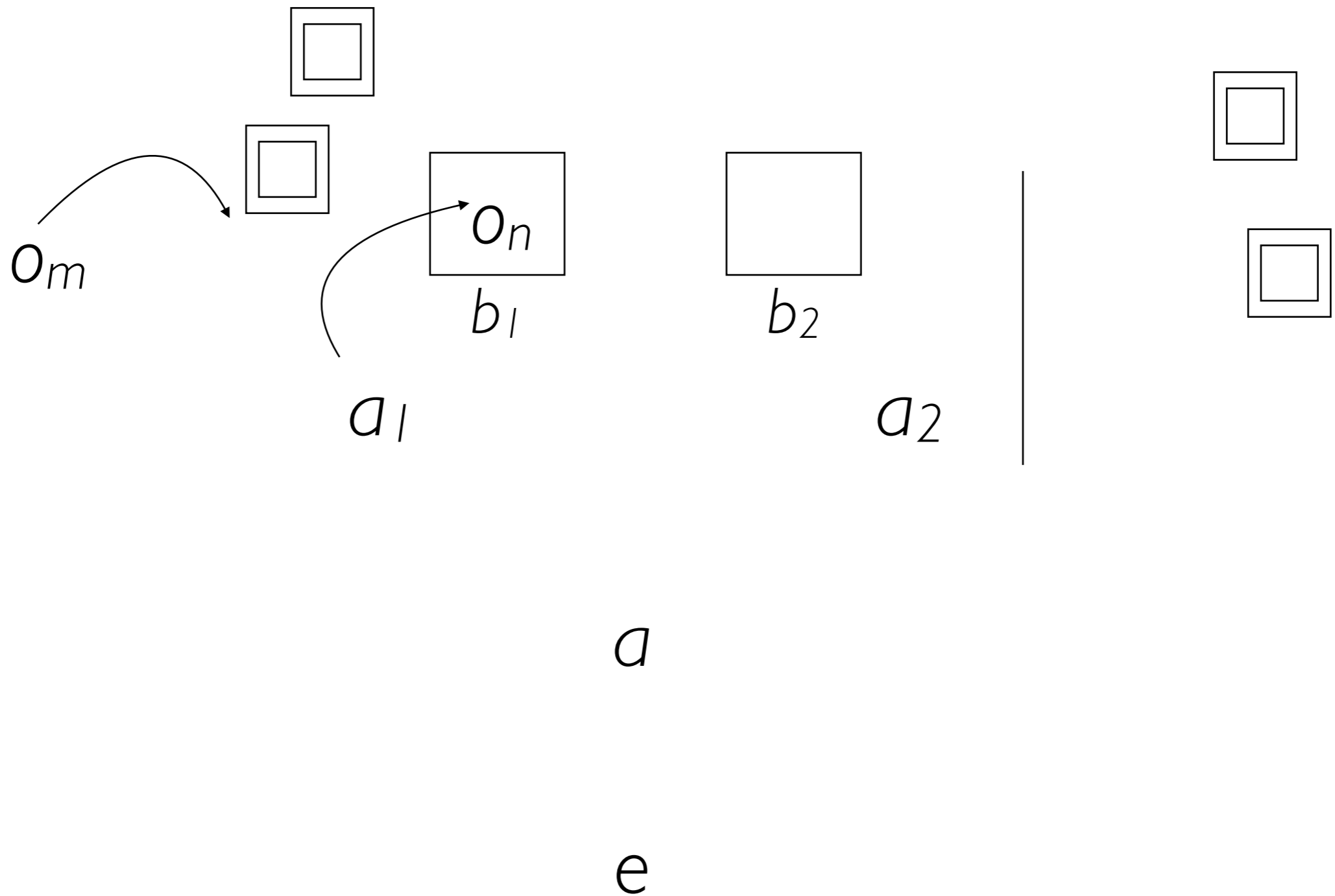
# Framework for FBT<sup>15</sup>

(ten timepoints)



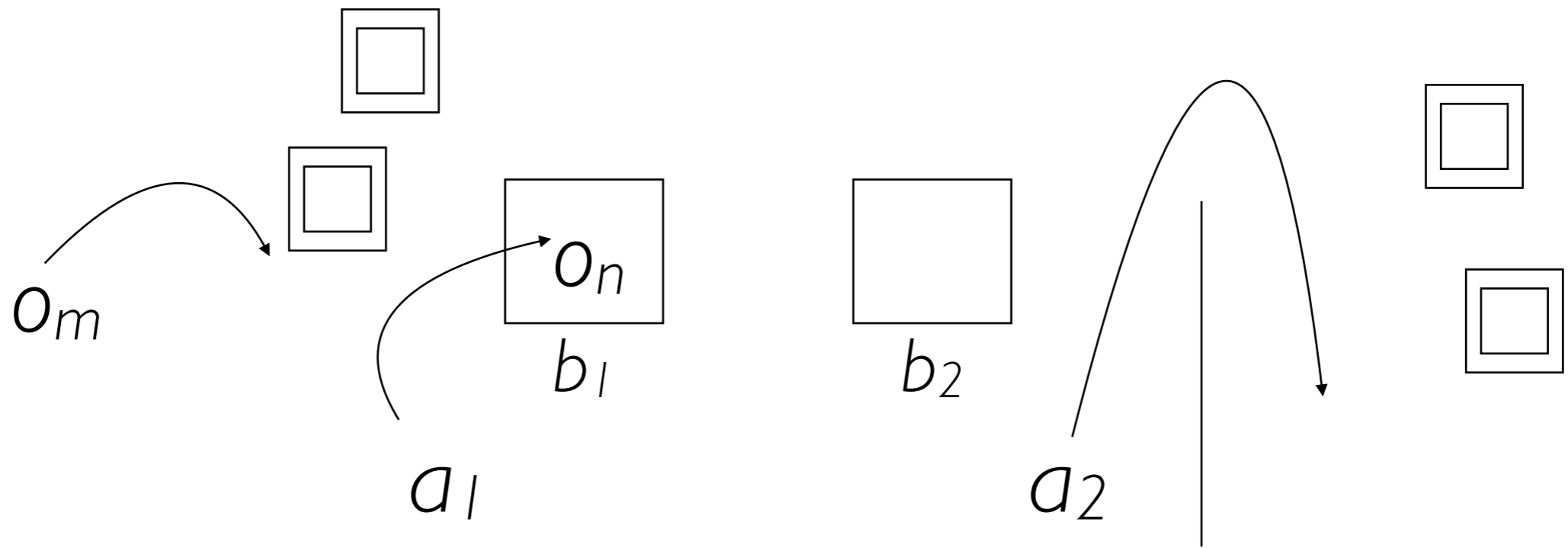
# Framework for FBT<sup>15</sup>

(ten timepoints)



# Framework for FBT<sup>15</sup>

(ten timepoints)

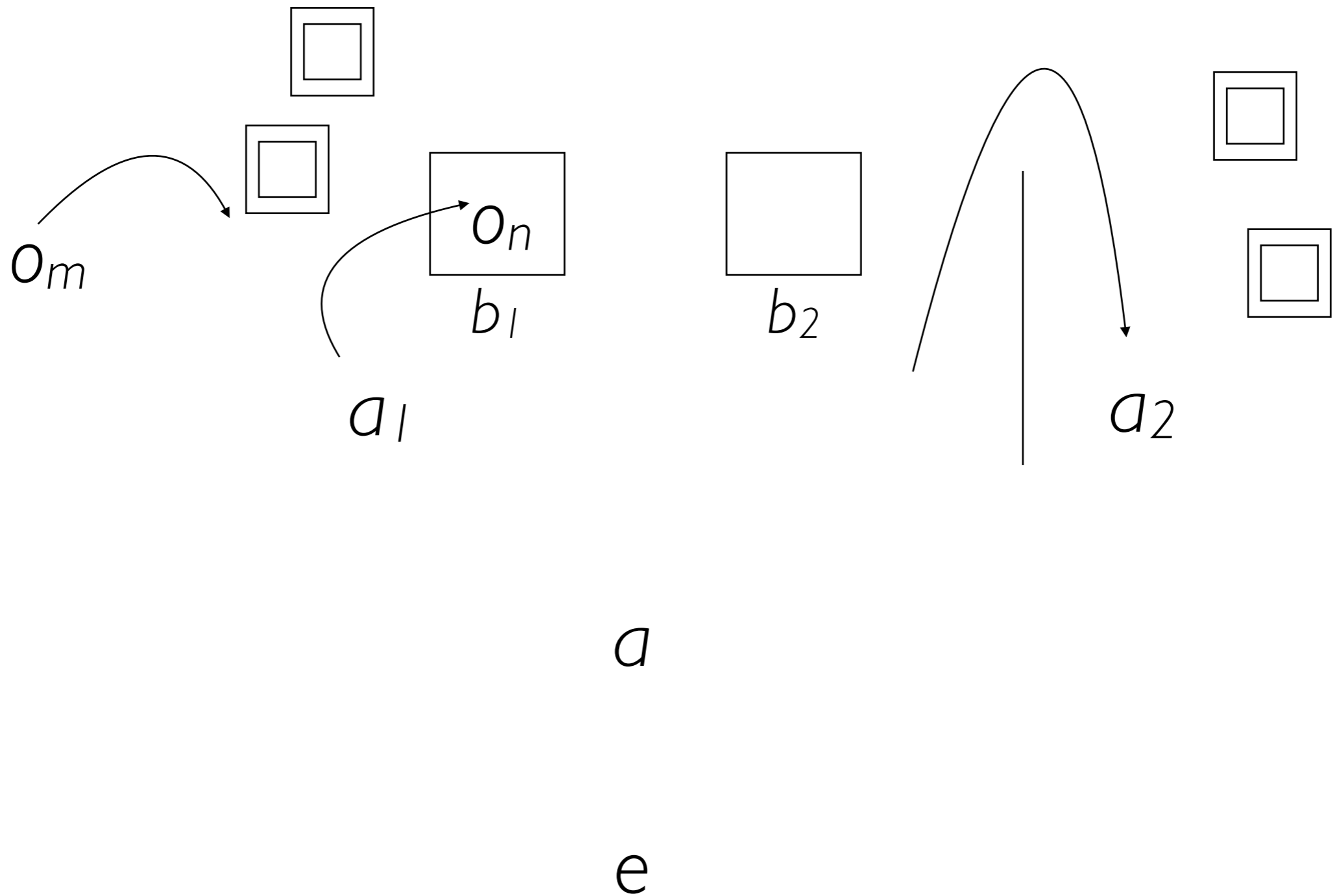


$a$

$e$

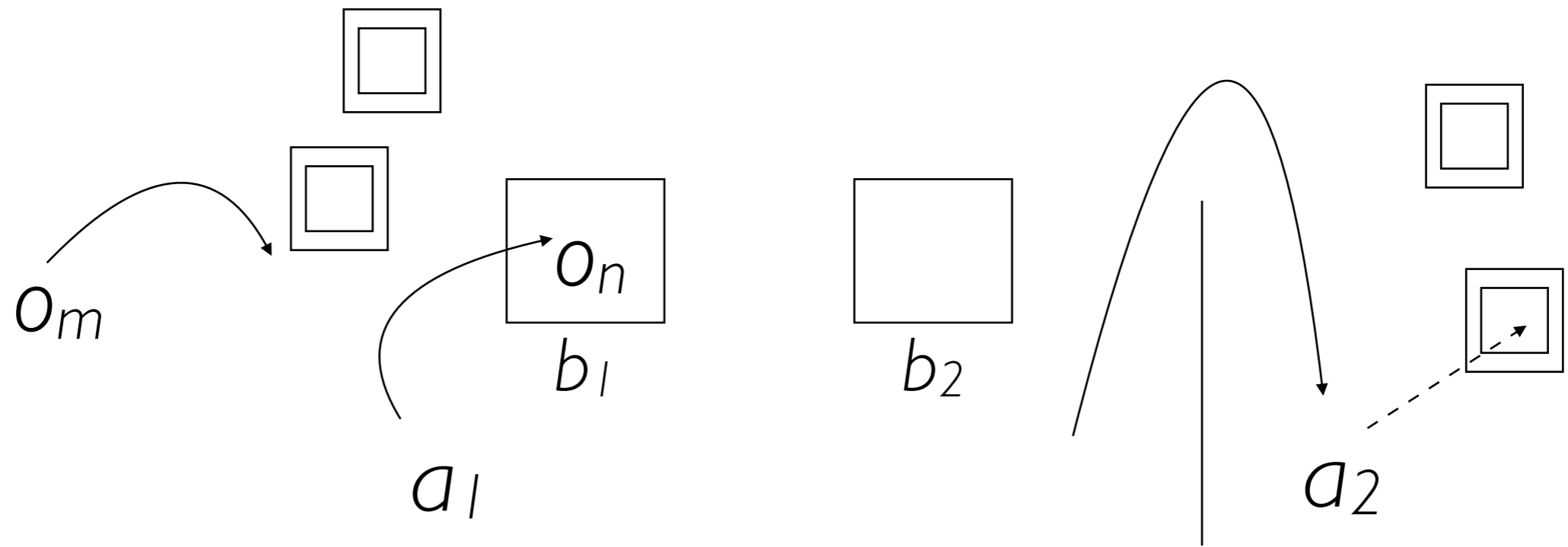
# Framework for FBT<sup>15</sup>

(ten timepoints)



# Framework for FBT<sup>1</sup><sub>5</sub>

(ten timepoints)

