

“The Wager,” Overview of the Book, and Gödel’s Completeness Theorem

Selmer Bringsjord

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Troy, New York 12180 USA

Intro to Formal Logic (With AI)
4/6/2026



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Note: This is a version of coverage of Gödel’s Completeness Theorem designed for those who’ve had at least one standard/standard-paced university-level course in formal logic.

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HyperGrader releases ...

AI vs. PI



AI vs. PI

AI has brought forth a dizzying acronym kaleidoscope:

AI, Strong AI, Weak AI, Strong Weak AI, AGI, HI, HLAI, ASI, ...
and SAI (Y. LeCun), and PI, HPI, API, SPI.



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Background Context ...

Could Artificial Intelligence Ever ... Match Personal Intelligence? Gödelian Essays on Minds vs. Machines


by Selmer Bringsjord

- Introduction (“The Wager”)
- Brief Preliminaries (elementary discrete math, incl. ZOL, FOL)
- The Completeness Theorem
- The First Incompleteness Theorem
- The Second Incompleteness Theorem
- The Speedup Theorem
- The Continuum-Hypothesis Theorem
- The Time-Travel Theorem
- Gödel’s “God Theorem”
- Could a Machine Match Gödel’s Genius?



Could Artificial Intelligence Ever ... Match Personal Intelligence? Gödelian Essays on Minds vs. Machines

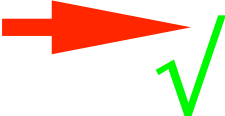
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Some Timeline Points



Some Timeline Points

1906 Brünn, Austria-Hungary



Some Timeline Points

1923 Vienna

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Some Timeline Points

Undergrad in seminar by Schlick

1923 Vienna

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Some Timeline Points

1929 Doctoral Dissertation: Proof of Completeness Theorem
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Some Timeline Points

1930 Announces (First) *Incompleteness* Theorem
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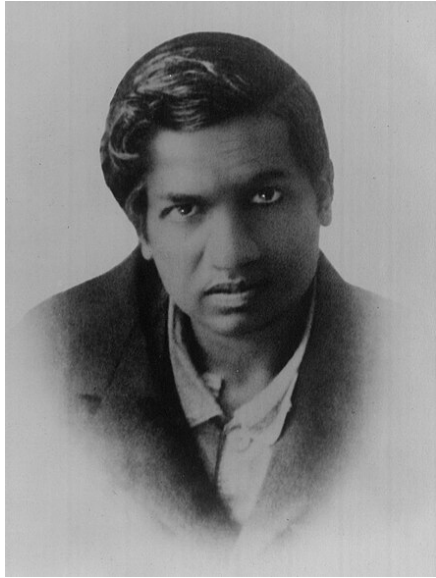
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“I have proved that syntax and semantics are fundamentally the same.”

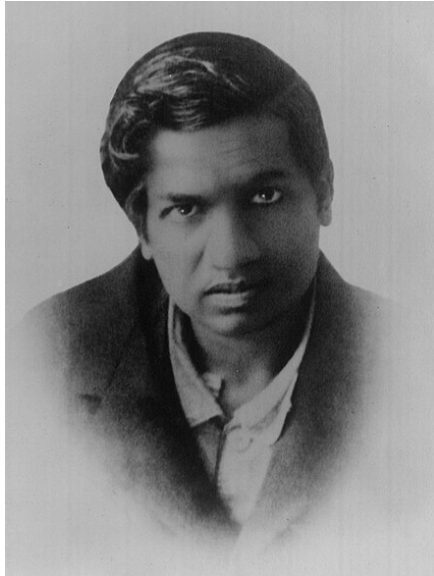


Ruminations on Srinivas Ramanujan

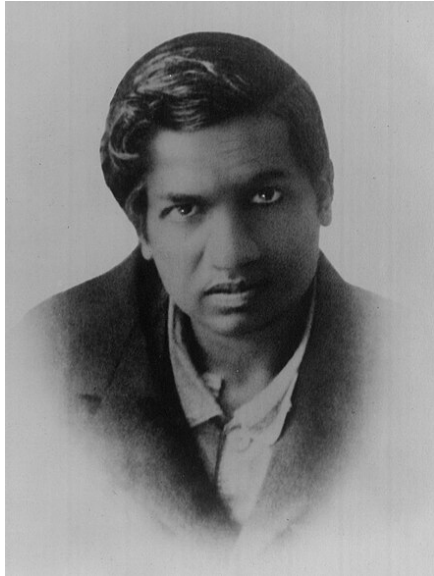
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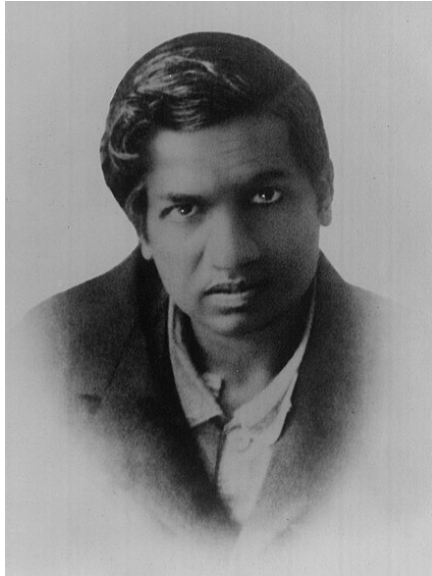


“ ϕ is true!”



