Exhortation; Truth Tables; FOL IV: Layered Quantification and Measuring Intelligence Using This Phenomenon

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Troy, New York 12180 USA

Intro to Formal Logic (With AI)
3/9/2026

R A I R
Rensselaer AI and Reasoning Lab

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Hypergraphical Tableaux/Truth Trees

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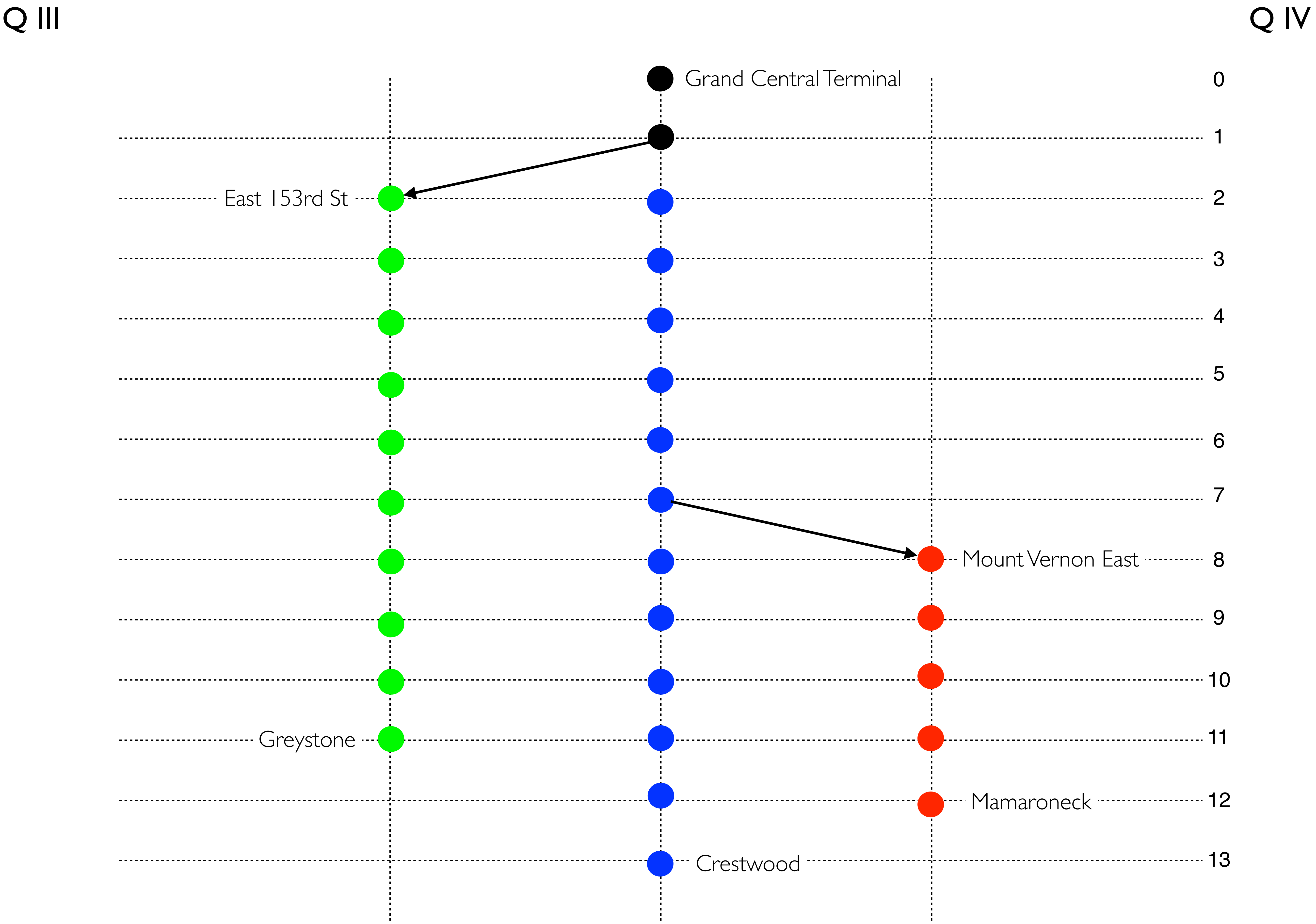
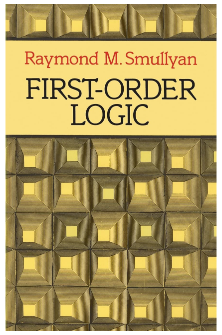
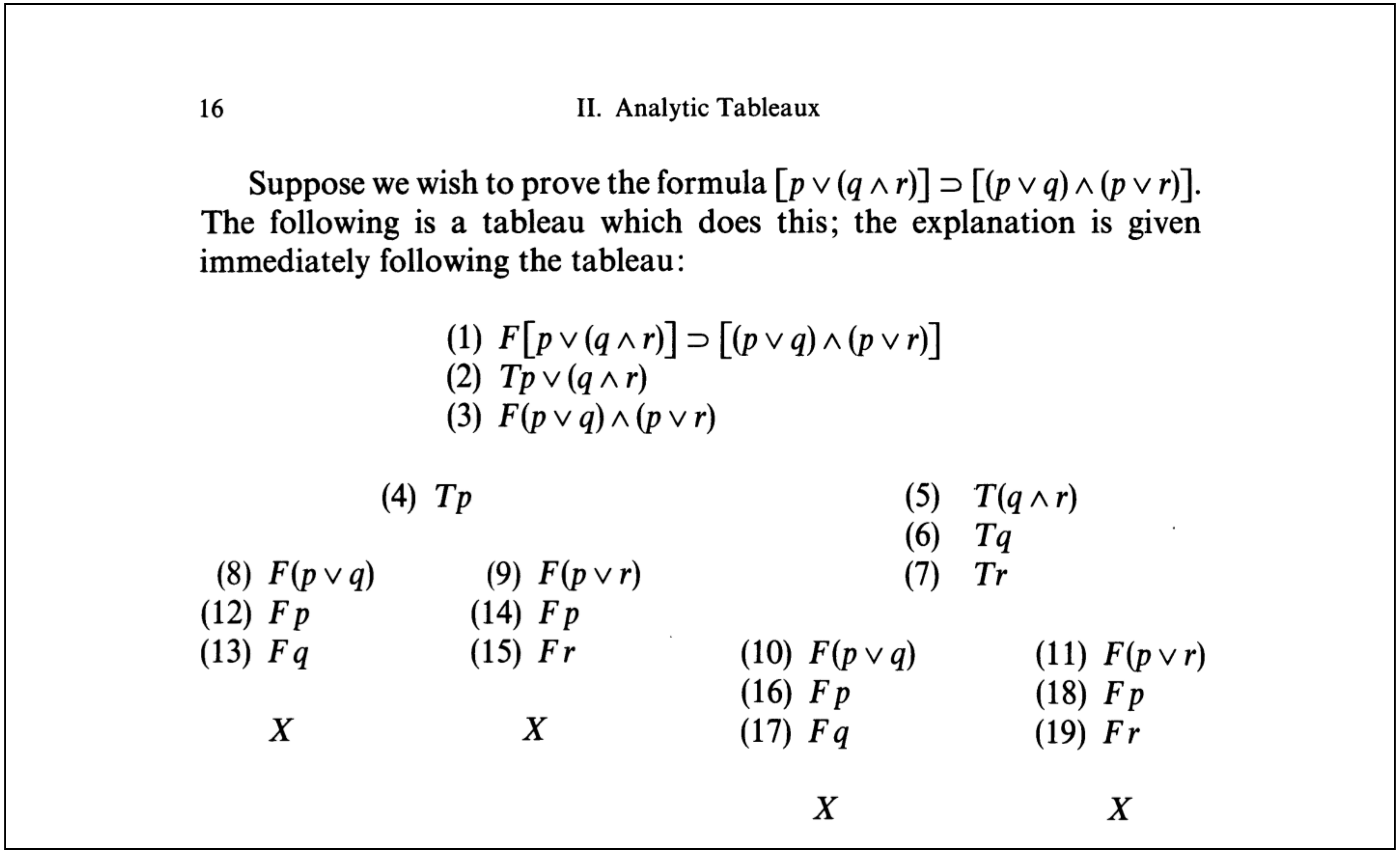
RAIR
Rensselaer AI and Reasoning Lab