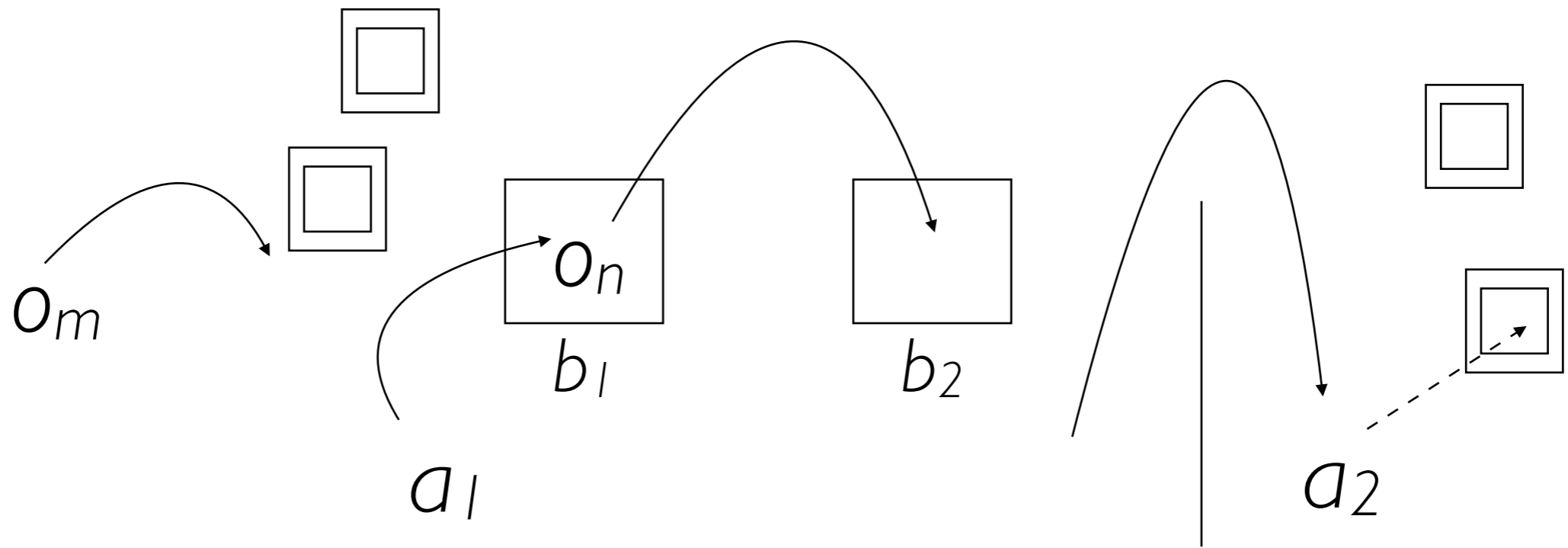


$a$

$e$

# Framework for FBT<sup>1</sup><sub>5</sub>

(ten timepoints)

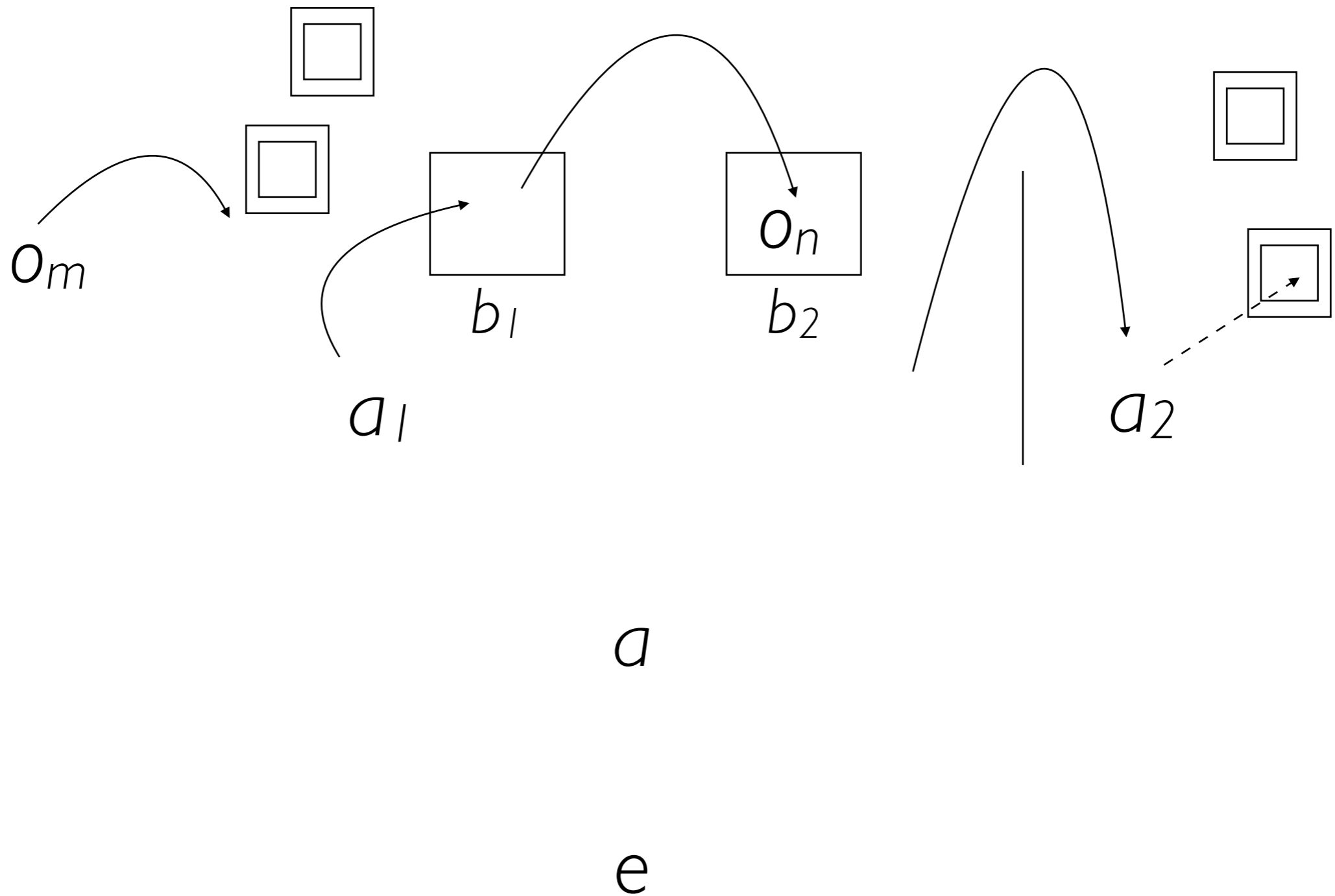


$a$

$e$

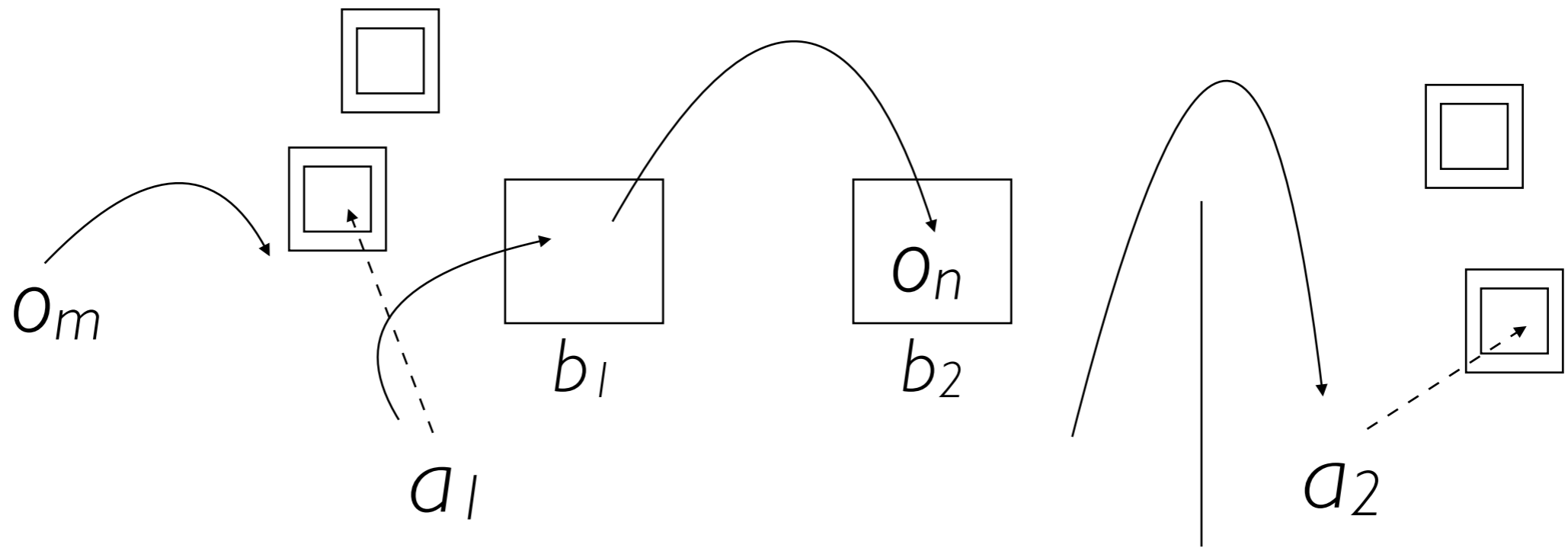
# Framework for FBT<sup>15</sup>

(ten timepoints)



# Framework for FBT<sup>1</sup><sub>5</sub>

(ten timepoints)

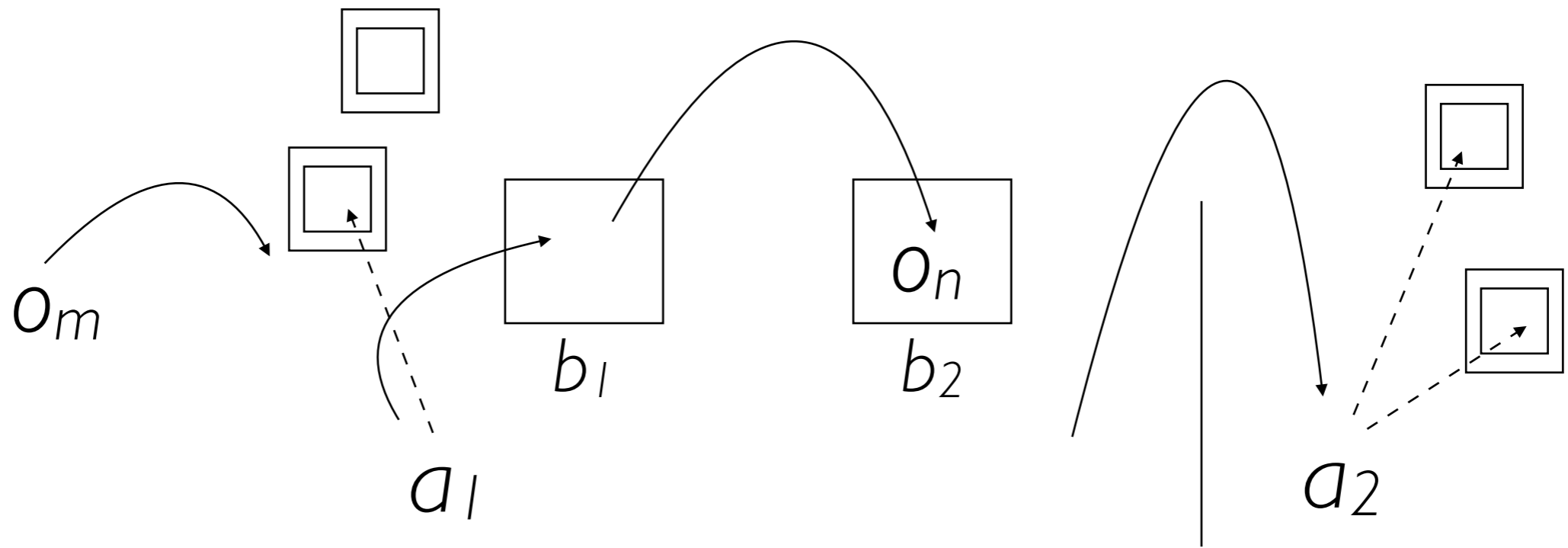


$a$

$e$

# Framework for FBT<sup>1</sup><sub>5</sub>

(ten timepoints)

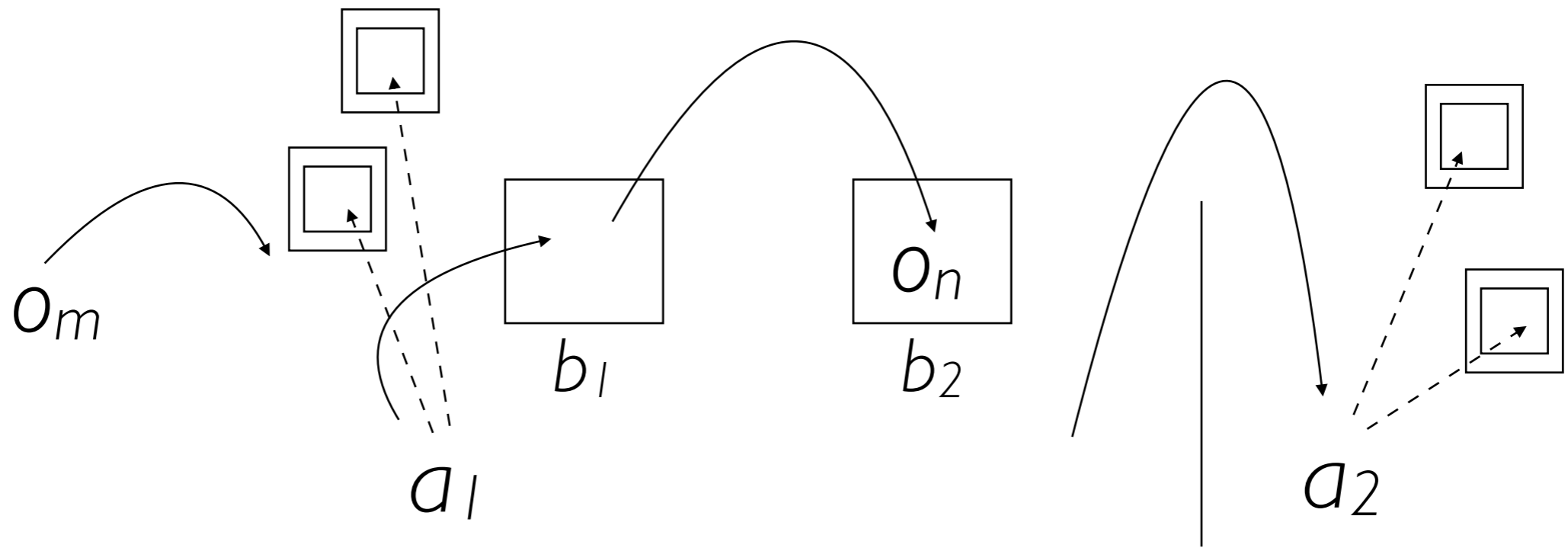


$a$

$e$

# Framework for FBT<sup>1</sup><sub>5</sub>

(ten timepoints)