" ϕ is true!"

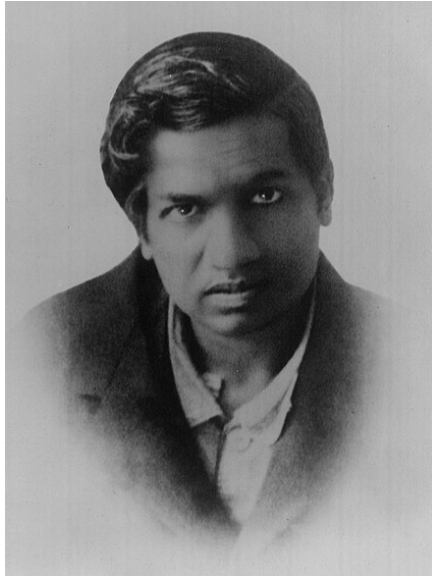
Should we trust R.?



“ ϕ is true!”

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Should *everyone* trust R.?

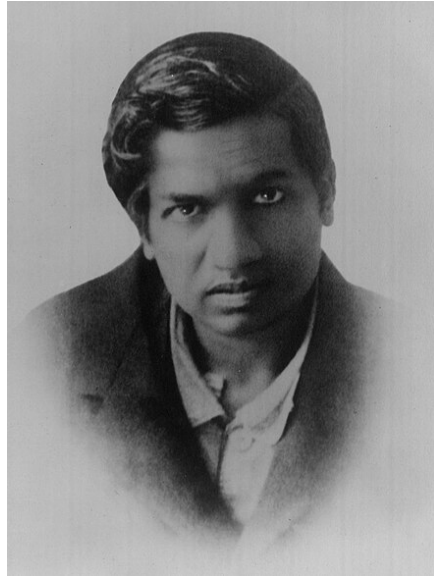


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Should *everyone* trust R.?

Let's assume the affirmative.



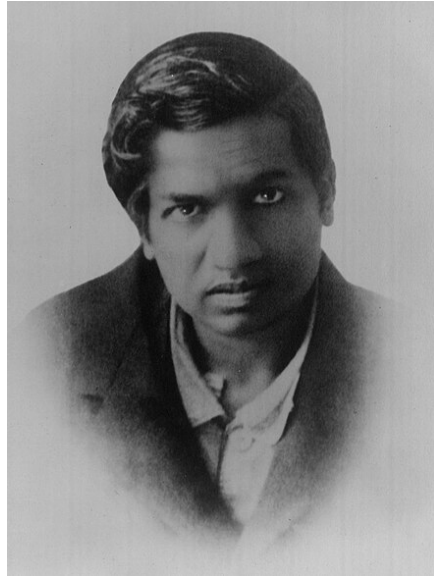
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Let's assume the affirmative.

We still need ... a proof!



“ ϕ is true!”

Should we trust R.?

Should *everyone* trust R.?

Let's assume the affirmative.

We still need ... a proof!

Or at least we need to know there *is* a proof!

**Preliminaries:
Propositional Calculus &
First-Order Logic**

...

Actually ...

$$\mathcal{L}_0 < \mathcal{L}_1 < \mathcal{L}_2 < \mathcal{L}_3 \dots$$

Actually ...

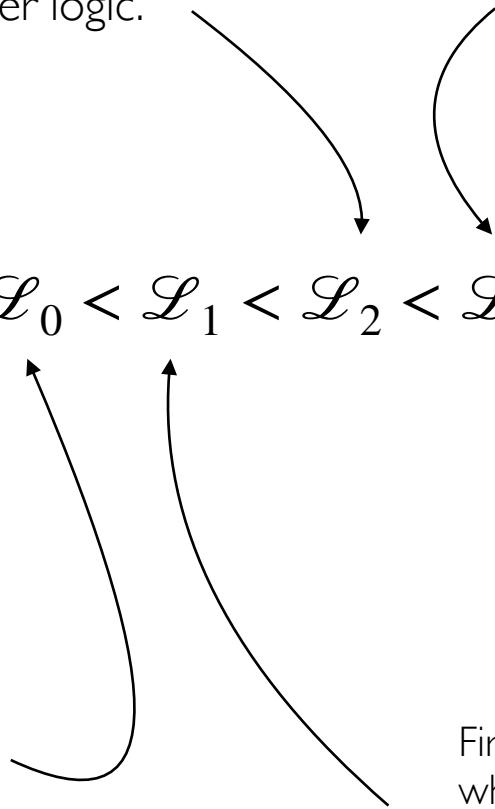
Second-order logic.

Third-order logic, which Gödel used for his “God Theorem.”

