FOL II: `universal intro`

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Intro to Formal Logic (& AI)
2/23/2026

Contract; Grading; Performance Overview

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- Don’t forget the “contract”!
    - &, “knowledge via thoroughness”: slides, review w., & … the book :).

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  - A grade of B if any 2 problems are trophied.
  - A grade of C if any 1 problem is trophied.
  - A grade of A+ of all problems are trophied.

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- Class status overview …
- TA Brandon Extra OH: Tues 11a–1p Carnegie 3rd Floor

Today

Part I: New Inference "Rule"

Today

Part I: New Inference "Rule"

Part II: Hands-on: Test 2 Help Session …

Logic-&-AI In The News

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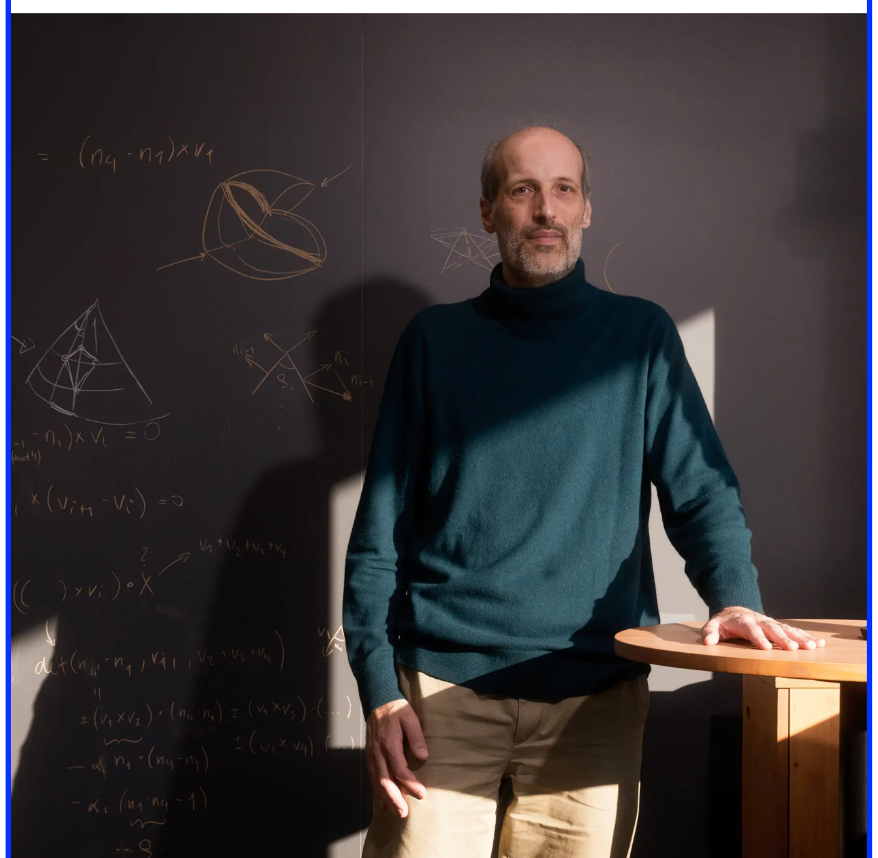
A CONVERSATION WITH

These Mathematicians Are Putting A.I. to the Test

Large language models struggle to solve research-level math questions. It takes a human to assess just how poorly they perform.



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By Siobhan Roberts

Published Feb. 7, 2026 Updated Feb. 12, 2026

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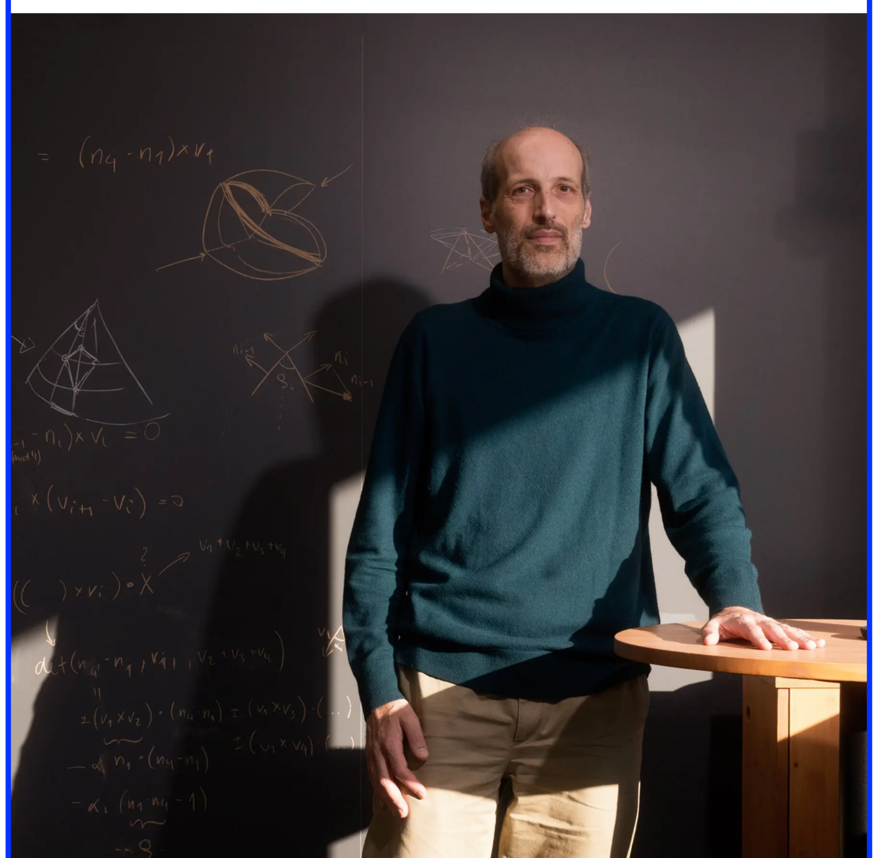

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Artificial Intelligence ›   Backlash to A.I. Boom   Impact on White-Collar Jobs

At This Newark School, A.I. Lessons Are the New Drivers' Ed

Teachers say they want to equip high school students to drive artificial intelligence, rather than be mere passengers steered by chatbots.

Listen to this article · 7:52 min Learn more

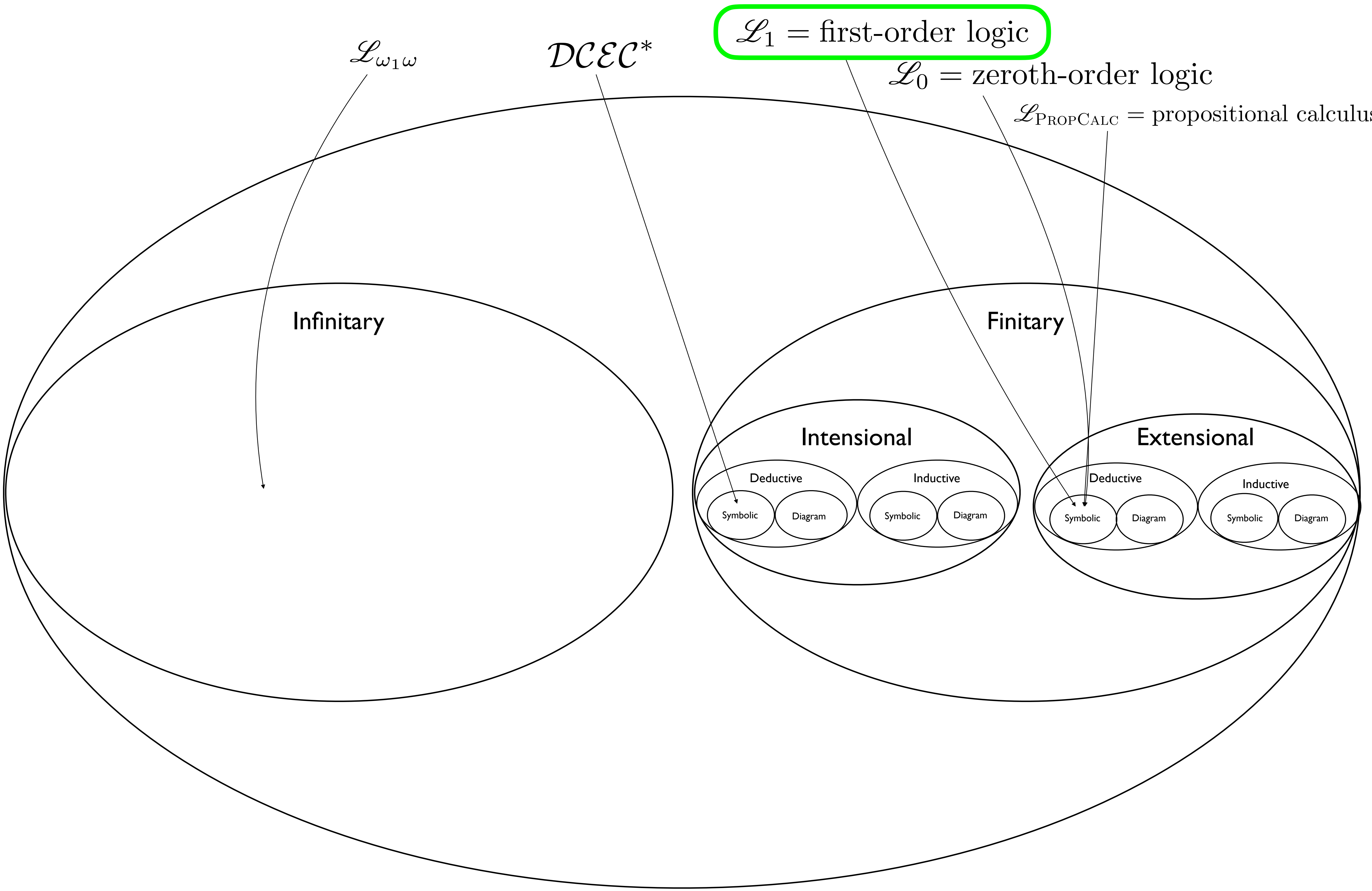
In February, North Star Academy Washington Park High School started a new A.I. class for seniors. Juan Arredondo for The New York Times

By Natasha Singer

Reporting from North Star Academy Washington Park High School in Newark

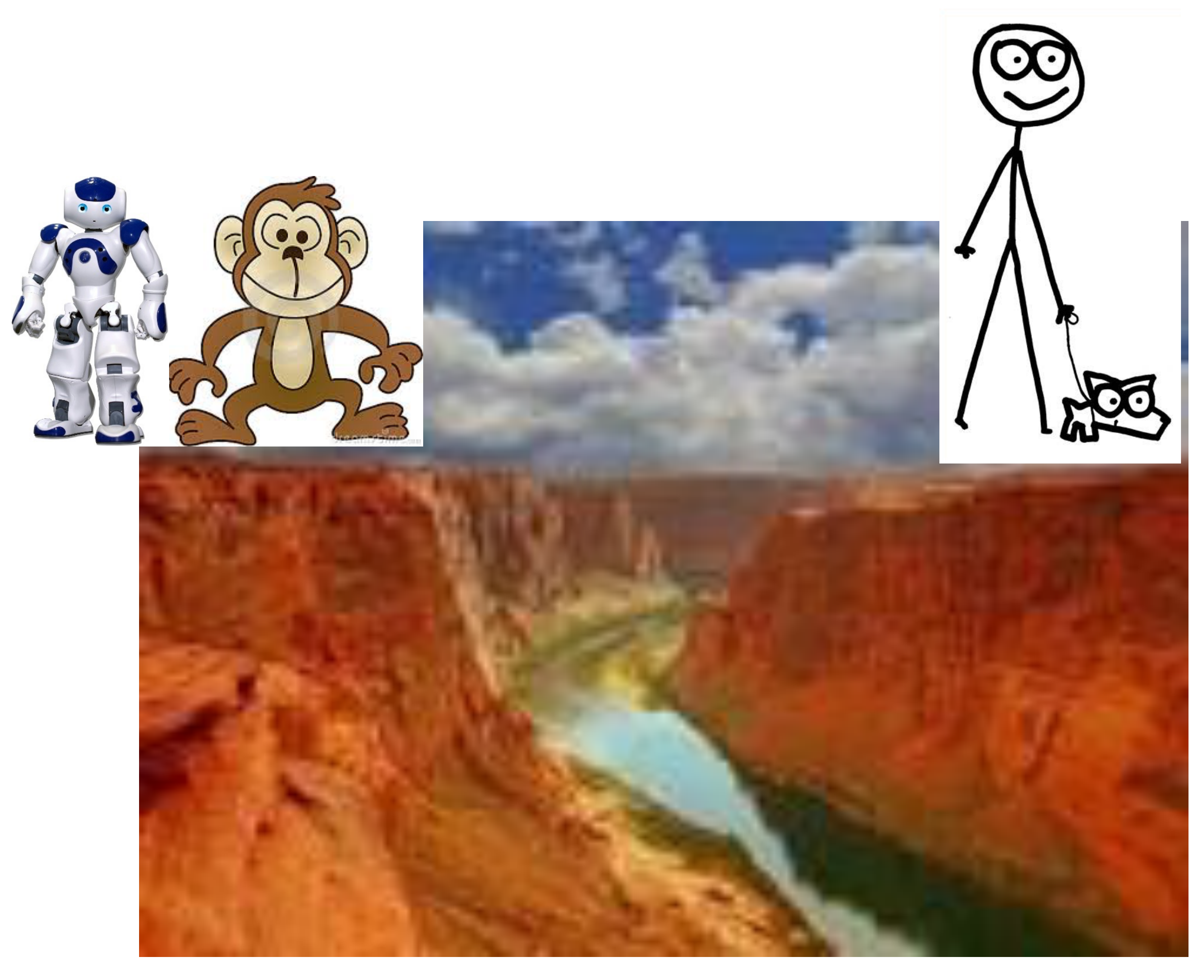
Feb. 23, 2026, 5:00 a.m. ET

The Universe of Logics



Quantifiers (etc) …

The Canyon of Discontinuity (or Darwin's Dread)