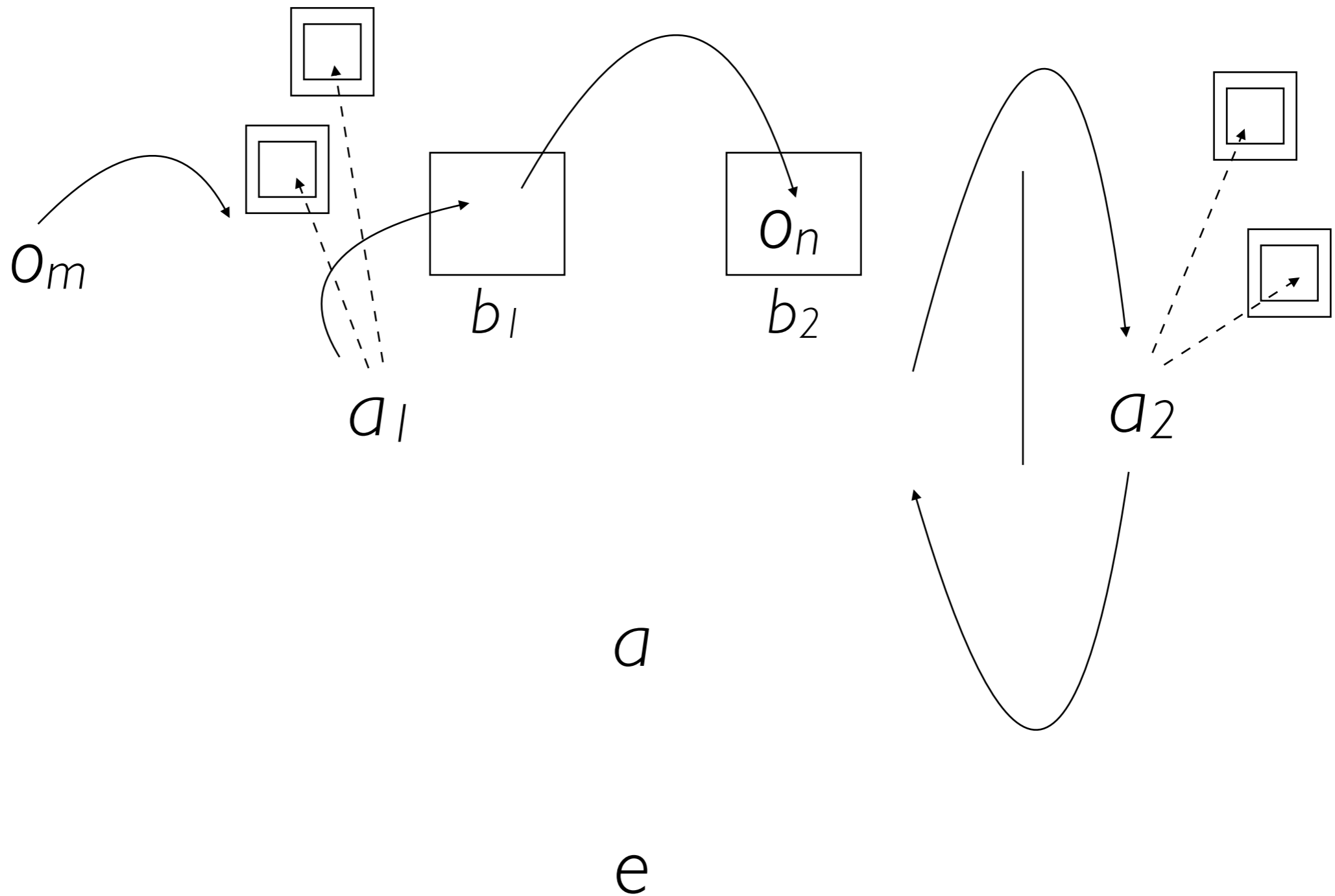


$a$

$e$

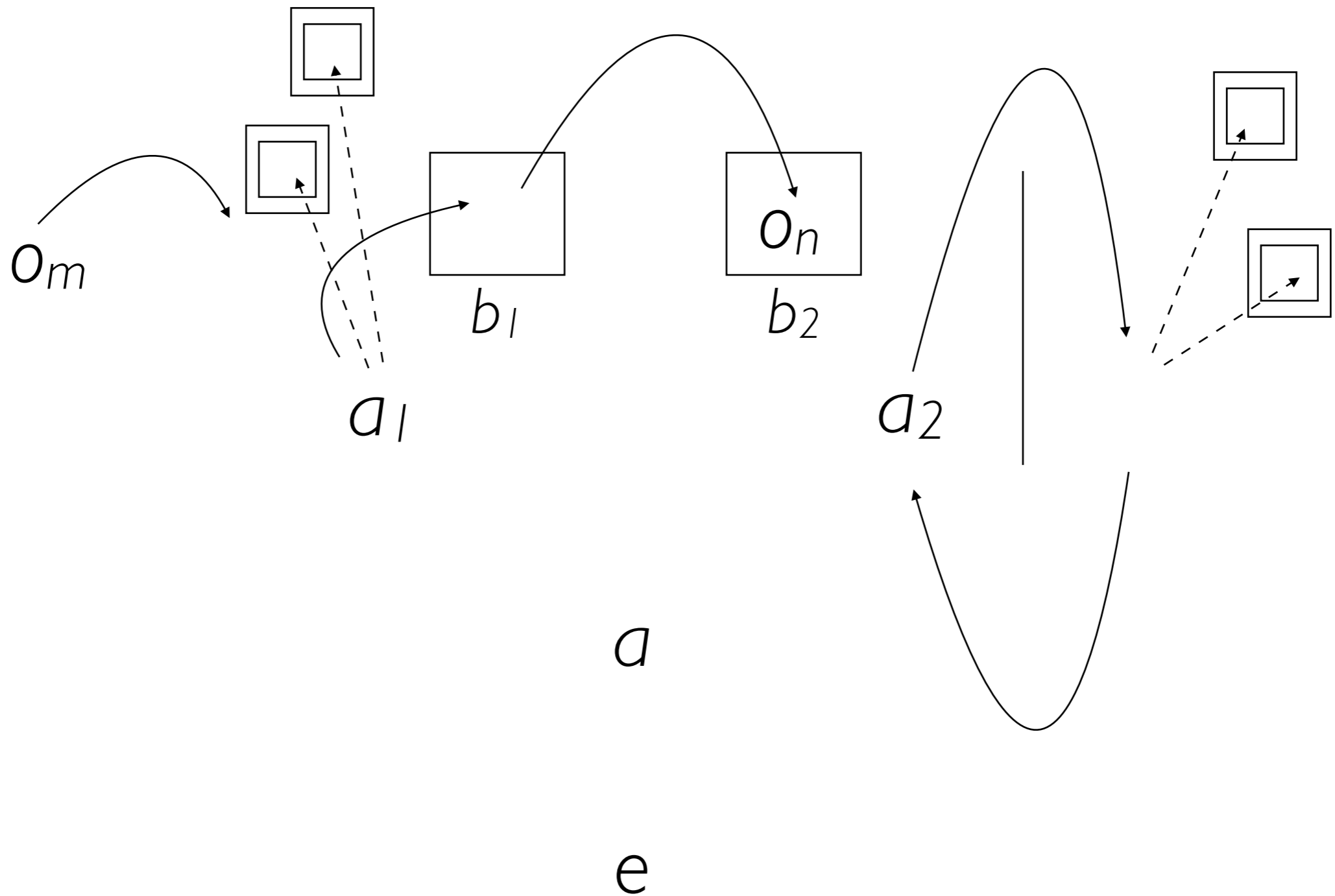
# Framework for FBT<sup>1</sup><sub>5</sub>

(ten timepoints)



# Framework for FBT<sup>1</sup><sub>5</sub>

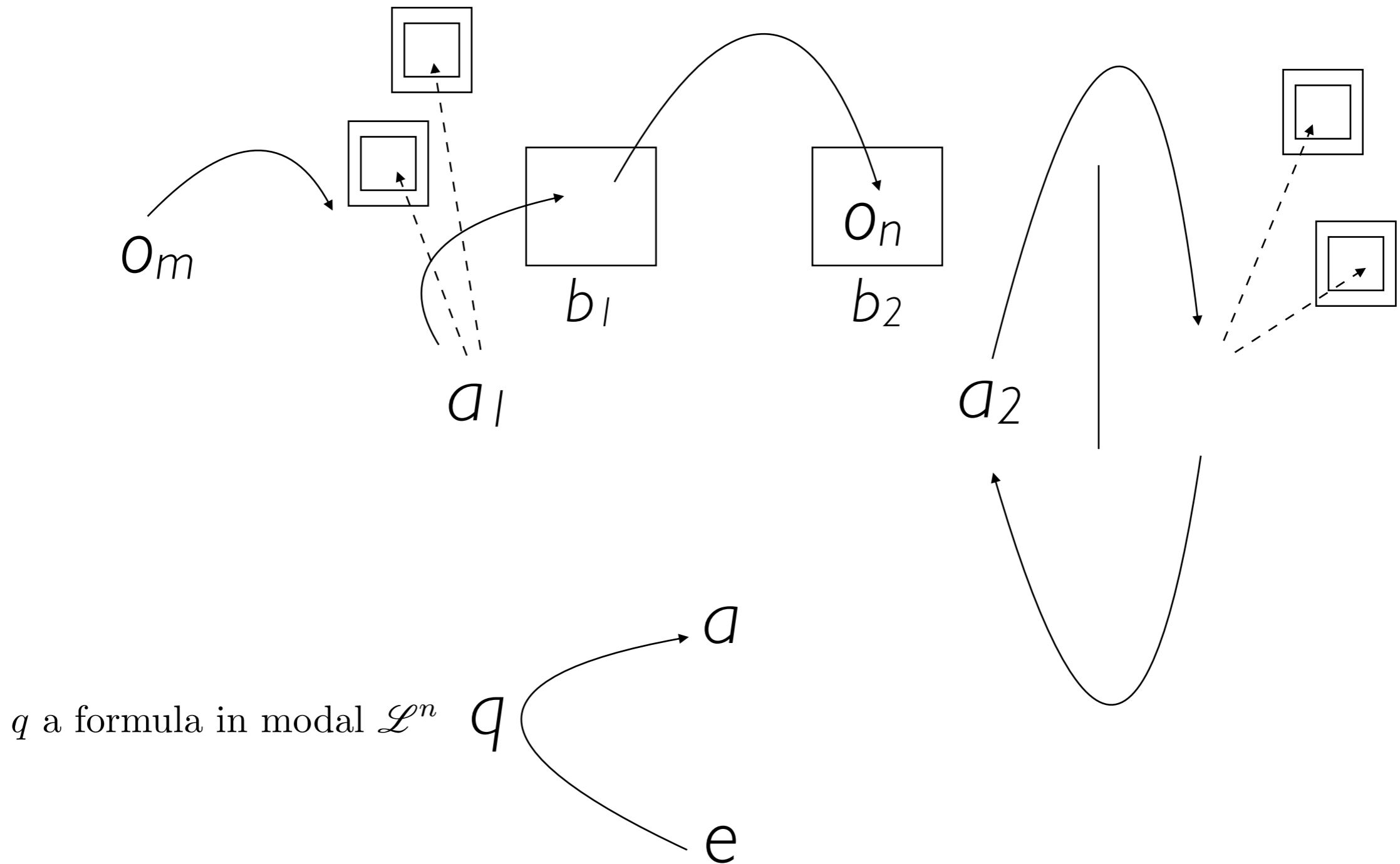
(ten timepoints)





# Framework for $\text{FBT}^1_5$

(ten timepoints)



# Humans Can Succeed

Neurobiologically normal, nurtured, educated, and sufficiently motivated humans can correctly answer any relevant query  $q$  for the infinite progression, and prove that their answer is correct. For the obvious subclass of queries (the form of which appear in the box below), they can prove and exploit the following lemma.

**Lemma:** Suppose  $\text{FBT}_k, k \in \mathbb{Z}^+$ , holds; (i.e. that level  $k$  of FBT holds). Then, if  $k$  is even,  $\mathbf{B}_2\mathbf{B}_1 \dots \mathbf{B}_2 \iota$ , where there are  $k + 1$  iterated  $\mathbf{B}_i$  operators; otherwise  $\mathbf{B}_1\mathbf{B}_2 \dots \mathbf{B}_1\mathbf{B}_2 \iota$ , where there again there are  $k + 1$  iterated  $\mathbf{B}_i$  operators.