Greystone ♿
Glenwood ♿
Yonkers ♿
Ludlow
Riverdale ♿
Hudson Rail Link
Spuyten Duyvil
Hudson Rail Link
Marble Hill ①
University Hts ♿
Morris Hts ♿
E 153 St ♿ B D 4
Harlem–125 Street ♿
4 5 6 6
Grand Central ♿
S 4 5 6 7
Grand Central Terminal · MNR
Grand Central Madison · LIRR
Metro-North
Crestwood ♿
Tuckahoe
Bronxville
Fleetwood ♿
Mt Vernon West ♿
Wakefield
Woodlawn 2 5
Williams Bridge
Fordham ♿
Tremont
Melrose
Mamaroneck
Larchmont ♿
New Rochelle ♿
Pelham
Mount Vernon East ♿
The Bronx
LaGuardia Airport
TIBBETTS BROOK PARK
VAN CORTLANDT PARK
BRONX ZOO
PELHAM BAY PARK
Amtrak
Amtrak
Amtrak
Amtrak
MAJOR DEEGAN EXPWY
GW BRIDGE I-95
I 87
I 95 NEW ENGLAND THRUWAY
BOSTON
POST RD


Q III
Q IV
Grand Central Terminal
East 153rd St
Greystone
Mount Vernon East
Mamaroneck
Crestwood
0
1
2
3
4
5
6
7
8
9
10
11
12
13





Suppose we wish to prove the formula $[p \vee (q \wedge r)] \supset [(p \vee q) \wedge (p \vee r)]$. The following is a tableau which does this; the explanation is given immediately following the tableau:

(1) $F[p \vee (q \wedge r)] \supset [(p \vee q) \wedge (p \vee r)]$
(2) $Tp \vee (q \wedge r)$
(3) $F(p \vee q) \wedge (p \vee r)$

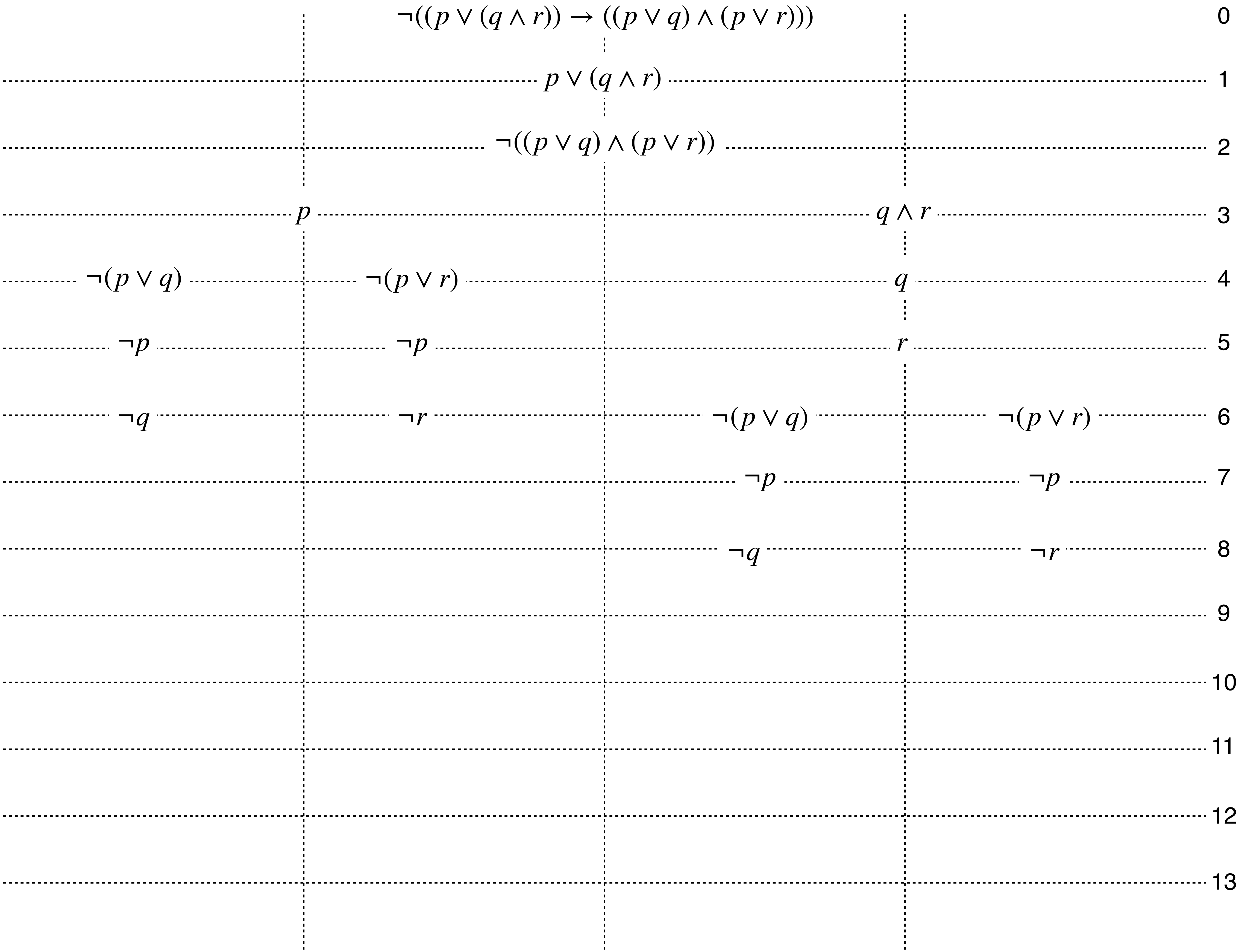
(4) Tp (5) $T(q \wedge r)$
(6) Tq
(7) Tr

(8) $F(p \vee q)$ (9) $F(p \vee r)$
(12) Fp (14) Fp
(13) Fq (15) Fr
(10) $F(p \vee q)$ (11) $F(p \vee r)$
(16) Fp (18) Fp
(17) Fq (19) Fr