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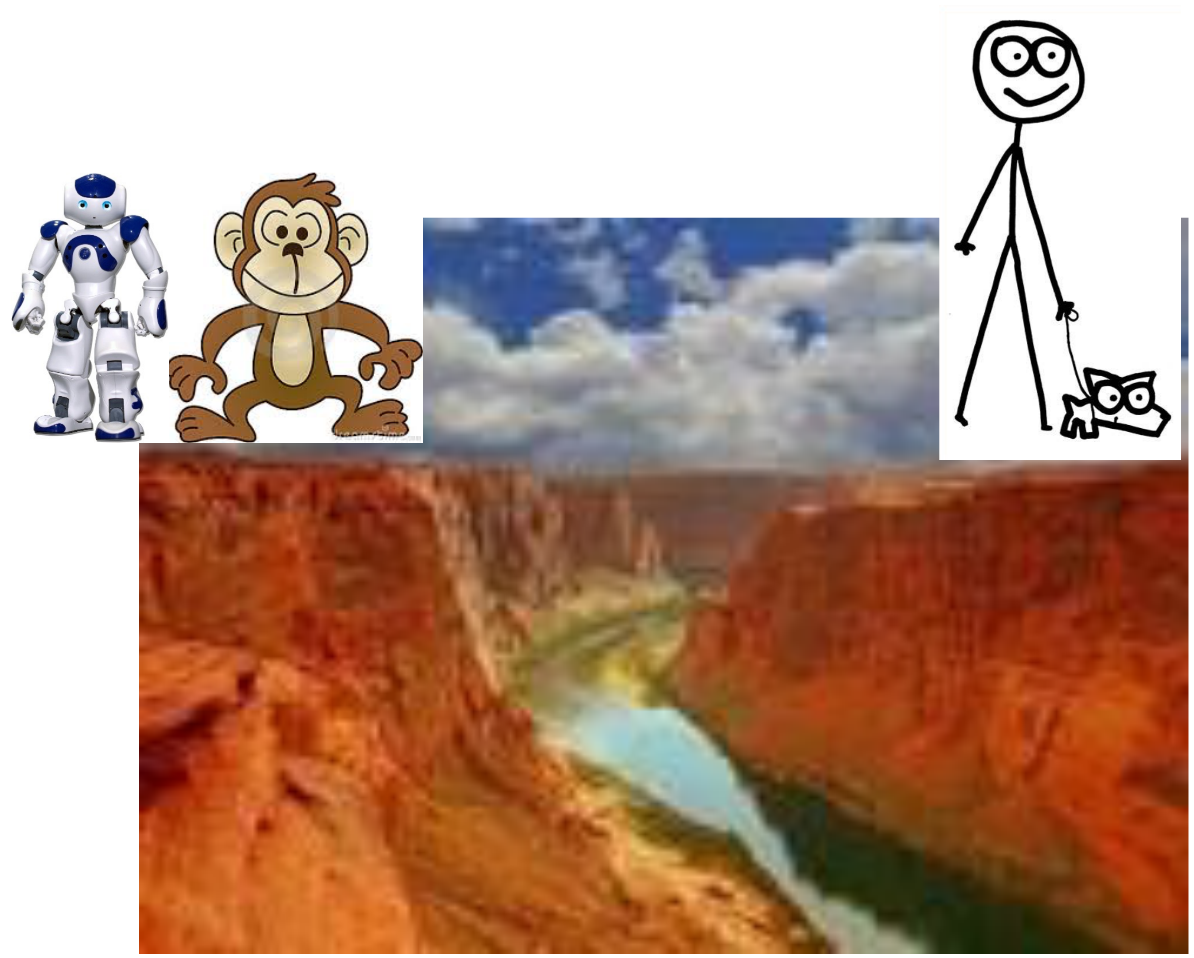

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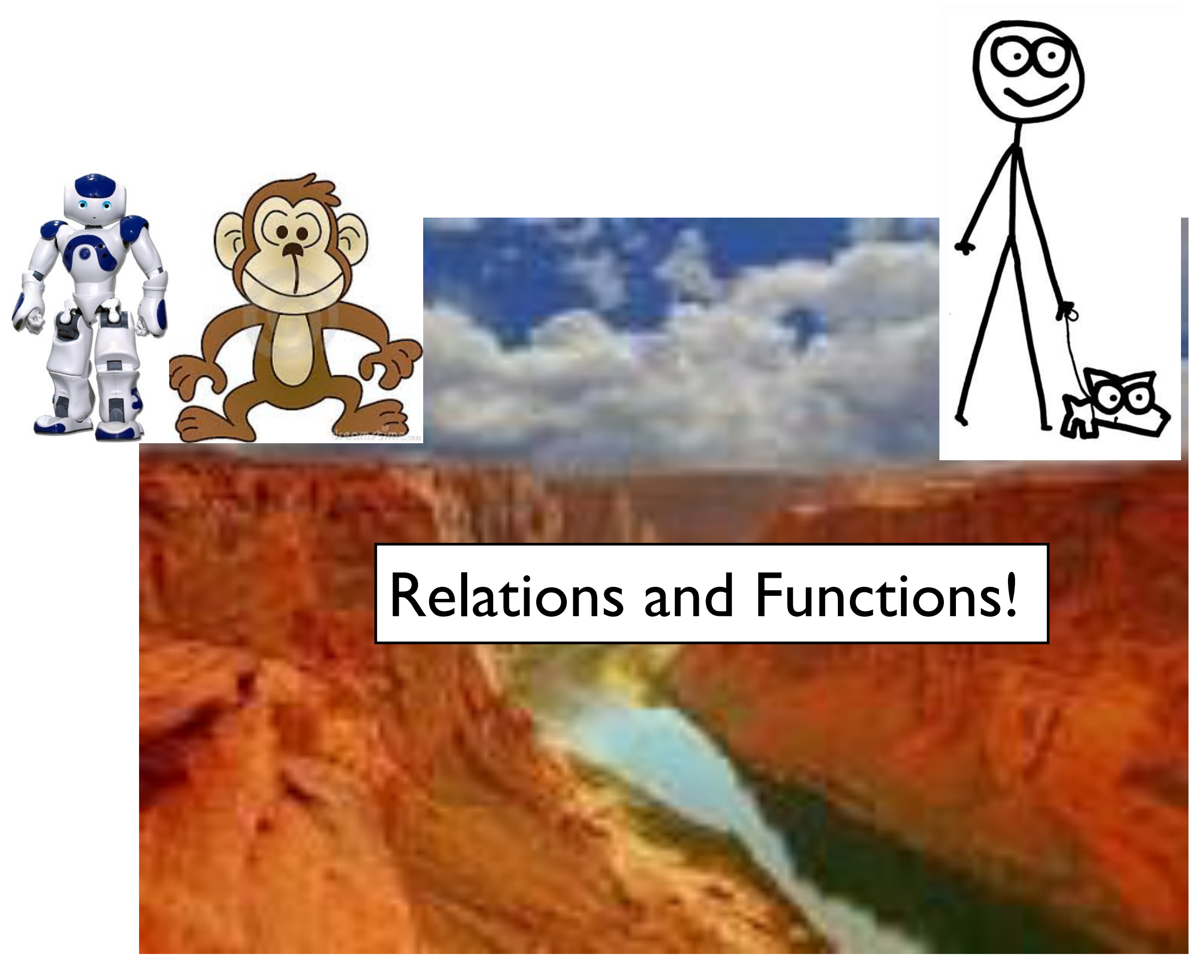

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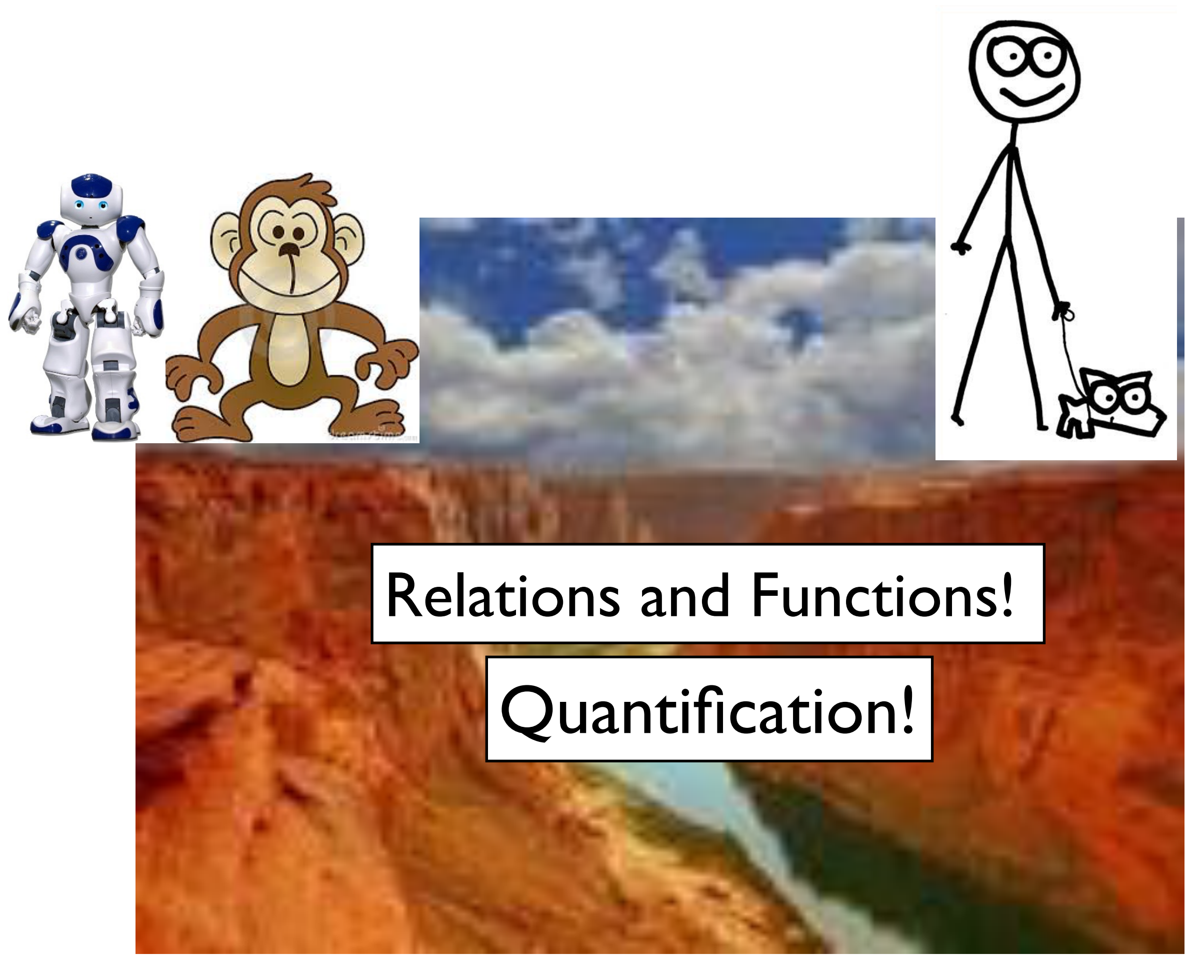