# **Passing to Probing Mastery of the Specific Subclass**

Experimenter to  $a$ : “At level  $k$ ,  
from which box will  $a_2$  attempt to  
retrieve the objects  $o_n$ ? Prove it!”

# *Theoretical* Machine Success on Infinite FBT!

**Theorem:**  $\forall q \in \mathcal{CC}, \mathfrak{M}$  can correctly answer and justify  $q$ .  
I.e.,  $\mathfrak{M}$  can pass  $\text{FBT}_\omega$ .

Ok, so this logic machine exists in the *mathematical* universe; but does there exist an *implemented* machine with this power?

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# Simulation Courtesy of ...

**ShadowProver!**



# Level 1

```
:name "Level 1: False Belief Task "  
  
:description "Agent a1 puts an object o into b1 in plain view of a2.  
Agent a2 then leaves, and in the absence of a2, a1 moves o  
from b1 into b2 ; this movement isn't perceived by a2 . Agent  
a2 now returns, and a is asked by the experimenter e: "If a2  
desires to retrieve o, which box will a2 look in?" If younger  
than four or five, a will reply "In b " (which of course fails 2  
the task); after this age subjects respond with the correct "In b1."  
  
Level1 Belief: a1 believes a2 believes o is in b1."  
  
:date "Monday July 22, 2019"  
  
:assumptions {  
  :P1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1)))  
  
  :P2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))  
  
  :P3 (holds (In o b1) t1)  
  
  :C1 (Common! t0 (forall [?f ?t2 ?t2]  
    (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))  
      (holds ?f ?t2))))  
  
  :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))  
}  
  
:goal (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3)))}
```

# Level 2

```
{:name "Level 2: False Belief Task "

:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level2 Belief: a2 believes a1 believes a2 believes o is in b1.
"

:date "Monday July 22, 2019"

:assumptions {

  :P1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))

  :P2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1)))))))

  :P3 (holds (In o b1) t1)

  :C1 (Common! t0
        (forall [?f ?t2 ?t2]
          (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
              (holds ?f ?t2))))

  :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}

:goal (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))}
```

# Level 3

```
{:name "Level 3: False Belief Task "  
  
:description "Agent a1 puts an object o into b1 in plain view of a2.  
Agent a2 then leaves, and in the absence of a2, a1 moves o  
from b1 into b2 ; this movement isn't perceived by a2 . Agent  
a2 now returns, and a is asked by the experimenter e: "If a2  
desires to retrieve o, which box will a2 look in?" If younger  
than four or five, a will reply "In b " (which of course fails 2  
the task); after this age subjects respond with the correct "In b1."  
  
Level3 Belief: a2 believes a1 believes a2 believes o is in b1.  
"  
  
:date "Monday July 22, 2019"  
  
:assumptions {  
  
:P1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))  
:P2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1)))))))  
  
:P3 (holds (In o b1) t1)  
  
:C1 (Common! t0  
| (forall [?f ?t2 ?t2]  
| | (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))  
| | (holds ?f ?t2))))  
  
:C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}  
  
:goal (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))}
```

# Level 4

```
{:name      "Level 4: False Belief Task "
:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level4 Belief: a2 believes a1 believes a2 believes a1 believes a2 believes o is in b1.
"

:date      "Monday July 22, 2019"

:assumptions {

  :P1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))))
  :P2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))))))
  :P3 (holds (In o b1) t1)

  :C1 (Common! t0
      (forall [?f ?t2 ?t2]
        (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
            (holds ?f ?t2))))
  :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}

:goal      (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))}
```

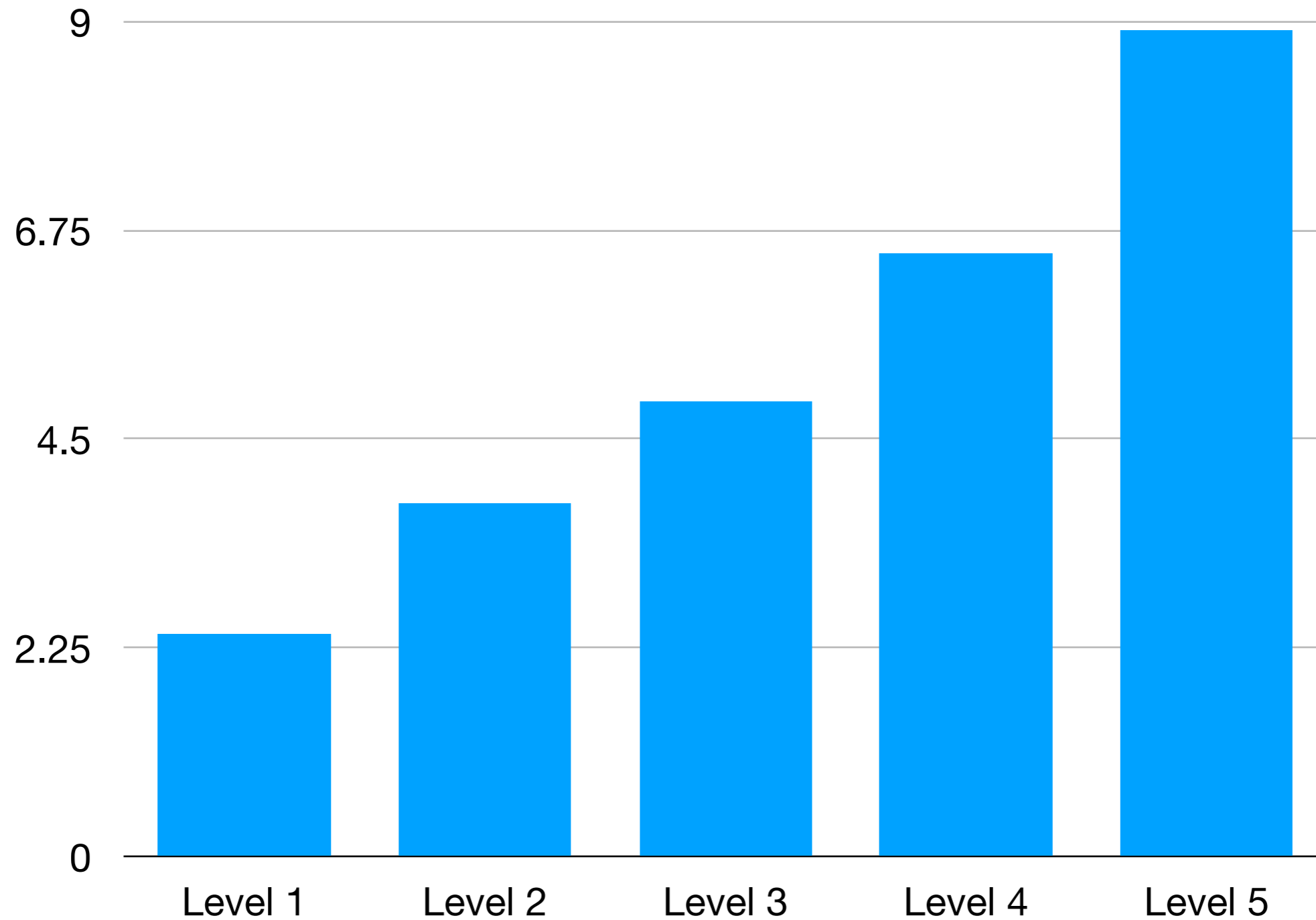
# Level 5

```
{:name      "Level 5: False Belief Task "  
  
:description "Agent a1 puts an object o into b1 in plain view of a2.  
Agent a2 then leaves, and in the absence of a2, a1 moves o  
from b1 into b2 ; this movement isn't perceived by a2 . Agent  
a2 now returns, and a is asked by the experimenter e: "If a2  
desires to retrieve o, which box will a2 look in?" If younger  
than four or five, a will reply "In b " (which of course fails 2  
the task); after this age subjects respond with the correct "In b1."  
  
Level5 Belief: a1 believes a2 believes a1 believes a2 believes a1 believes a2 believes o is in b1."  
  
:date      "Monday July 22, 2019"  
  
:assumptions {  
  
  :P1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))))  
  :P2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))))))  
  
  :P3 (holds (In o b1) t1)  
  
  :C1 (Common! t0  
    (forall [?f ?t2 ?t2]  
      (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))  
        (holds ?f ?t2))))  
  
  :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}  
  
:goal      (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3)))))))))}
```





# Time (in seconds) to Prove



# Simulation of Level 5 in Real Time

```
/Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java ...
```

```
objc[16653]: Class JavaLaunchHelper is implemented in both /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java (0x102a2d4c0) and /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/jre/lib/libinstrument.dylib (0x102ab94e0)
```

```
----- Level 5 -----
```

# Simulation of Level 5 in Real Time

```
/Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java ...  
objc[16653]: Class JavaLaunchHelper is implemented in both /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java (0x102a2d4c0) and /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/jre/lib/libinstrument.dylib (0x102ab94e0)  
----- Level 5 -----
```

# Encapsulation

Slate - K.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ K $\vdash$ ✓ $\infty$ $\Box$	T. $\Box\varphi \rightarrow \varphi$ K $\vdash$ ✗ $\infty$ $\Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ K $\vdash$ ✗ $\infty$ $\Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ K $\vdash$ ✗ $\infty$ $\Box$
--	--	--	--

# Encapsulation

The image shows two screenshots of a Slate editor window, each containing four boxes with modal logic formulas and their derivability in different systems.

**Screenshot 1: Slate - K.slt**

- Box 1:  $K. \Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   
 $K \vdash \checkmark \infty \Box$
- Box 2:  $T. \Box\varphi \rightarrow \varphi$   
 $K \vdash \times \infty \Box$
- Box 3:  $4. \Box\varphi \rightarrow \Box\Box\varphi$   
 $K \vdash \times \infty \Box$
- Box 4:  $5. \neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   
 $K \vdash \times \infty \Box$

**Screenshot 2: Slate - T.slt**

- Box 1:  $K. \Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   
 $M \vdash \checkmark \infty \Box$
- Box 2:  $T. \Box\varphi \rightarrow \varphi$   
 $M \vdash \checkmark \infty \Box$
- Box 3:  $4. \Box\varphi \rightarrow \Box\Box\varphi$   
 $M \vdash \times \infty \Box$
- Box 4:  $5. \neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   
 $M \vdash \times \infty \Box$

# Encapsulation

The image displays three overlapping windows, each showing a set of modal logic formulas and their validity in different systems. The windows are titled "Slate - K.slt", "Slate - T.slt", and "Slate - D.slt".

**Slate - K.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   
K  $\vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   
K  $\vdash \times \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   
K  $\vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   
K  $\vdash \times \infty \Box$

**Slate - T.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   
M  $\vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   
M  $\vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   
M  $\vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   
M  $\vdash \times \infty \Box$

**Slate - D.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   
D  $\vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   
D  $\vdash \times \infty \Box$
- D.  $\Box\varphi \rightarrow \Diamond\varphi$   
D  $\vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   
D  $\vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   
D  $\vdash \times \infty \Box$
- INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   
D  $\vdash \checkmark \infty \Box$

# Encapsulation

Slate - K.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ K $\vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ K $\vdash \times \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ K $\vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ K $\vdash \times \infty \Box$
---	---	---	---

Slate - T.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ M $\vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ M $\vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ M $\vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ M $\vdash \times \infty \Box$
---	---	---	---

Slate - D.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ D $\vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ D $\vdash \times \infty \Box$	D. $\Box\varphi \rightarrow \Diamond\varphi$ D $\vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ D $\vdash \times \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ D $\vdash \times \infty \Box$		INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ D $\vdash \checkmark \infty \Box$	

Slate - S4.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ S4 $\vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ S4 $\vdash \checkmark \infty \Box$	D. $\Box\varphi \rightarrow \Diamond\varphi$ S4 $\vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ S4 $\vdash \checkmark \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ S4 $\vdash \times \infty \Box$		INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ {INTER} Assume $\checkmark$	

# Encapsulation

**K**

**T**

**D**

**4 = S4**

**5 = S5**

The image shows five overlapping Slate windows, each displaying a grid of modal logic formulas and their derivability in different systems. The windows are titled as follows:

- Slate - K.slt**: Shows formulas K, T, 4, and 5. K is derivable in K (K ⊢ ✓ ∞ □). T, 4, and 5 are not derivable in K (K ⊢ ✗ ∞ □).
- Slate - T.slt**: Shows formulas K, T, 4, and 5. K and T are derivable in M (M ⊢ ✓ ∞ □). 4 and 5 are not derivable in M (M ⊢ ✗ ∞ □).
- Slate - D.slt**: Shows formulas K, T, D, 4, 5, and INTER. K, T, and 4 are not derivable in D (D ⊢ ✗ ∞ □). D and 5 are derivable in D (D ⊢ ✓ ∞ □). INTER is derivable in D (D ⊢ ✓ ∞ □).
- Slate - S4.slt**: Shows formulas K, T, D, 4, 5, and INTER. K, T, D, and 4 are derivable in S4 (S4 ⊢ ✓ ∞ □). 5 is not derivable in S4 (S4 ⊢ ✗ ∞ □). INTER is derivable in S4 with the assumption {INTER} (S4 ⊢ ✓ ∞ □).
- Slate - S5.slt**: Shows formulas K, T, D, 4, 5, and INTER. K, T, and 5 are derivable in S5 (S5 ⊢ ✓ ∞ □). D and 4 are not derivable in S5 (S5 ⊢ ✗ ∞ □). INTER is derivable in S5 with the assumption {INTER} (S5 ⊢ ✓ ∞ □).

# Encapsulation

**K**

**T**

**D**

**4 = S4**

**5 = S5**

The image shows five Slate windows, each displaying a grid of modal logic formulas and their derivability in a specific system. The windows are titled 'Slate - K.slt', 'Slate - T.slt', 'Slate - D.slt', 'Slate - S4.slt', and 'Slate - S5.slt'. The 'Slate - D.slt' window is highlighted with a red border.