X X

X X

$\vdash (p \lor (q \land r)) \to ((p \lor q) \land (p \lor r))$ **?**



$\neg((p \lor (q \land r)) \to ((p \lor q) \land (p \lor r)))$ 0

$p \lor (q \land r)$ 1

$\neg((p \lor q) \land (p \lor r))$ 2

p $\quad$ $q \land r$ 3

$\neg(p \lor q)$ $\quad$ $\neg(p \lor r)$ $\quad$ q 4

$\neg p$ $\quad$ $\neg p$ $\quad$ r 5

$\neg q$ $\quad$ $\neg r$ $\quad$ $\neg(p \lor q)$ $\quad$ $\neg(p \lor r)$ 6

$\neg p$ $\quad$ $\neg p$ 7

$\neg q$ $\quad$ $\neg r$ 8

9

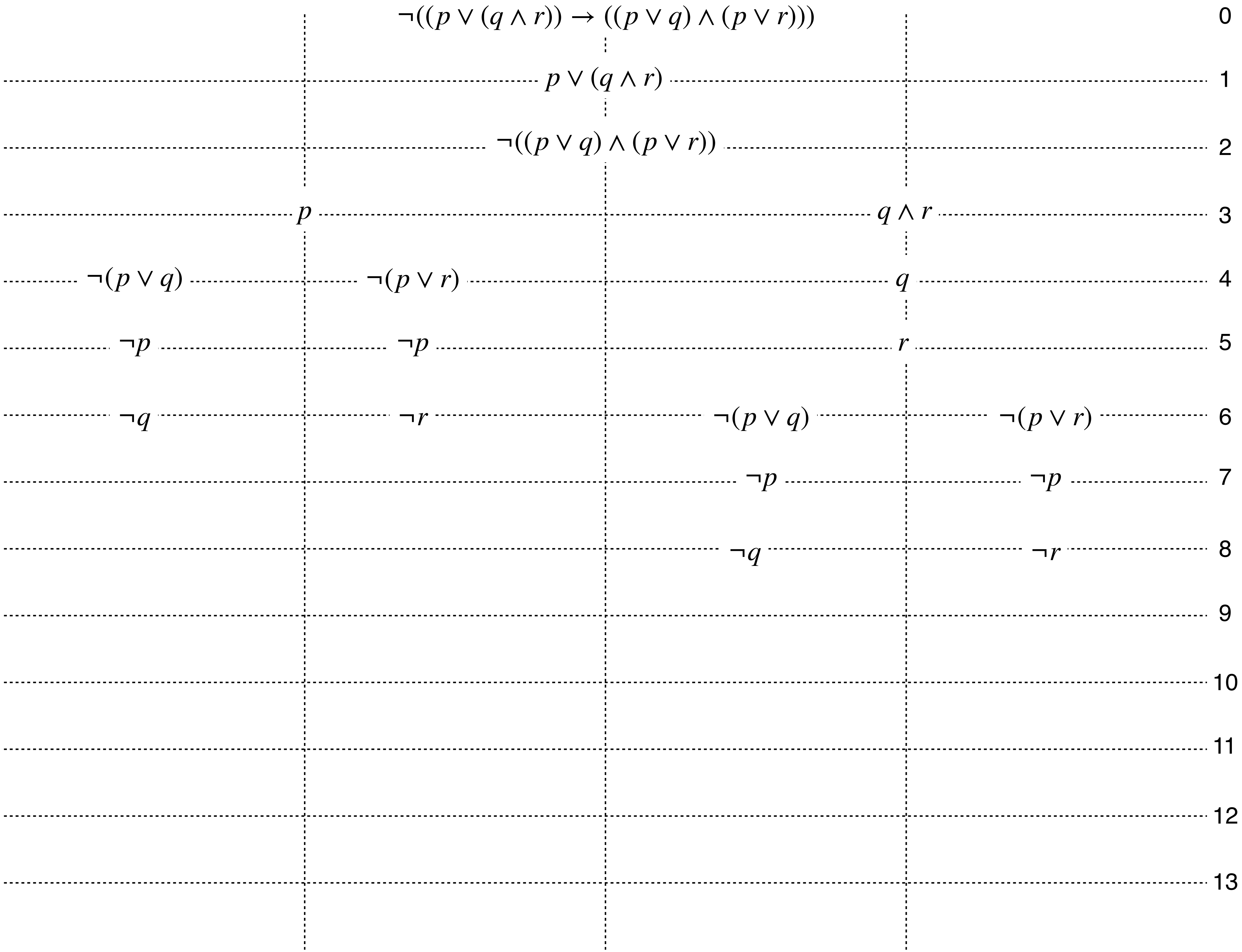
10

11

12

13

$\vdash (p \vee (q \wedge r)) \rightarrow ((p \vee q) \wedge (p \vee r))$



$\neg((p \vee (q \wedge r)) \rightarrow ((p \vee q) \wedge (p \vee r)))$ 0

$p \vee (q \wedge r)$ 1

$\neg((p \vee q) \wedge (p \vee r))$ 2

p $q \wedge r$ 3

$\neg(p \vee q)$ $\neg(p \vee r)$ q 4

$\neg p$ $\neg p$ r 5

$\neg q$ $\neg r$ $\neg(p \vee q)$ $\neg(p \vee r)$ 6

$\neg p$ $\neg p$ 7

$\neg q$ $\neg r$ 8

9

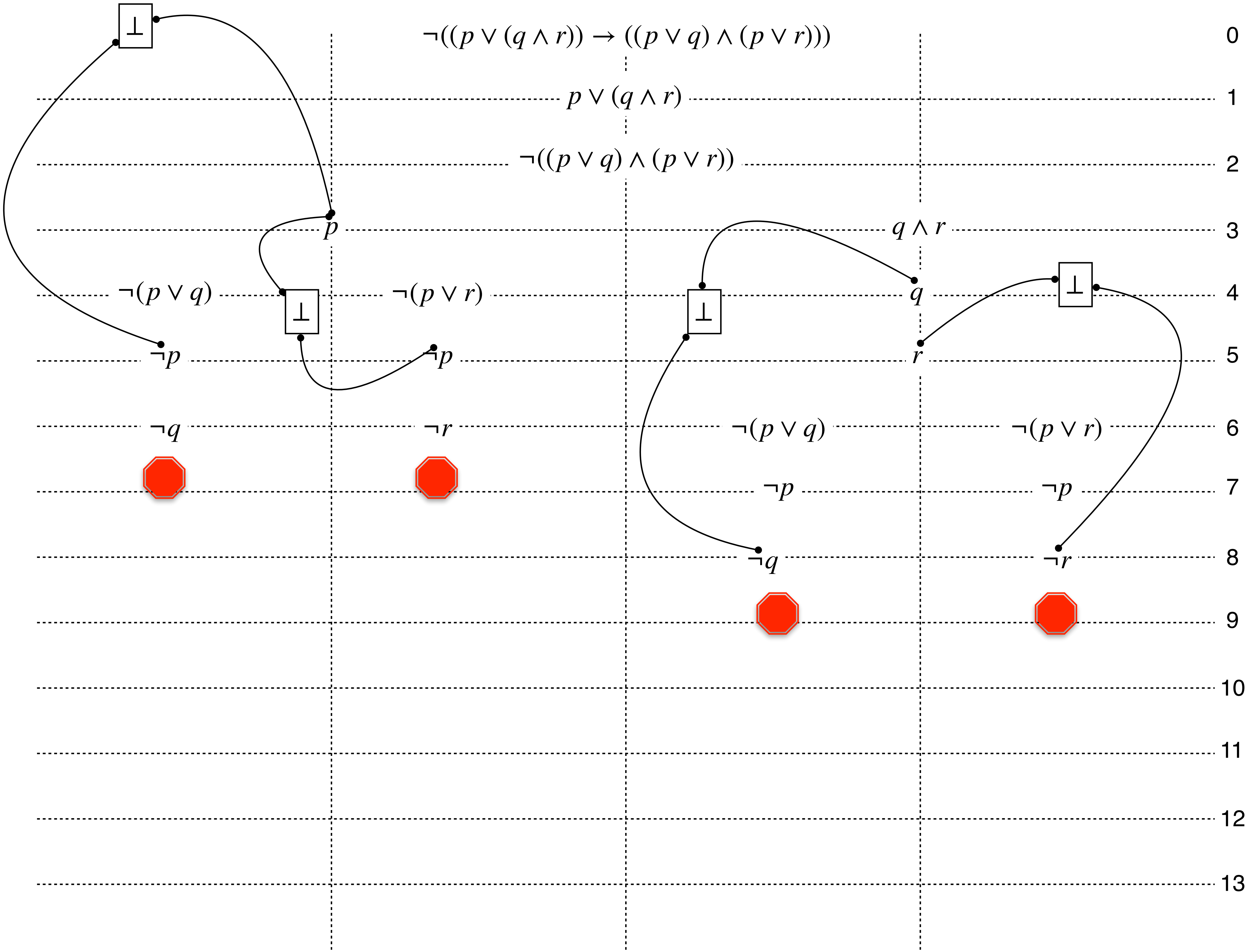
10

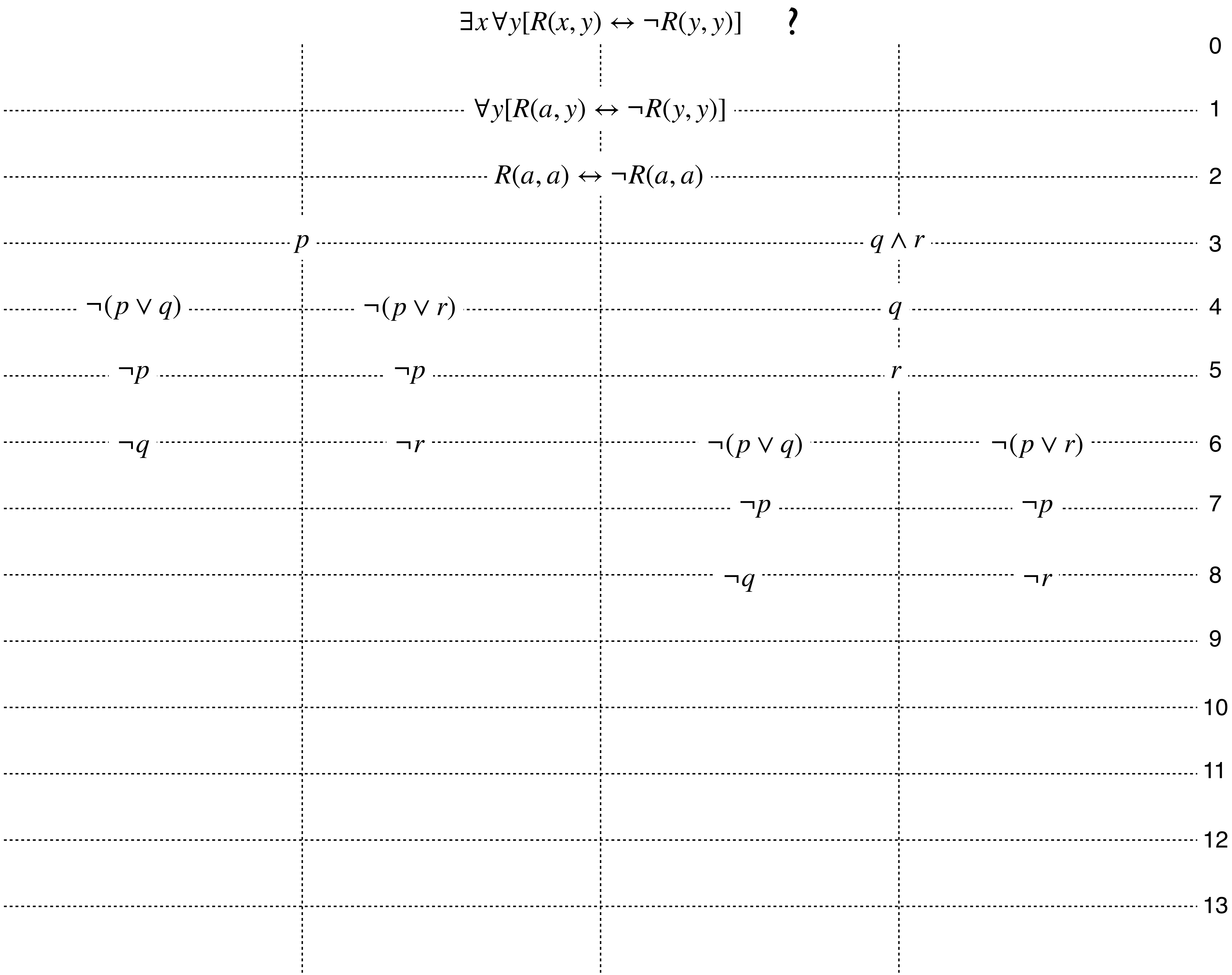
11

12

13

$\vdash (p \lor (q \land r)) \to ((p \lor q) \land (p \lor r))$





$\exists x \forall y[R(x,y) \leftrightarrow \neg R(y,y)]$?