$\mathcal{L}_0 < \mathcal{L}_1 < \mathcal{L}_2 < \mathcal{L}_3 \dots$

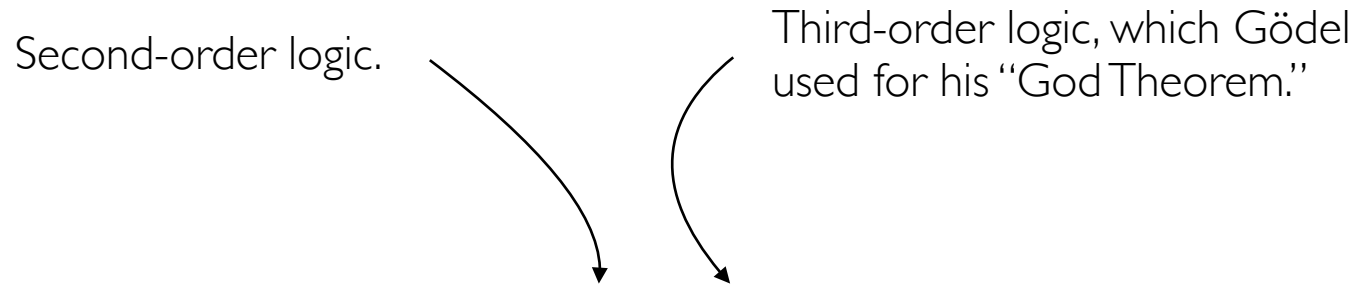
Zero-order logic; subsumes the propositional calculus.

First-order logic; this is what the Completeness Theorem is about: i.e., this logic is complete.



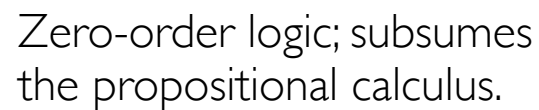
Actually ...

Second-order logic. Third-order logic, which Gödel used for his “God Theorem.”

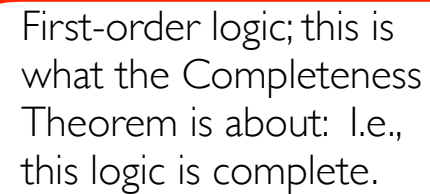


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Actually ...

This logic is *not* complete.**

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This logic is *not* complete.**

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Zero-order logic; subsumes the propositional calculus.

First-order logic; this is what the Completeness Theorem is about: i.e., this logic is complete.

**At least in its “complete” form. There are complete *fragments* of \mathcal{L}_2 .



R&W's Axiomatization of the Propositional Calculus

$$A1 \quad (\phi \vee \phi) \rightarrow \phi$$

$$A2 \quad \phi \rightarrow (\phi \vee \psi)$$

$$A3 \quad (\phi \vee \psi) \rightarrow (\psi \vee \phi)$$

$$A4 \quad (\psi \rightarrow \chi) \rightarrow ((\phi \vee \psi) \rightarrow (\phi \vee \chi))$$



R&W's Axiomatization of the Propositional Calculus

Some hyperlogical wizard want to throw these into a HOL workspace, and prove a substantive theorem from them?

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All instances of these schemata are true no matter what the input (true or false). (Agreed?) And indeed every single formula in the propositional calculus that is true no matter what the permutation (as shown in a truth table, e.g.), can be proved (somehow) from these four axioms (using any standard collection of inference schemata). This, Gödel (& later, Newell & Simon, when modern AI was born!) knew, and could use.



R&W's Axiomatization of the Propositional Calculus

Some hyperlogical wizard want to throw these into a HOL workspace, and prove a substantive theorem from them?

- A1 $(\phi \vee \phi) \rightarrow \phi$ (if (or ϕ ϕ) ϕ)
- A2 $\phi \rightarrow (\phi \vee \psi)$ (if ϕ (or ϕ ψ))
- A3 $(\phi \vee \psi) \rightarrow (\psi \vee \phi)$ (if (or ϕ ψ) (or ψ ϕ))
- A4 $(\psi \rightarrow \chi) \rightarrow ((\phi \vee \psi) \rightarrow (\phi \vee \chi))$ (if (if ψ χ) (if (or ϕ ψ) (or ϕ χ)))

All instances of these schemata are true no matter what the input (true or false). (Agreed?) And indeed every single formula in the propositional calculus that is true no matter what the permutation (as shown in a truth table, e.g.), can be proved (somehow) from these four axioms (using any standard collection of inference schemata). This, Gödel (& later, Newell & Simon, when modern AI was born!) knew, and could use.

Exercise I:

Verify that these are true-no-matter what in a truth tree in HyperSlate[®]; then prove using our rules for the prop. calc.; or perhaps better yet, have the oracle prove in HyperSlate[®].

$$(\phi \wedge \psi) \rightarrow (\psi \vee \chi)$$

$$\phi \rightarrow (\psi \rightarrow \phi)$$

Exercise 1:

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✓ Build **truth tree (semantic hypergraph, actually)**
showing this formula true no matter what the inputs.

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Proof:

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Build  **truth tree (semantic hypergraph, actually)**
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Proof:

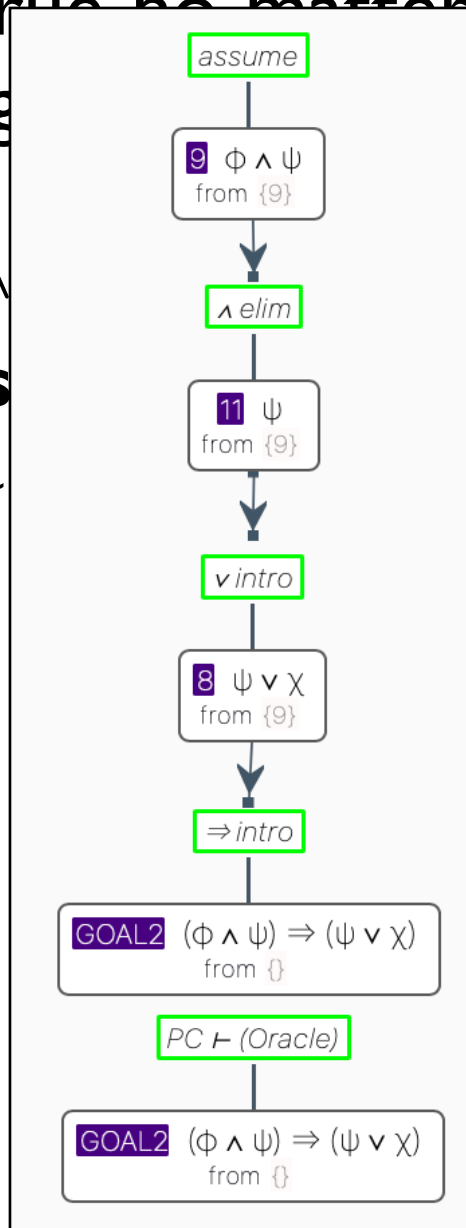
Exercise I:

Verify that these are true no matter what in a truth tree;
then prove using the prop. calc.

Build **truth tree** (showing this formula
($\phi \wedge \psi \Rightarrow (\psi \vee \chi)$)

Derivation graph, actually)
what the inputs.

Proof:



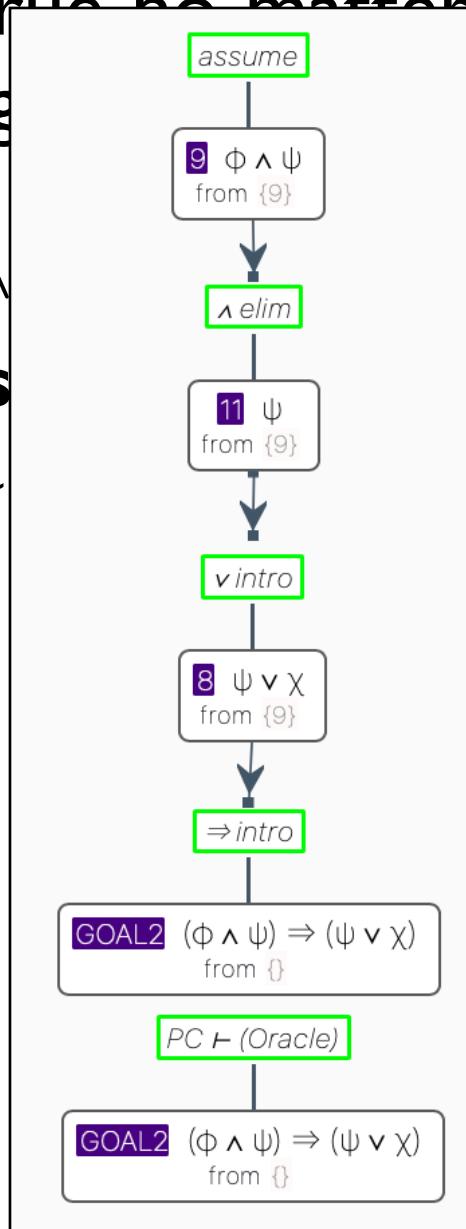
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then prove using the prop. calc.

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($\phi \wedge \psi$)

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(χ)

Proof:



resolution-based!

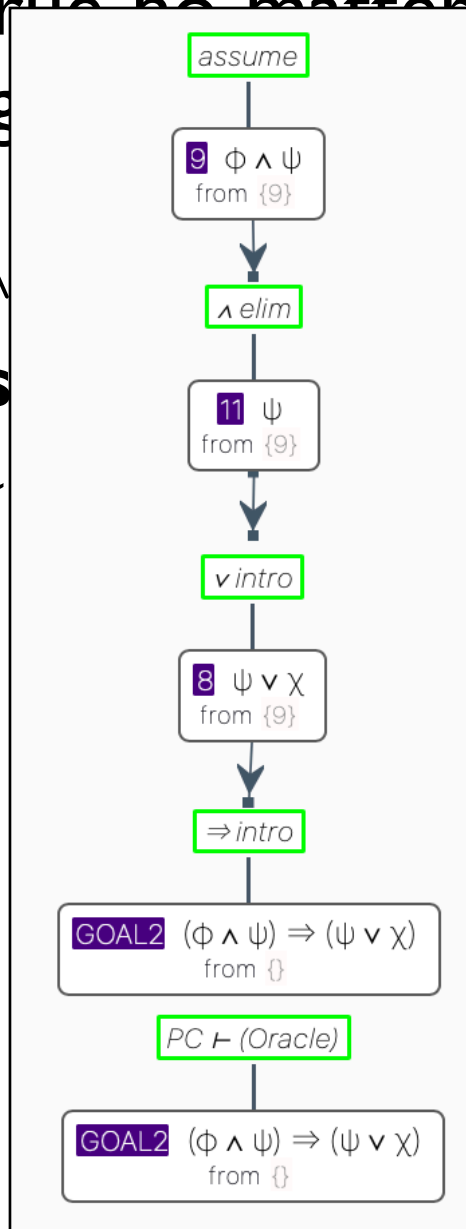
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(χ)
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what the inputs.