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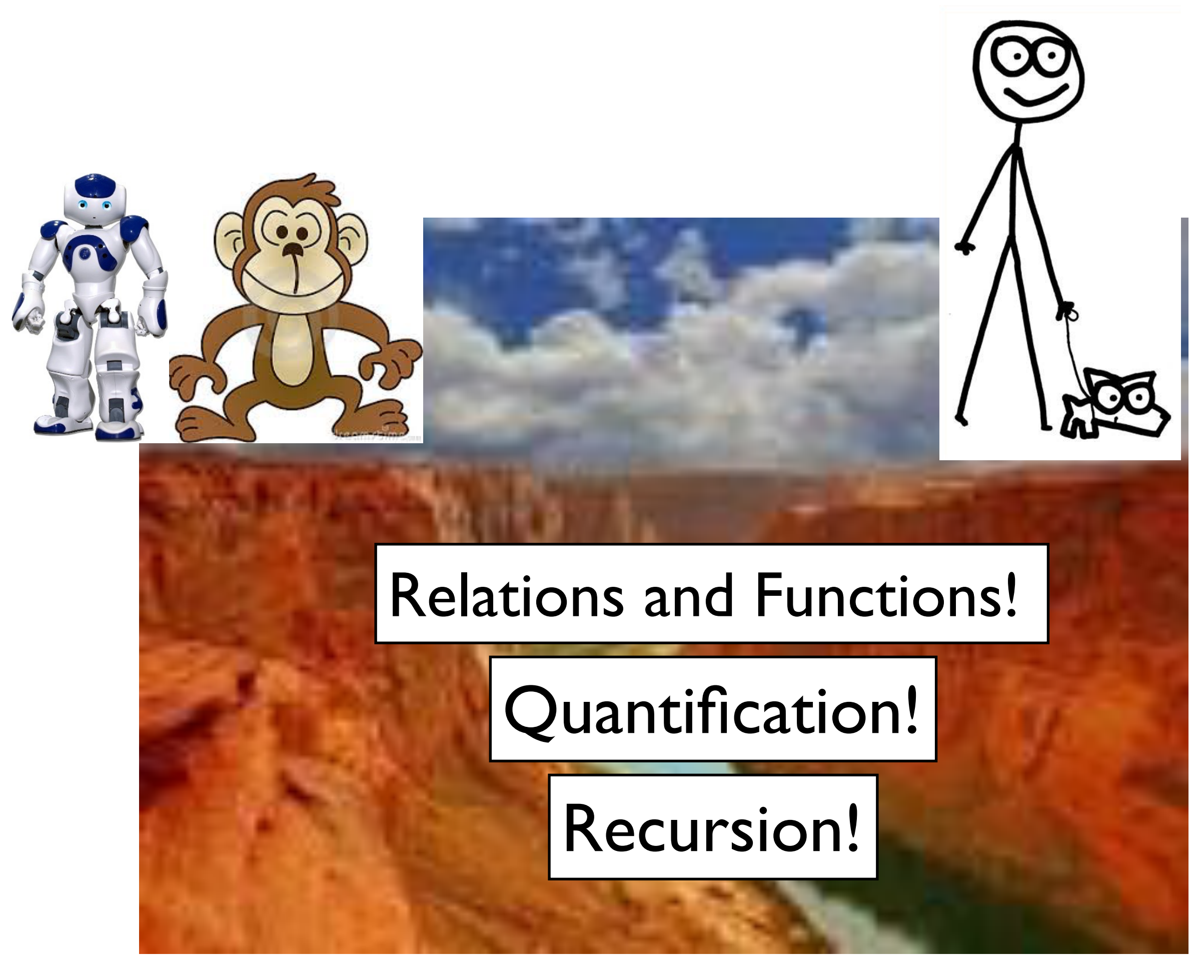

The Canyon of Discontinuity (or Darwin's Dread)

Quantification!

Karkooking Problem …

Everyone karkooks anyone who karkooks someone.

Alvin karkooks Bill.

Can you infer that everyone karkooks Bill?

ANSWER:

JUSTIFICATION:

Karkooking Problem …

Everyone karkooks anyone who karkooks someone.

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ANSWER:

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(Why oh why is Calc 1 so hard? Karkooking: Π_2 — & non-trivial. But the definition of a limit is … Π_3! How pray tell are students supposed to learn calculus without having a dedicated class in formal logic?!)

(Why oh why is Calc I so hard? Karkooking: Π_2 — & non-trivial. But the definition of a limit is … Π_3! How pray tell are students supposed to learn calculus without having a dedicated class in formal logic?!)

Definition: Finite Limits (Formal)

Let $f(x)$ be defined for all $x \neq a$ over an open interval containing a. Let L be a real number. Then

$$\lim_{x \to a} f(x) = L$$

if, for every $\varepsilon > 0$, there exists a $\delta > 0$, such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$.

This definition may seem rather complex from a mathematical point of view, but it becomes easier to understand if we break it down phrase by phrase. The statement itself involves something called a **universal quantifier** (for every $\varepsilon > 0$), an **existential quantifier** (there exists a $\delta > 0$), and, last, a **conditional statement** (if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$). Let's take a look at Table $2.5.1$, which breaks down the definition and translates each part.

Table $2.5.1$

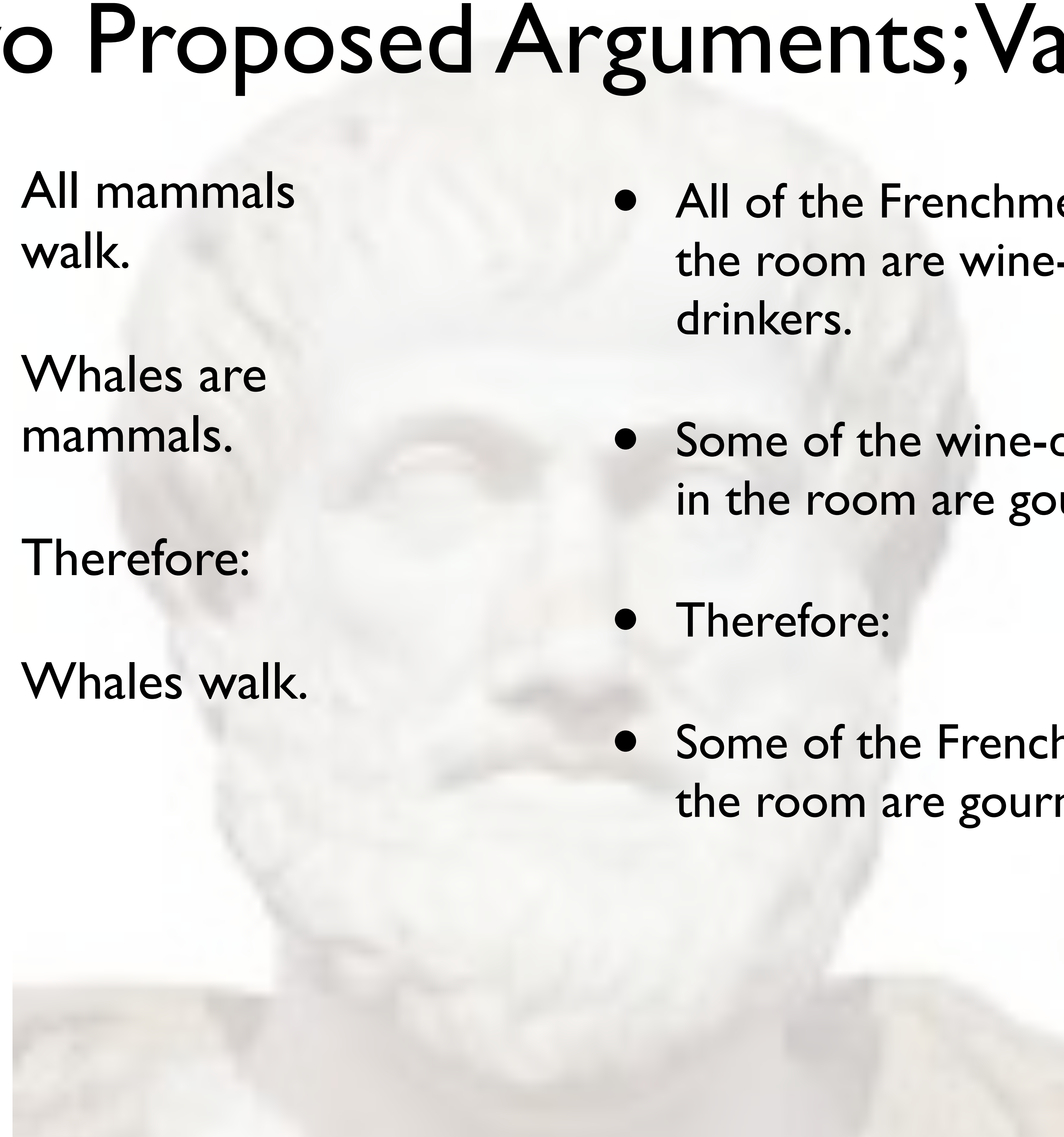
Definition	Translation
1. For every $\varepsilon > 0$,	1. For every positive distance ε from L,
2. there exists a $\delta > 0$,	2. There is a positive distance δ from a,
3. such that	3. such that
4. if $0 < \|x - a\| < \delta$, then $\|f(x) - L\| < \varepsilon$.	4. if x is closer than δ to a and $x \neq a$, then $f(x)$ is closer than ε to L.

Relations and Functions!

Quantification!

Recursion!

Two Proposed Arguments; Valid?



- All mammals walk.
- Whales are mammals.
- Therefore:
- Whales walk.

- All of the Frenchmen in the room are wine-drinkers.
- Some of the wine-drinkers in the room are gourmets.
- Therefore:
- Some of the Frenchmen in the room are gourmets.

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We can of course easily symbolize and settle the matter in HyperSlate® (PC oracle permitted now)! (Show this in a Pop problem.) Doing so is *impossible* in the prop calc, and likewise impossible in zeroth-order logic!

Two Proposed Arguments; Valid?

- All mammals walk. $\forall x[M(x) \rightarrow W(x)]$
- Whales are mammals. $\forall x(Wh(x) \rightarrow M(x))$
- Therefore:
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$\forall x(Wh(x) \rightarrow W(x))$

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- All of the Frenchmen in the room are wine-drinkers. $\forall x(F(x) \rightarrow W(x))$
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$\exists x(W(x) \land G(x))$

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s-expressions

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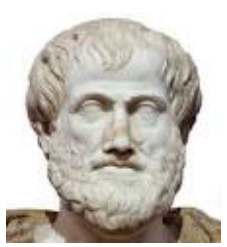
Bare-BonesBarbara

Historically speaking (recall) …


350 BC
~ 1890
2026
Euclid


350 BC
300 BC
~ 1890
2026
Euclid

350 BC | 300 BC | ~ 1890 | 2026



"I don't believe in magic! Why exactly is that so convincing? What the heck is he doing?!? I know! …"

Euclid

350 BC | 300 BC | ~ 1890 | 2026

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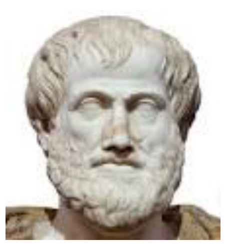


Organon

"I don't believe in magic! Why exactly is that so convincing? What the heck is he doing?!? I know! …"

“He’s using syllogisms!”

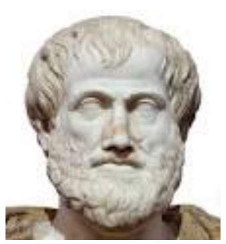
E.g.,

All As are Bs.
All Bs are Cs.

All As are Cs.