0

1 $\forall y[R(a,y) \leftrightarrow \neg R(y,y)]$

2 $R(a,a) \leftrightarrow \neg R(a,a)$

3 p | $q \wedge r$

4 $\neg(p \vee q)$ | $\neg(p \vee r)$ | q

5 $\neg p$ | $\neg p$ | r

6 $\neg q$ | $\neg r$ | $\neg(p \vee q)$ | $\neg(p \vee r)$

7 $\neg p$ | $\neg p$

8 $\neg q$ | $\neg r$

9

10

11

12

13

Exhortation …

Make sure you're up-to-date-ish now after Spring Break on HyperGrader®'s current (**Required** = Homework) Problems, currently due

Apr 20 2026 11:59pm NY time.

More FOL etc problems of course forthcoming shortly … but let's do one …

New in-class problem published, with needed tableau/truth-tree work …

Tableaux/Truth Trees vs. Truth Tables

Tableaux/Truth Trees vs. Truth Tables

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Violent breakage between tabular calculation and proof construction.

Tableaux/Truth Trees vs. ~~Truth Tables~~

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LAMA®'s hypergraphs/HyperLogic® achieves
seamless unification of proofs and trees, and provides
AI oracles for their construction and *certification*.

Tableaux/Truth Trees vs. ~~Truth Tables~~

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First very simple: truth-tree for *modus ponens* …

