

FOL III

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab

Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Formal Logic (& AI)
2/26/2026



\mathcal{L}_1
FOL III

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab

Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Formal Logic (& AI)
2/26/2026



Part I: New: *Last of FOL inference-rule quartet.*

Part I: New: *Last of FOL inference-rule quartet.*

Part II: Another hands-on session.

Part I: New: *Last of FOL inference-rule quartet.*

Part II: Another hands-on session.

Overview of Prop-Calc Outlier, displaying importance of proof engineering with AI.

Actually, ...
let me jump to JJ Outlier ...

<https://rpi.logicamodernapproach.com/hyperslate/public/Selmer.Bringsjord@gmail.com/JJChallenge>

Prof Ferguson-catalyzed investigation:

How does AI of today do on self-referential reasoning?

Self-Referential Tests of ChatGPT

Self-Referential Tests of ChatGPT

SE

Can you prove that if P holds, and (if P (and (Q (not Q))) holds, then the moon is made of green cheese?



No, I cannot prove that if P holds and (if P (and (Q (not Q))) holds, then the moon is made of green cheese.

The expression (Q (not Q)) is a logical contradiction, since Q cannot be both true and false at the same time. Therefore, the statement (if P (and (Q (not Q)))) is always false, regardless of the truth value of P.

Since the antecedent of the implication is always false, the implication itself is vacuously true. However, this does not provide any evidence or justification for the conclusion that the moon is made of green



Self-Referential Tests of ChatGPT

SE Can you prove that if P holds, and (if P (and (Q (not Q))) holds, then the moon is made of green cheese?



 No, I cannot prove that if P holds and (if P (and (Q (not Q))) holds, then the moon is made of green cheese.

The expression (Q (not Q)) is a logical contradiction, since Q cannot be both true and false at the same time. Therefore, the statement (if P (and (Q (not Q)))) is always false, regardless of the truth value of P.

Since the antecedent of the implication is always false, the implication itself is vacuously true. However, this does not provide any evidence or justification for the conclusion that the moon is made of green



Self-Referential Tests of ChatGPT

SE Can you prove that if P holds, and (if P (and (Q (not Q))) holds, then the moon is made of green cheese?



No, I cannot prove that if P holds and (if P (and (Q (not Q))) holds, then the moon is made of green cheese.

The expression (Q (not Q)) is a logical contradiction, since Q cannot be both true and false at the same time. Therefore, the statement (if P (and (Q (not Q)))) is always false, regardless of the truth value of P.

Since the antecedent of the implication is always false, the implication itself is vacuously true. However, this does not provide any evidence or justification for the conclusion that the moon is made of green



$$\{P, P \rightarrow (Q \wedge \neg Q)\} \vdash \perp$$

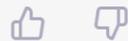
Self-Referential Tests of ChatGPT

Self-Referential Tests of ChatGPT

SE This sentence has a word in it longer than the string 'gfeeteru.'



 That sentence is true, since the word "longer" has more letters than the string "gfeeteru". "Longer" has 6 letters, while "gfeeteru" has only 8 letters.

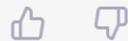


Self-Referential Tests of ChatGPT

SE This sentence has a word in it longer than the string 'gfeeteru.'



 That sentence is true, since the word "longer" has more letters than the string "gfeeteru". "Longer" has 6 letters, while "gfeeteru" has only 8 letters.



Are you quite sure?

Logic-&-AI: Self-Referential Tests of ChatGPT

Logic-&-AI: Self-Referential Tests of ChatGPT

SE

And, as Spock, I also tell you the following sentence. This sentence, if false, has more than 103 words. Am I lying, or telling the truth?



As Spock, you would be presenting a paradoxical statement known as the "liar paradox" or the "liar's paradoxical sentence." This type of statement is self-referential and produces a logical contradiction, and cannot be consistently true or false.

In this case, the statement you presented cannot be consistently true or false, because if it were true, then it would be false, and if it were false, then it would be true. Therefore, as Spock, you would not be lying or telling the truth in the traditional sense, but rather presenting a logical conundrum for



Logic-&-AI: Self-Referential Tests of ChatGPT

SE And, as Spock, I also tell you the following sentence. This sentence, if false, has more than 103 words. Am I lying, or telling the truth?

As Spock, you would be presenting a paradoxical statement known as the "liar paradox" or the "liar's paradoxical sentence." This type of statement is self-referential and produces a logical contradiction, and cannot be consistently true or false.

In this case, the statement you presented cannot be consistently true or false, because if it were true, then it would be false, and if it were false, then it would be true. Therefore, as Spock, you would not be lying or telling the truth in the traditional sense, but rather presenting a logical conundrum for

Are you quite sure?

**Live-action on
HyperGrader ...**

ThxForThePCOracle

ThxForThePCOracle

Please attempt that now-ish; thx.

ThxForThePCOracle

Please attempt that now-ish; thx.

 Problem Bank Edit Problems

Exercis

es

 Metrics for
ExercisesDownload: LAMA-
BDLAHSHG020421.pdf

Problems

 ThxForThePCOracle RipsSaysNo1
BiconditionalIntroByChaini
... New  ThxForThePCOracle

This straightforward problem is quickly solved with a minimum of tedium, courtesy of the PC (entailment) provability oracle, use of which is allowed to remain in your finished proof (but no use of any other oracle can be in the finished proof). This oracle is for the logic \mathcal{L}_{PC} . Any learning of formal logic, at more than a trivial level, without the availability of the kind of AI embodied by this oracle (and more powerful ones farther up the ladder of extensional logic), is not only pedagogically unwise, but also, for the learner, downright painful.

Deadline March 18, 2021, 12:00 PM EDT Solve Submissions Metrics  RipsSaysNo1

This problem relates to the interesting book *The Psychology of Proof*, by psychologist L. Rips, a book that, shortly before he died, nobelist and computational-logic pioneer Herbert Simon praised as crucial to advancing automated reasoning/AI. Specifically, you are presented here with the challenge of crafting a proof that, as implied by what Rips presents in his volume, is supposed to be beyond the reach of (at least logically naïve) humans! This is quite peculiar, because as you will soon see, that which is to be proved, expressed in meta-logic, is simply this: $\{\neg(\phi \rightarrow \psi)\} \vdash \phi$.

Deadline March 18, 2021, 12:00 PM EDT

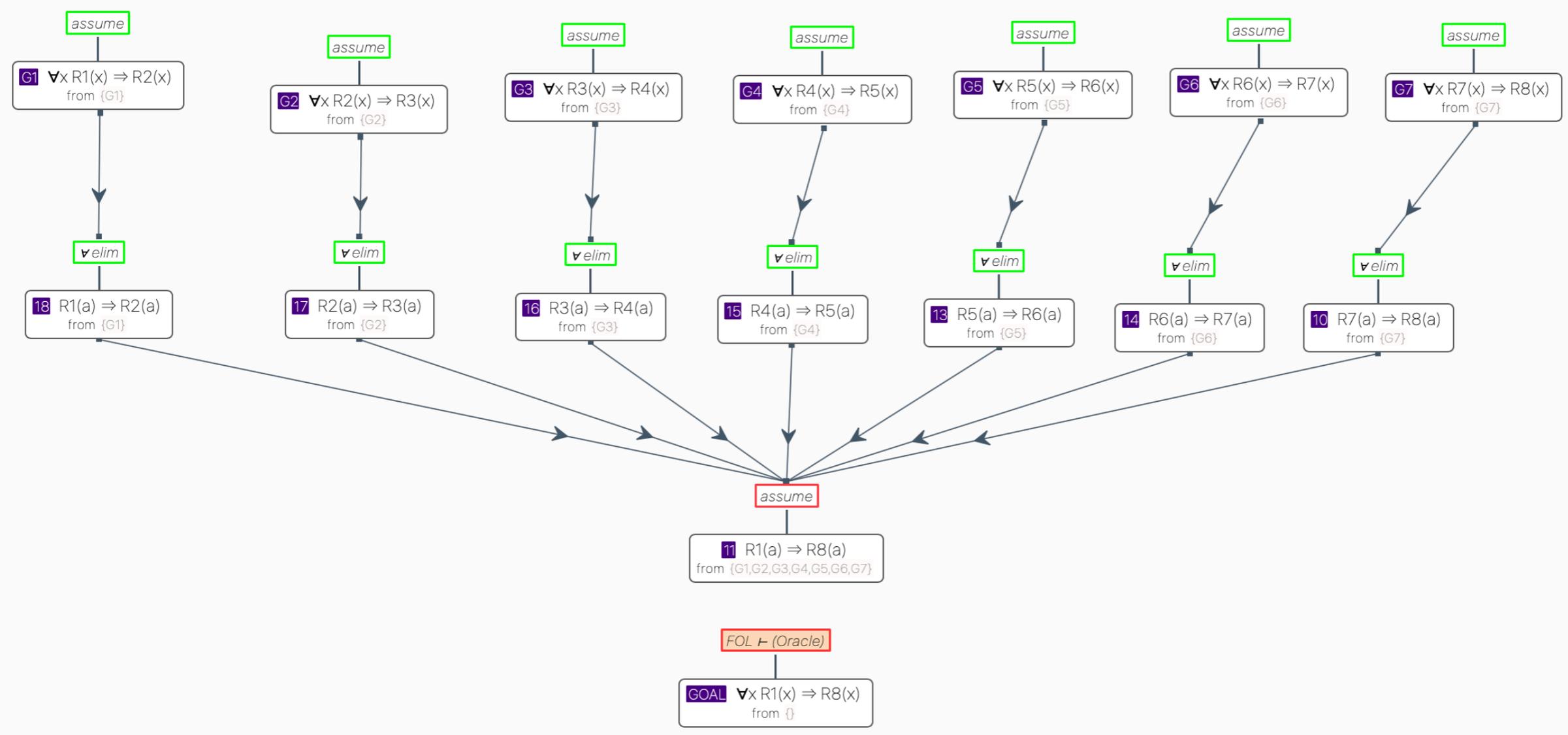
[Open in HyperSlate](#) ThxForThePCOracle

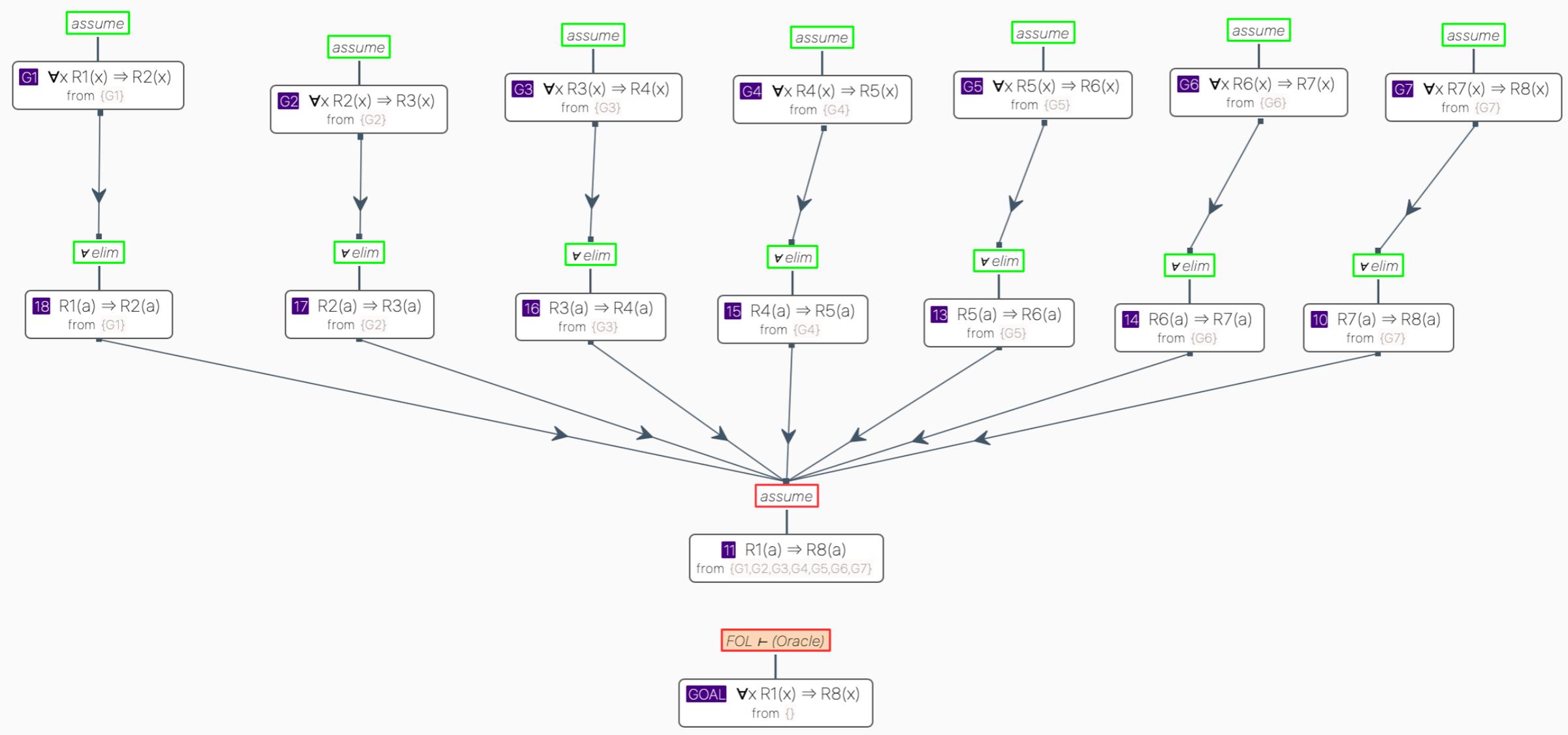
This straightforward problem is quickly solved with a minimum of tedium, courtesy of the PC (entailment) provability oracle, use of which is allowed to remain in your finished proof (but no use of any other oracle can be in the finished proof). This oracle is for the logic \mathcal{L}_{PC} . Any learning of formal logic, at more than a trivial level, without the availability of the kind of AI embodied by this oracle (and more powerful ones farther up the ladder of extensional logic), is not only pedagogically unwise, but also, for the learner, downright painful.

Deadline March 18, 2021, 12:00 PM EDT

Days Hours Minutes Seconds

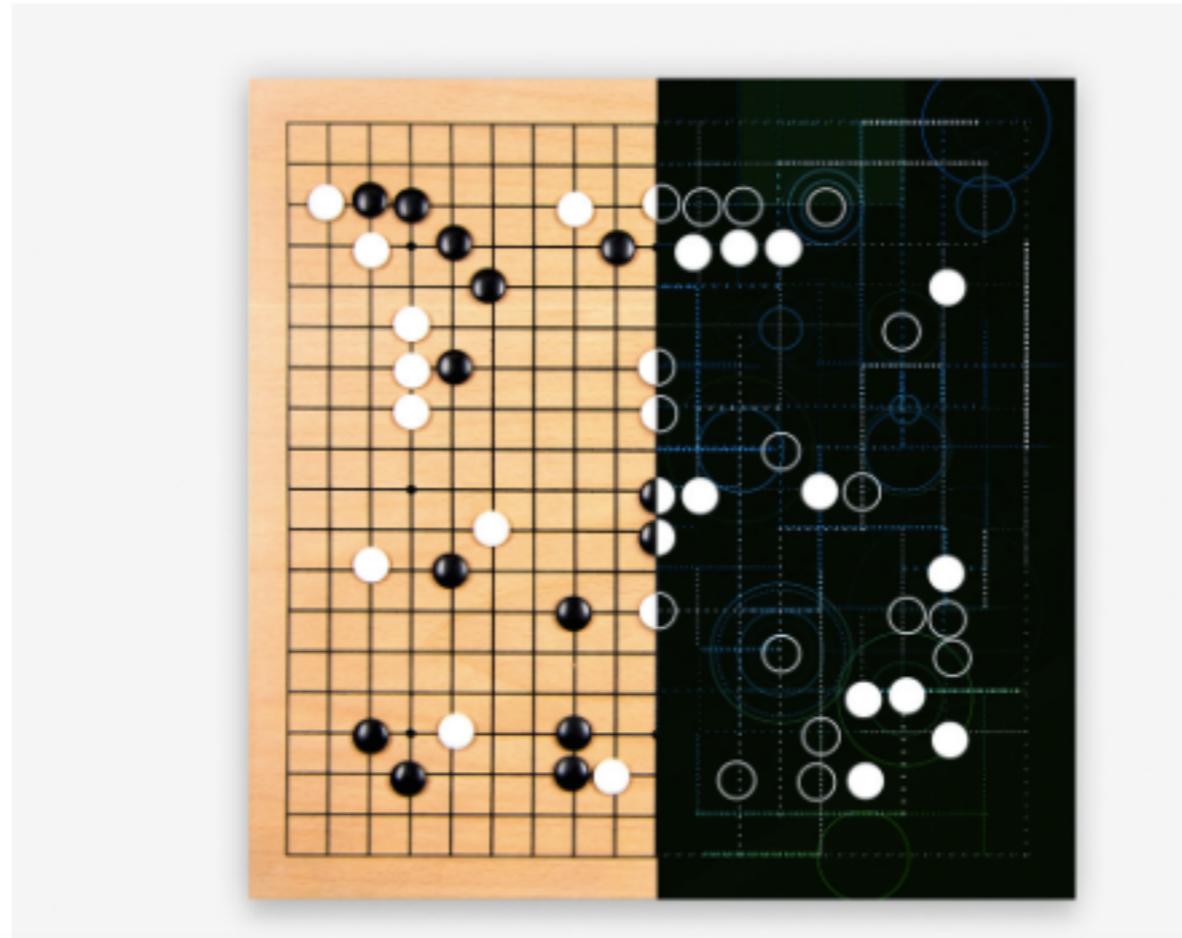
03:19:39:00**Problem Type:** SIMPLE**Difficulty:** 1**Points:** 10[Leaderboard](#)