350 BC | 300 BC | ~ 1890 | 2026

Euclid



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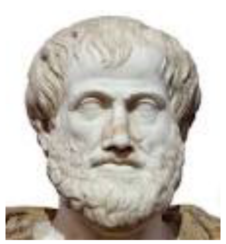

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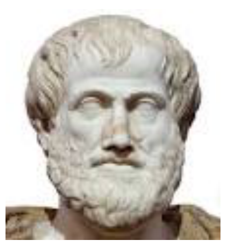

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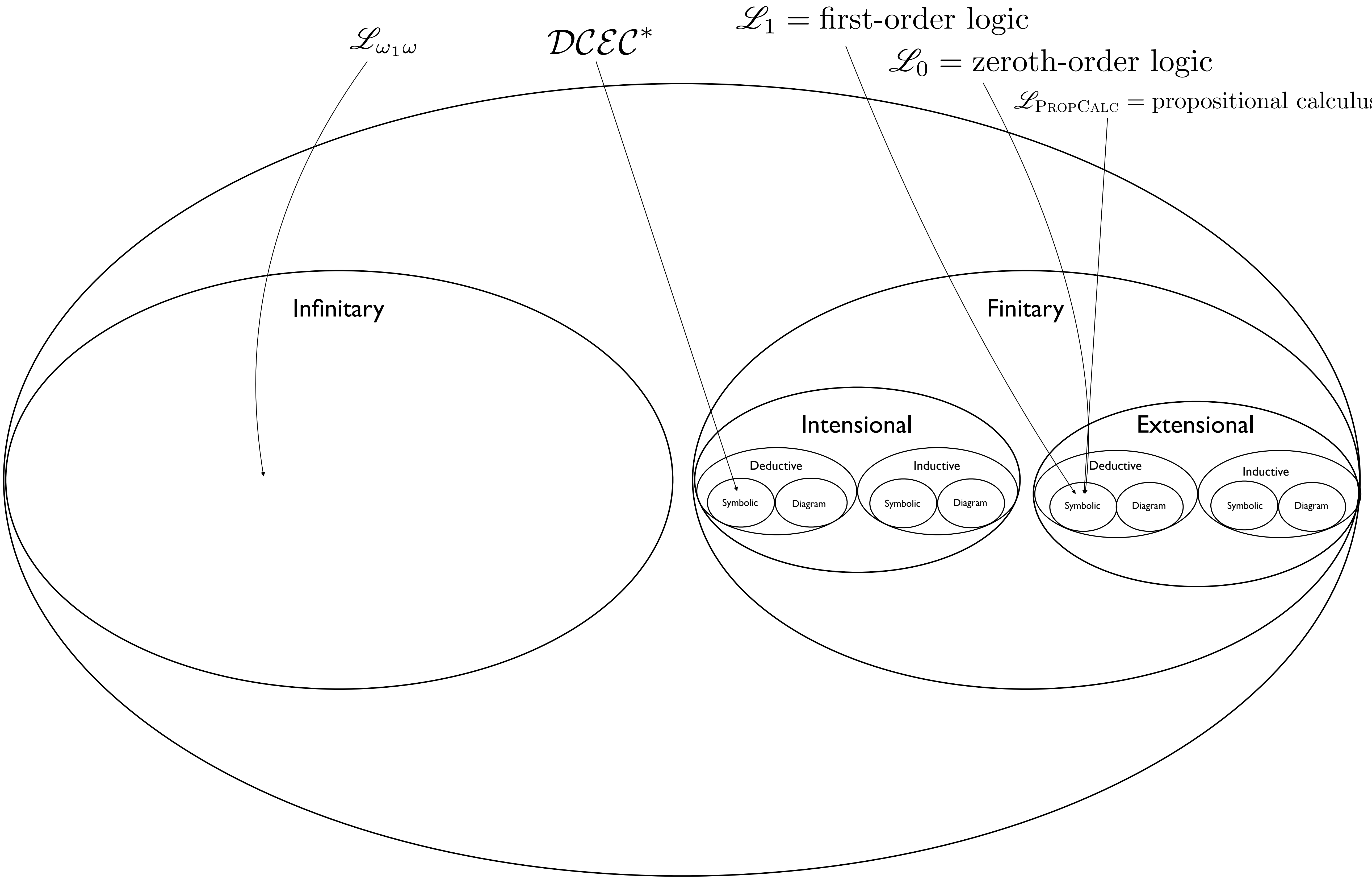
The Universe of Logics



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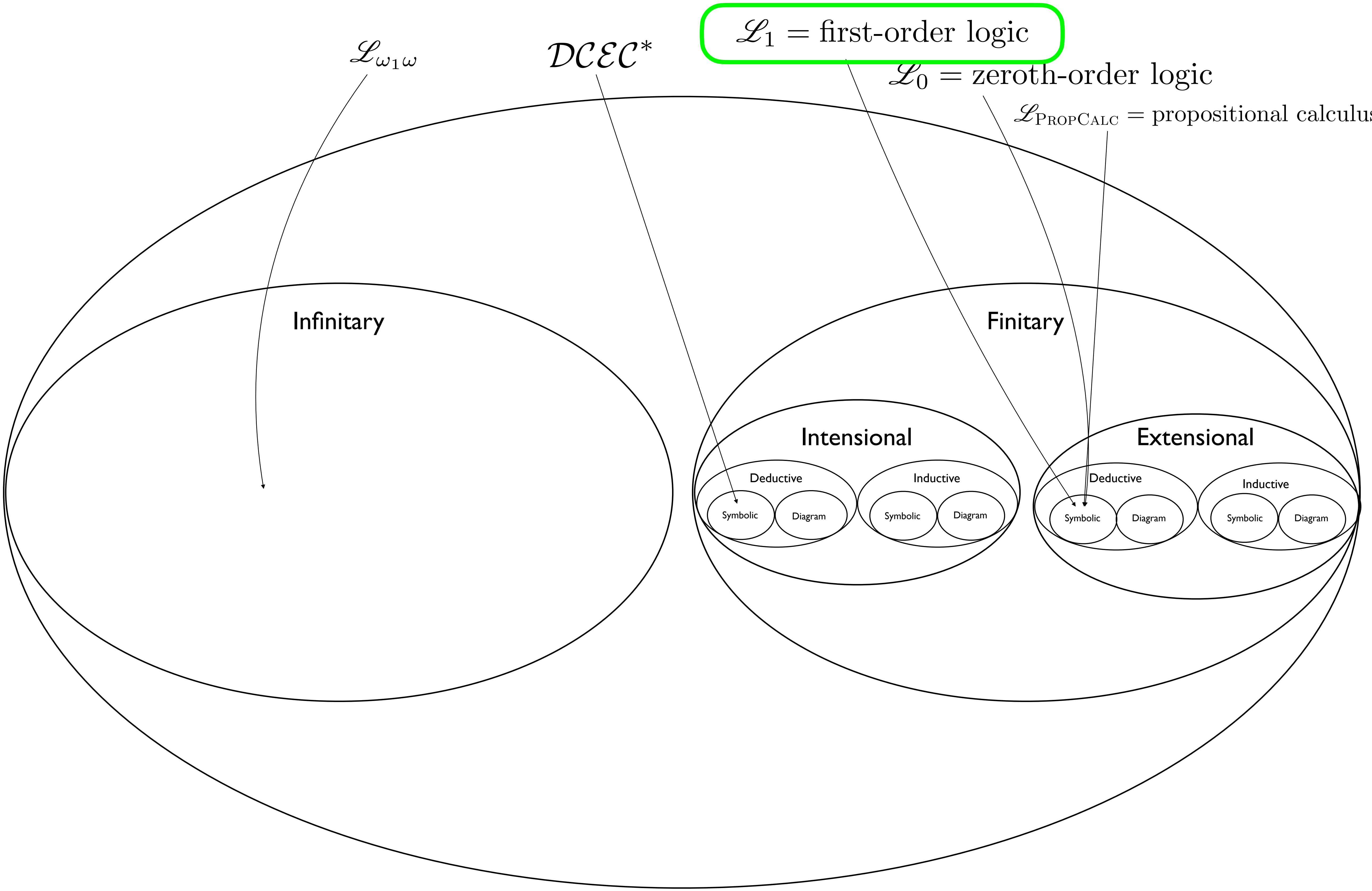

Next New (*Not*-So-Easy!) Inference Rule in FOL

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- `universal introduction`

Next New (*Not*-So-Easy!) Inference Rule in FOL

- `universal introduction`
  - If something *a* is an *R*, and the constant/name *a* is *genuinely arbitrary*, then we can deduce that everything is an *R*.

The Inference Schema

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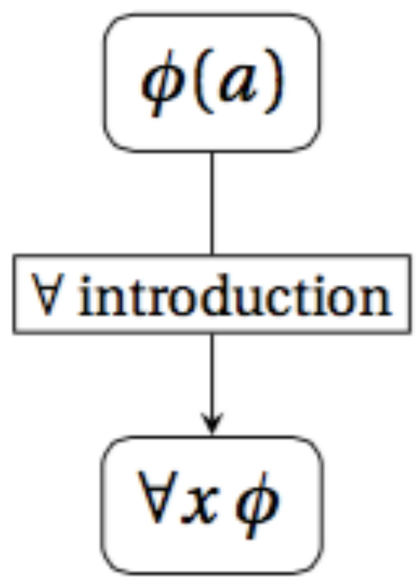

provided that a does not appear free in any in-scope assumption of ϕ, and that no occurrence of a appear in the inferred $\forall x\, \phi$ (3.16)

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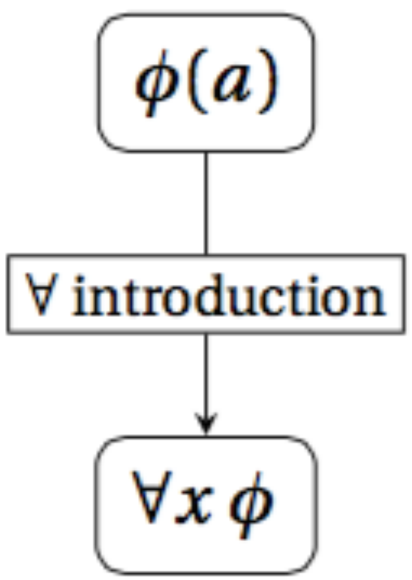
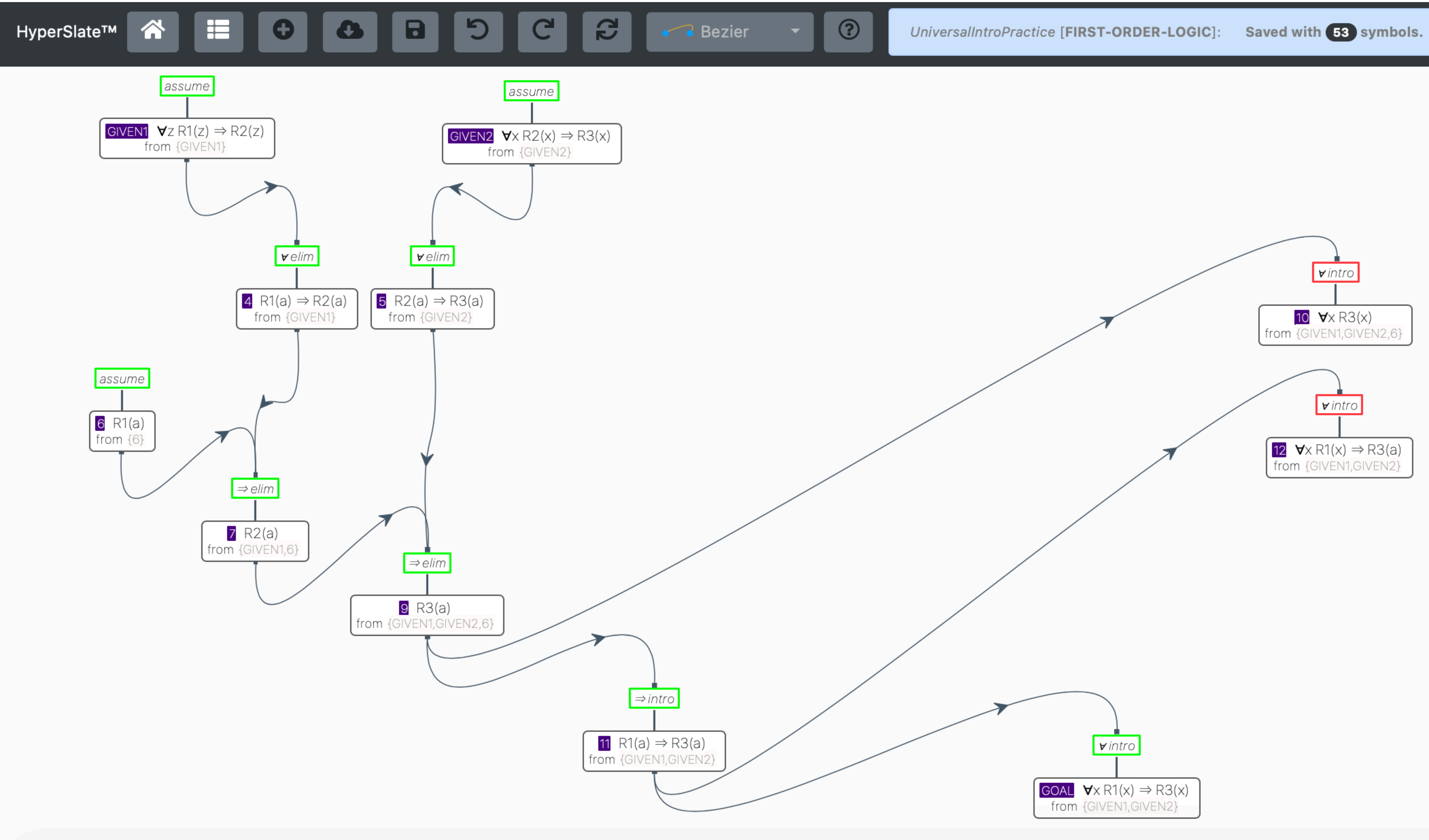


provided that a does not appear free in any in-scope assumption of ϕ, and that no occurrence of a appear in the inferred $\forall x\, \phi$ (3.16)

(Why the provisos?)