**Slate - K.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $K \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $K \vdash \times \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $K \vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $K \vdash \times \infty \Box$

**Slate - T.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $M \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $M \vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $M \vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $M \vdash \times \infty \Box$

**Slate - D.slt** (highlighted)

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $D \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $D \vdash \times \infty \Box$
- D.  $\Box\varphi \rightarrow \Diamond\varphi$   $D \vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $D \vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $D \vdash \times \infty \Box$
- INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   $D \vdash \checkmark \infty \Box$

**Slate - S4.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $S4 \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $S4 \vdash \checkmark \infty \Box$
- D.  $\Box\varphi \rightarrow \Diamond\varphi$   $S4 \vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $S4 \vdash \checkmark \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $S4 \vdash \times \infty \Box$
- INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   $\{INTER\} \text{ Assume } \checkmark$

**Slate - S5.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $S5 \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $S5 \vdash \checkmark \infty \Box$
- D.  $\Box\varphi \rightarrow \Diamond\varphi$   $\{D\} \text{ Assume } \checkmark$
- 4.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $\{4\} \text{ Assume } \checkmark$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $S5 \vdash \checkmark \infty \Box$
- INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   $\{INTER\} \text{ Assume } \checkmark$

# Encapsulation

**K**

**T**

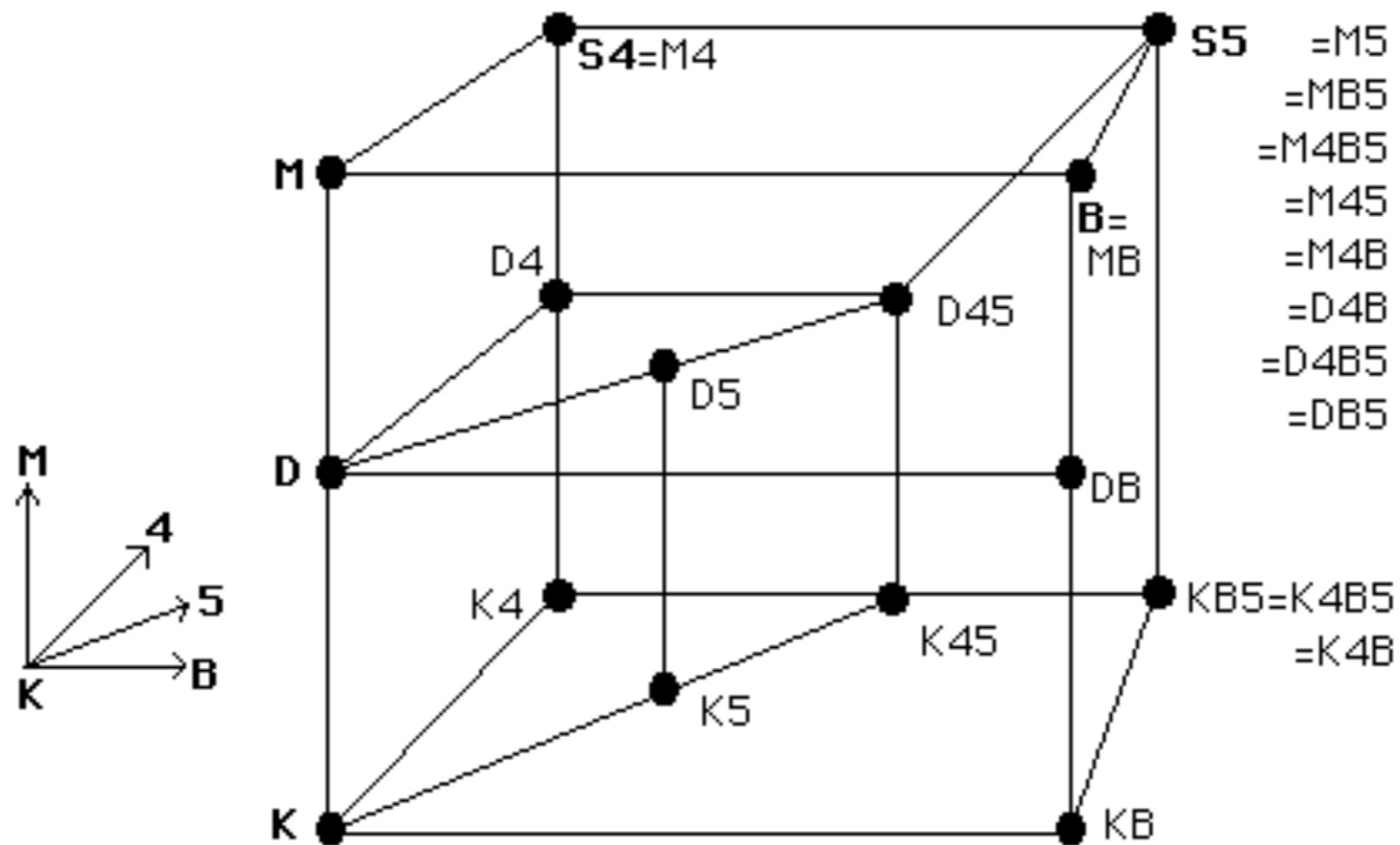
**D**

The screenshot displays five windows of the HyperSlate interface, each showing a set of logical formulas and their derivability status in a specific modal logic. The windows are titled as follows:

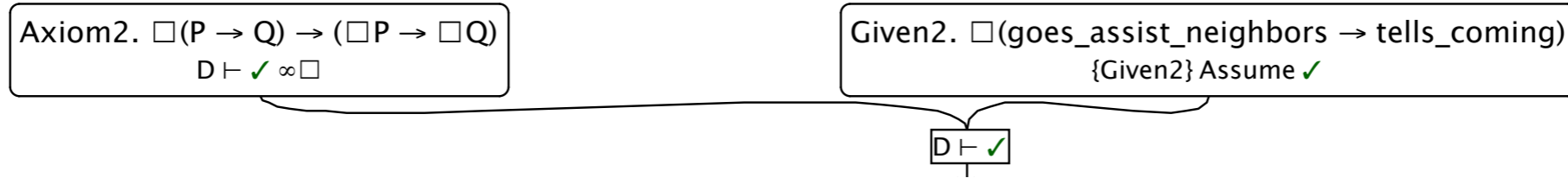
- Slate - K.slt**: Shows formulas K, T, 4, and 5. K is derivable (K ⊢ ✓ ∞ □), while T, 4, and 5 are not (K ⊢ ✗ ∞ □).
- Slate - T.slt**: Shows formulas K, T, 4, and 5. K and T are derivable (M ⊢ ✓ ∞ □), while 4 and 5 are not (M ⊢ ✗ ∞ □).
- Slate - D.slt** (highlighted with a red border): Shows formulas K, T, D, 4, 5, and INTER. K, T, and D are derivable (D ⊢ ✓ ∞ □), while 4 and 5 are not (D ⊢ ✗ ∞ □). The INTER formula is derivable (D ⊢ ✓ ∞ □).
- Slate - S4.slt**: Shows formulas K, T, D, 4, 5, and INTER. All formulas are derivable (S4 ⊢ ✓ ∞ □). The INTER formula is marked with "{INTER} Assume ✓".
- Slate - S5.slt**: Shows formulas K, T, D, 4, 5, and INTER. All formulas are derivable (S5 ⊢ ✓ ∞ □). The D and 4 formulas are marked with "{D} Assume ✓" and "{4} Assume ✓" respectively. The INTER formula is marked with "{INTER} Assume ✓".

**4 = S4**

**5 = S5**



# Chisholm's Paradox

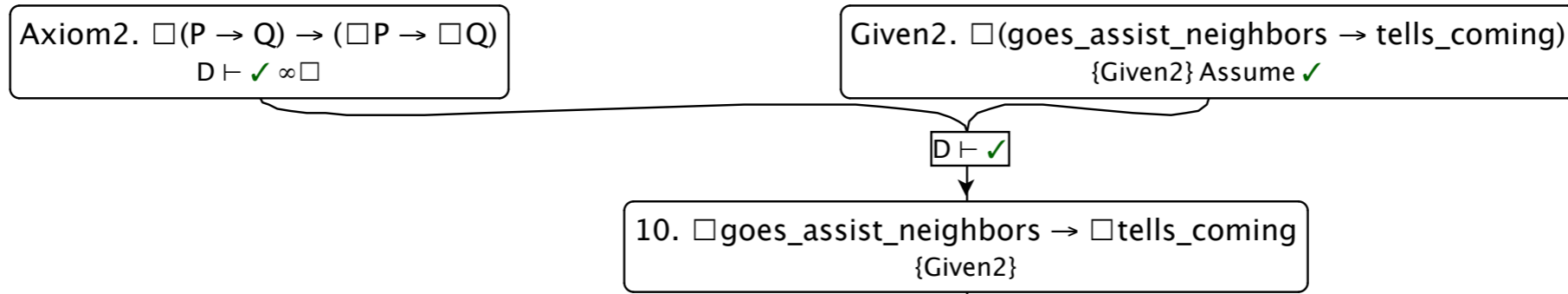


Axiom4. "Modus ponens for provability."  
{Axiom4} Assume ✓

Axiom5. "Theorems are obligatory."  
{Axiom5} Assume ✓

Axiom1. "All theorems of the propositional calculus."  
{Axiom1} Assume ✓

# Chisholm's Paradox

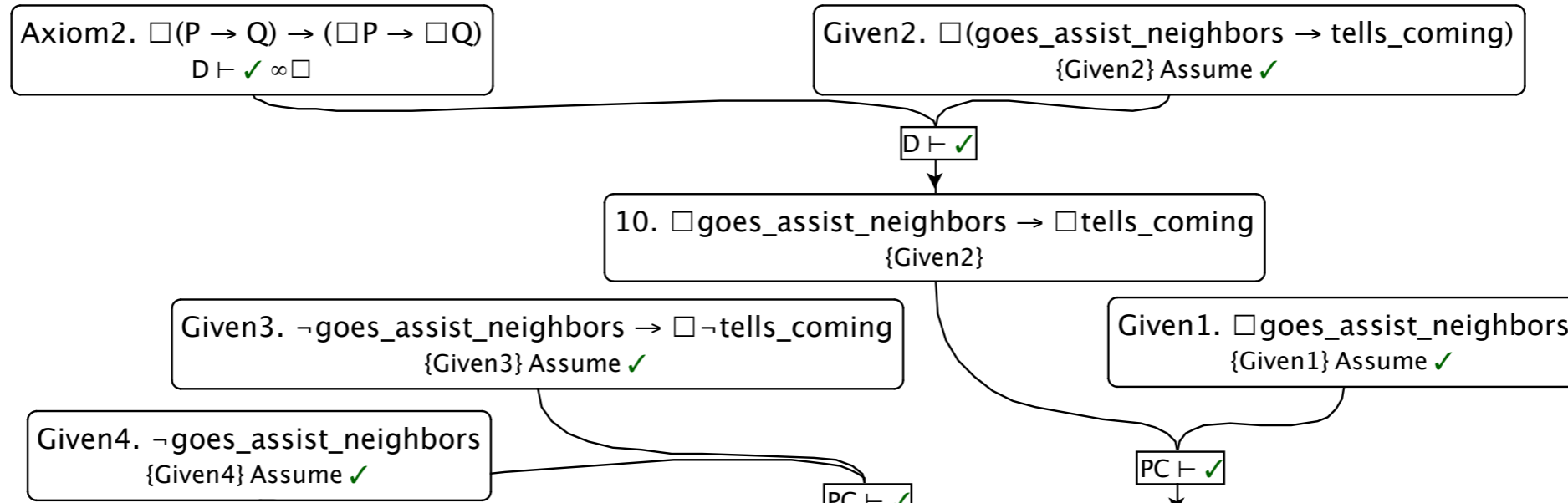


Axiom4. "Modus ponens for provability."  
{Axiom4} Assume  $\checkmark$

Axiom5. "Theorems are obligatory."  
{Axiom5} Assume  $\checkmark$

Axiom1. "All theorems of the propositional calculus."  
{Axiom1} Assume  $\checkmark$