Proof:



resolution-based!



As HyperSlate[®] Tutorial

The screenshot shows a Safari browser window displaying the HyperSlate web application. The address bar shows the URL `rpi.logicamodernapproach.com`. The browser's menu bar includes "Safari", "File", "Edit", "View", "History", "Bookmarks", "Window", and "Help". The page title is "RussellWhiteheadPropCalcAx [PROPOSITIONAL-CALCULUS]: Saved with 39 symbols." The interface features a toolbar with icons for home, list, add, save, undo, redo, and Bezier. The main workspace contains a logical proof diagram with the following components:

- AXIOM1**: $(\phi \vee \phi) \Rightarrow \phi$ from {AXIOM1}. Above it is a green box labeled "assume".
- AXIOM2**: $\phi \Rightarrow (\phi \vee \psi)$ from {AXIOM2}. Above it is a green box labeled "assume".
- AXIOM3**: $(\phi \vee \psi) \Rightarrow (\psi \vee \phi)$ from {AXIOM3}. Above it is a green box labeled "assume".
- AXIOM4**: $(\psi \Rightarrow \chi) \Rightarrow ((\phi \vee \psi) \Rightarrow (\phi \vee \chi))$ from {AXIOM4}. Above it is a green box labeled "assume".
- GOAL1**: $\phi \vee \neg \phi$ from {}. Above it is a green box labeled "PC ⊢ (Oracle)".
- GOAL2**: $(\phi \wedge \psi) \Rightarrow (\psi \vee \chi)$ from {}. Above it is a green box labeled "PC ⊢ (Oracle)".

The diagram shows a vertical chain of four axiom boxes on the left, with a goal box to their right. A second goal box is positioned below the first goal box. The interface is clean and professional, with a dark grey toolbar and a light grey workspace.

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The screenshot shows a web browser window with the URL `rpi.logicamodernapproach.com`. The browser's address bar and tabs are visible. The HyperSlate interface includes a toolbar with icons for home, list, add, save, undo, redo, and Bezier. The main workspace displays a logical proof diagram with the following components:

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The diagram shows a vertical chain of four axiom boxes on the left, with a goal box to their right. A second goal box is positioned below the first goal box. The interface also shows a status bar at the bottom indicating the document is saved with 39 symbols.

The Grammar of \mathcal{L}_0 = the Pure Predicate Calculus

Formula \Rightarrow *AtomicFormula*
| (*Formula* *Connective* *Formula*)
| \neg *Formula*

AtomicFormula \Rightarrow (*Predicate* *Term*₁ ... *Term*_k)
| (*Term* = *Term*)

Term \Rightarrow (*Function* *Term*₁ ... *Term*_k)
| *Constant*

Connective \Rightarrow \wedge | \vee | \rightarrow | \leftrightarrow

Predicate \Rightarrow P_1 | P_2 | P_3 ...

Constant \Rightarrow c_1 | c_2 | c_3 ...

Function \Rightarrow f_1 | f_2 | f_3 ...

Some Simple Examples

Formula \Rightarrow *AtomicFormula*
 | $(\text{Formula } \text{Connective } \text{Formula})$
 | $\neg \text{Formula}$

Sally likes Bill.
 (Likes sally bill)

AtomicFormula \Rightarrow $(\text{Predicate } \text{Term}_1 \dots \text{Term}_k)$
 | $(\text{Term} = \text{Term})$

Term \Rightarrow $(\text{Function } \text{Term}_1 \dots \text{Term}_k)$
 | *Constant*

Sally likes Bill and Bill likes Sally.

Sally likes Bill's mother.

Sally likes Bill only if Bill's mother is tall.

Matilda is Bill's super-smart mother.

5 plus 5 equals the number 10.

Connective \Rightarrow \wedge | \vee | \rightarrow | \leftrightarrow

Predicate \Rightarrow P_1 | P_2 | P_3 ...

Constant \Rightarrow c_1 | c_2 | c_3 ...

Function \Rightarrow f_1 | f_2 | f_3 ...

Lexicon

...

Can Roger be counted upon to declare: "Yes that sentence is okay!" whenever it's conforms to this grammar?

Some Simple Examples

Formula ⇒ *AtomicFormula*
 | *(Formula Connective Formula)*
 | \neg *Formula*

Sally likes Bill.
 (Likes sally bill)

AtomicFormula ⇒ *(Predicate Term₁ ... Term_k)*
 | *(Term = Term)*

Term ⇒ *(Function Term₁ ... Term_k)*
 | *Constant*

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Connective ⇒ \wedge | \vee | \rightarrow | \leftrightarrow

Likes
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If Sally likes Bill then Sally likes Bill.

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Function \Rightarrow f_1 | f_2 | f_3 ...

If Sally likes Bill then Sally likes Bill.

Sally likes Bill's mother, or not.

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If Sally likes Bill then Sally likes Bill.

Sally likes Bill's mother, or not.

Sally likes Bill and Bill likes Jane, only if Bill likes Jane.

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Bill's smart mother is a mother.

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Bill's smart mother is a mother.