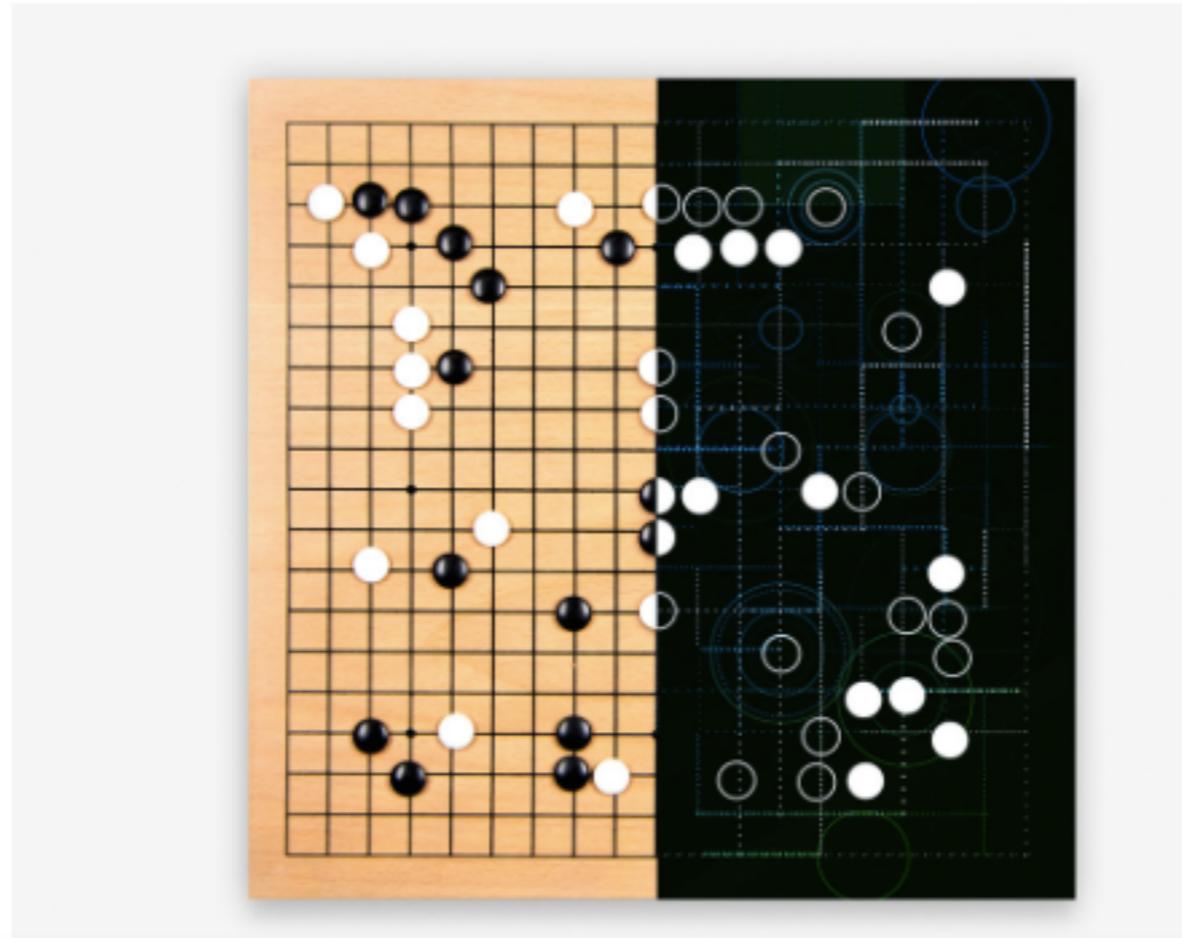


First Interlude re Formal Logic & Games ...