# Chisholm's Paradox

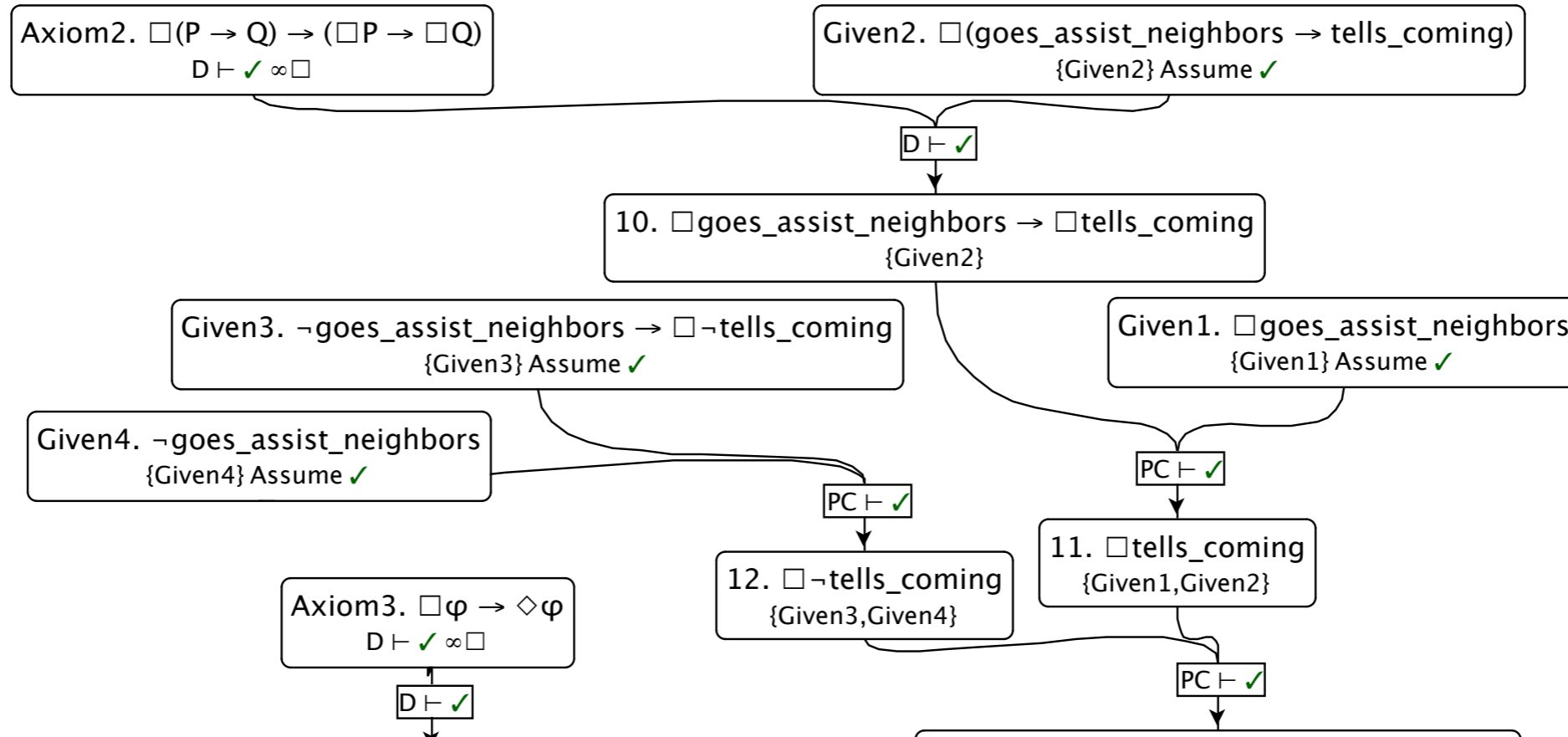


Axiom4. "Modus ponens for provability."  
 $\{\text{Axiom4}\} \text{ Assume } \checkmark$

Axiom5. "Theorems are obligatory."  
 $\{\text{Axiom5}\} \text{ Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."  
 $\{\text{Axiom1}\} \text{ Assume } \checkmark$

# Chisholm's Paradox

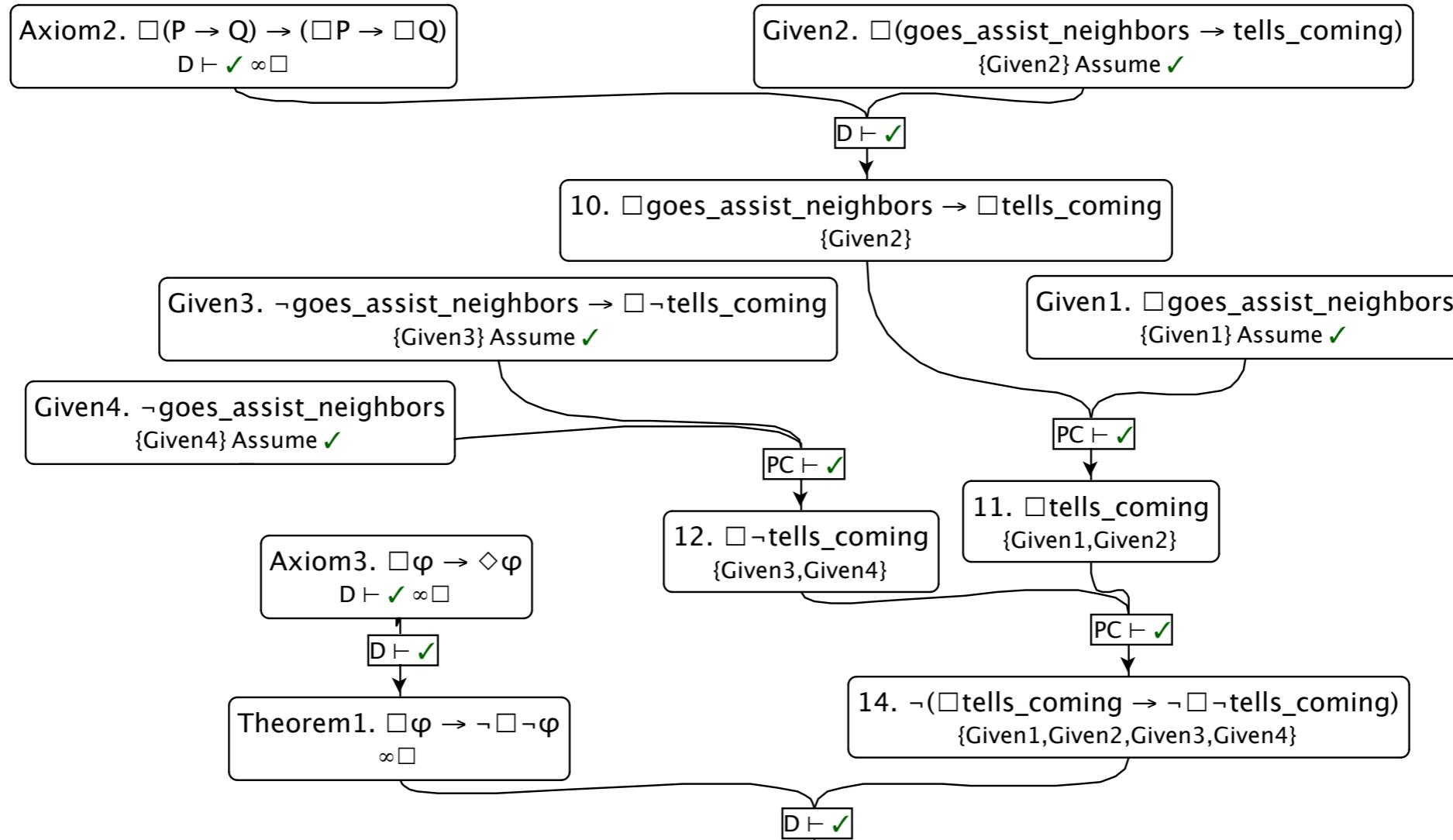


**Axiom4.** "Modus ponens for provability."  
 $\{\text{Axiom4}\} \text{ Assume } \checkmark$

**Axiom5.** "Theorems are obligatory."  
 $\{\text{Axiom5}\} \text{ Assume } \checkmark$

**Axiom1.** "All theorems of the propositional calculus."  
 $\{\text{Axiom1}\} \text{ Assume } \checkmark$

# Chisholm's Paradox

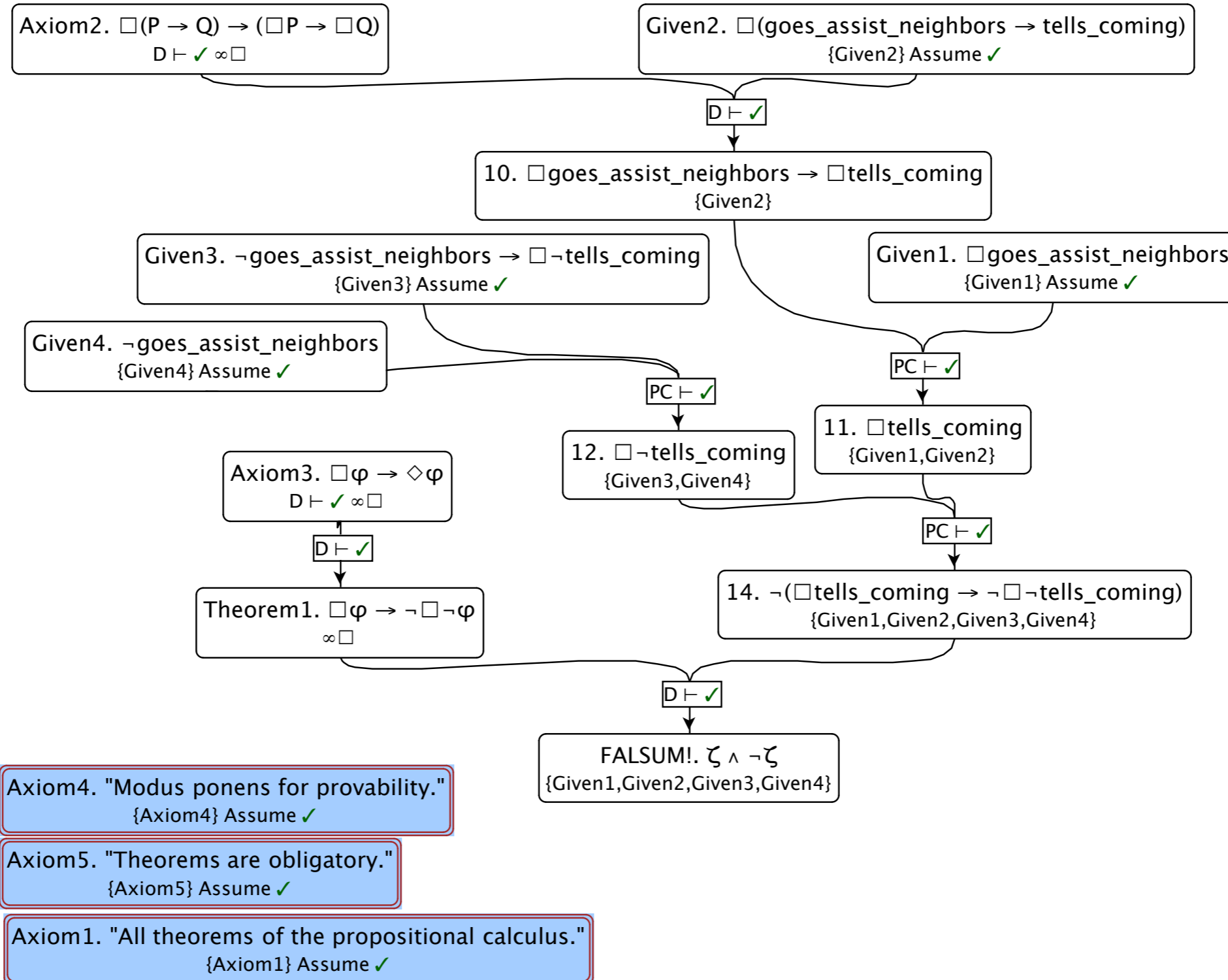


Axiom4. "Modus ponens for provability."  
 $\{\text{Axiom4}\} \text{Assume } \checkmark$

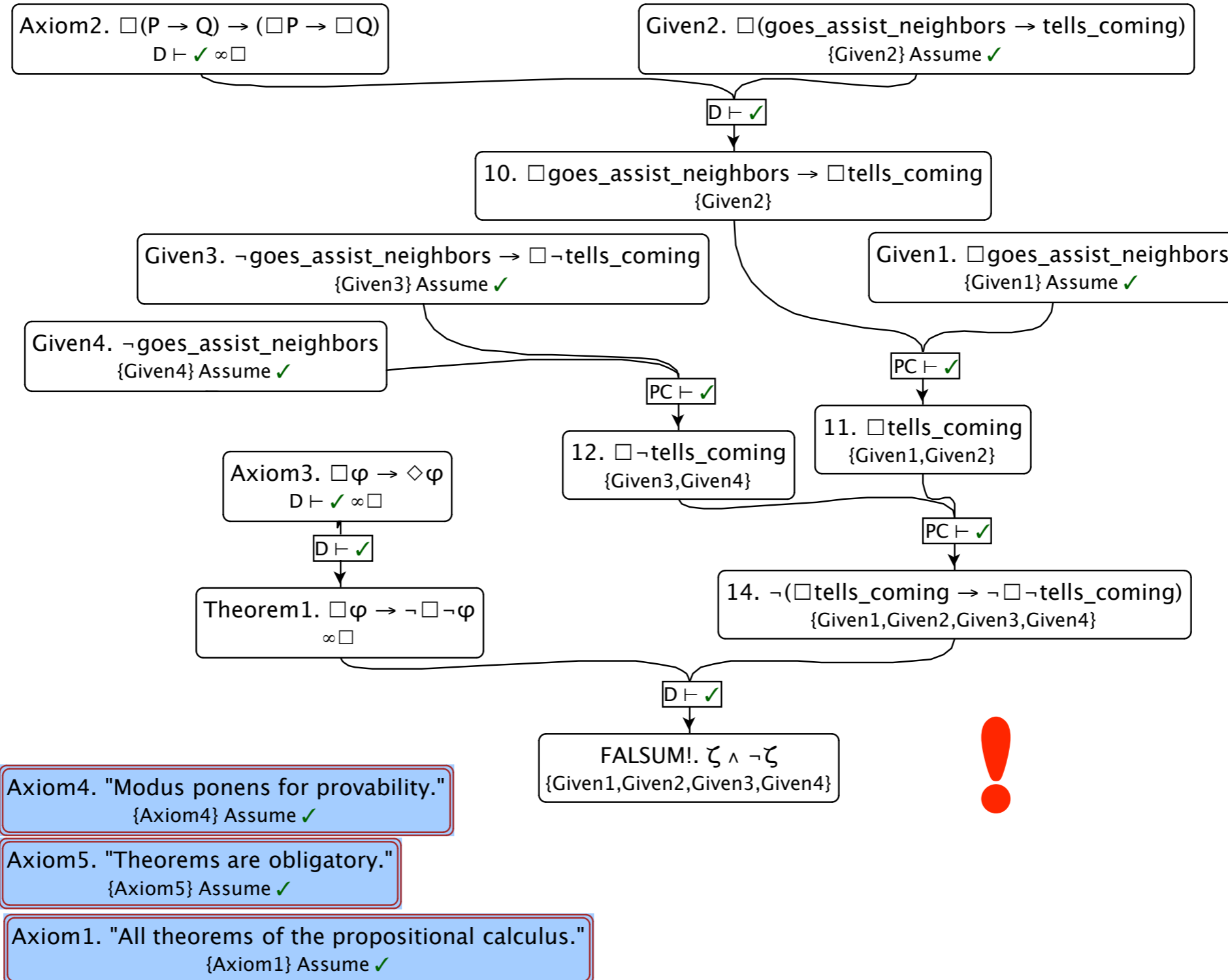
Axiom5. "Theorems are obligatory."  
 $\{\text{Axiom5}\} \text{Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."  
 $\{\text{Axiom1}\} \text{Assume } \checkmark$

# Chisholm's Paradox



# Chisholm's Paradox



# Review: Encapsulation

**K**

**T**

**D**

**4 = S4**

**5 = S5**

The screenshot displays five windows of the HyperSlate interface, each showing a set of modal logic formulas and their derivability in different systems. The systems are K, T, D, 4, 5, and INTER. Checkmarks (✓) indicate successful derivations, while red X's (✗) indicate failures.