Tableaux/Truth Trees vs. ~~Truth Tables~~

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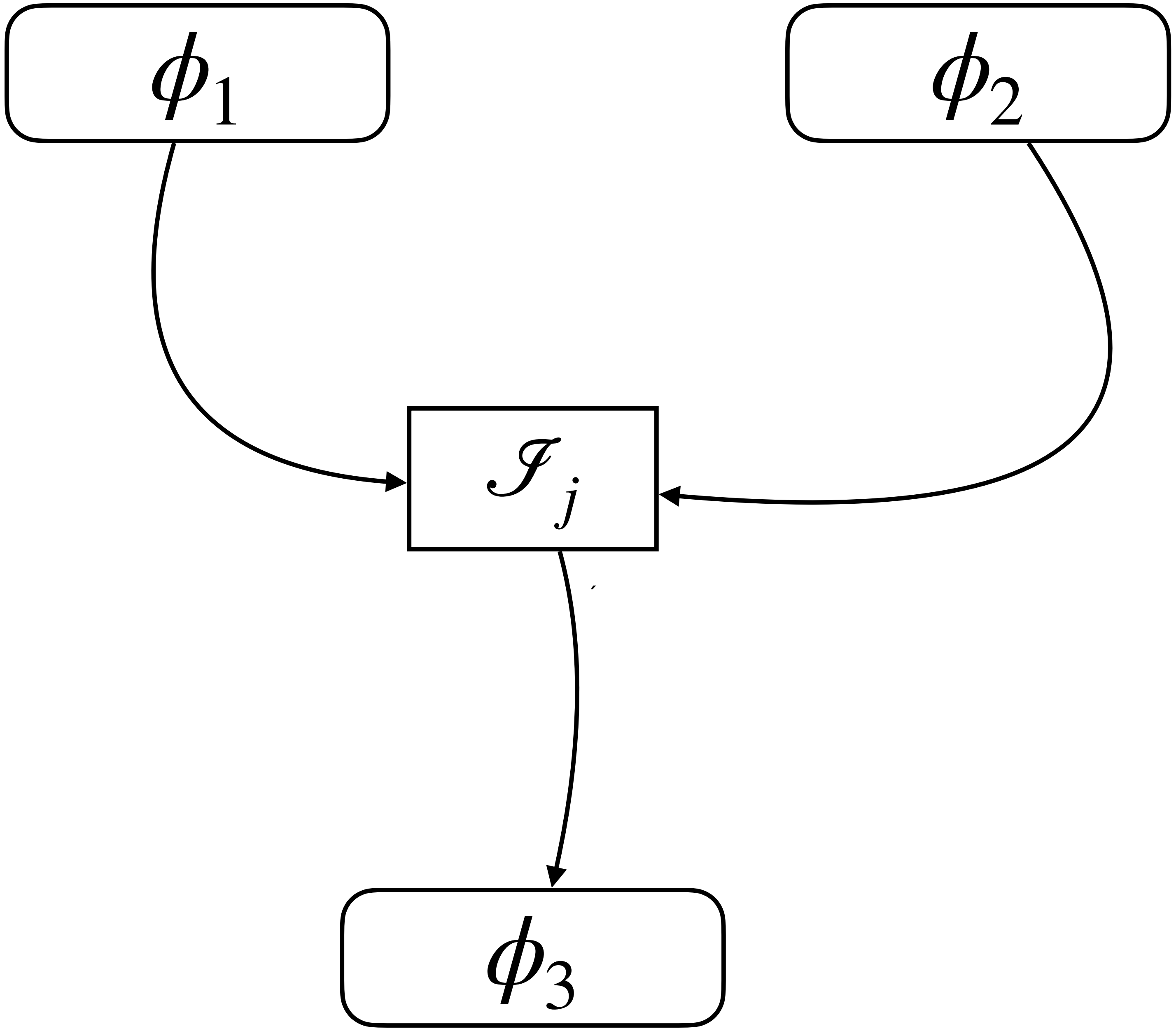