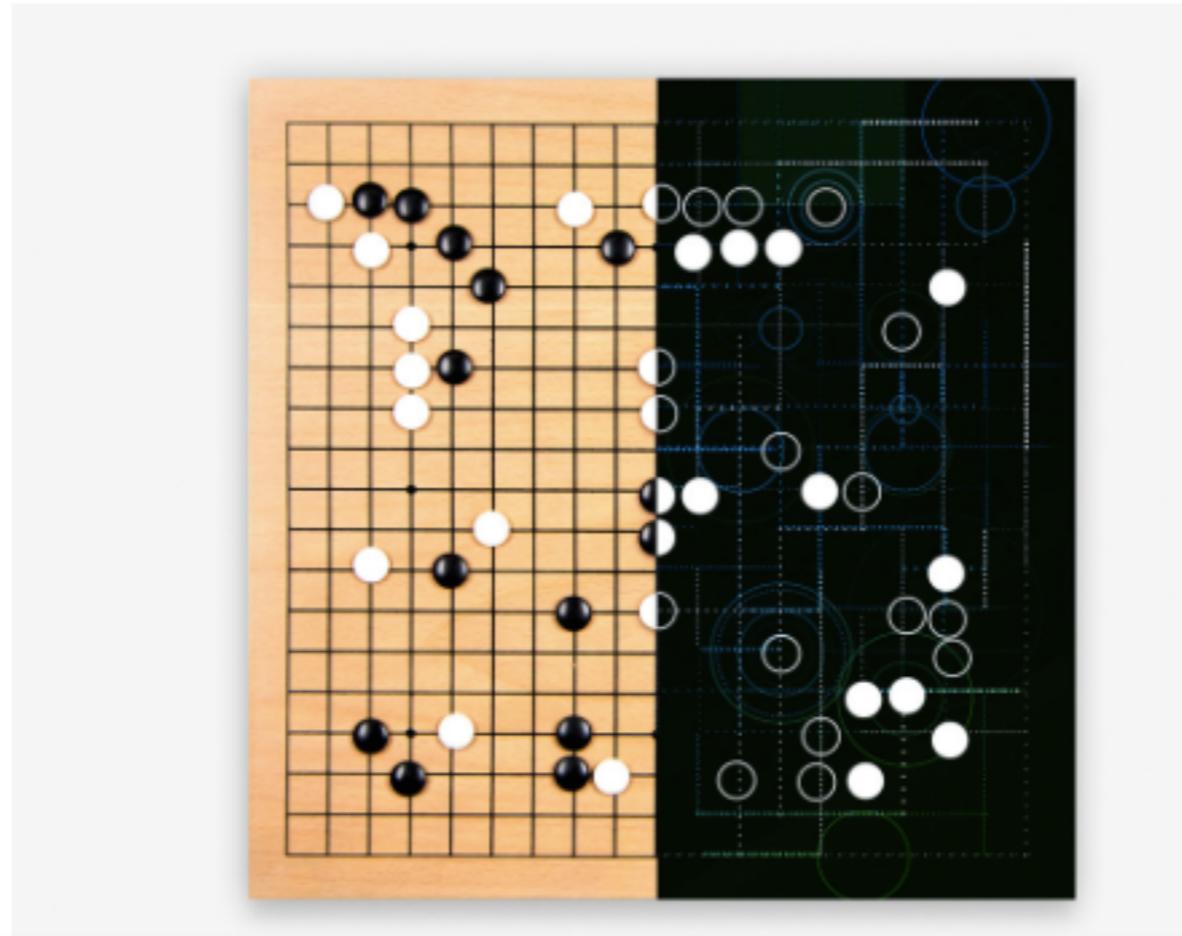
IN A HUGE BREAKTHROUGH, GOOGLE'S AI BEATS A TOP PLAYER AT THE GAME OF GO



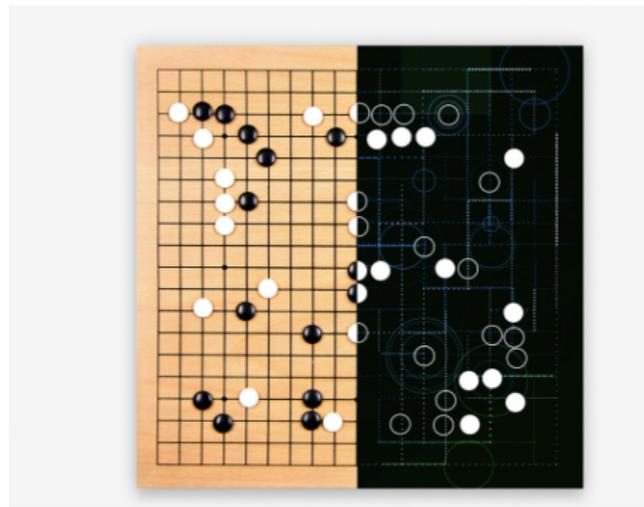
IN A HUGE BREAKTHROUGH, GOOGLE'S AI BEATS A TOP PLAYER AT THE GAME OF GO



IN A HUGE BREAKTHROUGH, GOOGLE'S AI BEATS A TOP PLAYER AT THE GAME OF GO



**IN A HUGE BREAKTHROUGH,
GOOGLE'S AI BEATS A TOP
PLAYER AT THE GAME OF GO**



IN A HUGE BREAKTHROUGH,
GOOGLE'S AI BEATS A TOP
PLAYER AT THE GAME OF GO



The *Entscheidungsproblem*

The *Entscheidungsproblem*

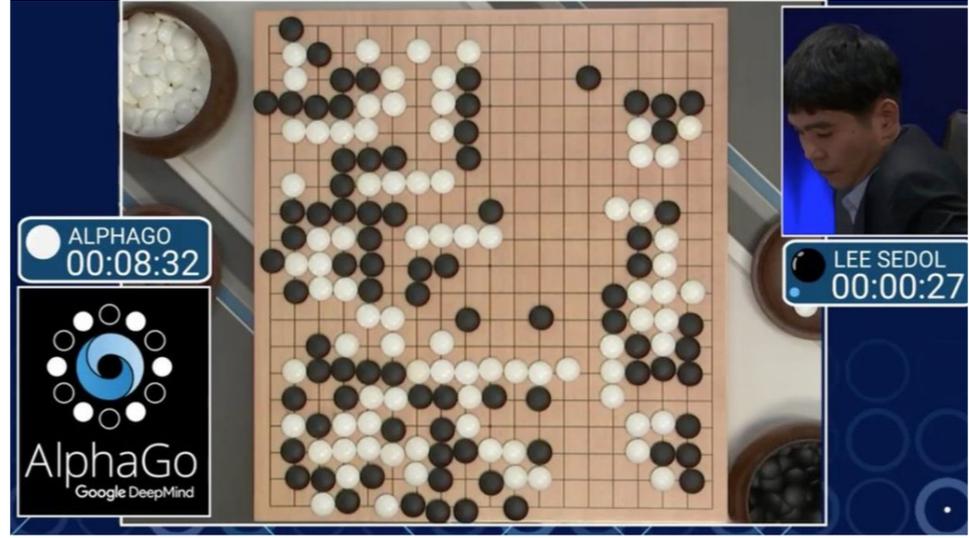
IN A HUGE BREAKTHROUGH,
GOOGLE'S AI BEATS A TOP
PLAYER AT THE GAME OF GO



The *Entscheidungsproblem*

IN A HUGE BREAKTHROUGH,
GOOGLE'S AI BEATS A TOP
PLAYER AT THE GAME OF GO

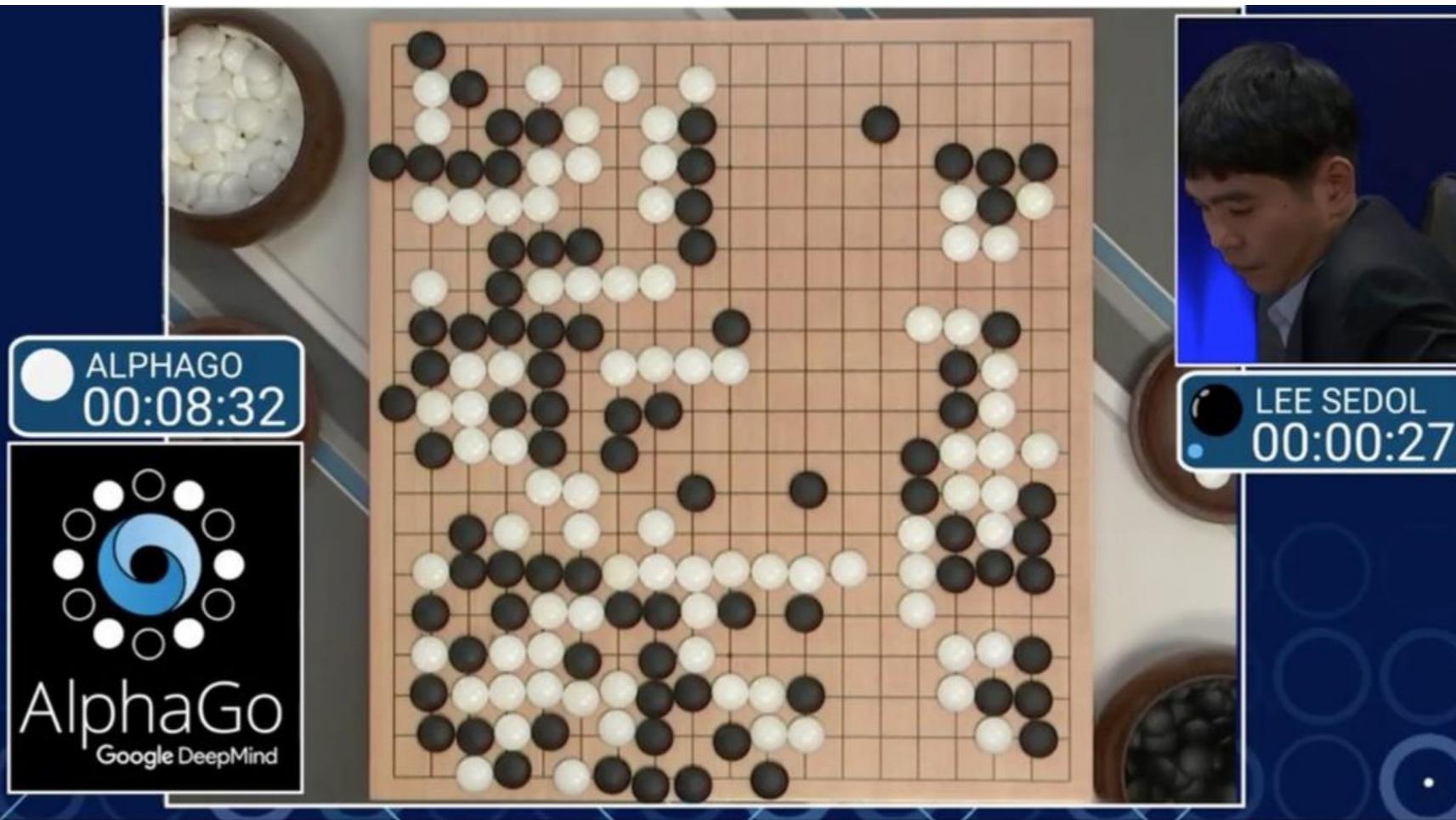




ALPHAGO
00:08:32



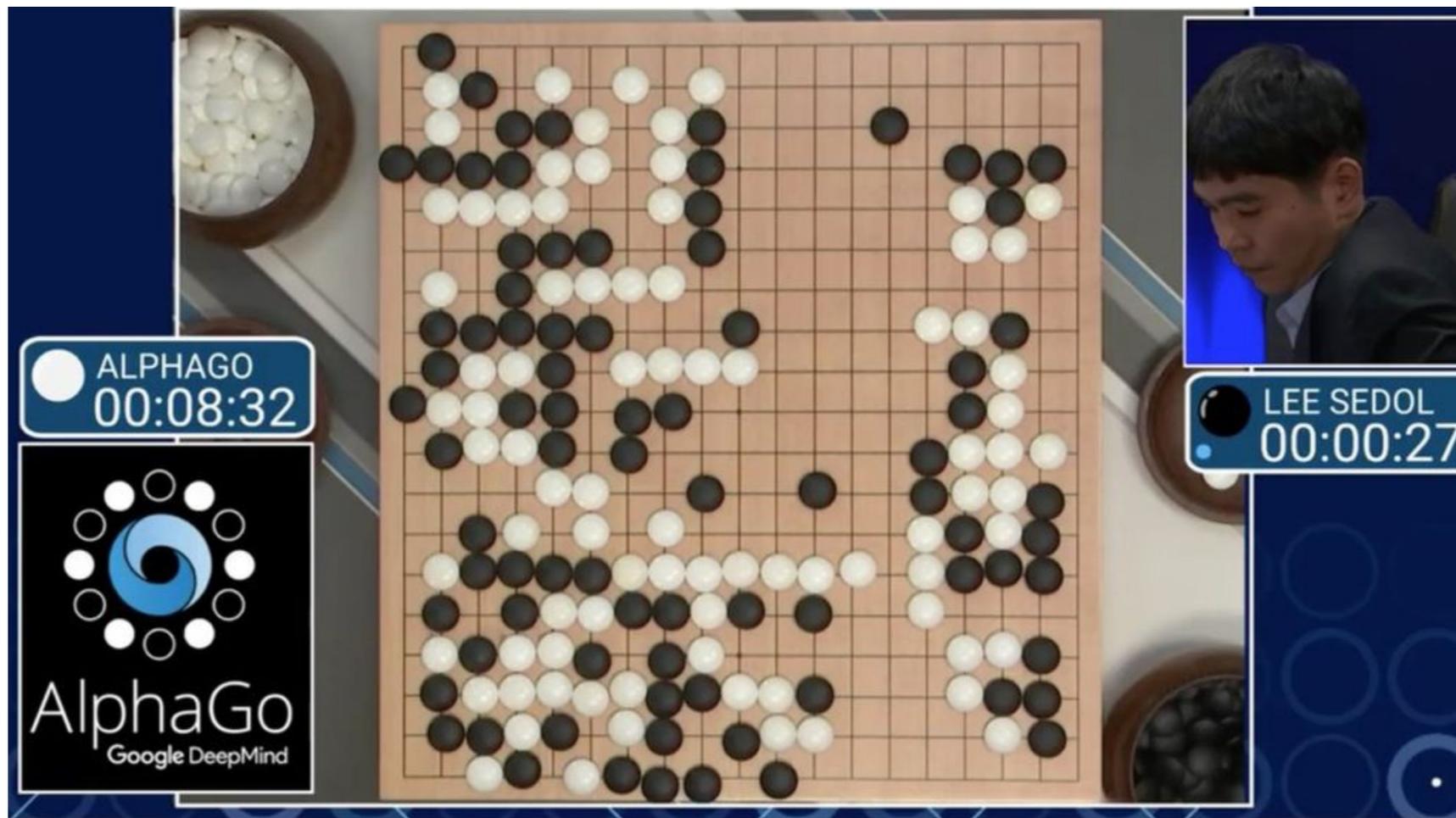
LEE SEDOL
00:00:27



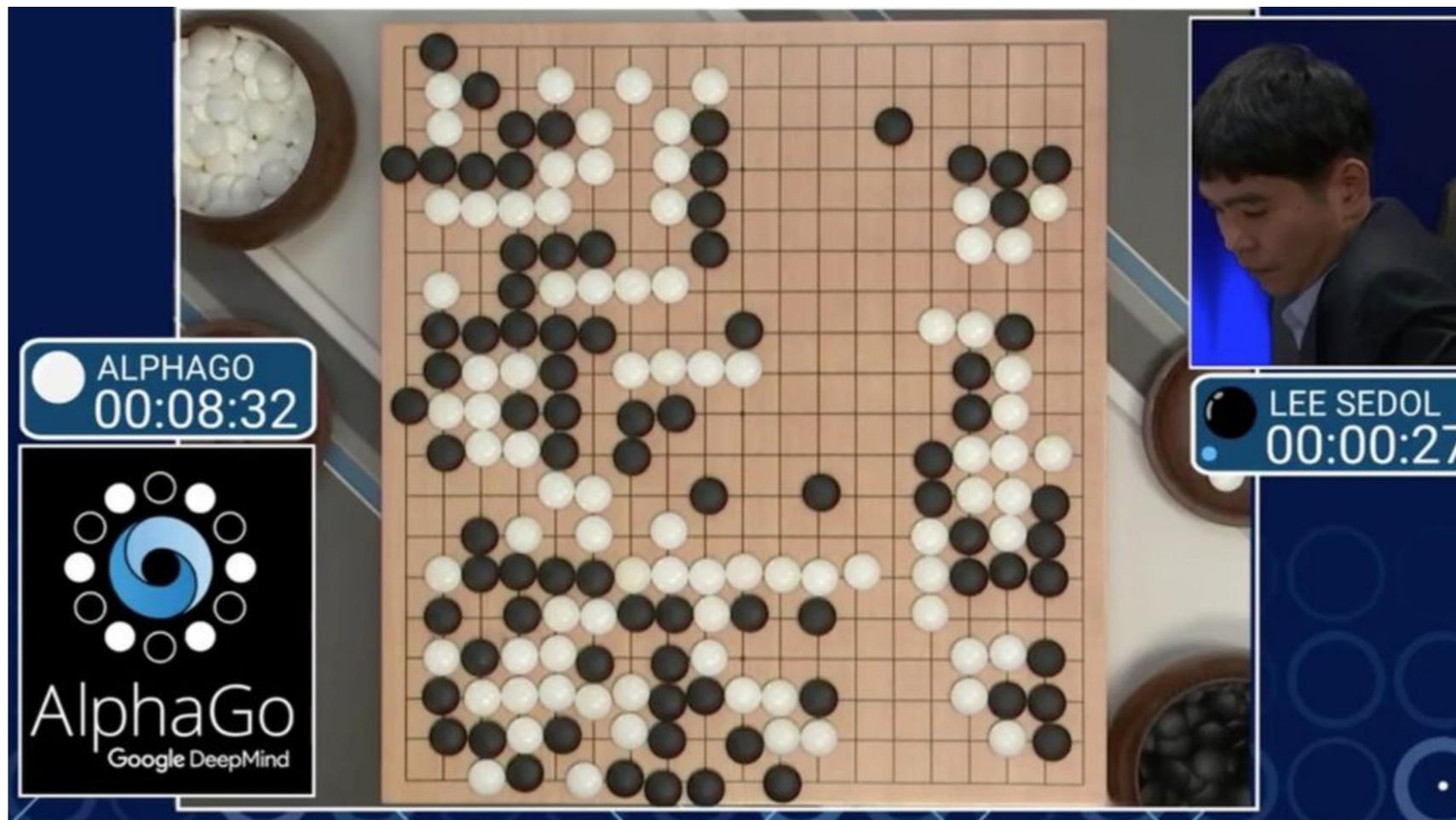
ALPHAGO
00:08:32



LEE SEDOL
00:00:27

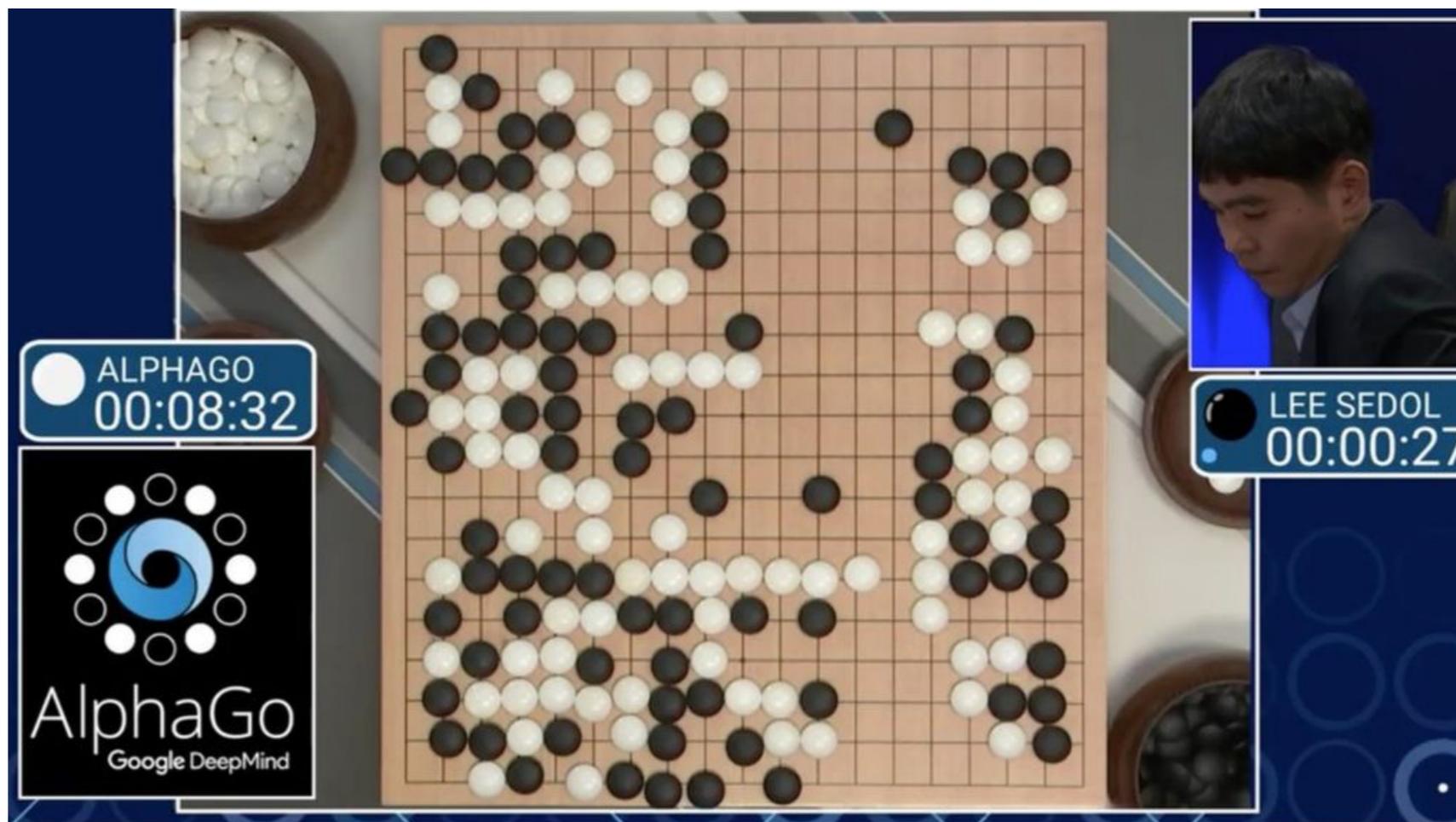


Praiseworthy *AI simplicter*, perhaps.



Praiseworthy *AI simpliciter*, perhaps.

But certainly *not AI = HI!*



Praiseworthy *AI simpliciter*, perhaps.

But certainly *not AI = HI!*

“AlphaGo, from the perspective of South, how many majuscule Roman letters are in black? Why do you say that?”

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive



Descartes

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive



Descartes



Leibniz

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive



Descartes



Leibniz



Church

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive



Descartes



Leibniz



Church



Gödel

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive



Descartes



Leibniz



Church



Gödel



Turing

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive



Descartes



Leibniz



Church



Gödel



Turing

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive



Descartes



Leibniz



Church



Gödel



Turing

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive



Descartes



Leibniz



Church



Gödel



Turing

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive



Descartes



Leibniz



Church



Gödel



Turing

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive



Descartes



Leibniz



Church



Gödel



Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human (& Above) Cognitive Power

Serious Human (& Above) Cognitive

The first (procedural) programmer!



Descartes



Leibniz



Church



Gödel



Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Serious Human (& Above) Cognitive

The first (procedural) programmer!



Descartes



Leibniz



Church



Gödel



Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy

The first (procedural) programmer!



Descartes



Leibniz



Church



Gödel



Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy

The first (procedural) programmer!



Descartes



Leibniz



Church



Gödel



Polynomial Hierarchy

Entscheidungsproblem

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy

The first (procedural) programmer!



Descartes



Leibniz



Church



Gödel



Polynomial Hierarchy

Entscheidungsproblem

$$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy

The first (procedural) programmer!



Descartes



Leibniz



Church



Gödel



⋮
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Entscheidungsproblem

Polynomial Hierarchy

$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

\mathcal{L}_1

Analytical Hierarchy

Arithmetical Hierarchy

The first (procedural) programmer!



Descartes



Leibniz



Church



Gödel



\vdots
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Entscheidungsproblem

Polynomial Hierarchy

$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

\mathcal{L}_1

Analytical Hierarchy

Arithmetical Hierarchy

The first (procedural) programmer!



Descartes



Leibniz



Church



Gödel



Go:AlphaGo



\vdots
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Entscheidungsproblem

Polynomial Hierarchy

$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

\mathcal{L}_1

Analytical Hierarchy

Arithmetical Hierarchy

The first (procedural) programmer!



Descartes



Leibniz



Church



Gödel



\vdots
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Jeopardy!:



Go:AlphaGo



Entscheidungsproblem

Polynomial Hierarchy

$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

\mathcal{L}_1

Analytical Hierarchy

Arithmetical Hierarchy

The first (procedural) programmer!



Descartes



Leibniz



Church



Gödel



\vdots
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Chess: Deep Blue
●

Jeopardy!:
●

Go: AlphaGo
●

Entscheidungsproblem

Polynomial Hierarchy

$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

\mathcal{L}_1

Analytical Hierarchy

Arithmetical Hierarchy

The first (procedural) programmer!



Descartes

Checkers: Chinook



Leibniz

Chess: Deep Blue



Church

Jeopardy!:



Gödel

Go: AlphaGo



\vdots
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Entscheidungsproblem

Polynomial Hierarchy

$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy



Descartes



Leibniz



Church



Gödel



Turing

⋮
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Entscheidungsproblem

Polynomial Hierarchy

Jeopardy!: **Watson**



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy



Descartes



Leibniz



Church



Turing

⋮
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Entscheidungsproblem

Polynomial Hierarchy

Jeopardy!: **Watson**



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy



Descartes



Leibniz



Turing

⋮
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Entscheidungsproblem

Polynomial Hierarchy

Jeopardy!: **Watson**



$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy



Leibniz



Turing

⋮
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Entscheidungsproblem

Polynomial Hierarchy

Jeopardy!: **Watson**



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy



Leibniz

⋮
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Entscheidungsproblem

Polynomial Hierarchy

Jeopardy!: **Watson**

Chess: Deep Blue
Checkers: Chinook
Go: AlphaGo

$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy

⋮
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Entscheidungsproblem

Polynomial Hierarchy

Jeopardy!: **Watson**

Chess: Deep Blue
Checkers: Chinook
Go: AlphaGo

$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Rengo Kriegspiel

usgo.org

Plated News SUNY System ...ess - Logon Ultra Hardwar...are Products Screen Door L...y Von Morris Screen Door ...d Von Morris Apple Amazon eBay Yahoo! .Mac



U.S. Go Congress | Crosstabs | Free E-Journal | Upcoming Events | Ratings | Kids & Teens | Latest Go News

AGA HOMEPAGE
+WHAT IS GO?
RATINGS
+MEMBERSHIP AND CHAPTERS
AGA CHAPTER EMAIL LIST
PROFESSIONALS
+PLAY GO
+TOURNAMENTS
+LEARN MORE
+TEACH OTHERS
+OUTREACH
+KIDS & TEENS
AMERICAN GO FOUNDATION
LATEST GO NEWS
+ABOUT THE AGA
DONATE TO THE AGA
AGA GO DATABASE
US GO CONGRESS
n Go Foundation ARCHIVE
+ADMINISTRATORS ONLY