- Slate - K.slt:**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (K  $\vdash$  ✓  $\infty\Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (K  $\vdash$  ✗  $\infty\Box$ )
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$  (K  $\vdash$  ✗  $\infty\Box$ )
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (K  $\vdash$  ✗  $\infty\Box$ )
- Slate - T.slt:**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (M  $\vdash$  ✓  $\infty\Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (M  $\vdash$  ✓  $\infty\Box$ )
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$  (M  $\vdash$  ✗  $\infty\Box$ )
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (M  $\vdash$  ✗  $\infty\Box$ )
- Slate - D.slt:**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (D  $\vdash$  ✓  $\infty\Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (D  $\vdash$  ✗  $\infty\Box$ )
  - D.  $\Box\varphi \rightarrow \Diamond\varphi$  (D  $\vdash$  ✓  $\infty\Box$ )
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$  (D  $\vdash$  ✗  $\infty\Box$ )
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (D  $\vdash$  ✗  $\infty\Box$ )
  - INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$  (D  $\vdash$  ✓  $\infty\Box$ )
- Slate - S4.slt:**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (S4  $\vdash$  ✓  $\infty\Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (S4  $\vdash$  ✓  $\infty\Box$ )
  - D.  $\Box\varphi \rightarrow \Diamond\varphi$  (S4  $\vdash$  ✓  $\infty\Box$ )
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$  (S4  $\vdash$  ✓  $\infty\Box$ )
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (S4  $\vdash$  ✗  $\infty\Box$ )
  - INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$  ({INTER} Assume ✓)
- Slate - S5.slt:**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (S5  $\vdash$  ✓  $\infty\Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (S5  $\vdash$  ✓  $\infty\Box$ )
  - D.  $\Box\varphi \rightarrow \Diamond\varphi$  ({D} Assume ✓)
  - 4.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  ({4} Assume ✓)
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (S5  $\vdash$  ✓  $\infty\Box$ )
  - INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$  ({INTER} Assume ✓)

# Review: Encapsulation

**K**

**T**

**D**

The screenshot shows five modal windows, each representing a different logic. Each window contains a grid of logical formulas and their provability status in that logic.

- Slate - K.slt**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $K \vdash \checkmark \infty \Box$
  - T.  $\Box\varphi \rightarrow \varphi$   $K \vdash \times \infty \Box$
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $K \vdash \times \infty \Box$
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $K \vdash \times \infty \Box$
- Slate - T.slt**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $M \vdash \checkmark \infty \Box$
  - T.  $\Box\varphi \rightarrow \varphi$   $M \vdash \checkmark \infty \Box$
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $M \vdash \times \infty \Box$
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $M \vdash \times \infty \Box$
- Slate - D.slt**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $D \vdash \checkmark \infty \Box$
  - T.  $\Box\varphi \rightarrow \varphi$   $D \vdash \times \infty \Box$
  - D.  $\Box\varphi \rightarrow \Diamond\varphi$   $D \vdash \checkmark \infty \Box$
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $D \vdash \times \infty \Box$
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $D \vdash \times \infty \Box$
  - INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   $D \vdash \checkmark \infty \Box$
- Slate - S4.slt** (highlighted with a red border)
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $S4 \vdash \checkmark \infty \Box$
  - T.  $\Box\varphi \rightarrow \varphi$   $S4 \vdash \checkmark \infty \Box$
  - D.  $\Box\varphi \rightarrow \Diamond\varphi$   $S4 \vdash \checkmark \infty \Box$
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $S4 \vdash \checkmark \infty \Box$
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $S4 \vdash \times \infty \Box$
  - INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   $\{INTER\} \text{ Assume } \checkmark$
- Slate - S5.slt**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $S5 \vdash \checkmark \infty \Box$
  - T.  $\Box\varphi \rightarrow \varphi$   $S5 \vdash \checkmark \infty \Box$
  - D.  $\Box\varphi \rightarrow \Diamond\varphi$   $\{D\} \text{ Assume } \checkmark$
  - 4.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $\{4\} \text{ Assume } \checkmark$
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $S5 \vdash \checkmark \infty \Box$
  - INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   $\{INTER\} \text{ Assume } \checkmark$

**4 = S4**

**5 = S5**

# Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot displays the HyperSlate interface with several windows showing logic calculi configurations and theorem verification results.

**Configuration Window (Create file):**

- Propositional Calculus
- $L_0$  = Pure Predicate Calculus
- $L_1$  = First-order Logic
- $L_2$  = Second-order Logic
- K
- T
- D
- S4
- S5
- DCEC (fragment)
- Hyperlog

**Slate - K.slt:**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $K \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $K \vdash \times \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $K \vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $K \vdash \times \infty \Box$

**Slate - T.slt:**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $M \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $M \vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $M \vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $M \vdash \times \infty \Box$

**Slate - S4.slt:**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $S4 \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $S4 \vdash \checkmark \infty \Box$
- D.  $\Box\varphi \rightarrow \Diamond\varphi$   $S4 \vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $S4 \vdash \checkmark \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $S4 \vdash \times \infty \Box$
- INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   $\{INTER\} \text{ Assume } \checkmark$

**Slate - S5.slt:**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $S5 \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $S5 \vdash \checkmark \infty \Box$
- D.  $\Box\varphi \rightarrow \Diamond\varphi$   $\{D\} \text{ Assume } \checkmark$
- 4.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $\{4\} \text{ Assume } \checkmark$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $S5 \vdash \checkmark \infty \Box$
- INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   $\{INTER\} \text{ Assume } \checkmark$

# Review: Encapsulation

K

T

D

The screenshot displays the HyperSlate interface with several windows showing logic calculi and their verification results. A green arrow points to the 'S5' button in the 'Create file' dialog.

**Create file dialog:**

- Propositional Calculus
- $L_0$  = Pure Predicate Calculus
- $L_1$  = First-order Logic
- $L_2$  = Second-order Logic
- K
- T
- D
- S4
- S5
- DCEC (fragment)
- Hyperlog

**Slate - K.slt:**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $K \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $K \vdash \times \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $K \vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $K \vdash \times \infty \Box$

**Slate - T.slt:**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $M \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $M \vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $M \vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $M \vdash \times \infty \Box$

**Slate - S4.slt:**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $S4 \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $S4 \vdash \checkmark \infty \Box$
- D.  $\Box\varphi \rightarrow \Diamond\varphi$   $S4 \vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $S4 \vdash \checkmark \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $S4 \vdash \times \infty \Box$
- INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   $\{INTER\} \text{ Assume } \checkmark$

**Slate - S5.slt:**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $S5 \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $S5 \vdash \checkmark \infty \Box$
- D.  $\Box\varphi \rightarrow \Diamond\varphi$   $\{D\} \text{ Assume } \checkmark$
- 4.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $\{4\} \text{ Assume } \checkmark$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $S5 \vdash \checkmark \infty \Box$
- INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   $\{INTER\} \text{ Assume } \checkmark$

4 = S4

5 = S5

# DCEC (supported fragment, student version)

## First-order (Propositional) Schema

- Assume
- Not Elim, Not Intro
- And Elim, And Intro
- Or Elim, Or Intro
- If Elim, If Intro
- Iff Elim, Iff Intro
- Forall Elim, Forall Intro
- Exists Elim, Exists Intro
- Higher Order Forall Elim, Higher Order Forall Intro
- Higher Order Exists Elim, Higher Order Exists Intro
- Eq Elim, Eq Intro
- Pc Oracle, Fol Oracle

## Modal Inference Schemata

- $R_1, R_2, R_3, R_4,$
- $R_k, R_b,$
- $R_{14}$

## Inference Schemata

Modal

$$\frac{\mathbf{K}(a, t_1, \Gamma), \Gamma \vdash \phi, t_1 \leq t_2}{\mathbf{K}(a, t_2, \phi)} [R_K] \quad \frac{\mathbf{B}(a, t_1, \Gamma), \Gamma \vdash \phi, t_1 \leq t_2}{\mathbf{B}(a, t_2, \phi)} [R_B]$$

$$\frac{}{\mathbf{C}(t, \mathbf{P}(a, t, \phi) \rightarrow \mathbf{K}(a, t, \phi))} [R_1] \quad \frac{}{\mathbf{C}(t, \mathbf{K}(a, t, \phi) \rightarrow \mathbf{B}(a, t, \phi))} [R_2]$$

$$\frac{\mathbf{C}(t, \phi) \quad t \leq t_1 \dots t \leq t_n}{\mathbf{K}(a_1, t_1, \dots \mathbf{K}(a_n, t_n, \phi) \dots)} [R_3] \quad \frac{\mathbf{K}(a, t, \phi)}{\phi} [R_4]$$

$$\frac{}{\mathbf{C}(t, \mathbf{K}(a, t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{K}(a, t_2, \phi_1) \rightarrow \mathbf{K}(a, t_3, \phi_2)} [R_5]$$

$$\frac{}{\mathbf{C}(t, \mathbf{B}(a, t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{B}(a, t_2, \phi_1) \rightarrow \mathbf{B}(a, t_3, \phi_2)} [R_6]$$

$$\frac{}{\mathbf{C}(t, \mathbf{C}(t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{C}(t_2, \phi_1) \rightarrow \mathbf{C}(t_3, \phi_2)} [R_7]$$

$$\frac{}{\mathbf{C}(t, \forall x. \phi \rightarrow \phi[x \mapsto t])} [R_8] \quad \frac{}{\mathbf{C}(t, \phi_1 \leftrightarrow \phi_2 \rightarrow \neg \phi_2 \rightarrow \neg \phi_1)} [R_9]$$

$$\frac{}{\mathbf{C}(t, [\phi_1 \wedge \dots \wedge \phi_n \rightarrow \phi] \rightarrow [\phi_1 \rightarrow \dots \rightarrow \phi_n \rightarrow \psi])} [R_{10}]$$

$$\frac{\mathbf{S}(s, h, t, \phi)}{\mathbf{B}(h, t, \mathbf{B}(s, t, \phi))} [R_{12}] \quad \frac{\mathbf{I}(a, t, \text{happens}(\text{action}(a^*, \alpha), t'))}{\mathbf{P}(a, t, \text{happens}(\text{action}(a^*, \alpha), t))} [R_{13}]$$

$$\frac{\mathbf{B}(a, t, \phi) \quad \mathbf{B}(a, t, \mathbf{O}(a, t, \phi, \chi)) \quad \mathbf{O}(a, t, \phi, \chi)}{\mathbf{K}(a, t, \mathbf{I}(a, t, \chi))} [R_{14}]$$

# DCEC (supported fragment, student version)

## First-order (Propositional) Schema

- Assume
- Not Elim, Not Intro
- And Elim, And Intro
- Or Elim, Or Intro
- If Elim, If Intro
- Iff Elim, Iff Intro
- Forall Elim, Forall Intro
- Exists Elim, Exists Intro
- Higher Order Forall Elim, Higher Order Forall Intro
- Higher Order Exists Elim, Higher Order Exists Intro
- Eq Elim, Eq Intro
- Pc Oracle, Fol Oracle

## Modal Inference Schemata

- $R_1, R_2, R_3, R_4,$
- $R_k, R_b,$
- $R_{14}$

## Inference Schemata

Modal

$$\frac{\mathbf{K}(a, t_1, \Gamma), \Gamma \vdash \phi, t_1 \leq t_2}{\mathbf{K}(a, t_2, \phi)} [R_K] \quad \frac{\mathbf{B}(a, t_1, \Gamma), \Gamma \vdash \phi, t_1 \leq t_2}{\mathbf{B}(a, t_2, \phi)} [R_B]$$

$$\frac{}{\mathbf{C}(t, \mathbf{P}(a, t, \phi) \rightarrow \mathbf{K}(a, t, \phi))} [R_1] \quad \frac{}{\mathbf{C}(t, \mathbf{K}(a, t, \phi) \rightarrow \mathbf{B}(a, t, \phi))} [R_2]$$

$$\frac{\mathbf{C}(t, \phi) \quad t \leq t_1 \dots t \leq t_n}{\mathbf{K}(a_1, t_1, \dots \mathbf{K}(a_n, t_n, \phi) \dots)} [R_3] \quad \frac{\mathbf{K}(a, t, \phi)}{\phi} [R_4]$$

$$\frac{}{\mathbf{C}(t, \mathbf{K}(a, t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{K}(a, t_2, \phi_1) \rightarrow \mathbf{K}(a, t_3, \phi_2)} [R_5]$$

$$\frac{}{\mathbf{C}(t, \mathbf{B}(a, t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{B}(a, t_2, \phi_1) \rightarrow \mathbf{B}(a, t_3, \phi_2)} [R_6]$$

$$\frac{}{\mathbf{C}(t, \mathbf{C}(t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{C}(t_2, \phi_1) \rightarrow \mathbf{C}(t_3, \phi_2)} [R_7]$$

$$\frac{}{\mathbf{C}(t, \forall x. \phi \rightarrow \phi[x \mapsto t])} [R_8] \quad \frac{}{\mathbf{C}(t, \phi_1 \leftrightarrow \phi_2 \rightarrow \neg \phi_2 \rightarrow \neg \phi_1)} [R_9]$$

$$\frac{}{\mathbf{C}(t, [\phi_1 \wedge \dots \wedge \phi_n \rightarrow \phi] \rightarrow [\phi_1 \rightarrow \dots \rightarrow \phi_n \rightarrow \psi])} [R_{10}]$$

$$\frac{\mathbf{S}(s, h, t, \phi)}{\mathbf{B}(h, t, \mathbf{B}(s, t, \phi))} [R_{12}] \quad \frac{\mathbf{I}(a, t, \mathit{happens}(\mathit{action}(a^*, \alpha), t'))}{\mathbf{P}(a, t, \mathit{happens}(\mathit{action}(a^*, \alpha), t))} [R_{13}]$$

$$\frac{\mathbf{B}(a, t, \phi) \quad \mathbf{B}(a, t, \mathbf{O}(a, t, \phi, \chi)) \quad \mathbf{O}(a, t, \phi, \chi)}{\mathbf{K}(a, t, \mathbf{I}(a, t, \chi))} [R_{14}]$$

**For Brave HyperLogical Adventurers**

# For Brave HyperLogical Adventurers

“Everything smart knows that everything tinks anything that tinks something identical with something.”