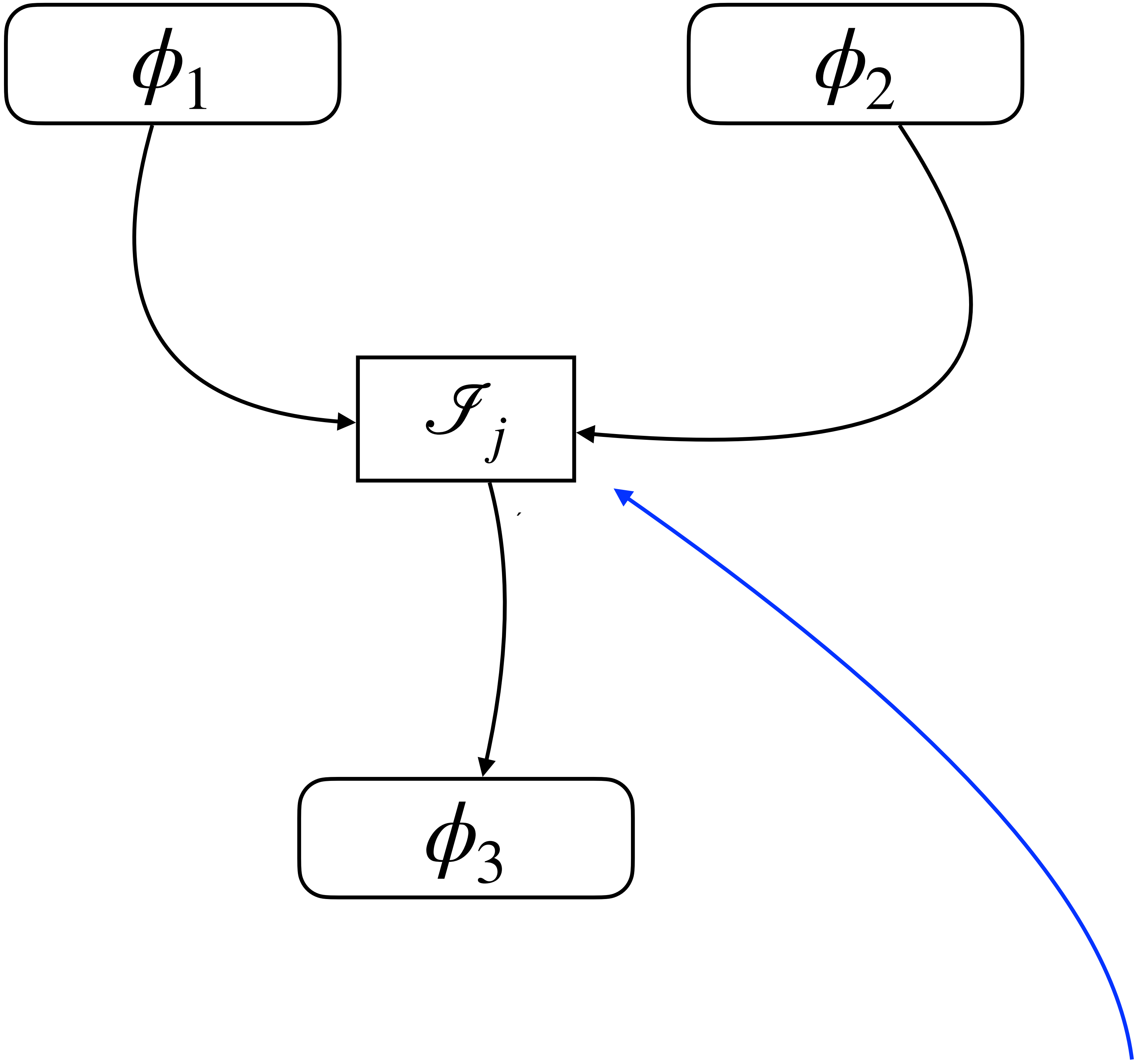
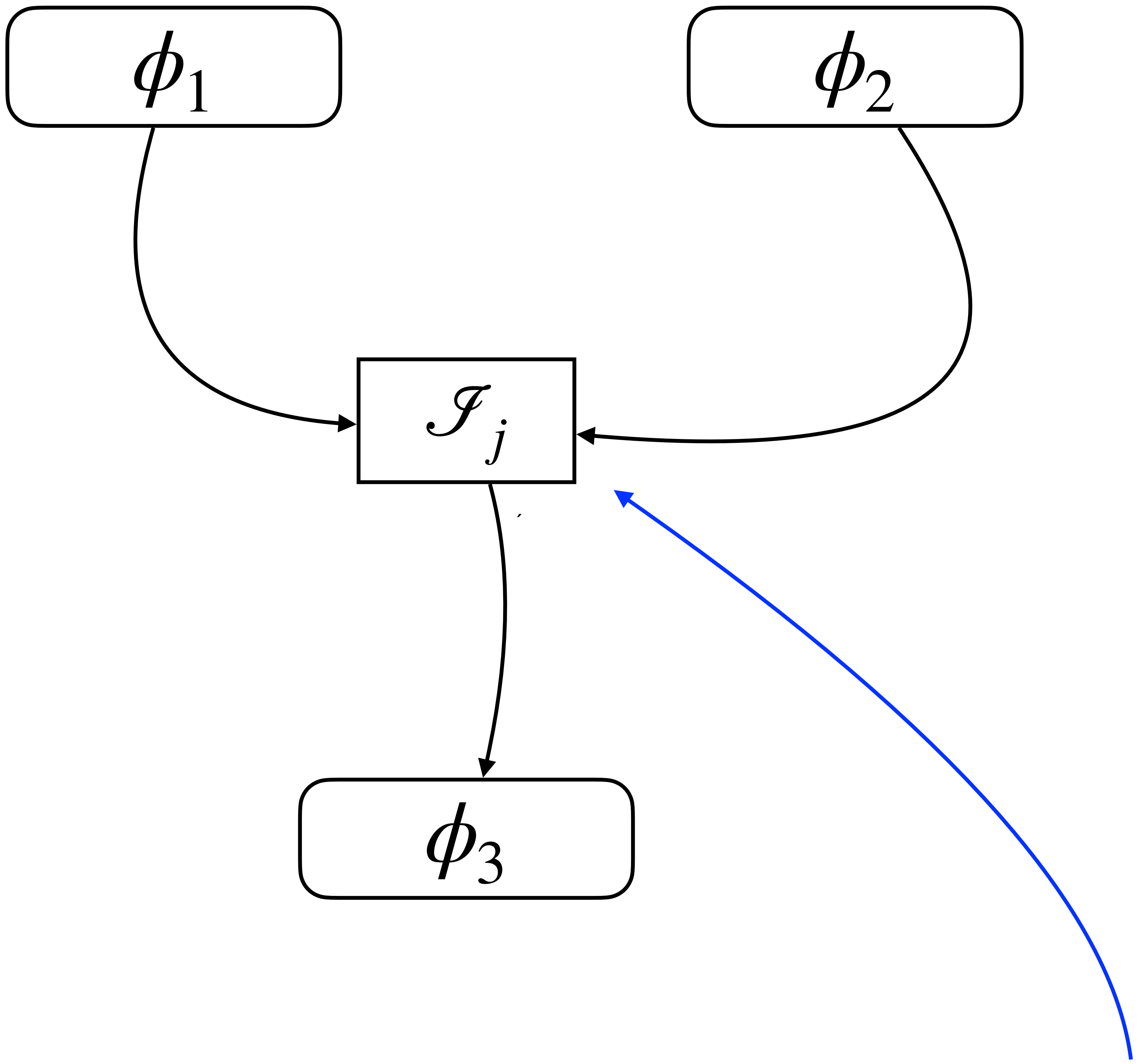
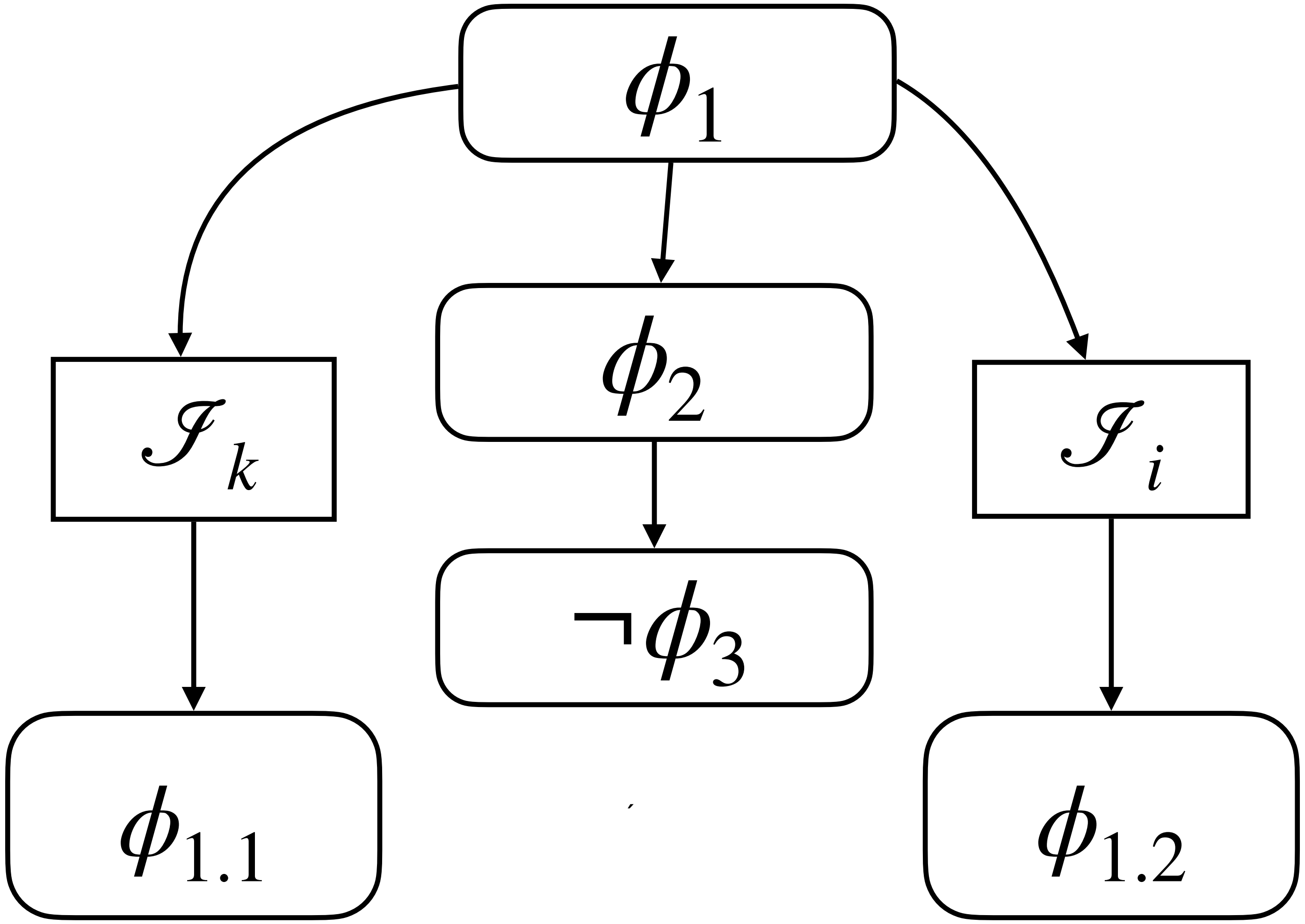
First very simple: truth-tree for *modus ponens* …