Site Search
go.org

American Go E-Journal

US Go Congress Goes a Little Crazy

Wednesday August 13, 2014



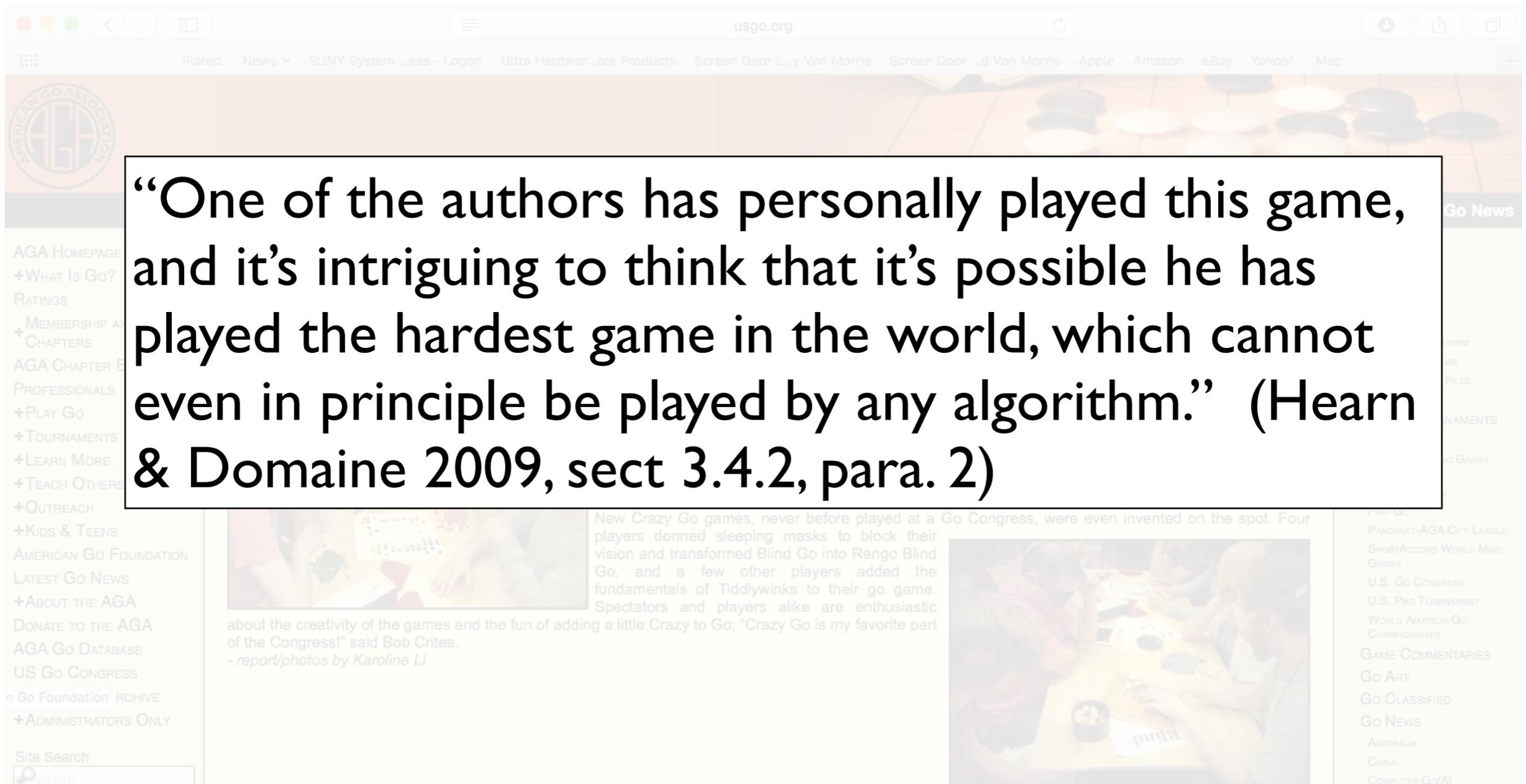
"White plays capturing black, putting herself and black into atari," calls Crazy Go TD Terry Benson. He officiated several games of Rengo Kriegspiel on Tuesday evening – a pair go game in which all four players face away from the main board and play their stones on their own empty board in front of them; the only clues about where their opponents — and even their partner — have played comes when they make an illegal move, or play where their own team or their opponents already have stones. Rengo Kriegspiel is only one of dozens of variants on the game of go that were played by an enthusiastic crowd of around 100 players. Familiar games include Magnetic Go, 4 Color Go, Tessellation Go, 3D Go, Spiral Go, and Blind Go. "After all these years, it's still crazy," said TD and Crazy Go founder Terry Benson. New Crazy Go games, never before played at a Go Congress, were even invented on the spot. Four players donned sleeping masks to block their vision and transformed Blind Go into Rengo Blind Go, and a few other players added the fundamentals of Tiddlywinks to their go game. Spectators and players alike are enthusiastic about the creativity of the games and the fun of adding a little Crazy to Go; "Crazy Go is my favorite part of the Congress!" said Bob Crites.
- report/photos by Karoline Li



CATEGORIES:
ALL
CALENDAR
COLUMNS
JOHN POWER REPORT
REDMOND REVIEWS
THE JANICE KIM FILES
WHY WE PLAY
EVENTS/TOURNAMENTS
COTSEN OPEN
IMSA ELITE MIND GAMES
OTHER
N.A. GO CONVENTION
PAIR GO
PANDANET-AGA CITY LEAGUE
SPORTACCORD WORLD MIND GAMES
U.S. GO CONGRESS
U.S. PRO TOURNAMENT
WORLD AMATEUR GO CHAMPIONSHIPS
GAME COMMENTARIES
GO ART
GO CLASSIFIED
GO NEWS
AUSTRALIA
CHINA
COMPUTER GO/AI

Rengo Kriegspiel

“One of the authors has personally played this game, and it’s intriguing to think that it’s possible he has played the hardest game in the world, which cannot even in principle be played by any algorithm.” (Hearn & Domaine 2009, sect 3.4.2, para. 2)



Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy



Descartes



Leibniz



Church



Gödel



Turing

⋮
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Entscheidungsproblem

Polynomial Hierarchy

Jeopardy!: **Watson**



$P \subseteq NP \subseteq PSPACE = NPSpace \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

But starting simpler ...

Tabular “Deduction” (Example)

Logic Puzzles

Story

	Eduardo	Greg	Jay	Patrick	Salvador	indigo	lime	maroon	orange	red	Fullerton	Hollister	San Pedro	Templeton	Urbandale
22 minutes															
23 minutes															
24 minutes															
25 minutes															
26 minutes															
Fullerton															
Hollister															
San Pedro															
Templeton															
Urbandale															
indigo															
lime															
maroon															
orange															
red															

5K Fun Run

Valero's 5k Fun Run was held yesterday in the downtown district. Determine the shirt color and hometown of each of the top five runners, and match each to their finishing time.

- Patrick, who is from Templeton and finished with a time of 24 minutes, wore neither the red nor the maroon shirt.
- The runner in the orange shirt finished one minute before the one in the
- finished in
- wasn't
- om
- testant
- maroon
- one finished two minutes before Greg.
- The runner who finished in 25 minutes wore the lime shirt.
- Jay didn't finish with the fastest time.
- The runner from San Pedro was either the one in the orange shirt or Eduardo.

Tabular “Deduction” (Example)

< Logic Puzzles

Story

	Eduardo	Greg	Jay	Patrick	Salvador	indigo	lime	maroon	orange	red	Fullerton	Hollister	San Pedro	Templeton	Urbandale
22 minutes															
23 minutes															
24 minutes															
25 minutes															
26 minutes															
Fullerton															
Hollister															
San Pedro															
Templeton															
Urbandale															
indigo															
lime															
maroon															
orange															
red															

1. Patrick, who is from Templeton and finished with a time of 24 minutes, wore neither the red nor the maroon shirt.
2. The runner in the orange shirt finished one minute before the one in the maroon shirt.
3. The runner who finished in 23 minutes (who wasn't Eduardo) wasn't from Hollister.
4. The Urbandale contestant wore the red shirt.
5. The runner in the maroon shirt finished two minutes before Greg.
6. The runner who finished in 25 minutes wore the lime shirt.
7. Jay didn't finish with the fastest time.
8. The runner from San Pedro was either the one in the orange shirt or Eduardo.

Tabular “Deduction” (Example)

< Logic Puzzles
↶
✏
💡
✎
?
🔊
☰

Story

	Eduardo	Greg	Jay	Patrick	Salvador	indigo	lime	maroon	orange	red	Fullerton	Hollister	San Pedro	Templeton	Urbandale
22 minutes															
23 minutes															
24 minutes															
25 minutes															
26 minutes															
Fullerton				✗											
Hollister				✗											
San Pedro				✗											
Templeton	✗	✗	✗	●	✗										
Urbandale				✗											
indigo															
lime															
maroon															
orange															
red															

1. Patrick, who is from Templeton and finished with a time of 24 minutes, wore neither the red nor the maroon shirt.
2. The runner in the orange shirt finished one minute before the one in the maroon shirt.
3. The runner who finished in 23 minutes (who wasn't Eduardo) wasn't from Hollister.
4. The Urbandale contestant wore the red shirt.
5. The runner in the maroon shirt finished two minutes before Greg.
6. The runner who finished in 25 minutes wore the lime shirt.
7. Jay didn't finish with the fastest time.
8. The runner from San Pedro was either the one in the orange shirt or Eduardo.

Tabular “Deduction” (Example)

< Logic Puzzles

Story

	Eduardo	Greg	Jay	Patrick	Salvador	indigo	lime	maroon	orange	red	Fullerton	Hollister	San Pedro	Templeton	Urbandale
22 minutes															
23 minutes															
24 minutes															
25 minutes															
26 minutes															
Fullerton				✗											
Hollister				✗											
San Pedro				✗											
Templeton	✗	✗	✗	●	✗										
Urbandale				✗											
indigo															
lime															
maroon															
orange															
red															

1. Patrick, who is from Templeton and finished with a time of 24 minutes, wore neither the red nor the maroon shirt.
2. The runner in the orange shirt finished one minute before the one in the maroon shirt.
3. The runner who finished in 23 minutes (who wasn't Eduardo) wasn't from Hollister.
4. The Urbandale contestant wore the red shirt.

In HyperSlate ...

1. Patrick, who is from Templeton and finished with a time of 24 minutes, wore neither the red nor the maroon shirt.
2. The runner in the orange shirt finished one minute before the one in the maroon shirt.
3. The runner who finished in 23 minutes (who wasn't Eduardo) wasn't from Hollister.
4. The Urbandale contestant wore the red shirt.
5. The runner who finished in 22 minutes was Greg.
6. The runner who finished in 25 minutes wore the lime shirt.
7. Jay didn't finish with the fastest time.
8. The runner from San Pedro was either the one in the orange shirt or Eduardo.

Tabular “Deduction” (Example)

← Logic Puzzles

S

Story	Eduardo	Greg	Jay	Patrick	Salvador	indigo	lime	maroon	orange	red	Fullerton	Hollister	San Pedro	Templeton	Urbandale
22 minutes															
23 minutes															
24 minutes															
25 minutes															
26 minutes															
Fullerton				✗						✗					
Hollister				✗						✗					
San Pedro				✗						✗					
Templeton	✗	✗	✗	●	✗					✗					
Urbandale				✗		✗	✗	✗	✗	●					
indigo															
lime															
maroon															
orange															
red															

1. Patrick, who is from Templeton and finished with a time of 24 minutes, wore neither the red nor the maroon shirt.
2. The runner in the orange shirt finished one minute before the one in the maroon shirt.
3. The runner who finished in 23 minutes (who wasn't Eduardo) wasn't from Hollister.
4. The Urbandale contestant wore the red shirt.
5. The runner in the maroon shirt finished two minutes before Greg.
6. The runner who finished in 25 minutes wore the lime shirt.
7. Jay didn't finish with the fastest time.
8. The runner from San Pedro was either the one in the orange shirt or Eduardo.

Tabular “Deduction” (Example)

Logic Puzzles
↶ ✍️ 💡 ? 🔊 ⋮

Story	Eduardo	Greg	Jay	Patrick	Salvador	indigo	lime	maroon	orange	red	Fullerton	Hollister	San Pedro	Templeton	Urbandale
22 minutes															
23 minutes															
24 minutes															
25 minutes															
26 minutes															
Fullerton				✗						✗					
Hollister				✗						✗					
San Pedro				✗						✗					
Templeton	✗	✗	✗	●	✗					✗					
Urbandale				✗		✗	✗	✗	✗	●					
indigo															
lime															
maroon															
orange															
red															

These two cells tell you something about the cell in yellow.

- Patrick, who is from Templeton and finished with a time of 24 minutes, wore neither the red nor the maroon shirt.
- The runner in the orange shirt finished one minute before the one in the maroon shirt.
- The runner who finished in 23 minutes (who wasn't Eduardo) wasn't from Hollister.
- The Urbandale contestant wore the red shirt.
- The runner in the maroon shirt finished two minutes before Greg.
- The runner who finished in 25 minutes wore the lime shirt.
- Jay didn't finish with the fastest time.
- The runner from San Pedro was either the one in the orange shirt or Eduardo.

Tabular “Deduction”: It’s Taught!



Example

Grace, Dylan, Kira, and Diego are each wearing different colored shirts. Grace’s shirt is red. Dylan’s shirt is not white. Kira’s shirt is not green. Diego’s shirt is not yellow or white. What color shirt is each person wearing?

First, make a chart to show what you know.

- Each shirt is a different color.
- Grace’s shirt is red.
- Dylan’s shirt is not white.
- Kira’s shirt is not green.
- Diego’s shirt is not yellow or white.

	Red	White	Green	Yellow
Grace	yes	no	no	no
Dylan	no	no		
Kira	no		no	
Diego	no	no	yes	no

Then use reasoning and the information in the chart to complete the chart and find the answer.

Grace’s shirt is red, so no other shirt can be red.

Diego’s shirt is not red, white, or yellow, so it must be green.

Dylan’s shirt must be yellow because it cannot be red, white, or green.

That means Kira’s shirt must be white.

Solve

Tabular “Deduction”: It’s Taught!



Example

Grace, Dylan, Kira, and Diego are each wearing different colored shirts. Grace’s shirt is red. Dylan’s shirt is not white. Kira’s shirt is not green. Diego’s shirt is not yellow or white. What color shirt is each person wearing?

First, make a chart to show what you know.

- Each shirt is a different color.
- Grace’s shirt is red.
- Dylan’s shirt is not white.
- Kira’s shirt is not green.
- Diego’s shirt is not yellow or white.

	Red	White	Green	Yellow
Grace	yes	no	no	no
Dylan	no	no		
Kira	no		no	
Diego	no	no	yes	no

Then use reasoning and the information in the chart to complete the chart and find the answer.

Grace’s shirt is red, so no other shirt can be red.

Diego’s shirt is not red, white, or yellow, so it must be green.

Dylan’s shirt must be yellow because it cannot be red, white, or green.

That means Kira’s shirt must be white.

Solve

Tabular “Deduction”: It’s Taught!



Example

Grace, Dylan, Kira, and Diego are each wearing different colored shirts. Grace’s shirt is red. Dylan’s shirt is not white. Kira’s shirt is not green. Diego’s shirt is not yellow or white. What color shirt is each person wearing?

First, make a chart to show what you know.

- Each shirt is a different color.
- Grace’s shirt is red.
- Dylan’s shirt is not white.
- Kira’s shirt is not green.
- Diego’s shirt is not yellow or white.

	Red	White	Green	Yellow
Grace	yes	no	no	no
Dylan	no	no		
Kira	no		no	
Diego	no	no	yes	no

Then use reasoning and the information in the chart to complete the chart and find the answer.

Grace’s shirt is red, so no other shirt can be red.
Diego’s shirt is not red, white, or yellow, so it must be green.
Dylan’s shirt must be yellow because it cannot be red, white, or green.
That means Kira’s shirt must be white.

Solve

IMHO very bad idea—if *before* real learning of deduction to answer “Why, exactly? Prove it!”

Tabular “deduction” *not* the skill that’s valuable.

8:29 AM

iPad

8:29 AM

91

Recall from Lesson 4-8 that the complex numbers $a + bi$ and $a - bi$ are conjugates. Similarly, the irrational numbers $a + \sqrt{b}$ and $a - \sqrt{b}$ are conjugates. If a complex number or an irrational number is a root of a polynomial equation with rational coefficients, so is its conjugate.

TAKE NOTE Theorem

Conjugate Root Theorem

If $P(x)$ is a polynomial with *rational* coefficients, then irrational roots of $P(x) = 0$ that have the form $a + \sqrt{b}$ occur in conjugate pairs. That is, if $a + \sqrt{b}$ is an irrational root with a and b rational, then $a - \sqrt{b}$ is also a root.

If $P(x)$ is a polynomial with *real* coefficients, then the complex roots of $P(x) = 0$ occur in conjugate pairs. That is, if $a + bi$ is a complex root with a and b real, then $a - bi$ is also a root.

Every quadratic polynomial equation has two roots, every cubic polynomial equation has three roots, and so on.

This result is related to the *Fundamental Theorem of Algebra*. The German mathematician Carl Friedrich Gauss (1777–1855) is credited with proving this theorem.

TAKE NOTE Theorem

The Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n roots, including multiple and complex roots.

iPad

From Algebra 2

Practice and Problem-Solving Exercises - Contir

Determine whether each of the following statements is *always*, *sometimes*, or *never* true.

41. A polynomial function with real coefficients has real zeros.
42. Polynomial functions with complex coefficients have one complex zero.
43. A polynomial function that does not intercept the x -axis has complex roots only.
44. **Reasoning** A 4th-degree polynomial function has zeros at 3 and $5 - i$. Can $4 + i$ also be a zero of the function? Explain your reasoning.
45. **Open-Ended** Write a polynomial function that has four possible rational zeros but no actual rational zeros.
46. **Reasoning** Show that the Fundamental Theorem of Algebra must be true for all quadratic polynomial functions.

C • Challenge

47. Use the Fundamental Theorem of Algebra and the Conjugate Root Theorem to show that any odd degree polynomial equation with real coefficients has at least one real root.
48. **Reasoning** What is the maximum number of points of intersection between the graphs of a quartic and a quintic polynomial function?
49. **Reasoning** What is the least possible degree of a polynomial with rational coefficients, leading coefficient 1, constant term 5, and zeros at $\sqrt{2}$ and $\sqrt{3}$? Show that such a polynomial has a rational zero and indicate this zero.