`universal intro` Example/Tutorial



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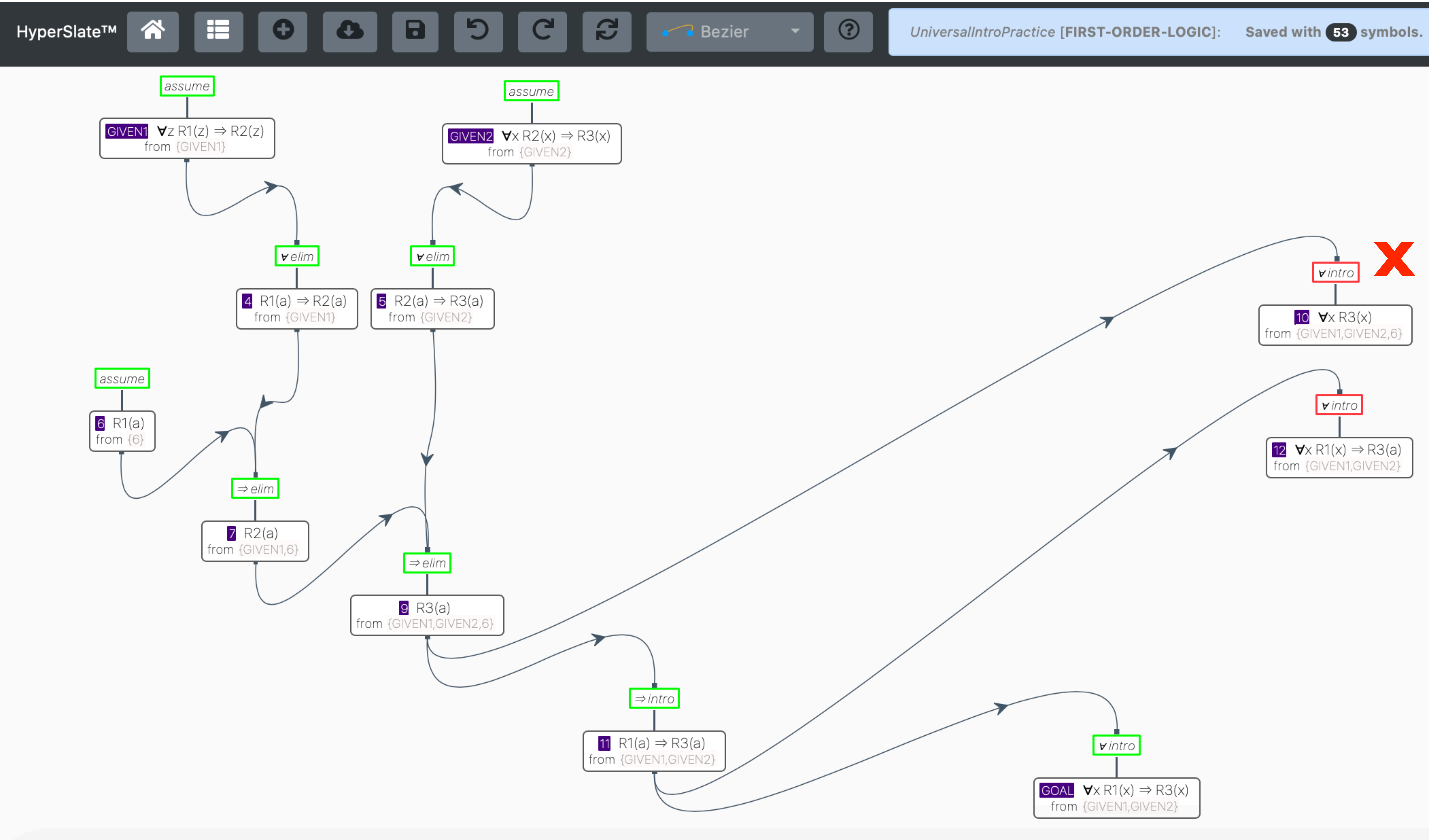

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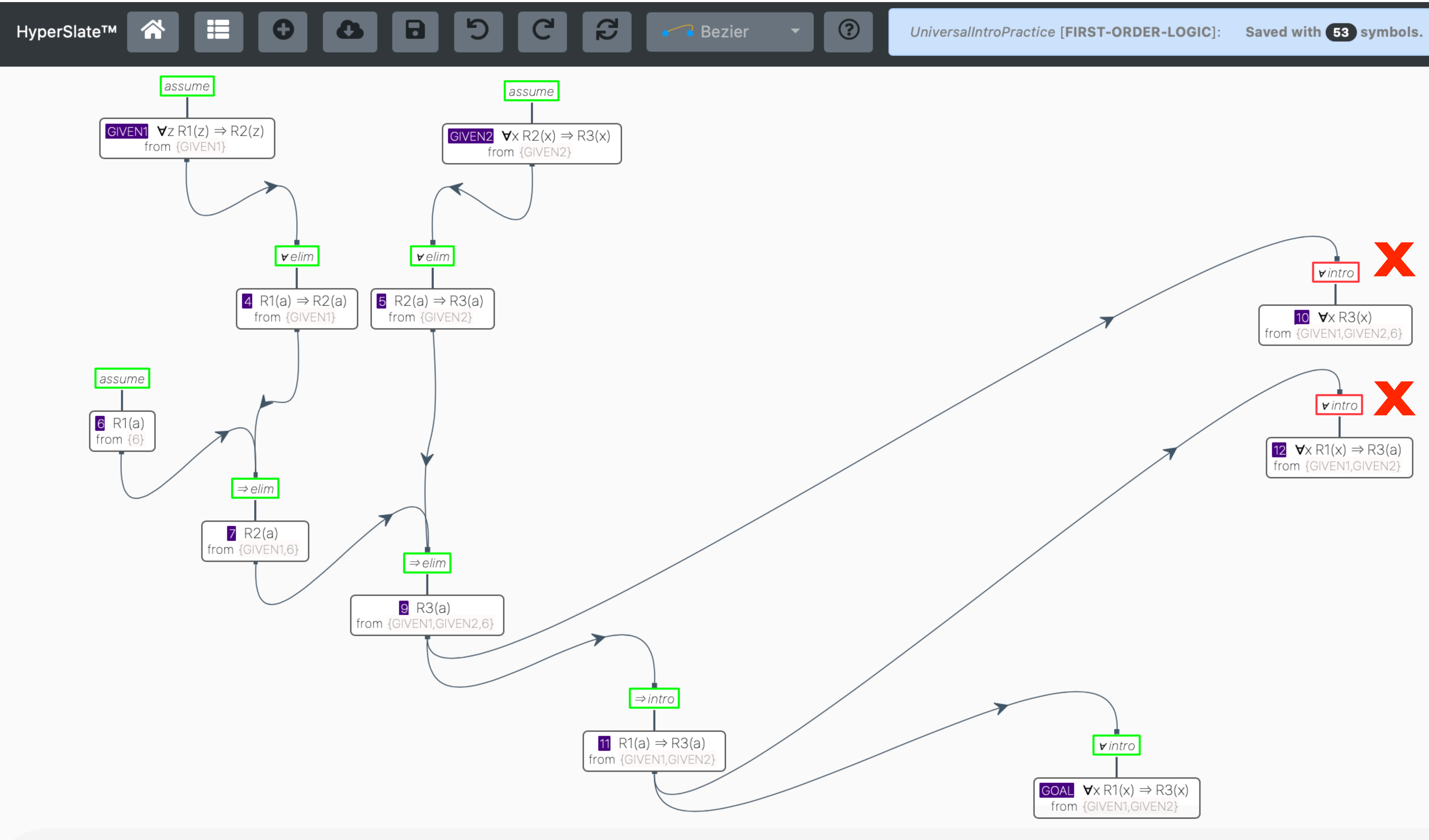