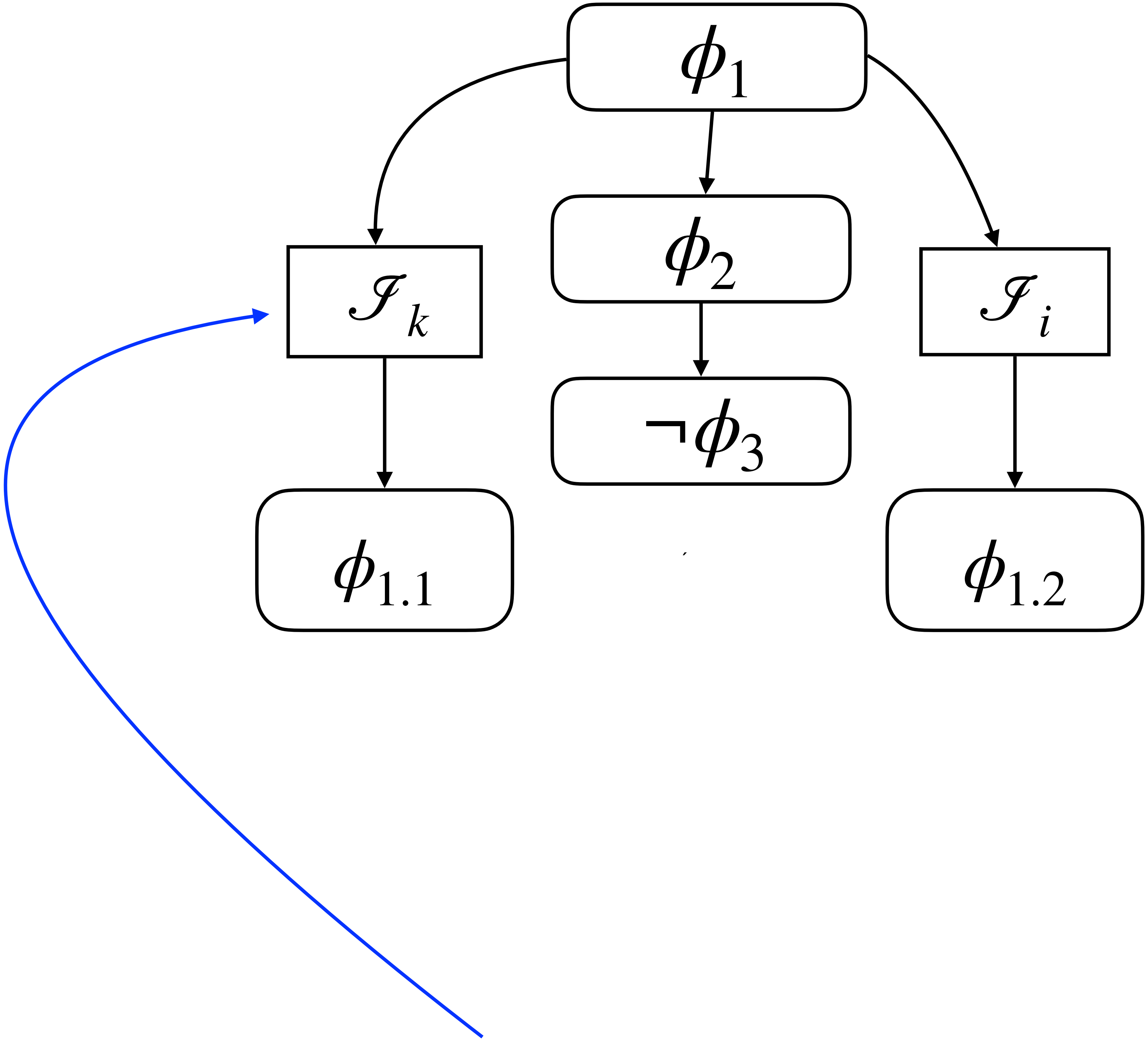

ϕ_1
ϕ_2
$\mathscr{J}_j$
ϕ_3


ϕ_1
ϕ_2
$\mathscr{I}_j$
ϕ_3



from our collection of natural-deduction inference rules/schemata


ϕ_1
$\mathcal{I}_k$
ϕ_2
$\mathcal{I}_i$
$\neg\phi_3$
$\phi_{1.1}$
$\phi_{1.2}$


ϕ_1
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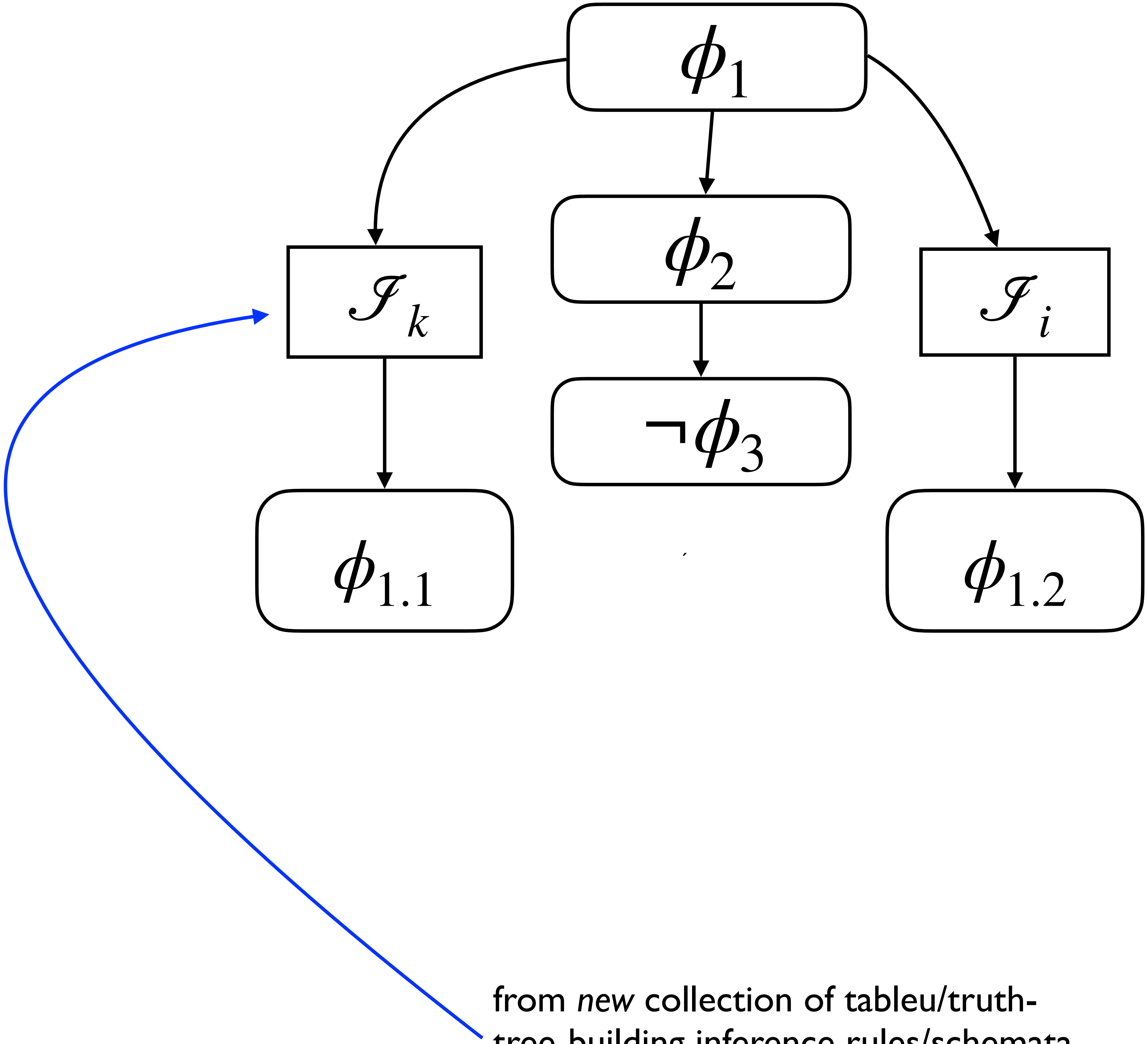
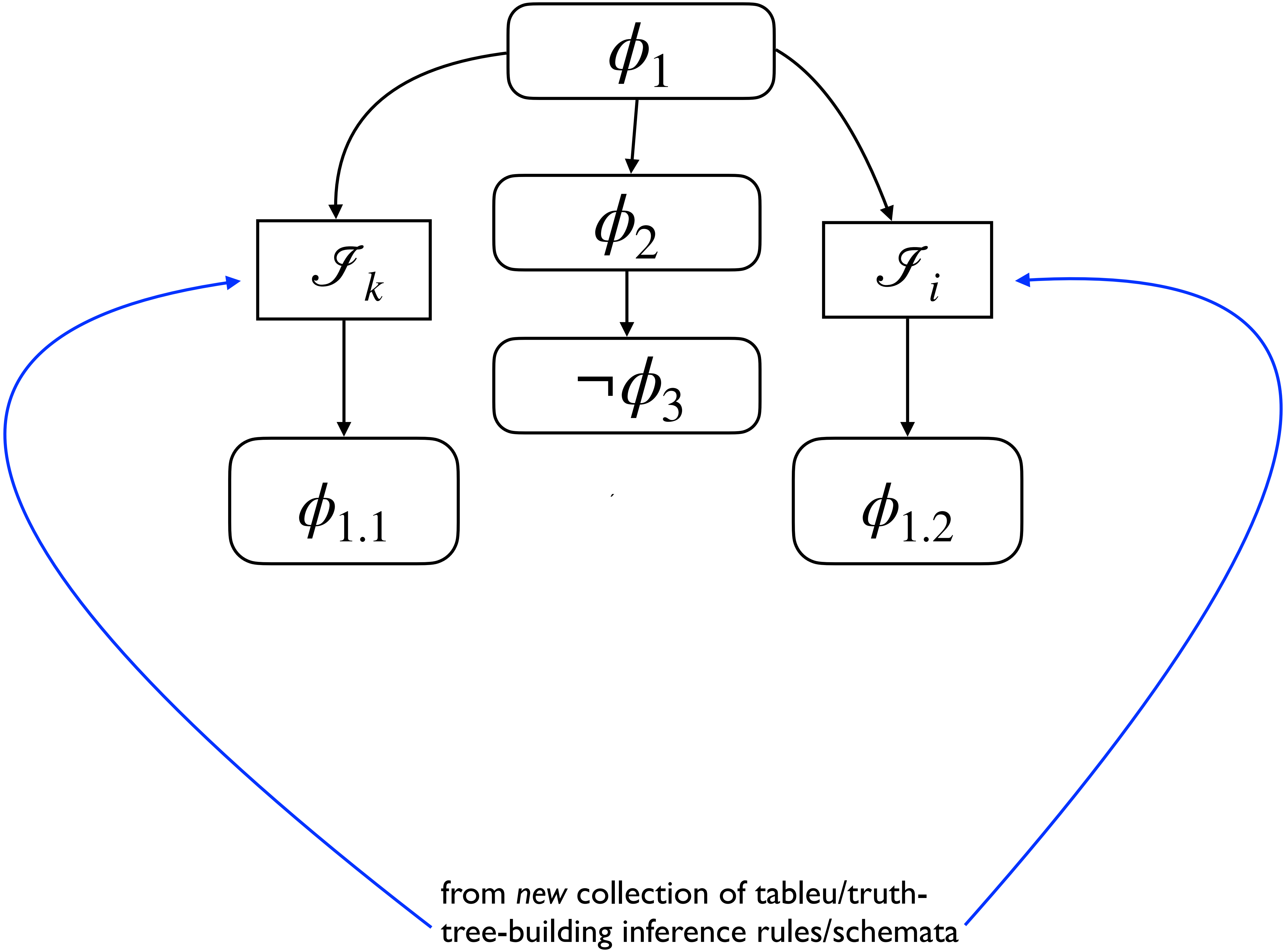