Tabular “deduction” *not* the skill that’s valuable.

8:29 AM

iPad

8:29 AM

91

Recall from Lesson 4-8 that the complex numbers $a + bi$ and $a - bi$ are conjugates. Similarly, the irrational numbers $a + \sqrt{b}$ and $a - \sqrt{b}$ are conjugates. If a complex number or an irrational number is a root of a polynomial equation with rational coefficients, so is its conjugate.

TAKE NOTE Theorem

Conjugate Root Theorem

If $P(x)$ is a polynomial with *rational* coefficients, then irrational roots of $P(x) = 0$ that have the form $a + \sqrt{b}$ occur in conjugate pairs. That is, if $a + \sqrt{b}$ is an irrational root with a and b rational, then $a - \sqrt{b}$ is also a root.

If $P(x)$ is a polynomial with *real* coefficients, then the complex roots of $P(x) = 0$ occur in conjugate pairs. That is, if $a + bi$ is a complex root with a and b real, then $a - bi$ is also a root.

Every quadratic polynomial equation has two roots, every cubic polynomial equation has three roots, and so on.

This result is related to the *Fundamental Theorem of Algebra*. The German mathematician Carl Friedrich Gauss (1777–1855) is credited with proving this theorem.

TAKE NOTE Theorem

The Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n roots, including multiple and complex roots.

iPad

From Algebra 2

Practice and Problem-Solving Exercises - Contir

Determine whether each of the following statements is *always*, *sometimes*, or *never* true.

41. A polynomial function with real coefficients has real zeros.
42. Polynomial functions with complex coefficients have one complex zero.
43. A polynomial function that does not intercept the x -axis has complex roots only.
44. **Reasoning** A 4th-degree polynomial function has zeros at 3 and $5 - i$. Can $4 + i$ also be a zero of the function? Explain your reasoning.
45. **Open-Ended** Write a polynomial function that has four possible rational zeros but no actual rational zeros.
46. **Reasoning** Show that the Fundamental Theorem of Algebra must be true for all quadratic polynomial functions.

C • Challenge

47. Use the Fundamental Theorem of Algebra and the Conjugate Root Theorem to show that any odd degree polynomial equation with real coefficients has at least one real root.
48. **Reasoning** What is the maximum number of points of intersection between the graphs of a quartic and a quintic polynomial function?
49. **Reasoning** What is the least possible degree of a polynomial with rational coefficients, leading coefficient 1, constant term 5, and zeros at $\sqrt{2}$ and $\sqrt{3}$? Show that such a polynomial has a rational zero and indicate this zero.

The Game of LogiNim \mathcal{L}

In HyperSlate®

The Game of LogiNim \mathcal{L} in HyperSlate®

Selmer Bringsjord

Motalen LLC

0220210900NY

A *logician directed acyclic hypergraph*, hereafter simply a *hypergraph*, is a pair

$$\mathcal{H} := \langle N, A \rangle$$

where

- each *node* $\nu \in N$ contains some formula $\phi \in \mathcal{L}$, where \mathcal{L} is a background formal language, possibly including a label l_ϕ for the node, and the set of all assumptions on which inference of ϕ may rely;
- each *arc* $a \in A$ is a pair composed of a label l_σ for some $\sigma \in \mathcal{I}$, a collection of inference schemata, and one of $\{r, g\}$;
- arcs are directed; and
- no *cycles* are permitted.

The Game of LogiNim \mathcal{L}

In HyperSlate®

The Game of LogiNim \mathcal{L} in HyperSlate®

Selmer Bringsjord

Motalen LLC

0220210900NY

A *logician directed acyclic hypergraph*, hereafter simply a *hypergraph*, is a pair

$$\mathcal{H} := \langle N, A \rangle$$

where

- each *node* $\nu \in N$ contains some formula $\phi \in \mathcal{L}$, where \mathcal{L} is a background formal language, possibly including a label l_ϕ for the node, and the set of all assumptions on which inference of ϕ may rely;
- each *arc* $a \in A$ is a pair composed of a label l_σ for some $\sigma \in \mathcal{I}$, a collection of inference schemata, and one of $\{r, g\}$;
- arcs are directed; and
- no *cycles* are permitted.

Back to FOL ...

Our Final New Inference Rule in FOL

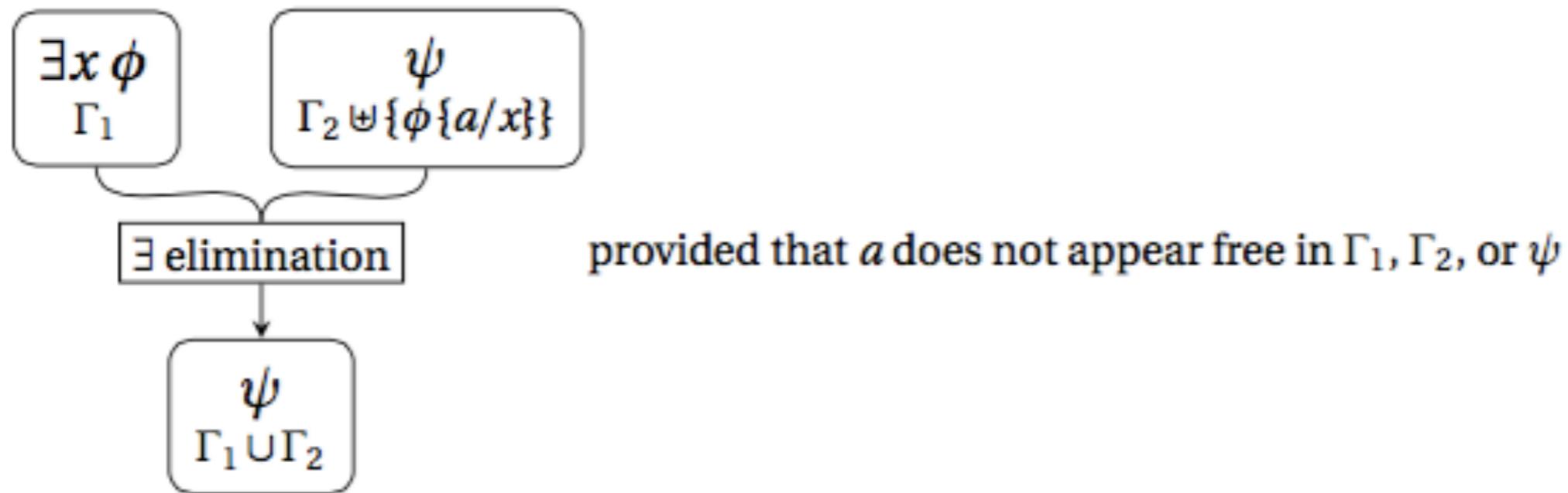
Our Final New Inference Rule in FOL

- existential elimination (intuitively put):

Our Final New Inference Rule in FOL

- existential elimination (intuitively put):
 - If we know that (i) there's something x which is an R , and (ii) on the supposition that a is an arbitrary representative (a “witness”) of such an x we can prove P , then we are permitted to deduce P from (i) alone.

existential elimination, precise version:

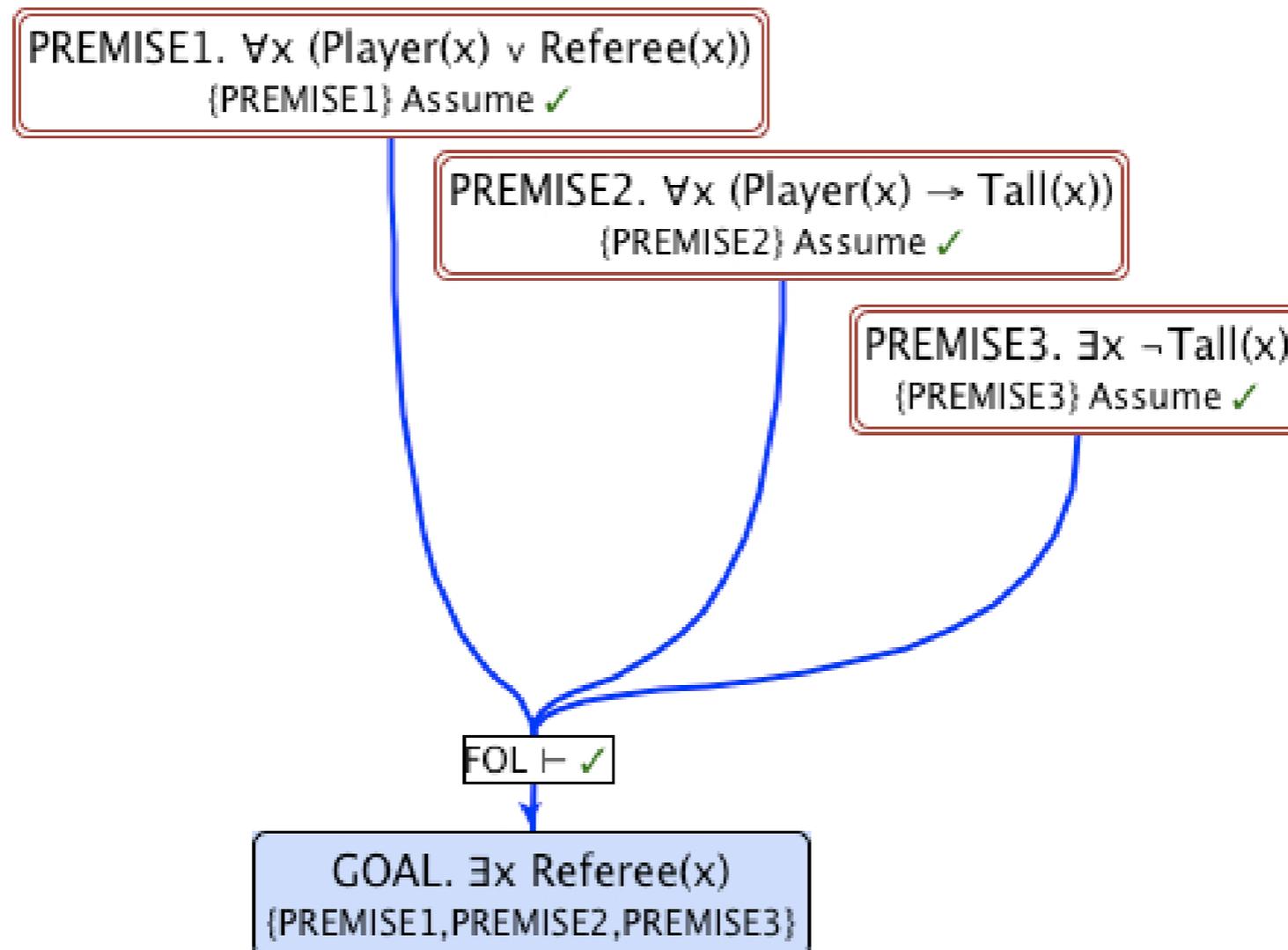


Starting State of Workspace

(Assumes a domain of e.g. players on a
March-madness basketball court.)

Starting State of Workspace

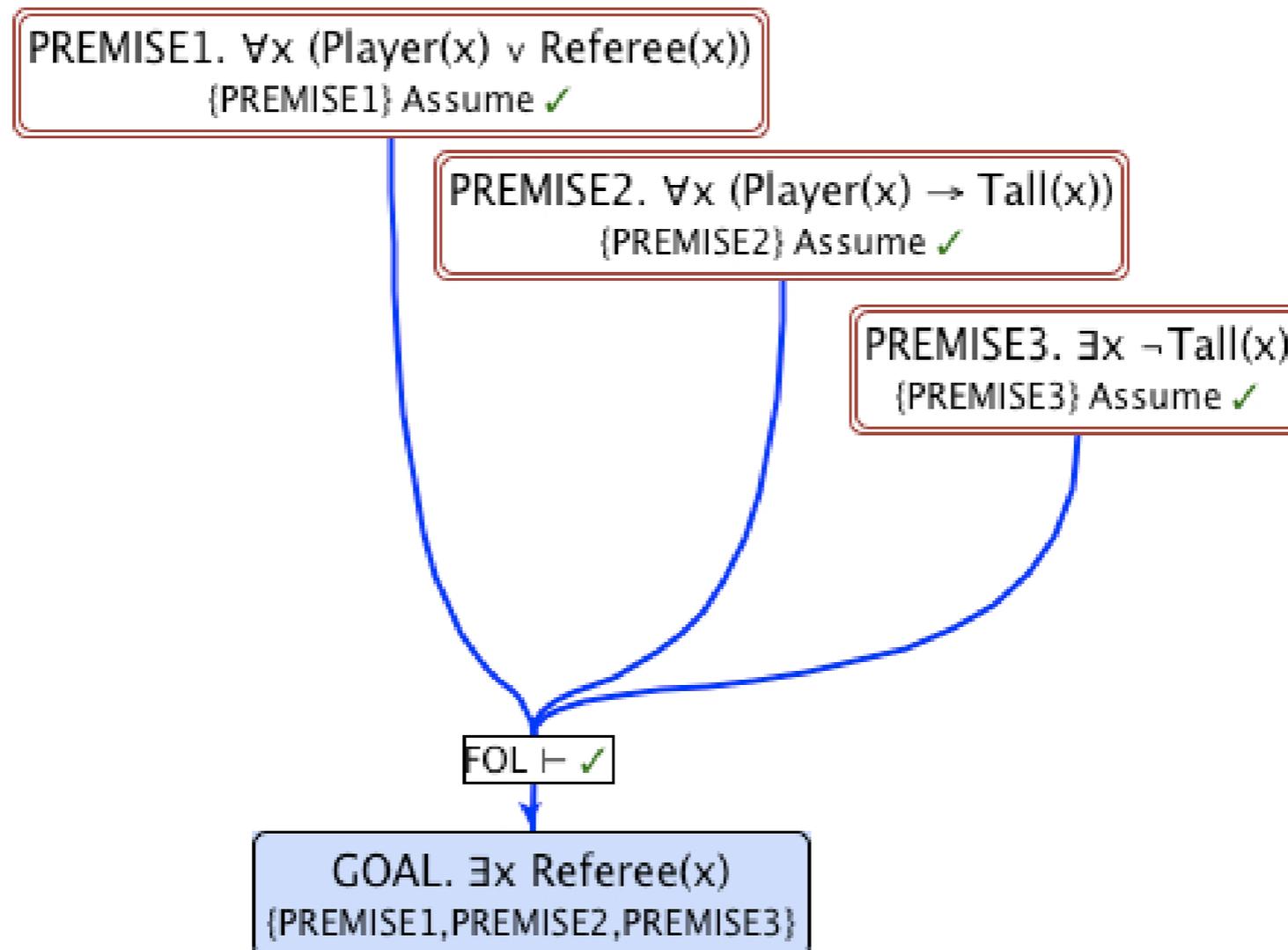
(Assumes a domain of e.g. players on a March-madness basketball court.)



Starting State of Workspace

(Assumes a domain of e.g. players on a March-madness basketball court.)

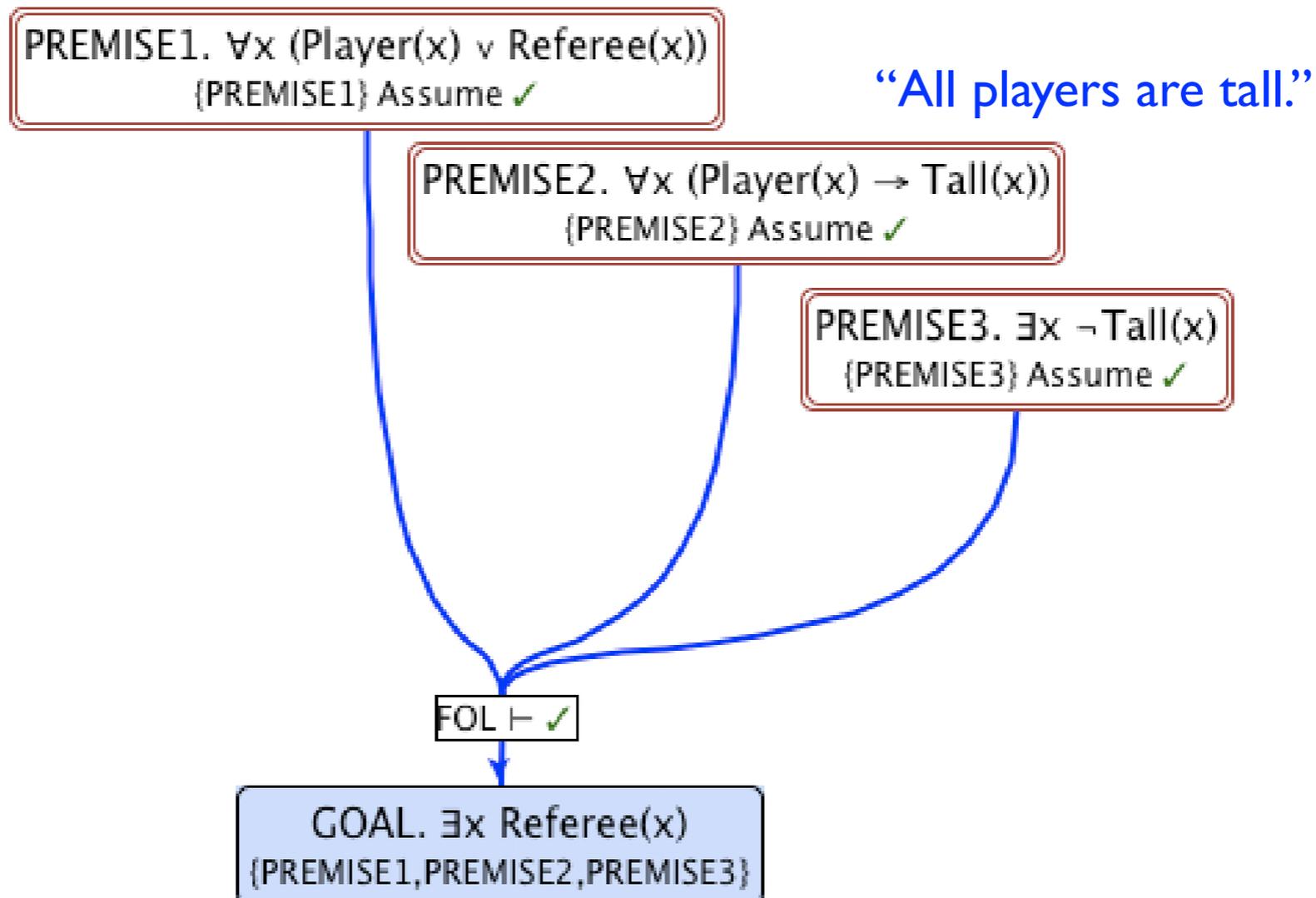
“Each and every thing is either a player or a referee.”