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Slightly More Complicated Examples

Formula \Rightarrow *AtomicFormula*
 | *(Formula Connective Formula)*
 | \neg *Formula*

AtomicFormula \Rightarrow *(Predicate Term₁ ... Term_k)*
 | *(Term = Term)*

Term \Rightarrow *(Function Term₁ ... Term_k)*
 | *Constant*

Connective \Rightarrow \wedge | \vee | \rightarrow | \leftrightarrow

Predicate \Rightarrow P_1 | P_2 | P_3 ...
Constant \Rightarrow c_1 | c_2 | c_3 ...
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Add Two Quantifiers to the Pure Predicate Calculus,

Which Yields \mathcal{L}_1 First-order Logic = Predicate Calculus *simpliciter*

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The Shoulders Available to Gödel for Standing Upon

...

Completeness Theorem for The Propositional Calculus

Let Γ be a set $\{\phi_1, \phi_2, \dots\}$ of formulae in the the propositional calculus. Then either all of Γ are satisfiable, or the conjunction up to and including the point k (i.e. $\phi_1 \wedge \phi_2 \wedge \dots \wedge \phi_k$) of failure is refutable.

Completeness Theorem for The Propositional Calculus

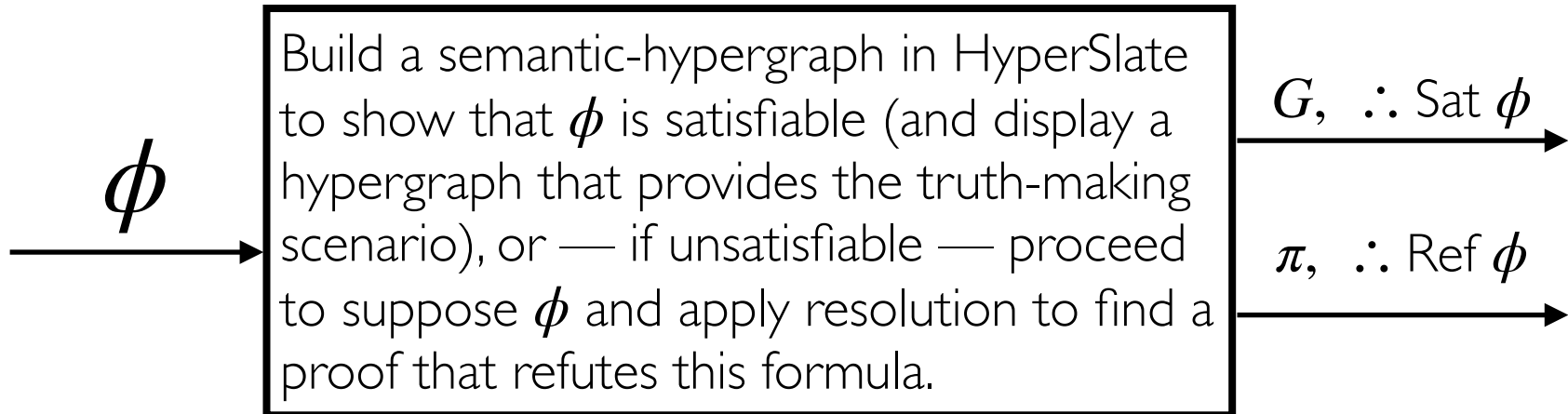
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Let Γ be a set $\{\phi_1, \phi_2, \dots\}$ of formulae in the the propositional calculus. Then either all of Γ can be simultaneously true in some scenario, or the conjunction up to and including the point k (i.e. $\phi_1 \wedge \phi_2 \wedge \dots \wedge \phi_k$) of failure is **refutable** (i.e. $\vdash \neg(\phi_1 \wedge \phi_2 \wedge \dots \wedge \phi_k)$).

Completeness Theorem (prop calc):
Either ϕ is satisfiable, or ϕ is refutable.



What does the F-O
Completeness Theorem
say?

...

Completeness Theorem as an Equation

In first-order logic: NECESSARY TRUTH = PROVABILITY.

Completeness Theorem, More Precisely Put

For every first-order statement ϕ : if ϕ is a necessary or absolute truth (i.e. true in any scenario whatsoever), then ϕ is provable.

And the version Gödel targeted,
and proved:

For every first-order statement ϕ : Either ϕ is true in some scenario, or ϕ is refutable (= it's negation $\neg\phi$ can be proved).

GCT

The Proof-Sketch

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Prove the equivalence.

Can you prove Gödel's version for the propositional calculus in HyperSlate?

The Proof-Sketch

To prove the theorem in the case of first-order logic ($= \mathcal{L}_1$), we need to show that given any set Γ of formulae in first-order logic, either there's a scenario on which every member of this set is true; otherwise, there is a refutation of the set, i.e. a proof from the set to an outright contradiction $\phi \wedge \neg\phi$. We can accomplish this by finding a procedure \mathcal{P} that first takes the set in question and goes hunting for a scenario that does the trick. If the scenario is found, we're done. But, if such a scenario *can't* be found, then our procedure moves on to find a proof of a contradiction from Γ !

How?! The procedure \mathcal{P} is the building out of a semantic hypergraph! If all the branches in the graph close, then the finding of a proof of a contradiction uses **resolution**, and the **resolution guarantee**. The guarantee is that if you have a set of formulae that can't be true in any scenario, resolution applied to the set finds a contradiction $\perp = \zeta \wedge \neg\zeta = \{\}$. **QED**

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See LAMA-BDLAHHGS.