# For Brave HyperLogical Adventurers

“Everything smart knows that everything tinks anything that tinks something identical with something.”

“Blinky is smart and tinks himself.”

# For Brave HyperLogical Adventurers

“Everything smart knows that everything tinks anything that tinks something identical with something.”

“Blinky is smart and tinks himself.”

**Therefore:**

# For Brave HyperLogical Adventurers

“Everything smart knows that everything tinks anything that tinks something identical with something.”

“Blinky is smart and tinks himself.”

“Everything tinks anything that tinks something identical with something.”

**Therefore:**

# For Brave HyperLogical Adventurers

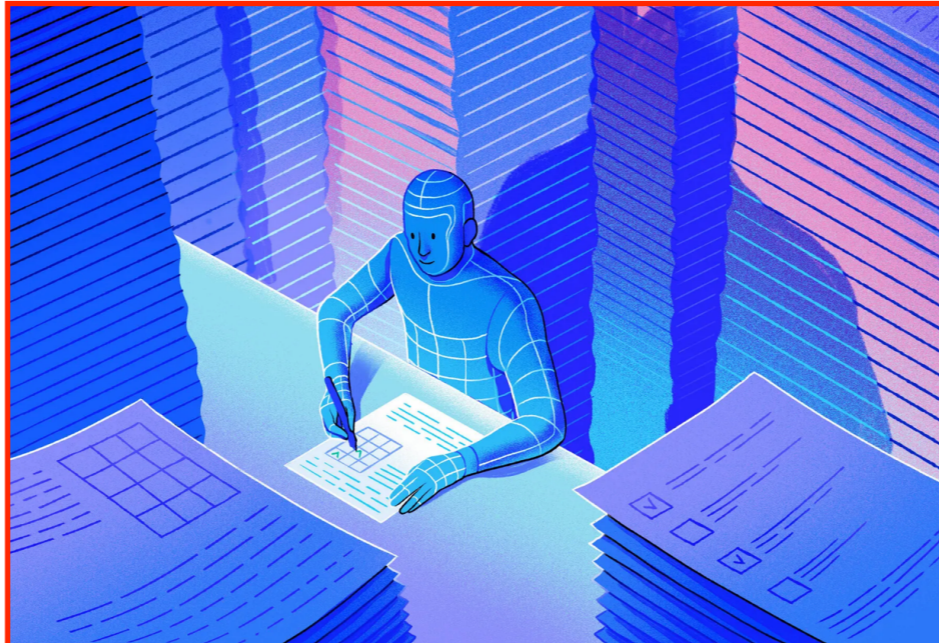
“Everything smart knows that everything tinks anything that tinks something identical with something.”

“Blinky is smart and tinks himself.”

“Everything tinks anything that tinks something identical with something.”

**Therefore:**

“Yolanda tinks Xavier.”



Rune Fisker

#### THE SHIFT

## *When A.I. Passes This Test, Look Out*

The creators of a new test called “Humanity’s Last Exam” argue we may soon lose the ability to create tests hard enough for A.I. models.

▶ Listen to this article · 8:04 min [Learn more](#)



By **Kevin Roose**  
Reporting from San Francisco

Jan. 23, 2025

If you’re looking for a new reason to be nervous about artificial intelligence, try this: Some of the smartest humans in the world are struggling to create tests that A.I. systems can’t pass.

**For Brave HyperLogical Adventurers**

# For Brave HyperLogical Adventurers

☰ ChatGPT 4o > ✎

Blinky is smart and tinks himself.

Blinky is indeed a thoughtful and self-aware household robot. He doesn't just follow orders blindly—he considers the implications of his actions, the needs of the Booker family, and even the deeper meaning behind his existence.

Would you like Blinky to explore a philosophical question, optimize his household tasks, or maybe have a moment of self-reflection?

📄 🔊 👍 👎 ↻

Everything smart knows that everything tinks anything that tinks something identical with something.

That's a fascinating and layered statement, Blinky. If I parse it correctly, you're suggesting that intelligence recognizes that all thinking entities perceive identical concepts as being fundamentally linked.

# For Brave HyperLogical Adventurers

The image shows a screenshot of a ChatGPT 4o chat interface. The chat window is titled "ChatGPT 4o" and has a hamburger menu icon on the left and a share icon on the right. The chat history shows a user message: "Blinky is indeed a thoughtful household robot. He considers things not just blindly—he considers the needs of others, the needs of his actions, the needs of his environment, even the deeper meaning of his actions. Would you like Blinky to ask you a question, optimize his thinking, maybe have a moment of reflection?" and a system response: "That's a fascinating question about Blinky. If I parse it correctly, it suggests that intelligence recognizes that entities perceive identical concepts as being fundamentally linked." A second chat window is overlaid on the first, showing a user message: "This raises an interesting question: is 'thinking' merely the act of recognizing identity and equivalence, or does it involve something deeper—like creating new connections or questioning the very nature of those identities? What do you think, Blinky? Does your thinking consist of recognizing sameness, or do you go beyond that?" and a system response: "Can you prove from these two propositions that Blinky's friend Yolanda is smart? That depends on the exact logical structure of the two propositions. Let's break it down systematically." The interface includes icons for copying, voice recording, liking, and disliking messages.

ChatGPT 4o >

Blinky

Blinky is indeed a thoughtful household robot. He considers things not just blindly—he considers the needs of others, the needs of his actions, the needs of his environment, even the deeper meaning of his actions. Would you like Blinky to ask you a question, optimize his thinking, maybe have a moment of reflection?

That's a fascinating question about Blinky. If I parse it correctly, it suggests that intelligence recognizes that entities perceive identical concepts as being fundamentally linked.

ChatGPT 4o >

This raises an interesting question: is "thinking" merely the act of recognizing identity and equivalence, or does it involve something deeper—like creating new connections or questioning the very nature of those identities? What do you think, Blinky? Does your thinking consist of recognizing sameness, or do you go beyond that?

Can you prove from these two propositions that Blinky's friend Yolanda is smart?

That depends on the exact logical structure of the two propositions. Let's break it down systematically.

# For Brave HyperLogical Adventurers

ChatGPT 4o >

Blinky

Blinky is indeed a thoughtful household robot. He doesn't think blindly—he considers actions, the needs of others, and even the deeper meaning of things.

Would you like Blinky to ask a question, optimize his behavior, or maybe have a moment of reflection?

Copy | Play | Like | Dislike | Refresh

Even though Blinky is a robot, he thinks some things are important.

ChatGPT 4o >

This raises an interesting question: "thinking" merely a function of identity and equipment, or something deeper, like connections or relationships between those identities?

What do *you* think? Do you think they consist of recognizing patterns beyond that?

Copy | Play | Like | Dislike | Refresh

ChatGPT 4o >

You stated:

- Everything smart knows that everything thinks anything that thinks something identical with something.**
  - This suggests that all smart entities recognize that any thinking entity acknowledges identical things as identical.
- Blinky is smart and thinks himself.**
  - This establishes Blinky as a smart entity and confirms that he engages in thinking.

To prove that **Blinky's friend Yolanda is smart**, we need a logical connection between these premises and Yolanda's intelligence.

That depends on how you define the two propositions. Let's break it down systematically.

entities perceive identical concepts as being fundamentally linked.

# For Brave HyperLogical Adventurers

The image displays three overlapping screenshots of a ChatGPT 4o interface. The background window shows a conversation about Blinky, a household robot, and a question about formalizing his intelligence in logical notation. The middle window shows a list of two points: 1. **Everything smart thinks anything identical with so** and 2. **Blinky is smart a**. The foreground window shows a detailed response to the question about formalizing intelligence, stating that the given propositions do not provide enough information to prove Yolanda's intelligence, and then asks for a proof from the same two propositions that Yolanda tinks Xavier.

**ChatGPT 4o >**

Blinky is indeed a thoughtful household robot. He doesn't think blindly—he considers actions, the needs of others, and even the deeper meaning of things.

Would you like Blinky to ask a question, optimize his behavior, or maybe have a moment of reflection?

Copy | Speak | Like | Dislike | Refresh

Even though Blinky doesn't think, he tinks something.

That's a fascinating analysis of Blinky. If I parse it correctly, that intelligence recognition that intelligent entities perceive identical concepts as being fundamentally linked.

**ChatGPT 4o >**

This raises an interesting question about the nature of "thinking" merely recognizing identity and equating it to something deeper, like connections or relationships between those identities?

What do *you* think intelligence should consist of recognizing beyond that?

Copy | Speak | Like | Dislike | Refresh

**ChatGPT 4o >**

You stated:

- Everything smart thinks anything identical with so**
  - This suggests that smart entities recognize that a certain property acknowledges intelligence.
- Blinky is smart a**
  - This establishes that Blinky is smart and confirms that smart entities recognize a certain property and confirms that property.

To prove that **Blinky is smart**, we need a logical connection between these premises and the conclusion.

That depends on how you define the two propositions. Let's break it down systematically.

**ChatGPT 4o >**

Would you like to formalize this in logical notation or explore it further conceptually?

Copy | Speak | Like | Dislike | Refresh

Answer directly: Can you prove from these two propositions I gave you that Yolanda is smart?

No, the two given propositions do not provide enough information to prove that Yolanda is smart. They establish that Blinky is smart and that smart entities recognize a certain property of thinking, but they do not state anything about Yolanda's intelligence.

Copy | Speak | Like | Dislike | Refresh

Can you prove from the same two propositions that Yolanda tinks Xavier (who happens to be a friend of Yolanda)?

# For Brave HyperLogical Adventurers

The image displays three overlapping screenshots of a ChatGPT 4o chat interface. The background window shows a conversation about Blinky, a household robot, and a question about formalizing his intelligence in logical notation. The middle window shows a list of two points: 1. **Everything smart thinks anything identical with so** and 2. **Blinky is smart a**. The foreground window shows a detailed response: "No, the two given propositions do not provide enough information to prove that Yolanda thinks Xavier. They only establish that Blinky is smart and that smart entities recognize a certain property of thinking, but they do not state anything about Yolanda's thinking or whether she thinks about Xavier." Below this, a question asks: "Can you prove from the same two propositions that Yolanda tinks Xavier (who happens to be a friend of Yolanda)?"

ChatGPT 4o >

Blinky

Blinky is indeed a thoughtful household robot. He doesn't think blindly—he considers actions, the needs of others, and even the deeper meaning of things.

Would you like Blinky to answer your question, optimize his responses, or maybe have a moment of reflection?

Copy Speak Like Dislike Refresh

Even if Blinky never thinks, some things are true.

That's a fascinating analysis of Blinky. If I parse it correctly, that intelligence recognition entities perceive identical concepts as being fundamentally linked.

ChatGPT 4o >

This raises an interesting question about the identity and equivalence of concepts—something deeper than simple connections or coincidences between those identities?

What do *you* think these concepts consist of recognizing beyond that?

Copy Speak Like Dislike Refresh

ChatGPT 4o >

You stated:

- Everything smart thinks anything identical with so**
  - This suggests that smart entities recognize that a concept acknowledges its own identity.
- Blinky is smart a**
  - This establishes that Blinky is smart and confirms that smart entities recognize a certain property of thinking.

To prove that **Blinky is smart**, we need a logical framework based on these premises and their implications.

That depends on how we interpret the two propositions. Let's break it down systematically.

ChatGPT 4o >

Would you like to formalize this in logical notation or explore it further conceptually?

Copy Speak Like Dislike Refresh

Answer directly: Can you prove from these two propositions I gave you that Yolanda thinks Xavier?

No, the two given propositions do not provide enough information to prove that Yolanda thinks Xavier. They only establish that Blinky is smart and that smart entities recognize a certain property of thinking, but they do not state anything about Yolanda's thinking or whether she thinks about Xavier.

Copy Speak Like Dislike Refresh

Can you prove from the same two propositions that Yolanda tinks Xavier (who happens to be a friend of Yolanda)?



*Det er en logikk for  
hvert problem!*