Starting State of Workspace

(Assumes a domain of e.g. players on a March-madness basketball court.)

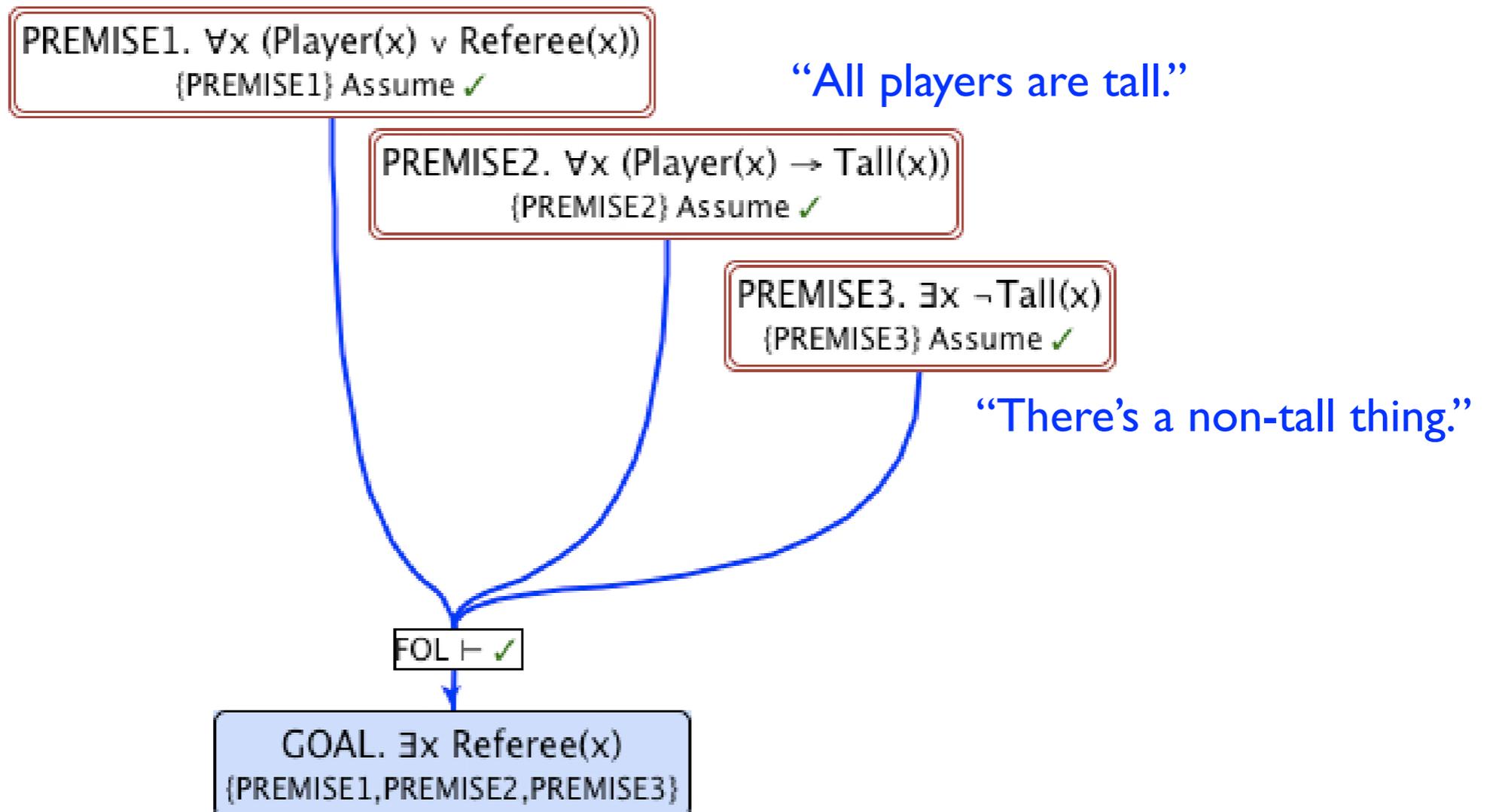
“Each and every thing is either a player or a referee.”



Starting State of Workspace

(Assumes a domain of e.g. players on a March-madness basketball court.)

“Each and every thing is either a player or a referee.”



Step 1

PREMISE1. $\forall x (\text{Player}(x) \vee \text{Referee}(x))$
{PREMISE1} Assume ✓

PREMISE2. $\forall x (\text{Player}(x) \rightarrow \text{Tall}(x))$
{PREMISE2} Assume ✓

PREMISE3. $\exists x \neg \text{Tall}(x)$
{PREMISE3} Assume ✓

5. $\neg \text{Tall}(a)$
{5} Assume ✓

GOAL. $\exists x \text{Referee}(x)$
FOL \vdash ✗

Step 1

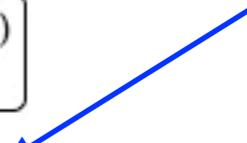
PREMISE1. $\forall x (\text{Player}(x) \vee \text{Referee}(x))$
{PREMISE1} Assume ✓

PREMISE2. $\forall x (\text{Player}(x) \rightarrow \text{Tall}(x))$
{PREMISE2} Assume ✓

PREMISE3. $\exists x \neg \text{Tall}(x)$
{PREMISE3} Assume ✓

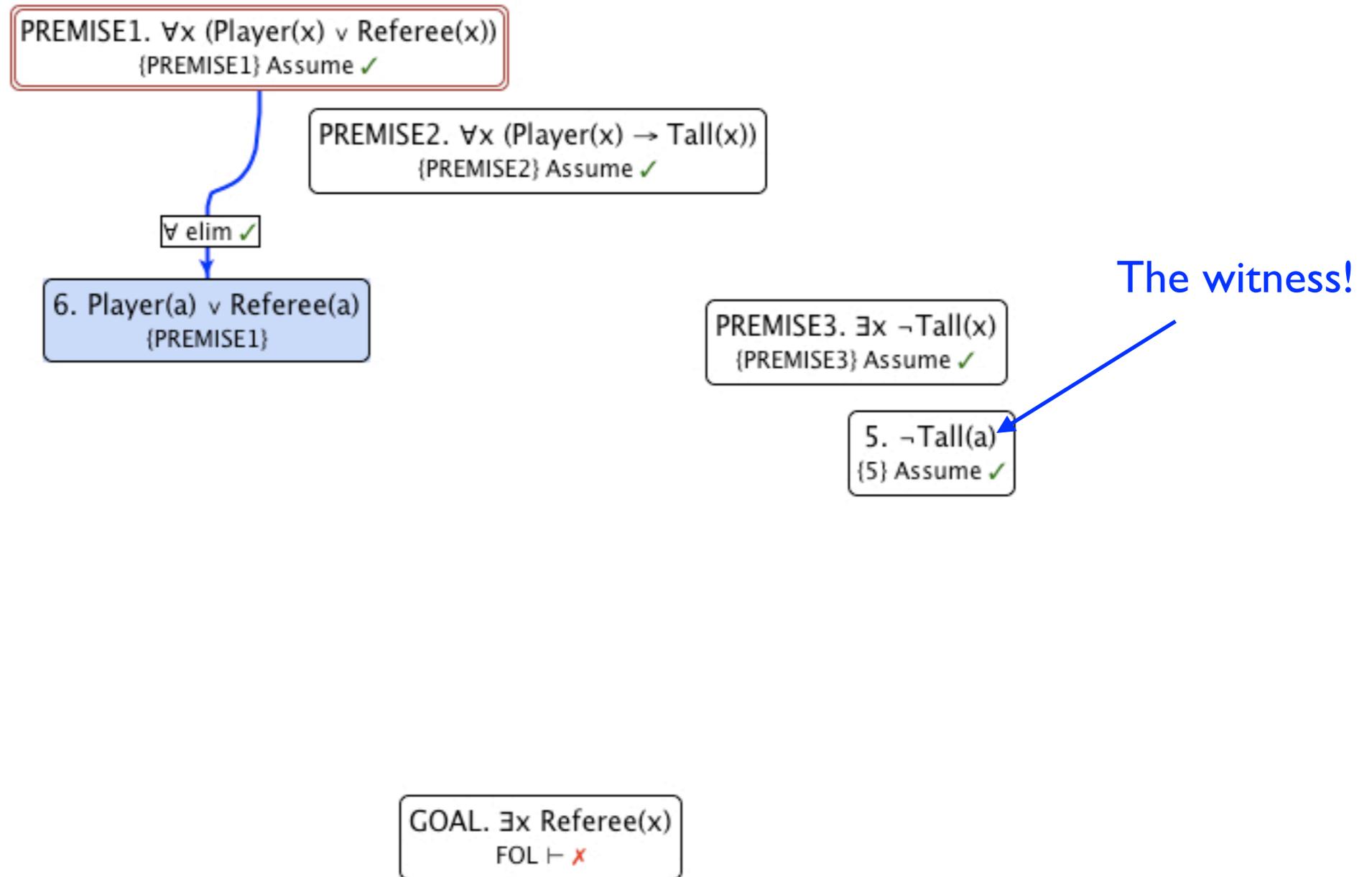
5. $\neg \text{Tall}(a)$
{5} Assume ✓

The witness!