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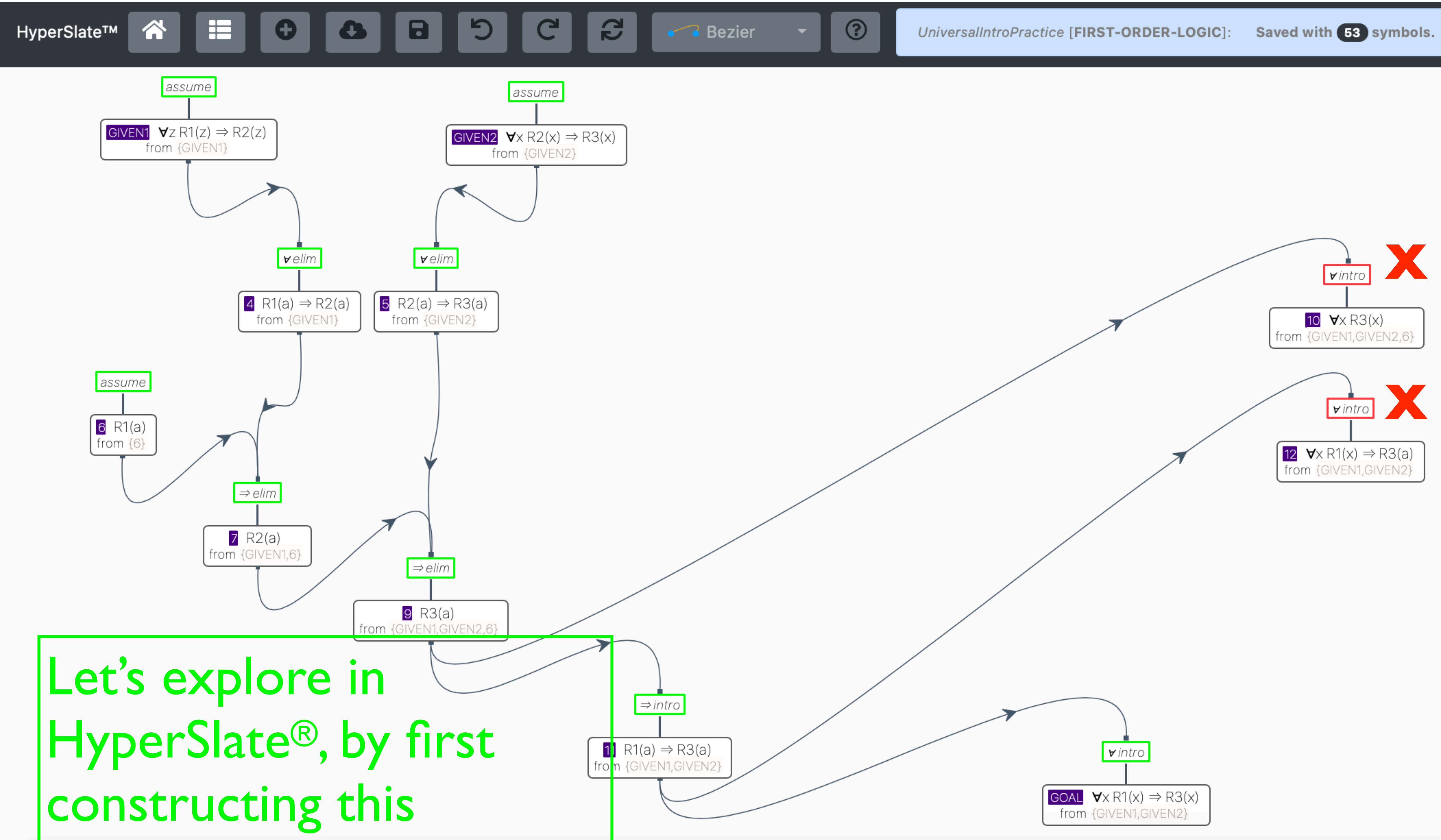
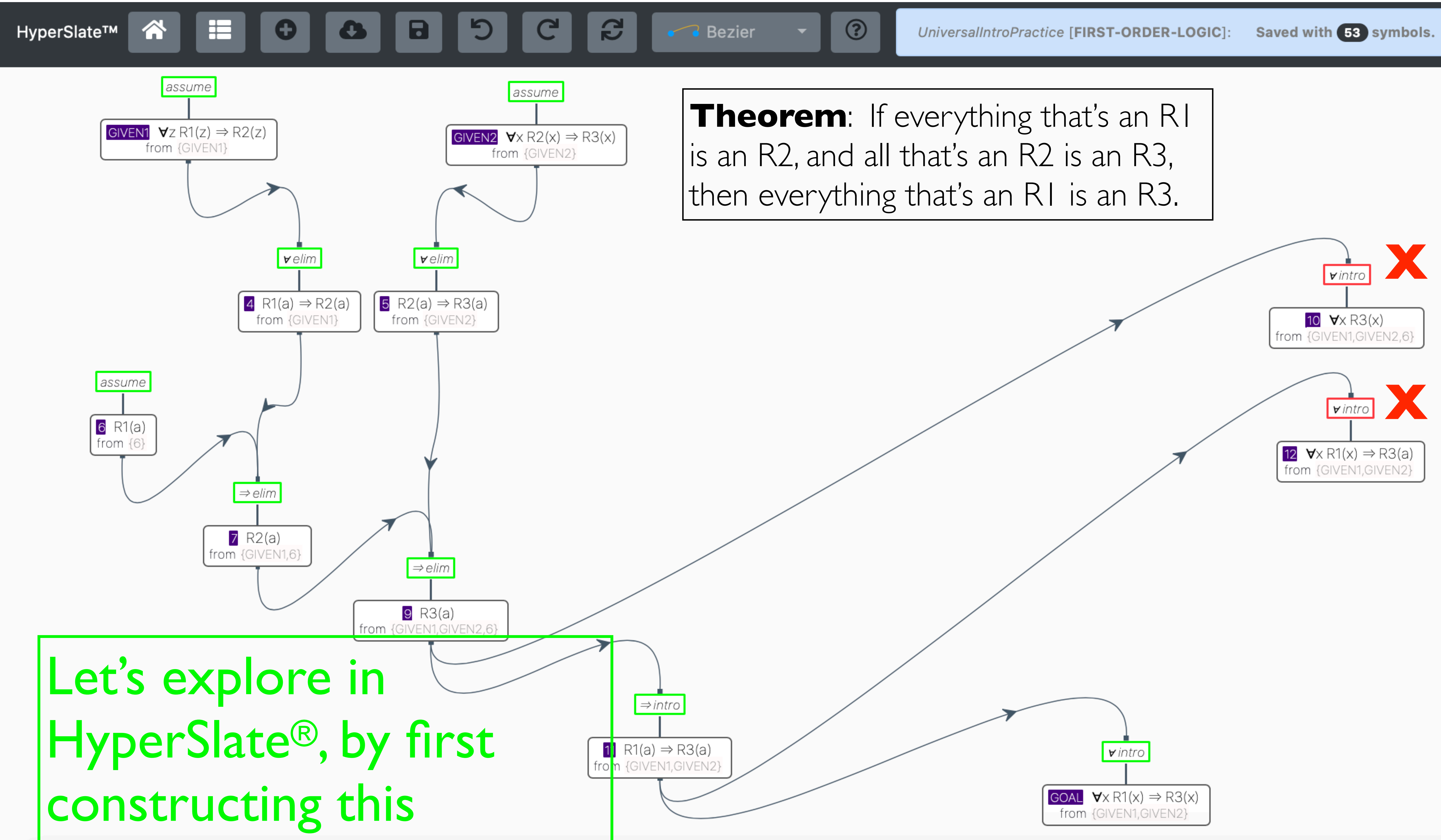


Let's explore in HyperSlate®, by first constructing this example from scratch …

universal intro Example/Tutorial



Theorem: If everything that's an R1 is an R2, and all that's an R2 is an R3, then everything that's an R1 is an R3.

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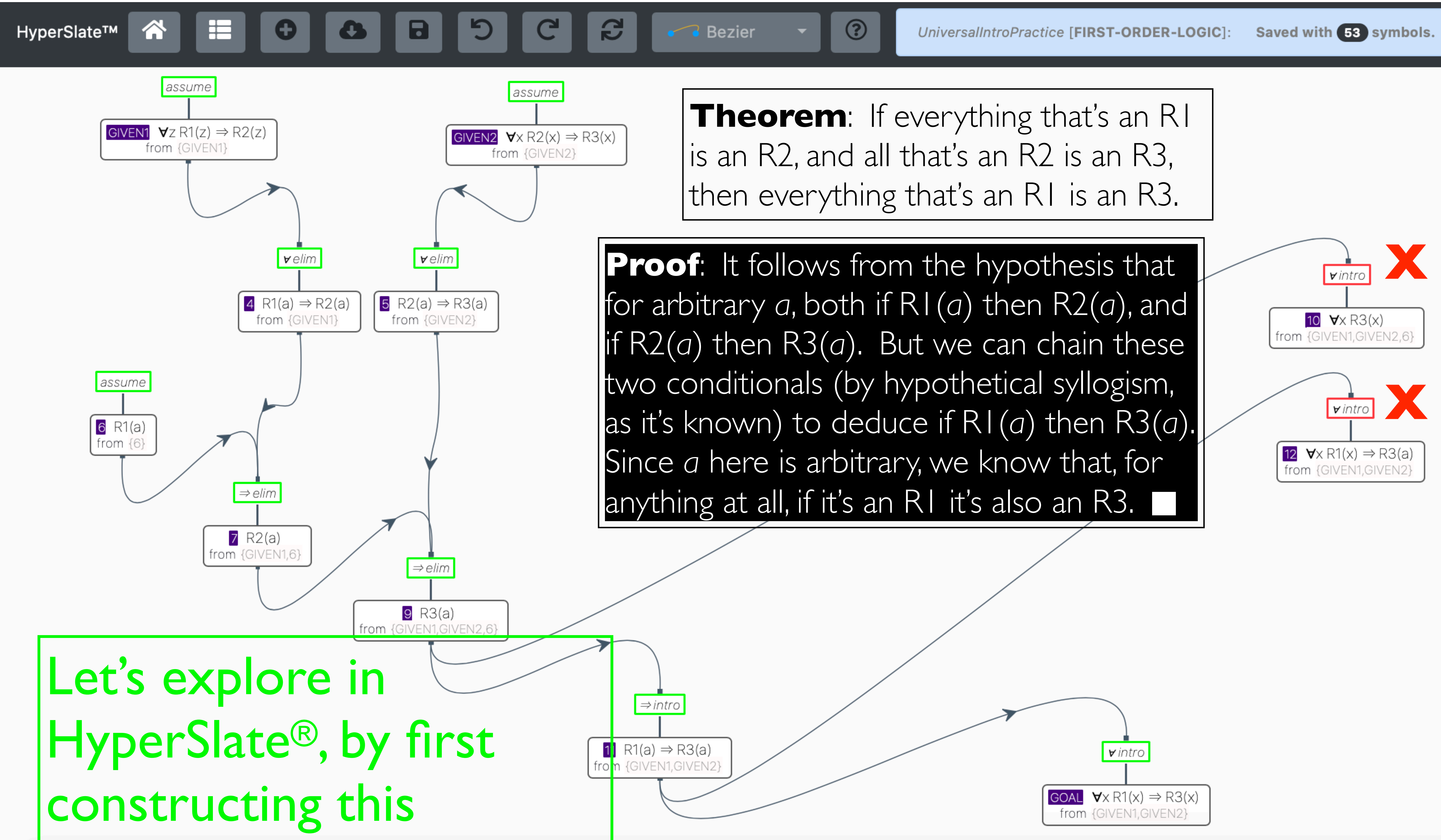
`universal intro` Example/Tutorial



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Proof: It follows from the hypothesis that for arbitrary *a*, both if R1(*a*) then R2(*a*), and if R2(*a*) then R3(*a*). But we can chain these two conditionals (by hypothetical syllogism, as it's known) to deduce if R1(*a*) then R3(*a*). Since *a* here is arbitrary, we know that, for anything at all, if it's an R1 it's also an R3. ■

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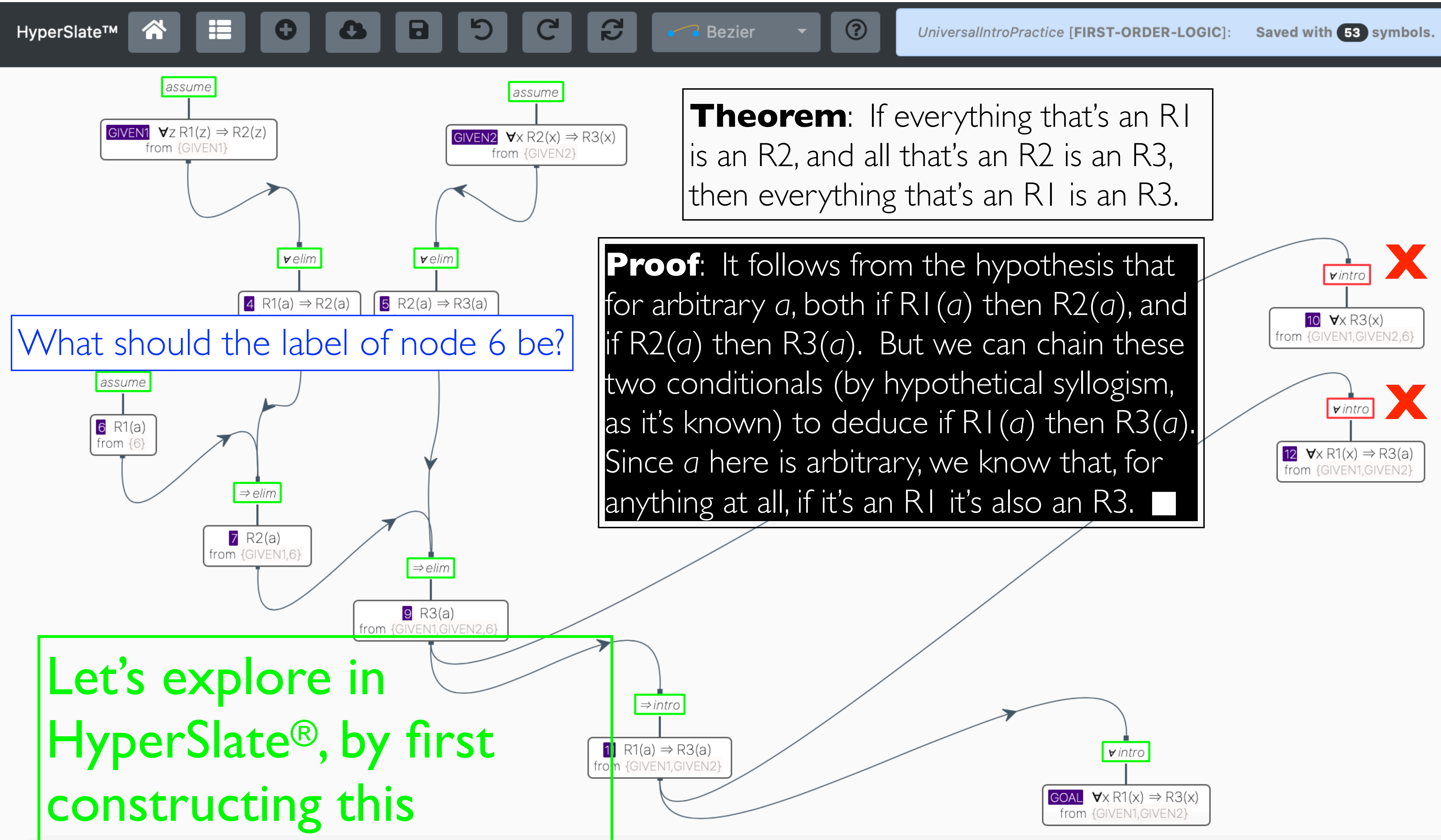

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What should the label of node 6 be?