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Gödel as Giant: Some Evidence

THE DISCOVERY OF MY COMPLETENESS PROOFS

LEON HENKIN

Dedicated to my teacher, Alonzo Church, in his 91st year.

§1. Introduction. This paper deals with aspects of my doctoral dissertation¹ which contributed to the early development of model theory. What was of use to later workers was less the results of my thesis, than the method by which I proved the completeness of first-order logic—a result established by Kurt Gödel in *his* doctoral thesis 18 years before.²

The ideas that fed my discovery of this proof were mostly those I found in the teachings and writings of Alonzo Church. This may seem curious, as his work in logic, and his teaching, gave great emphasis to the constructive character of mathematical logic, while the model theory to which I contributed is filled with theorems about very large classes of mathematical structures, whose proofs often by-pass constructive methods.

Another curious thing about my discovery of a new proof of Gödel's completeness theorem, is that it arrived in the midst of my efforts to prove an entirely different result. Such "accidental" discoveries arise in many parts of scientific work. Perhaps there are regularities in the conditions under which such "accidents" occur which would interest some historians, so I shall try to describe in some detail the accident which befell me.

Received November 17, 1995, and in revised form, January 4, 1996.

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$$\forall x_1, x_2, \dots, x_k \exists y_1, y_2, \dots, y_m \phi(x_1, x_2, \dots, x_k, y_1, y_2, \dots, y_m)$$

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How? By ingenious tree-building, which starts by creating an enumeration of new constants $c = c_1, c_2, \dots$ that becomes our “universe of discourse”/“domain of quantification.” Note that from c we can algorithmically generate an enumeration of tuples $c^t = \langle c \rangle_1, \langle c \rangle_2, \dots$ of any finite size. (Those of size k will work for the x -variables, and those of size n will work for the y -variables.) And now we can build a BIG tree at the level of the pure predicate calculus, looking for either a scenario that makes our formula true by traveling with Buzz to infinity, or getting all branches closed, in which case we turn back to the resolution guarantee! Let's make sense of this by hand on paper ...

Making this Concrete Courtesy of HyperSlate[®]

The screenshot shows the HyperSlate web interface. At the top, there is a dark navigation bar with the HyperSlate logo and several icons for home, menu, add, cloud, lock, undo, redo, and refresh. A dropdown menu is set to 'Straight'. On the right, a light blue status bar displays 'GodelFormula1 [FIRST-ORDER-LOGIC]: Saved with 38 symbols.'

The main content area features a central box with a purple header 'SAME FORMULA FOR SAT TESTING' and the formula $\forall x: \exists y: L(x, y) \wedge \neg L(x, y)$. Below the formula, it says 'from {SAME FORMULA FOR SAT TESTING}'. A green box labeled 'assume' is connected to the top of the formula box by a vertical line.

Making this Concrete Courtesy of HyperSlate®

HyperSlate®

Home, List, Add, Cloud, Save, Undo, Redo, Refresh, Share, Straight, Help

GodelFormula1 [FIRST-ORDER-LOGIC]: Saved with 38 symbols.

assume









SAME FORMULA FOR SAT TESTING $\forall x: \exists y: L(x, y) \wedge \neg L(x, y)$
from {SAME FORMULA FOR SAT TESTING}

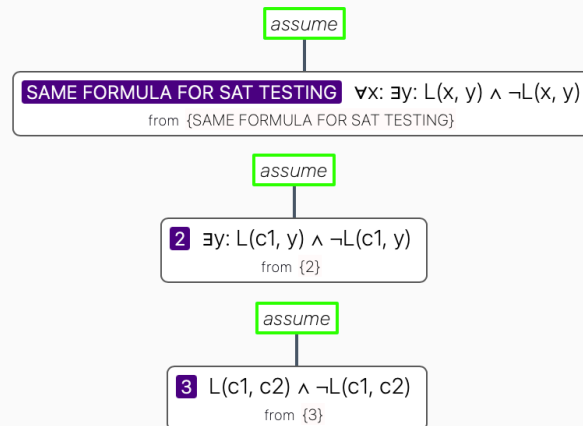
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HyperSlate[®]          Straight  GodelFormula1 [FIRST-ORDER-LOGIC]: Saved with 38 symbols.