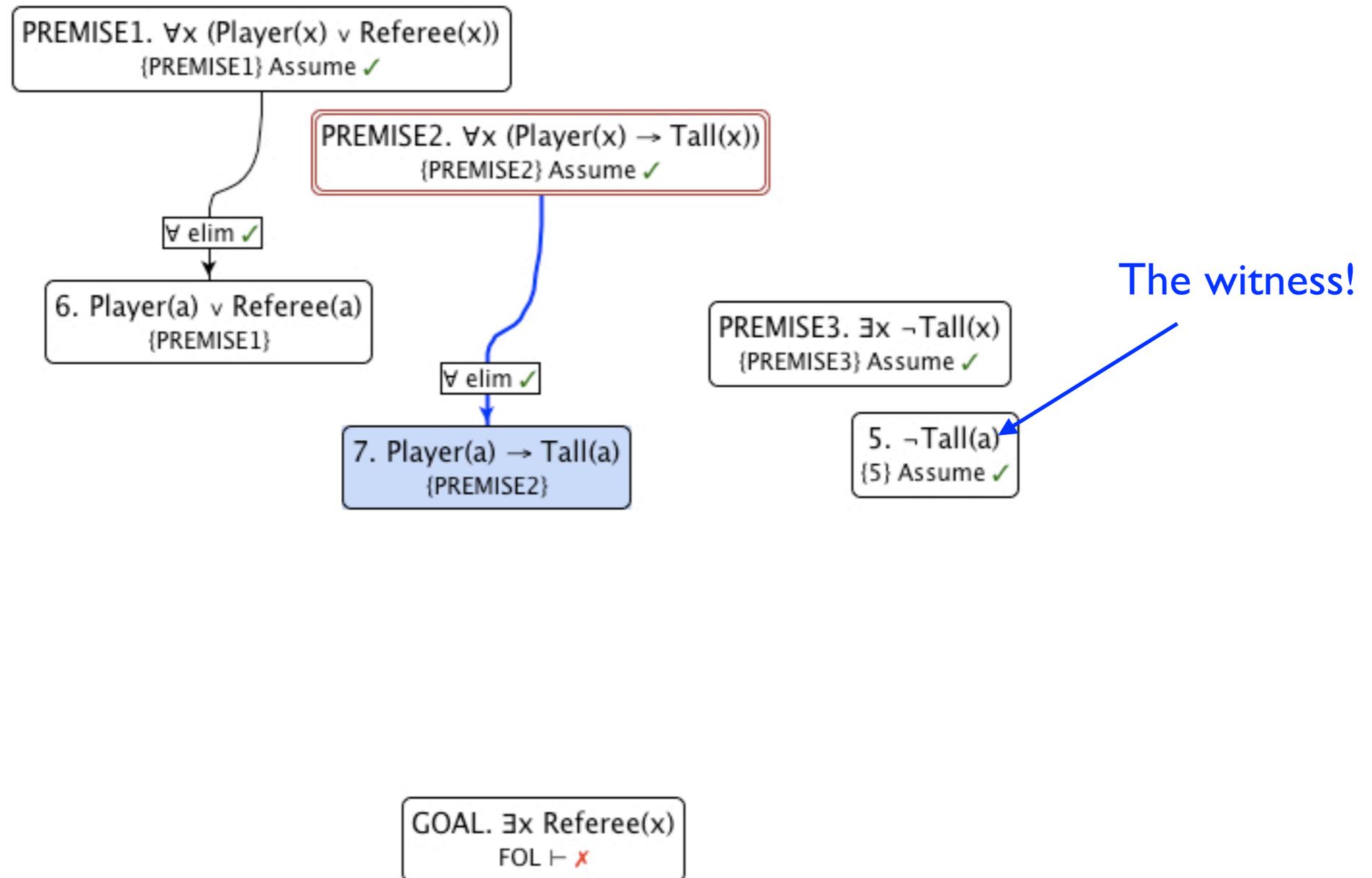


GOAL. $\exists x \text{Referee}(x)$
FOL ⊢ ✗

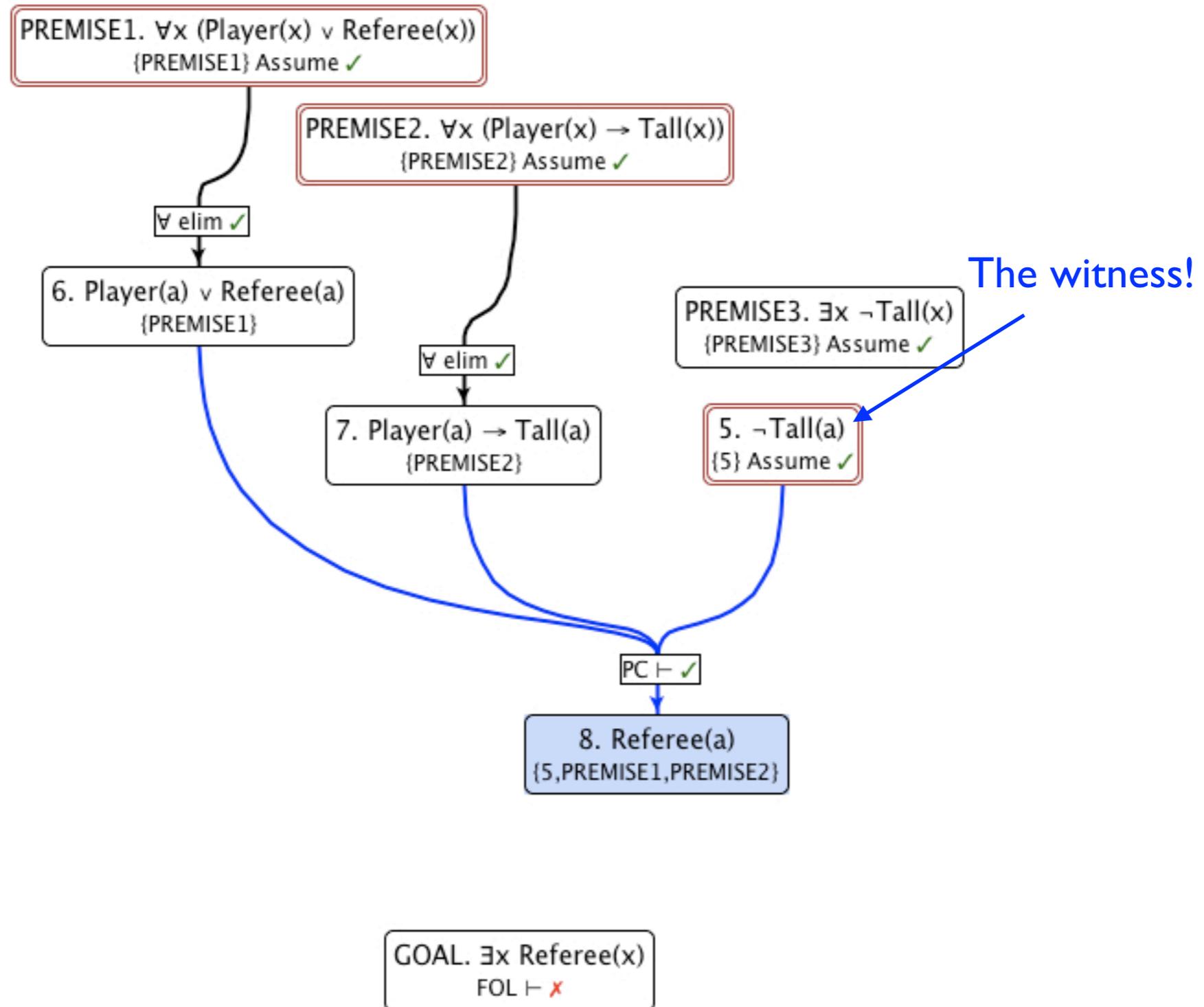
Step 2



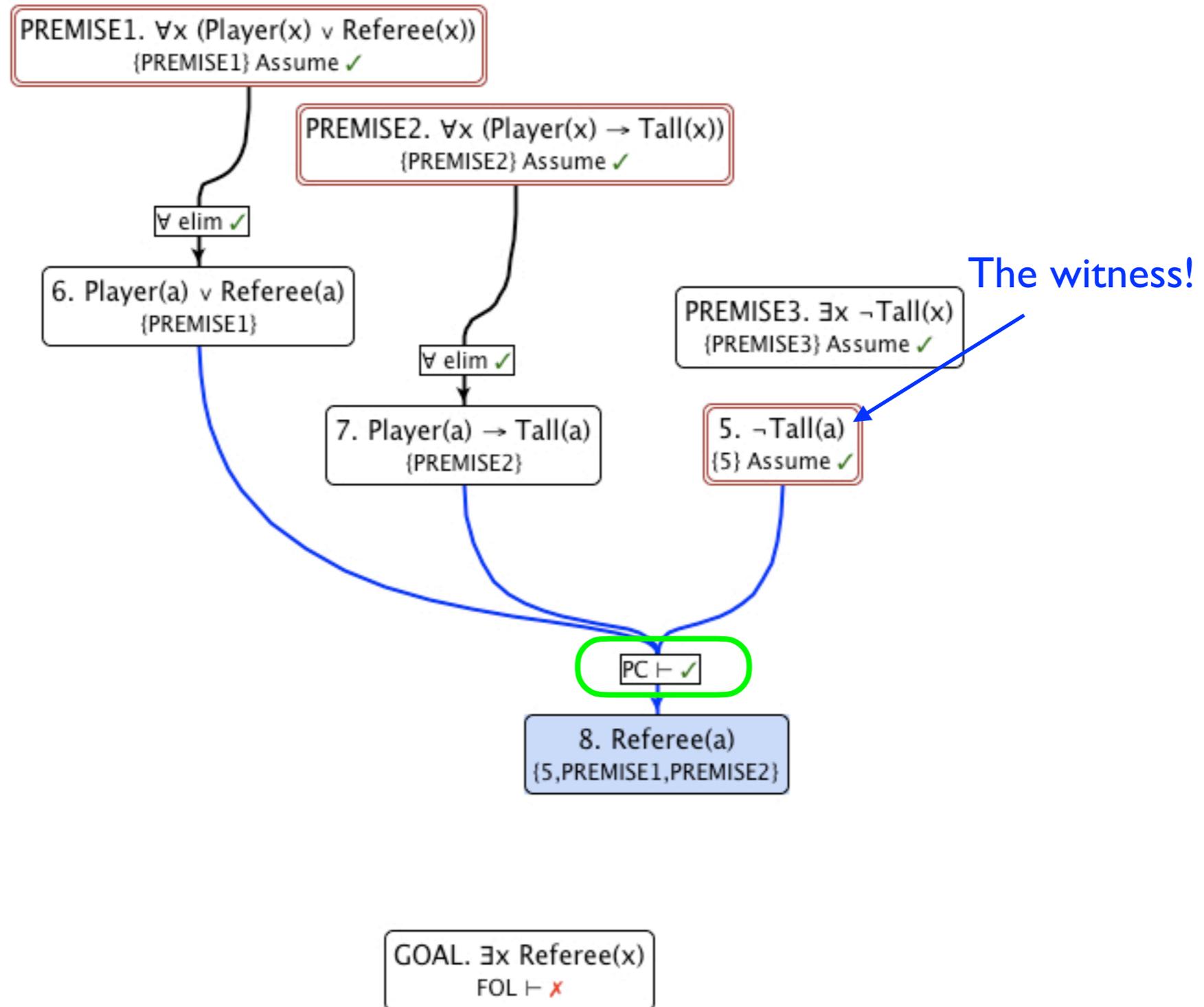
Step 3



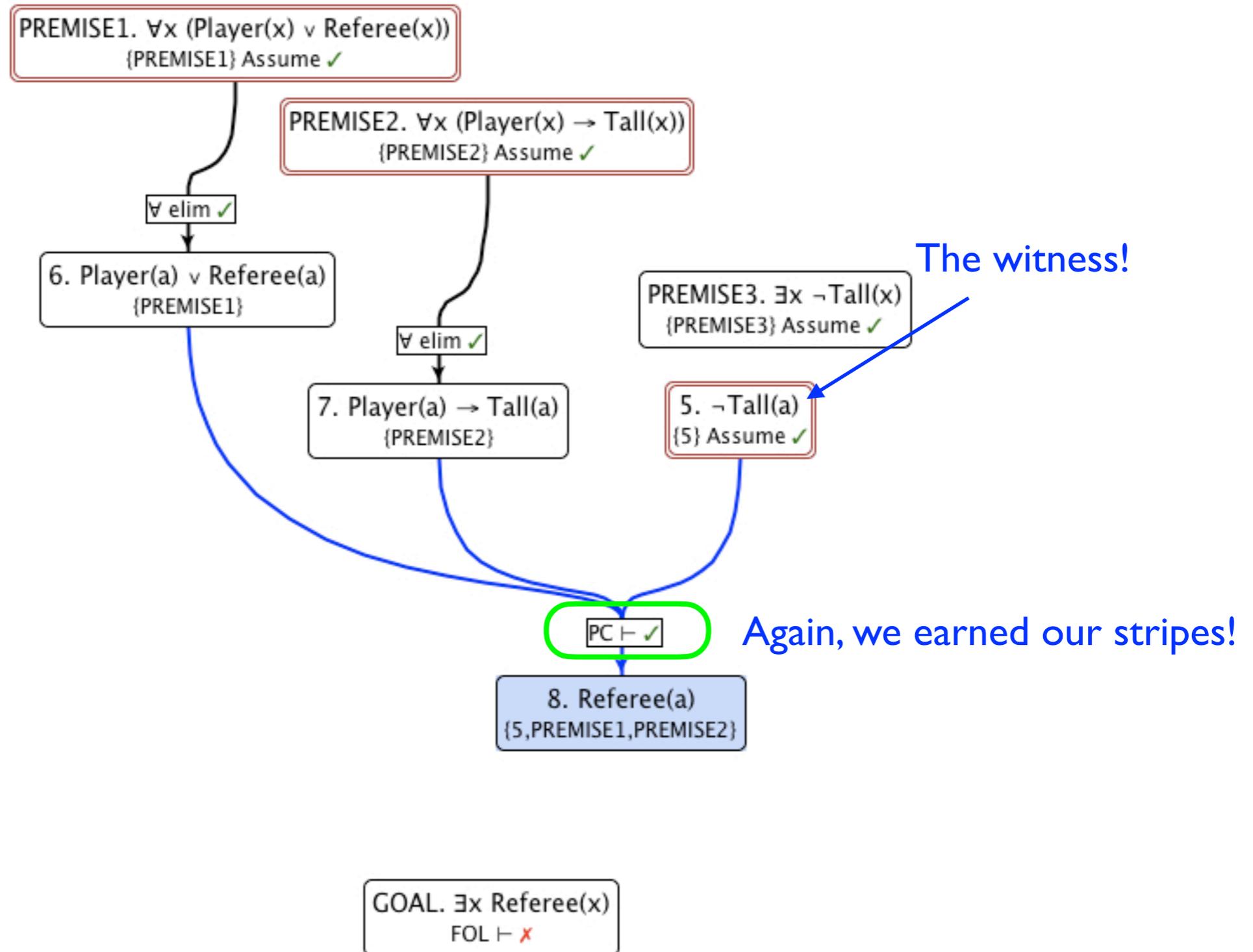
Step 4



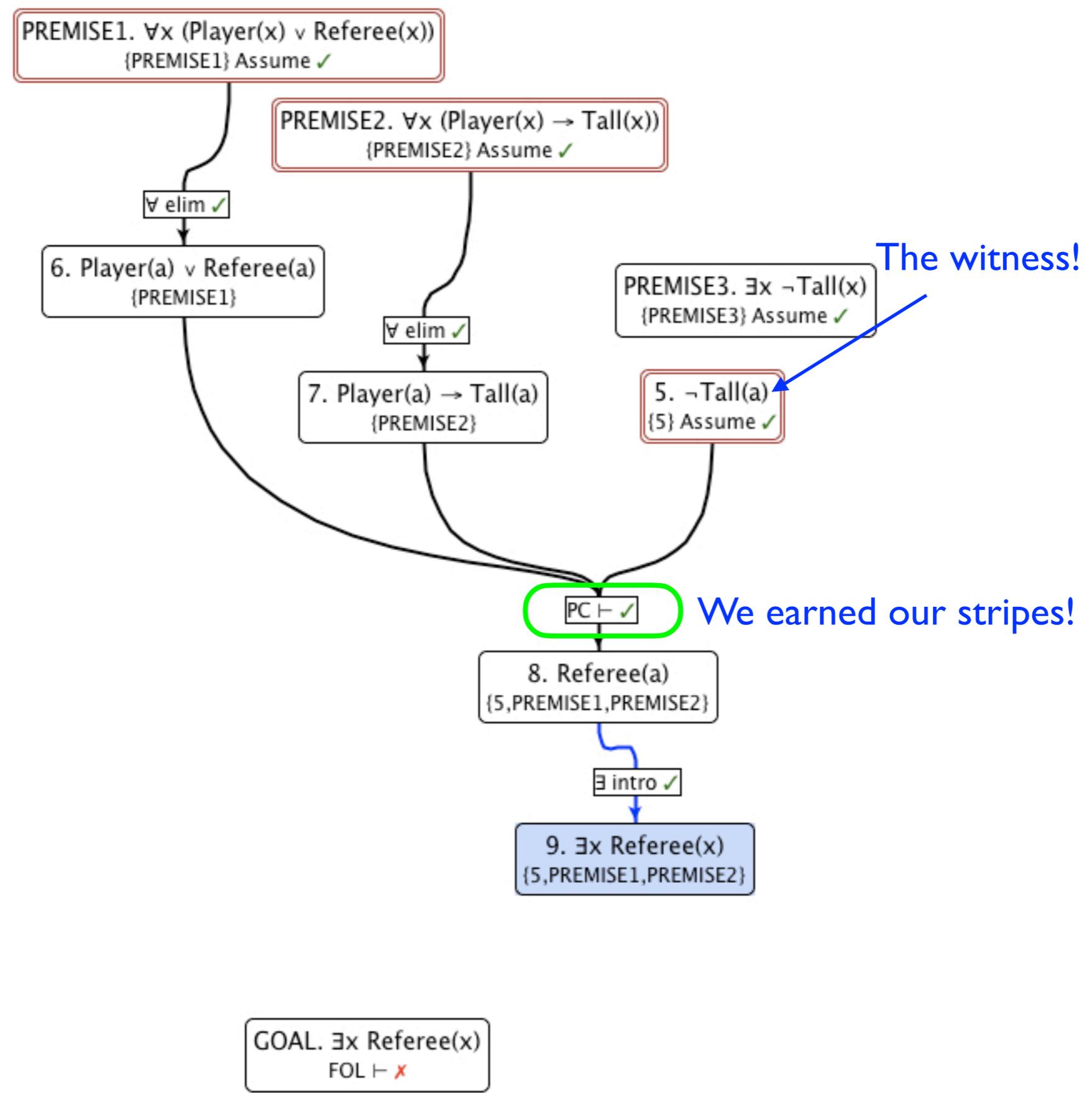
Step 4



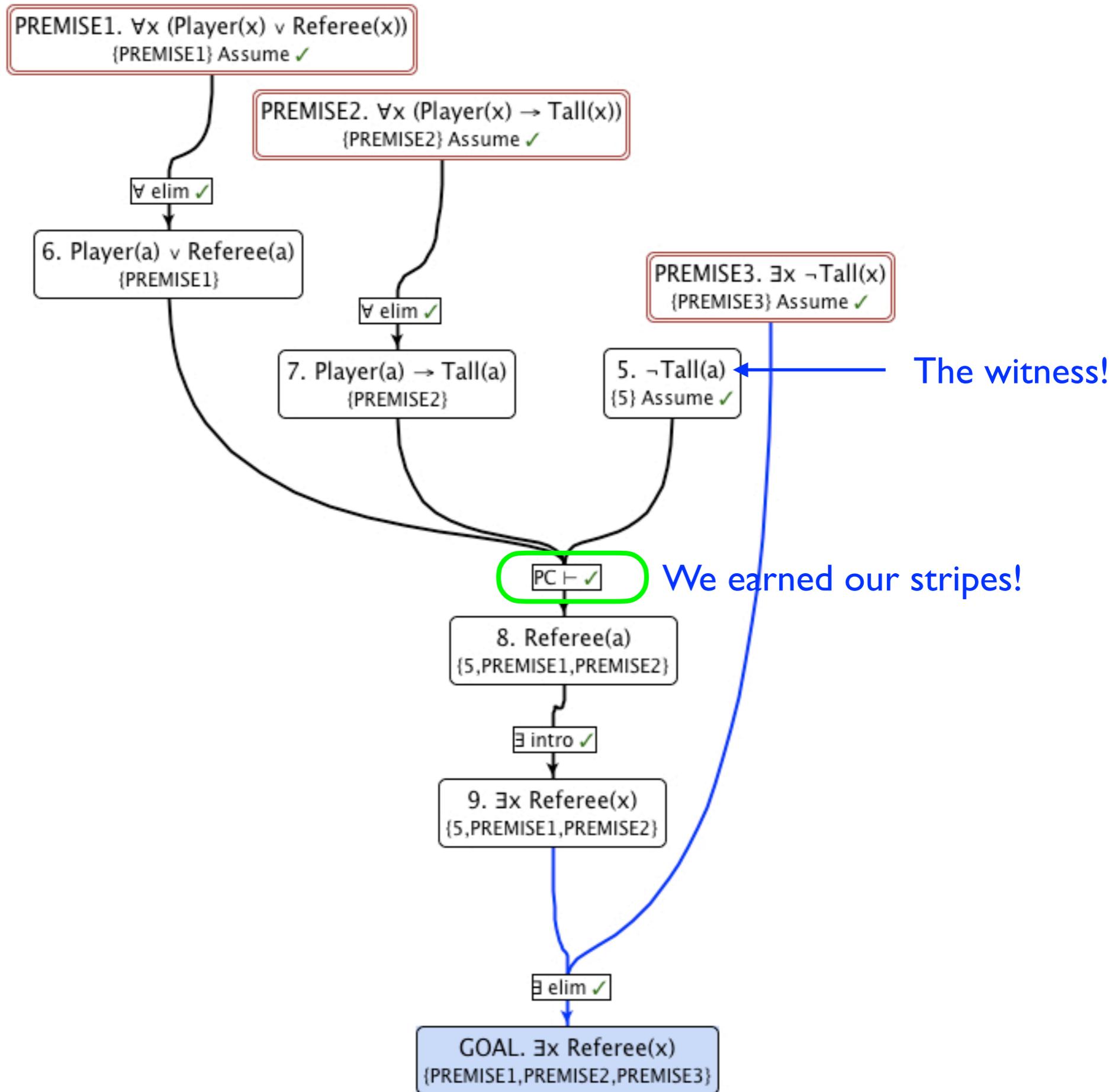
Step 4



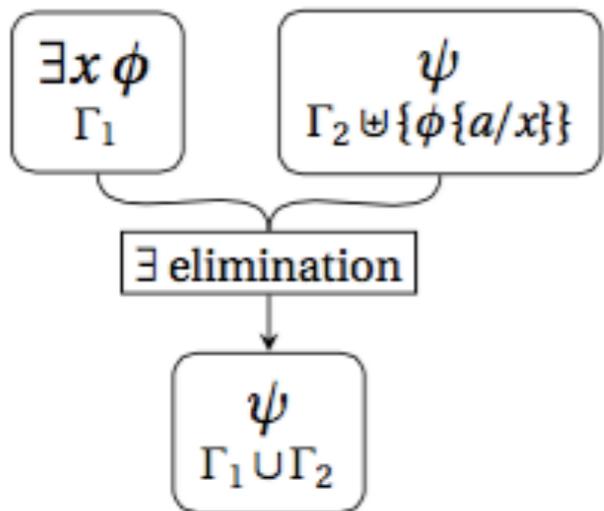
Step 5



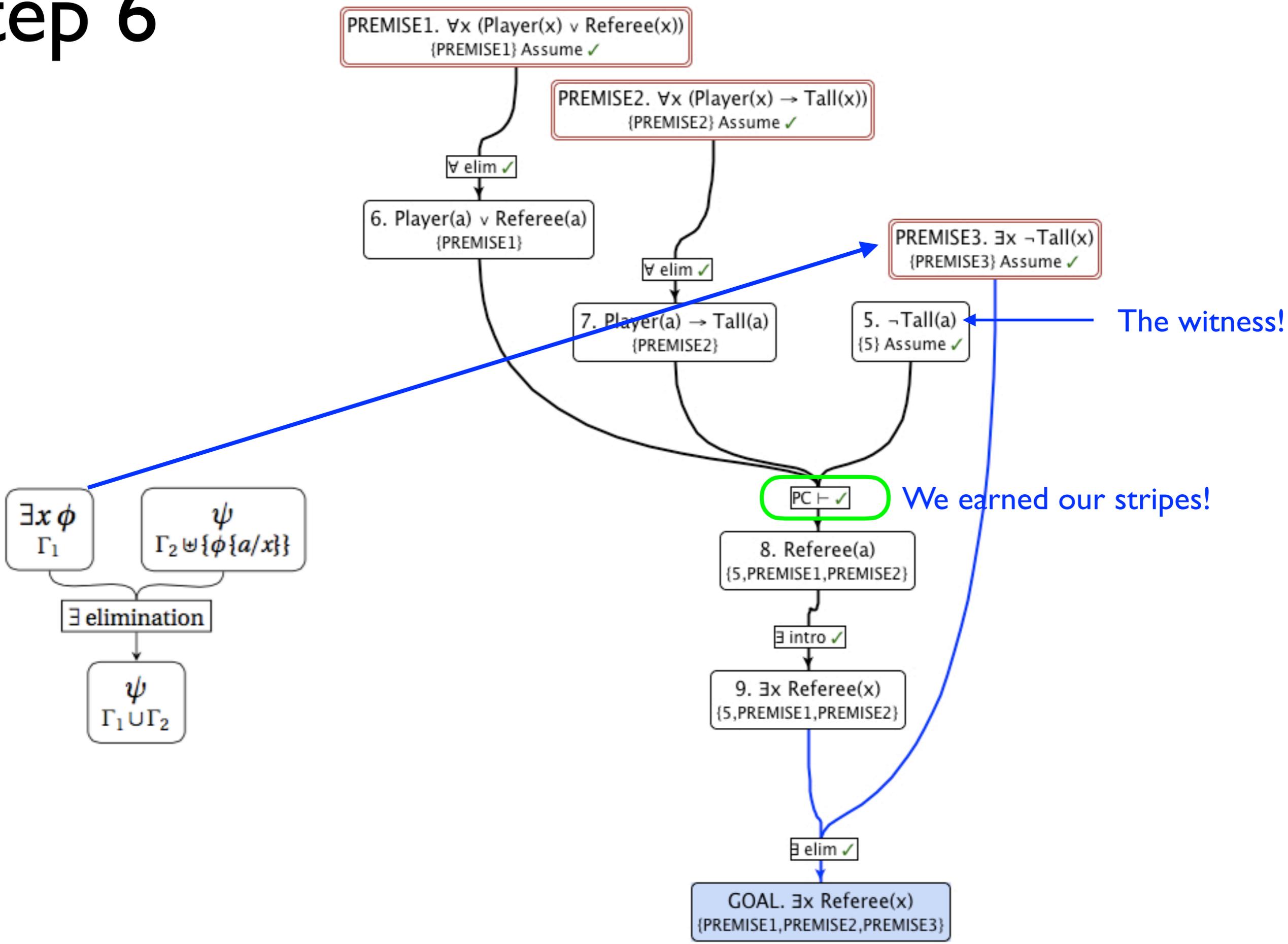
Step 6



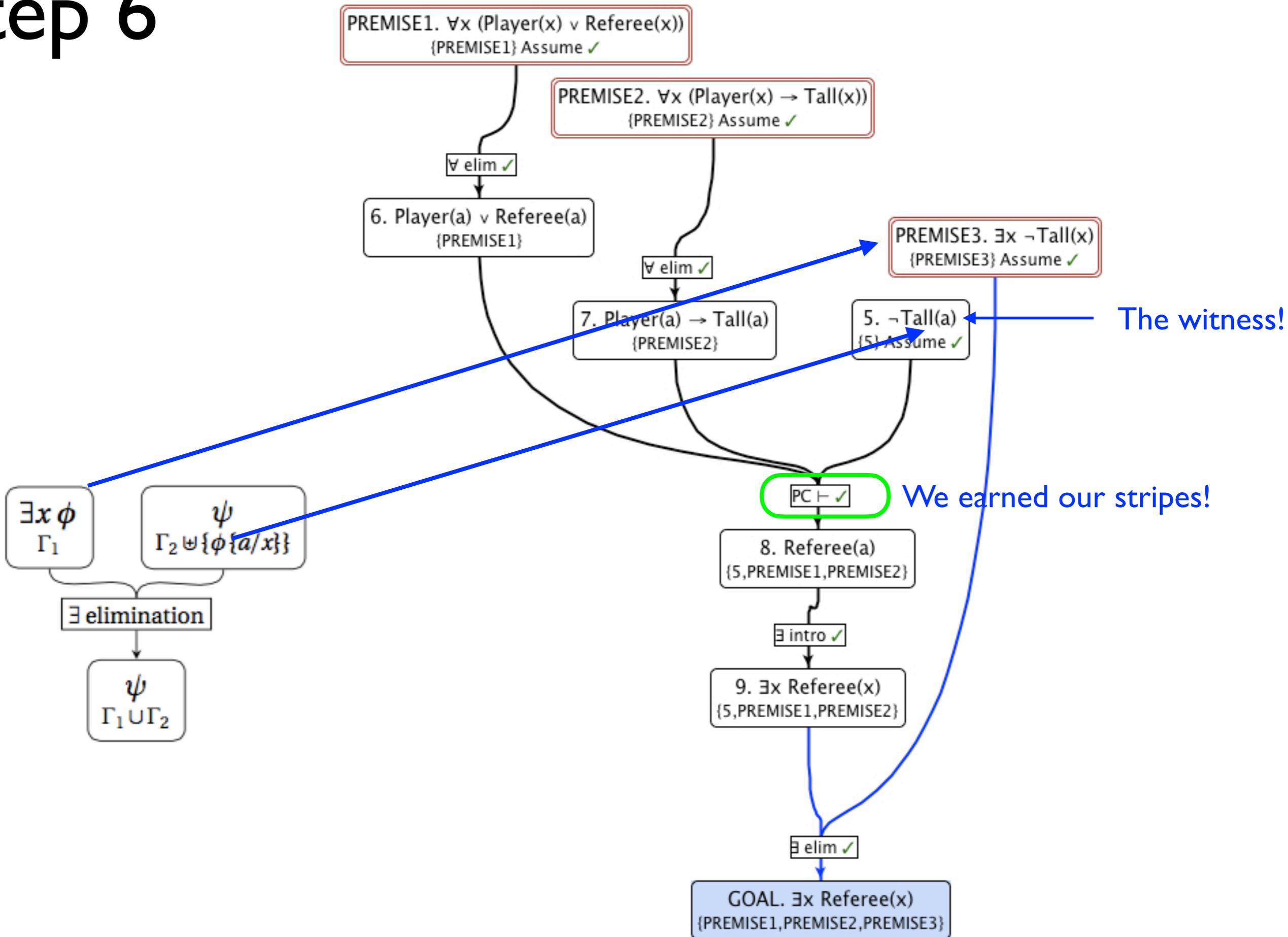
Step 6