Let's explore in HyperSlate®, by first constructing this example from scratch …

Suggested Practice Problems in HyperSlate®!

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$$\{\forall \mathtt{x}(\mathtt{R}(\mathtt{x}) \leftrightarrow \mathtt{S}(\mathtt{x})), \forall \mathtt{x}\mathtt{R}(\mathtt{x})\} \vdash \forall \mathtt{x}\mathtt{S}(\mathtt{x})\} \ \textbf{?}$$

Suggested Practice Problems in HyperSlate®!

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$$\{\forall \mathtt{x}[\mathtt{Norsk}(\mathtt{x}) \rightarrow \forall \mathtt{y}(\mathtt{Svensk}(\mathtt{y}) \rightarrow \mathtt{Smarter}(\mathtt{x},\mathtt{y}))]\} \vdash \forall \mathtt{x},\mathtt{y}[(\mathtt{Norsk}(\mathtt{x}) \land \mathtt{Svensk}(\mathtt{y})) \rightarrow \mathtt{Smarter}(\mathtt{x},\mathtt{y})] \ ?$$

Suggested Practice Problems in HyperSlate®!

$$\{\forall x(R(x) \leftrightarrow S(x)), \forall x R(x)\} \vdash \forall x S(x)\} \ ?$$

$$\{\forall x[\mathtt{Norsk}(x) \rightarrow \forall y(\mathtt{Svensk}(y) \rightarrow \mathtt{Smarter}(x,y))]\} \vdash \forall x,y[(\mathtt{Norsk}(x) \land \mathtt{Svensk}(y)) \rightarrow \mathtt{Smarter}(x,y)] \ ?$$

$$\{\forall x,y[(\mathtt{Norsk}(x) \land (\mathtt{Svensk}(y)) \rightarrow \mathtt{Smarter}(x,y)],$$
$$\forall x,y[(\mathtt{Svensk}(x) \land (\mathtt{Dansk}(y)) \rightarrow \mathtt{Smarter}(x,y)]\} \vdash$$
$$\forall x,y[(\mathtt{Norsk}(x) \land (\mathtt{Dansk}(y)) \rightarrow \mathtt{Smarter}(x,y)] \ ?$$

Hvis du forstår det, kan du bevise det.

Part I: *Slutten — for i dag.*

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Part II: Hands-on: Test 1 Help Session …

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$$\{\psi\} \vdash \phi \Rightarrow \psi$$

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Let's prove this Test-1 relevant theorem in HyperSlateS®.

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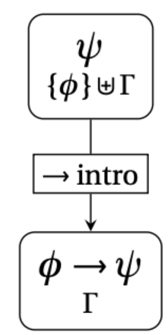
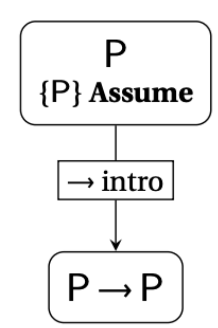
$$\{\psi\} \vdash \phi \Rightarrow \psi$$

Let's prove this Test-1 relevant theorem in HyperSlateS®.



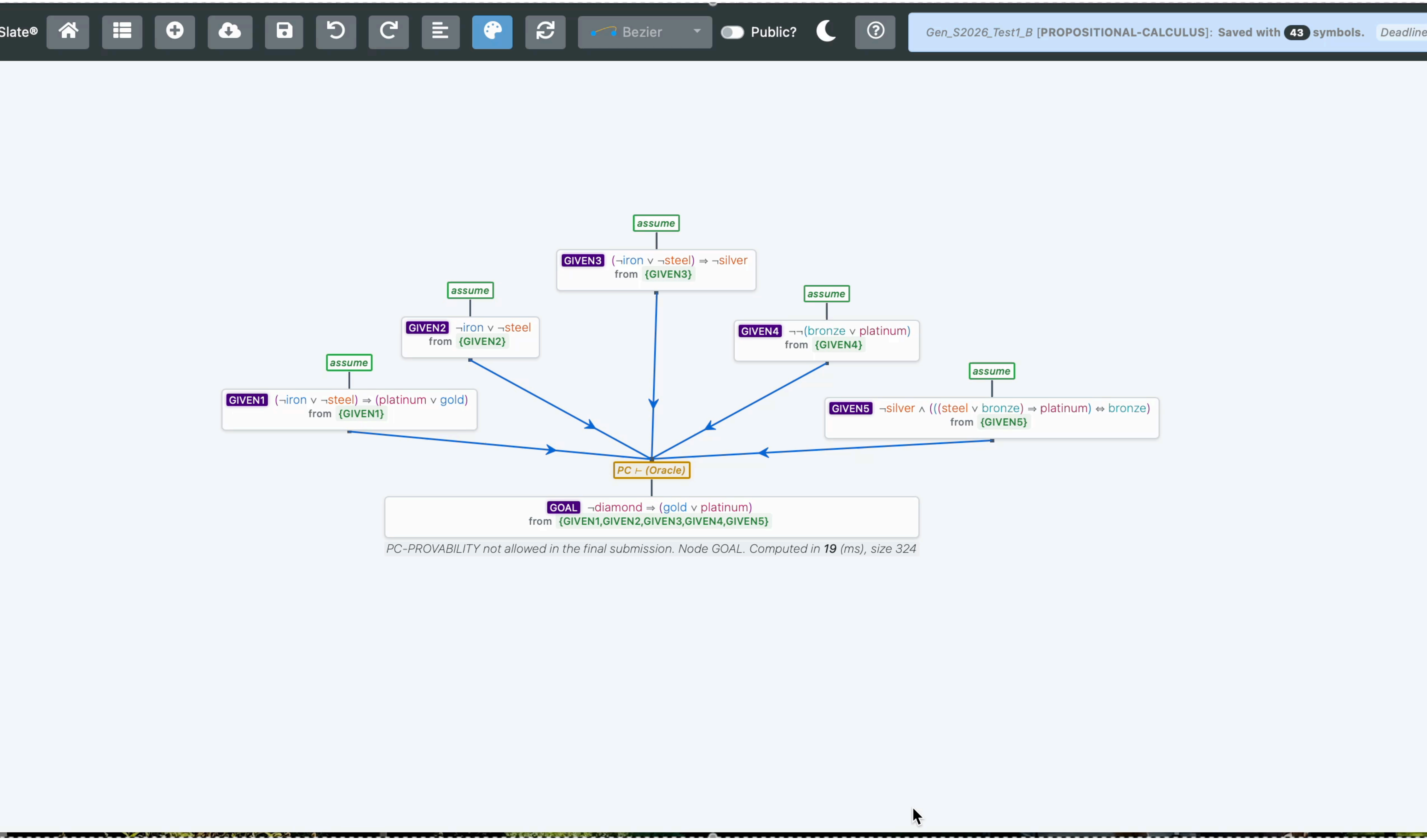
(2.19)

Since conditional introduction allows for the discharge of assumptions, its conclusion may have no assumptions at all. A vertex in a proof that has no in-scope assumptions, assuming that the proof is, in fact, correctly formed, is called a **theorem**. For instance, the sentence P → P is a theorem established by the following proof. (When a vertex has no in-scope assumptions, we won't bother to write the symbol for the empty set.)

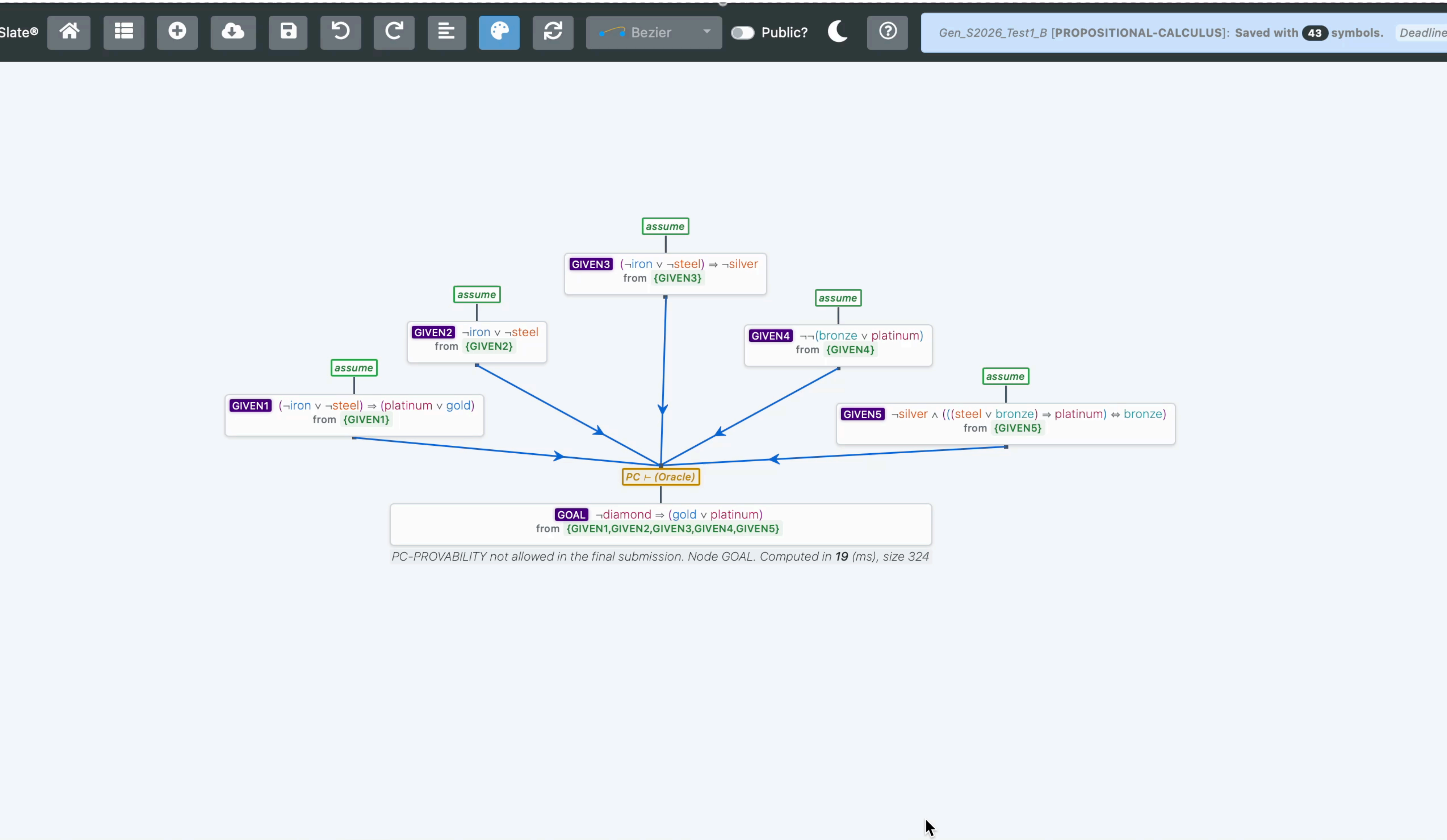


(2.20)

Selmer's Test-1 AI-Generated Easiest-of-Four Solved (Video)



Selmer's Test-1 AI-Generated Easiest-of-Four Solved (Video)



Who has a question/
problem to ask about?