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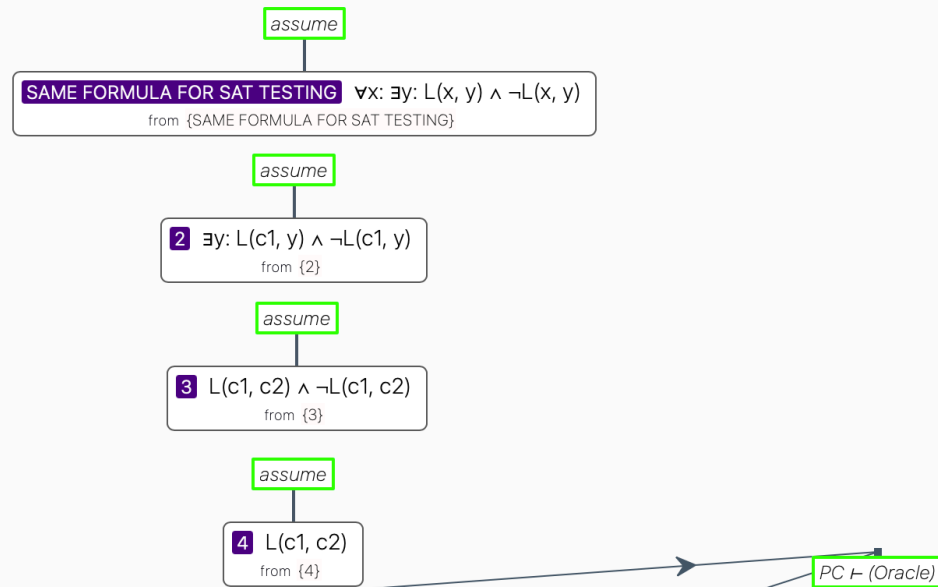
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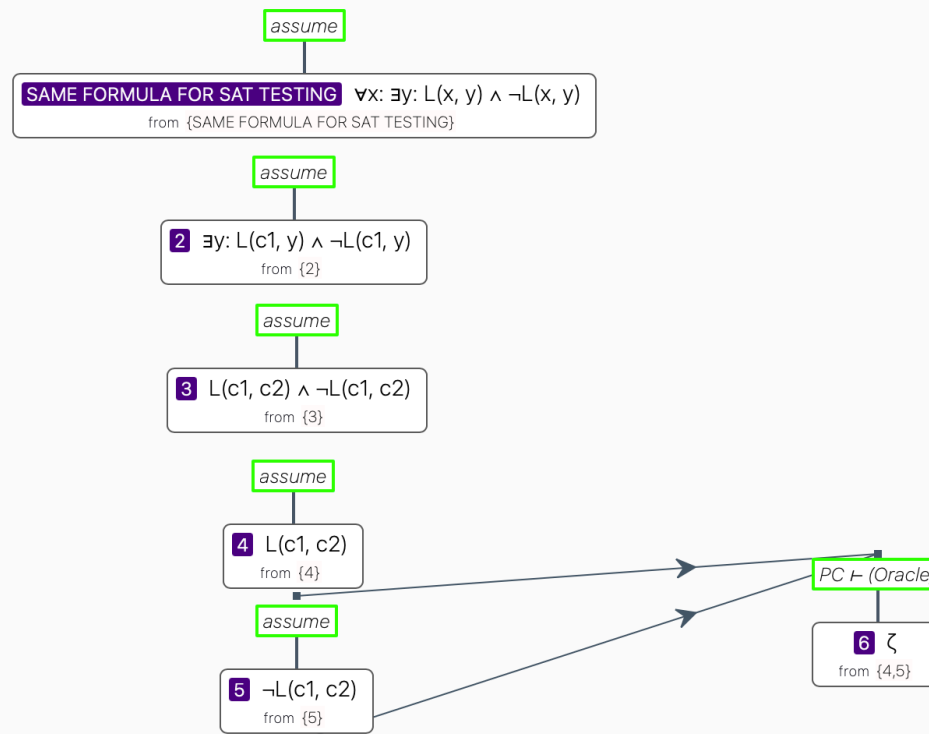
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The screenshot shows a Safari browser window displaying the HyperSlate web application. The address bar shows the URL `rpi.logicamodernapproach.com`. The browser tabs include "Sign in to hypergrader", "Editing GodelFormula1", and "teller 2ch8.pdf truth trees - Google Search". The HyperSlate interface features a toolbar with icons for home, list, add, save, undo, redo, and Bezier. A status bar at the top right indicates "GodelFormula1 [FIRST-ORDER-LOGIC]: Saved with 38 symbols." The main content area displays a logical proof diagram with the following steps:

- Root node: **assume** (green box)
- Node 1: **SAME FORMULA FOR SAT TESTING** $\forall x: \exists y: L(x, y) \wedge \neg L(x, y)$ from {SAME FORMULA FOR SAT TESTING}
- Node 2: **assume** (green box)
- Node 2: **2** $\exists y: L(c1, y) \wedge \neg L(c1, y)$ from {2}
- Node 3: **assume** (green box)
- Node 3: **3** $L(c1, c2) \wedge \neg L(c1, c2)$ from {3}
- Node 4: **assume** (green box)
- Node 4: **4** $L(c1, c2)$ from {4}
- Node 5: **assume** (green box)
- Node 5: **5** $\neg L(c1, c2)$ from {5}
- Node 6: **PC ⊢ (Oracle)** (green box)
- Node 6: **6** ζ from {4,5}

Arrows indicate dependencies: Node 4 and Node 5 both point to Node 6. The diagram is displayed on a desktop environment with a dock at the bottom containing various application icons.

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The main content area displays a logical proof diagram. The proof starts with an `assume` step leading to a box containing the formula $\forall x: \exists y: L(x, y) \wedge \neg L(x, y)$. Below this, a sequence of steps is shown:

- Step 2: $\exists y: L(c1, y) \wedge \neg L(c1, y)$ (from {2})
- Step 3: $L(c1, c2) \wedge \neg L(c1, c2)$ (from {3})
- Step 4: $L(c1, c2)$ (from {4})
- Step 5: $\neg L(c1, c2)$ (from {5})

Arrows from steps 4 and 5 point to a box labeled `PC ⊢ (Oracle)`. This box leads to a final step 6 containing the symbol ζ (from {4,5}).

The bottom of the image shows the macOS dock with various application icons.

slutten