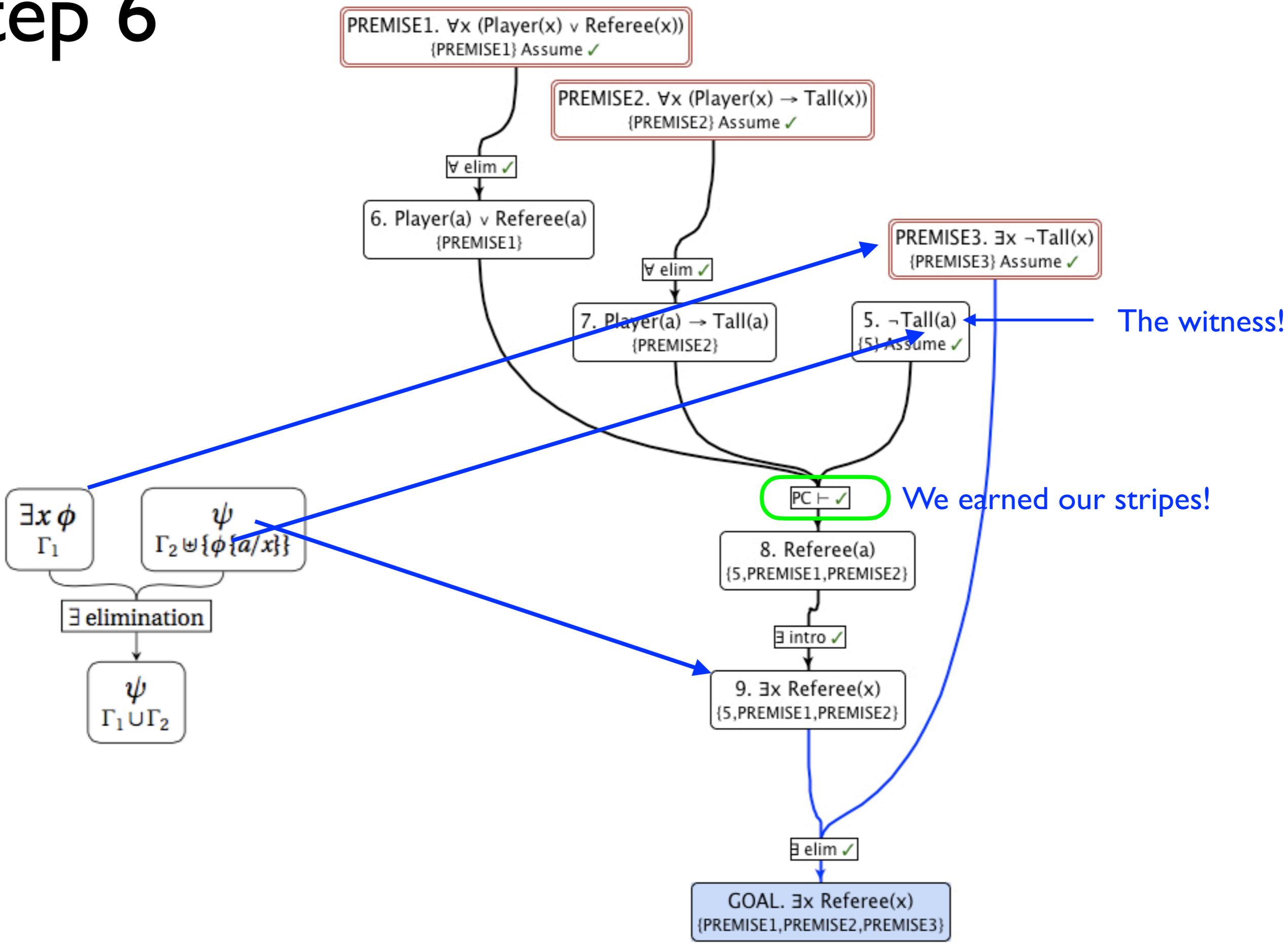
Step 6



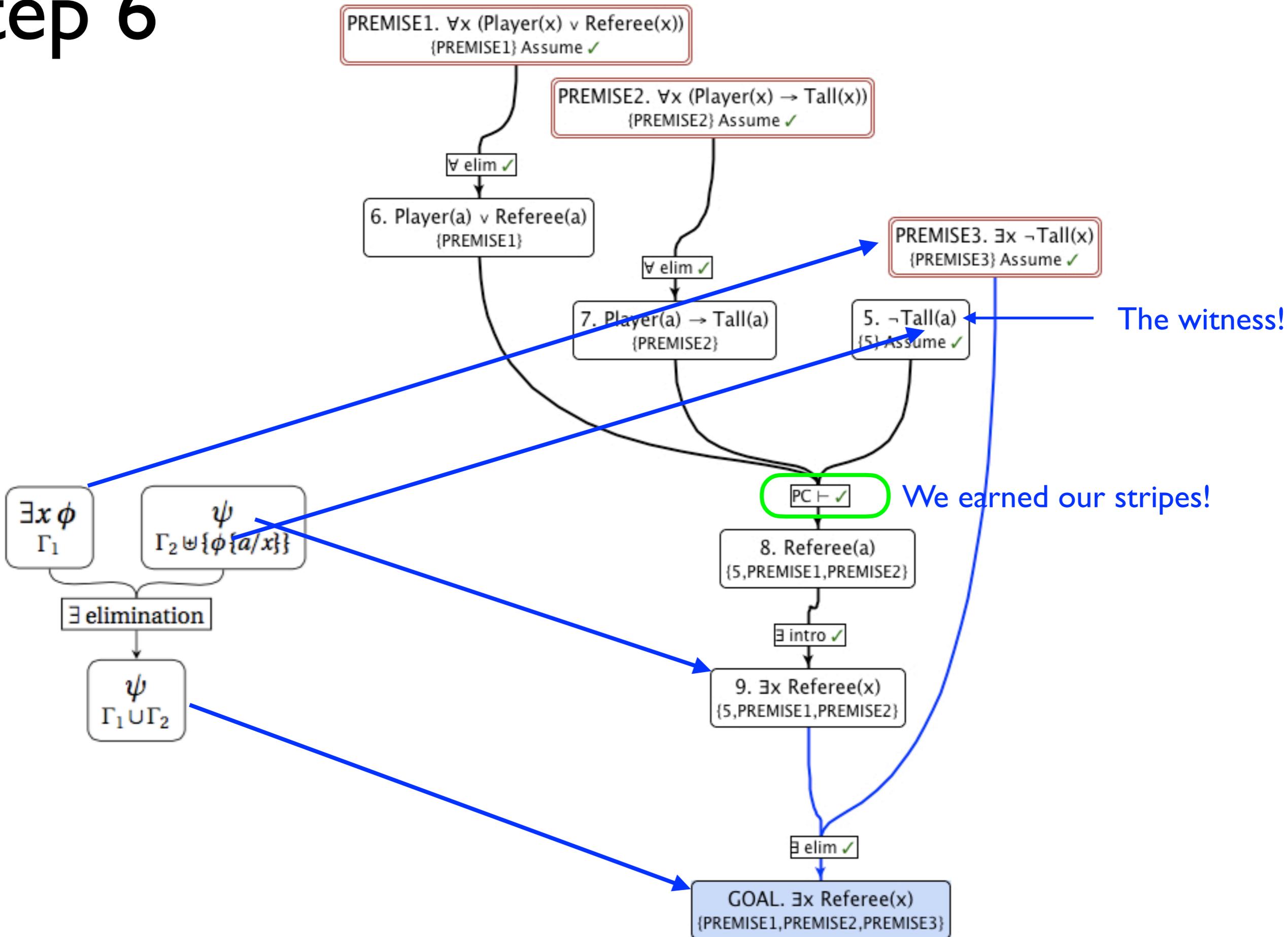
Step 6



Step 6



Step 6



Practice:

$\{\forall x(\text{Scared}(x) \leftrightarrow \text{Small}(x)), \exists x \neg \text{Scared}(x)\} \vdash \exists x \neg \text{Small}(x)$

$\{\exists x, y \text{Contiguous}(x, y), \forall x, y(\text{Contiguous}(x, y) \rightarrow \neg \text{SameCountry}(x, y))\} \vdash \exists x, y \neg \text{SameCountry}(x, y)$

*Hvis du forstår det, kan
du bevise det.*

*Hvis du forstår det, kan
du bevise det.*