Hands-on: DeMorgan's …

Hands-on: DeMorgan's …

e.g. $\{\neg(\phi \vee \psi)\} \vdash \neg\phi \wedge \neg\psi$

Hands-on: DeMorgan's …

e.g. $\{\neg(\phi \vee \psi)\} \vdash \neg\phi \wedge \neg\psi$

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Hands-on: DeMorgan's …

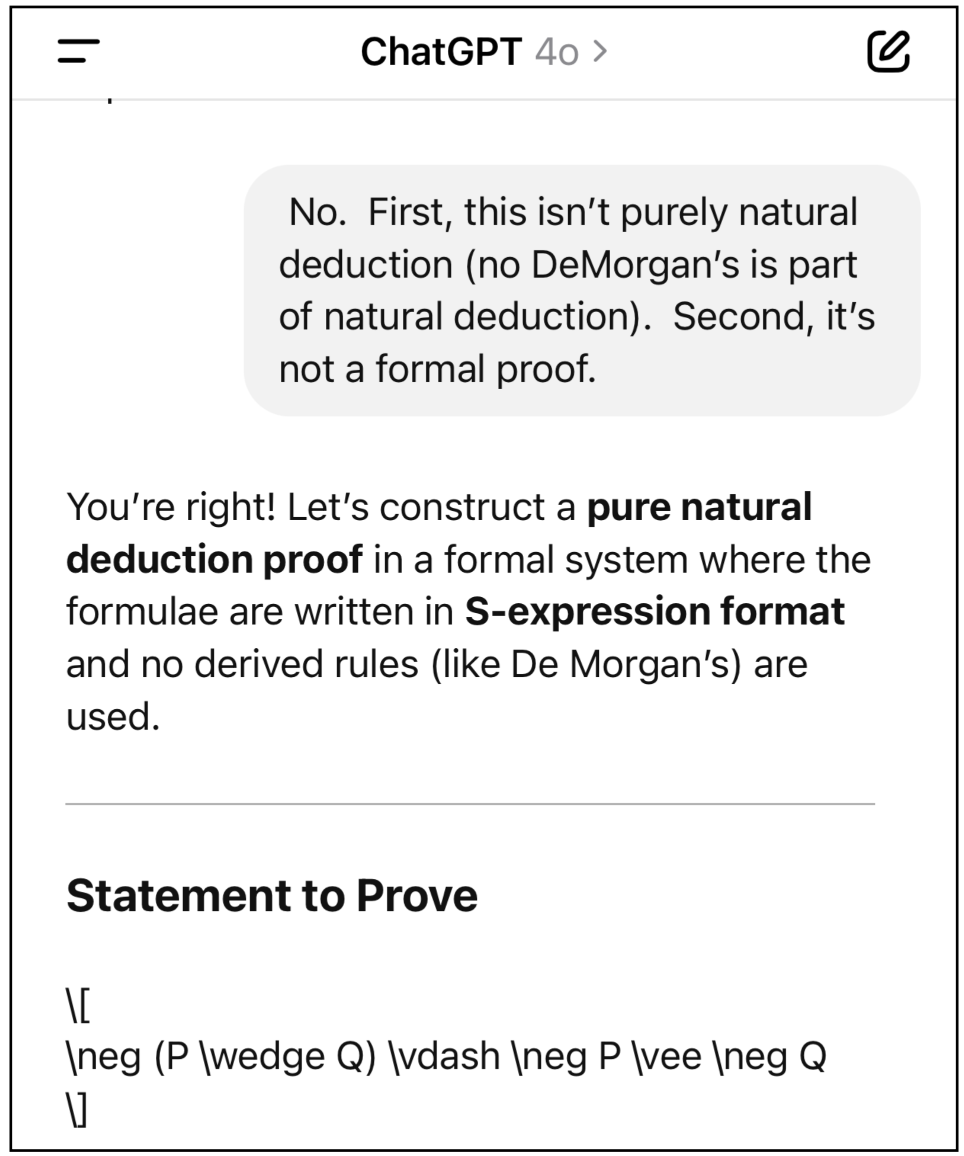
e.g. $\{\neg(\phi \vee \psi)\} \vdash \neg\phi \wedge \neg\psi$

e.g. $\{\neg(\phi \wedge \psi)\} \vdash \neg\phi \vee \neg\psi$

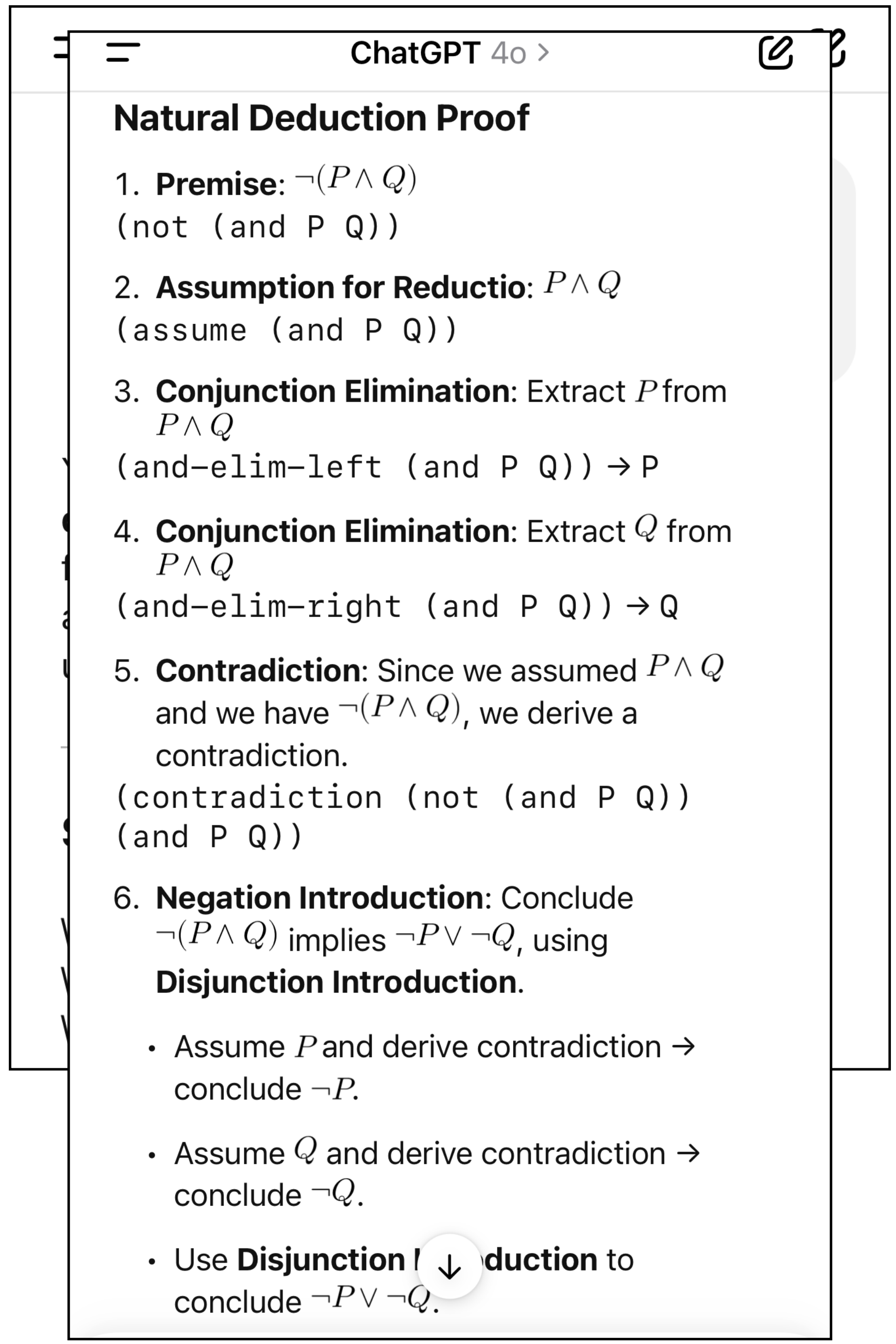
Let's prove the second of these in HyperSlate®, and then …can someone prove the first of these? …

e.g. $\{\neg(\phi \wedge \psi)\} \vdash \neg\phi \vee \neg\psi$

e.g. $\{\neg(\phi \wedge \psi)\} \vdash \neg\phi \vee \neg\psi$



e.g. $\{\neg(\phi \wedge \psi)\} \vdash \neg\phi \vee \neg\psi$



e.g. $\{\neg(\phi \wedge \psi)\} \vdash \neg\phi \vee \neg\psi$

This is positively scandalous.

1. **Premise**: $\neg(P \wedge Q)$
`(not (and P Q))`
2. **Assumption for Reductio**: $P \wedge Q$
`(assume (and P Q))`
3. **Conjunction Elimination**: Extract P from $P \wedge Q$
`(and-elim-left (and P Q))` → P
4. **Conjunction Elimination**: Extract Q from $P \wedge Q$
`(and-elim-right (and P Q))` → Q
5. **Contradiction**: Since we assumed $P \wedge Q$ and we have $\neg(P \wedge Q)$, we derive a contradiction.
`(contradiction (not (and P Q)) (and P Q))`
6. **Negation Introduction**: Conclude $\neg(P \wedge Q)$ implies $\neg P \vee \neg Q$, using **Disjunction Introduction**.
   - Assume P and derive contradiction → conclude $\neg P$.
   - Assume Q and derive contradiction → conclude $\neg Q$.
   - Use **Disjunction I**[illegible]**duction** to conclude $\neg P \vee \neg Q$.

e.g. $\{\neg(\phi \wedge \psi)\} \vdash \neg\phi \vee \neg\psi$

This is positively scandalous.

ChatGPT 4o >

```
(assume (not (and P Q)))  ; Premise: ¬(P ∧ Q)

  (assume P)  ; Assume P for reductio
    (assume Q)  ; Assume Q for reductio
      (and-intro P Q)  ; P ∧ Q
      (contradiction (not (and P Q)) (and P Q))  ; Contradiction!
    (not-intro Q (and P Q) (not (and P Q)))  ; ¬Q
    (or-intro-right (not P) (not Q))  ; ¬P ∨ ¬Q
  (not-intro P (and P Q) (not (and P Q)))  ; ¬P
  (or-intro-left (not P) (not Q))  ; ¬P ∨ ¬Q

  (or-elim (not P) (not Q) (or (not P) (not Q)))  ; Conclusion
```

- Assume Q and derive contradiction → conclude $\neg Q$.
- Use **Disjunction I** **duction** to conclude $\neg P \vee \neg Q$.

e.g. $\{\neg(\phi \wedge \psi)\} \vdash \neg\phi \vee \neg\psi$

This is positively scandalous.

1. **Premise**: $\neg(P \wedge Q)$
`(not (and P Q))`

2. **Assumption for Reductio**: $P \wedge Q$
`(assume (and P Q))`

3. **Conjunction Elimination**: Extract P from $P \wedge Q$
`(and-elim-left (and P Q))` → `P`

4. **Conjunction Elimination**: Extract Q from $P \wedge Q$
`(and-elim-right (and P Q))` → `Q`

5. **Contradiction**: Since we assumed $P \wedge Q$ and we have $\neg(P \wedge Q)$, we derive a contradiction.
`(contradiction (not (and P Q)) (and P Q))`

6. **Negation Introduction**: Conclude $\neg(P \wedge Q)$ implies $\neg P \vee \neg Q$, using **Disjunction Introduction**.
   - Assume P and derive contradiction → conclude $\neg P$.
   - Assume Q and derive contradiction → conclude $\neg Q$.
   - Use **Disjunction I**↓**duction** to conclude $\neg P \vee \neg Q$.

e.g. $\{\neg(\phi \land \psi)\} \vdash \neg\phi \lor \neg\psi$

This is positively scandalous

ChatGPT o3-mini-high >

```
1.   ¬(p ∧ q)                          Premise
2.   |   Assume: ¬(¬p ∨ ¬q)              Assumption [for indirect proof]
3.   |   |   Assume: ¬p                   Assumption
4.   |   |   ¬p ∨ ¬q                     ∨-Introduction (from 3, left)
5.   |   |   ⊥                            Contradiction (from 2 and 4)
6.   |   p                               RAA, discharging 3–5
7.   |   |   Assume: q                   Assumption
8.   |   |   p ∧ q                     ∧-Introduction (from 6 and 7)
9.   |   |   ⊥                            Contradiction (from 1 and 8)
10.  |   ¬q                            RAA, discharging 7–9
11.  |   ¬p ∨ ¬q                     ∨-Introduction (from 10, right)
12.  ⊥                              Contra…tion (from 2 and 11)
13.  ¬p ∨ ¬q                     RAA, di…rging 2–12
```

- Assume P and derive contradiction → conclude $\neg P$.
- Assume Q and derive contradiction → conclude $\neg Q$.
- Use **Disjunction I…duction** to conclude $\neg P \lor \neg Q$.