from *new* collection of tableu/truth-tree-building inference rules/schemata



ϕ_1

$\mathcal{I}_k$

ϕ_2

$\mathcal{I}_i$

$\neg\phi_3$

$\phi_{1.1}$

$\phi_{1.2}$

from *new* collection of tableu/truth-tree-building inference rules/schemata

$\{P \to Q, P\} \vdash Q$

GIVEN1. $P \to Q$

PC $\vdash$ ✗

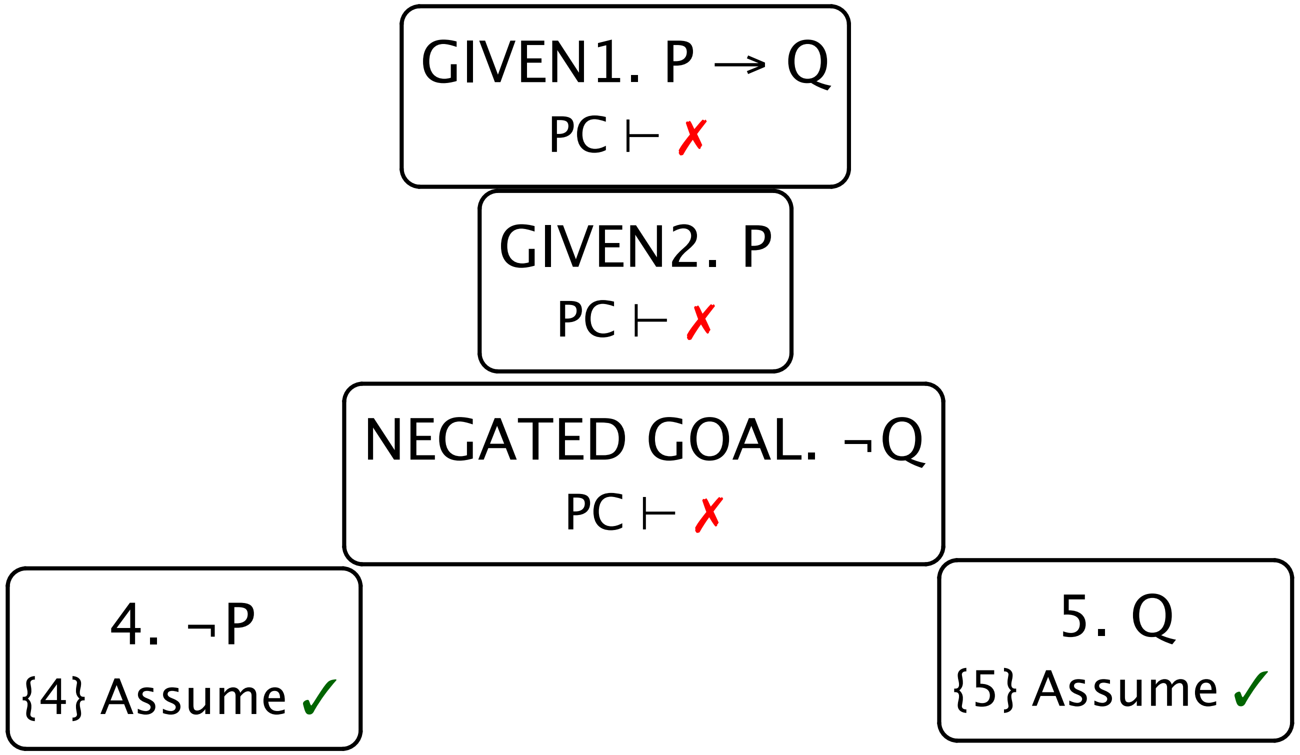
GIVEN2. P

PC $\vdash$ ✗

NEGATED GOAL. $\neg Q$

PC $\vdash$ ✗

$\{P \rightarrow Q, P\} \vdash Q$



GIVEN1. P → Q
PC ⊢ ✗

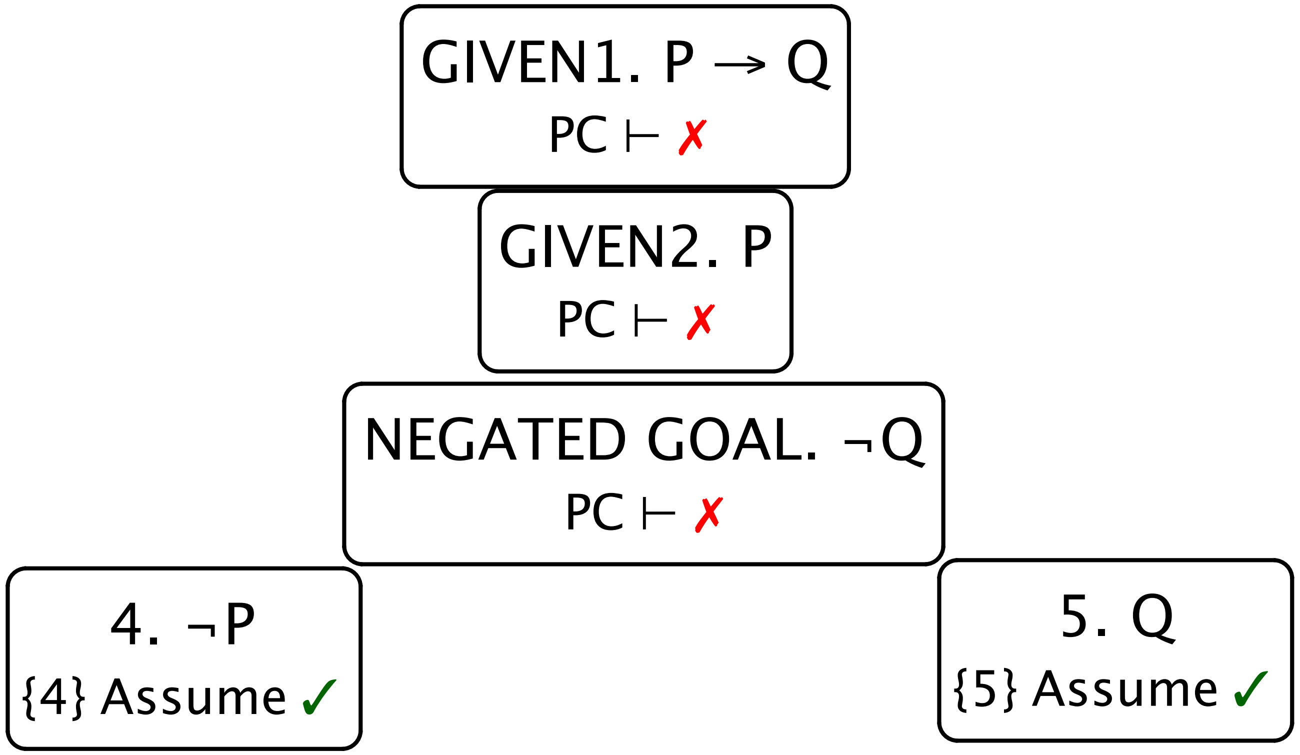
GIVEN2. P
PC ⊢ ✗

NEGATED GOAL. ¬Q
PC ⊢ ✗

4. ¬P
{4} Assume ✓

5. Q
{5} Assume ✓

$\{P \rightarrow Q, P\} \vdash Q$



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{4} Assume ✓

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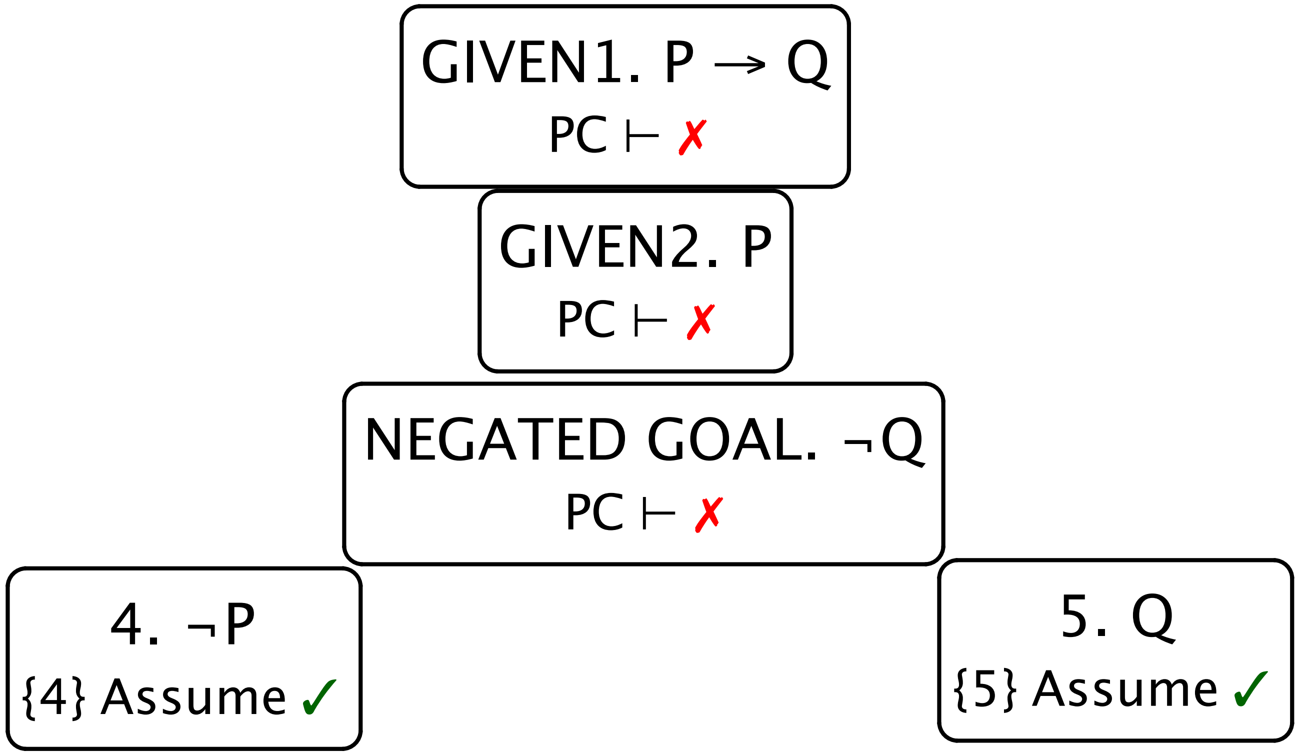
$\{P \to Q, P\} \vdash Q$



GIVEN1. $P \to Q$
PC $\vdash$ ✗

GIVEN2. P
PC $\vdash$ ✗

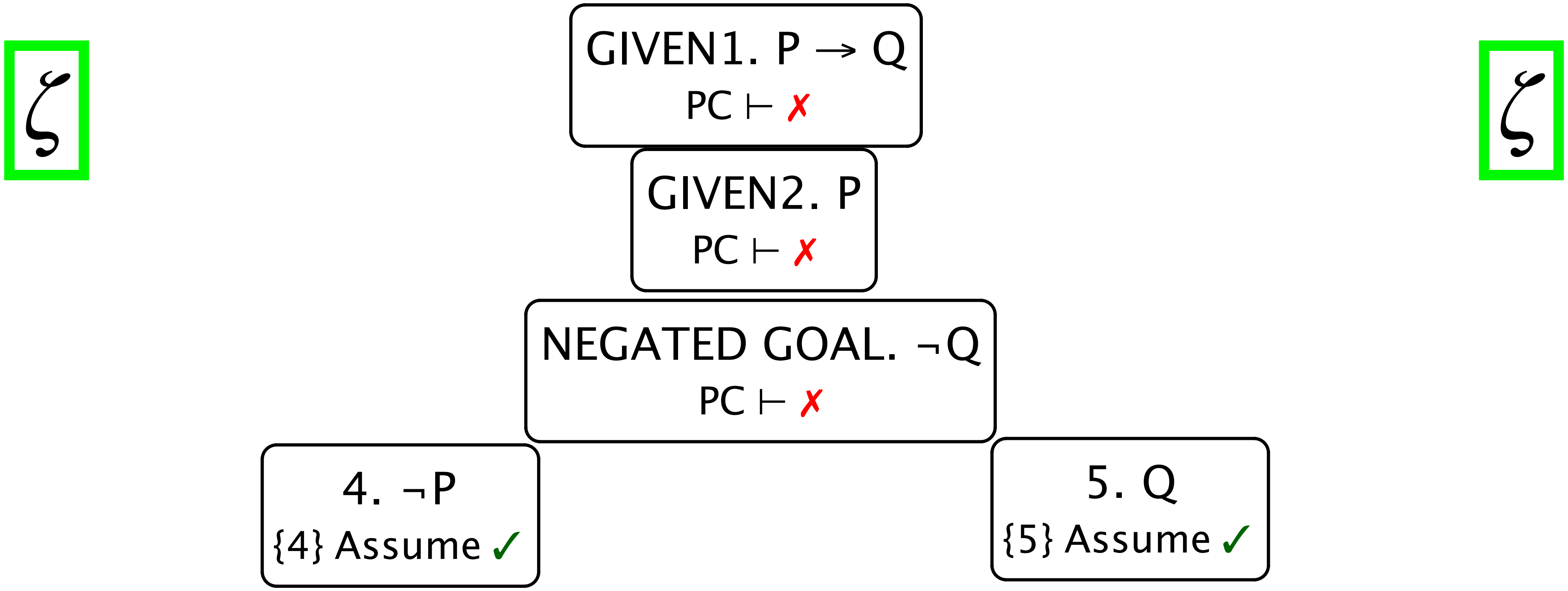
NEGATED GOAL. $\neg Q$
PC $\vdash$ ✗

4. $\neg P$
{4} Assume ✓

5. Q
{5} Assume ✓

Either way, a contradiction!

$\{P \rightarrow Q, P\} \vdash Q$



GIVEN1. $P \rightarrow Q$
PC $\vdash$ ✗

GIVEN2. P
PC $\vdash$ ✗

NEGATED GOAL. $\neg Q$
PC $\vdash$ ✗

4. $\neg P$
{4} Assume ✓

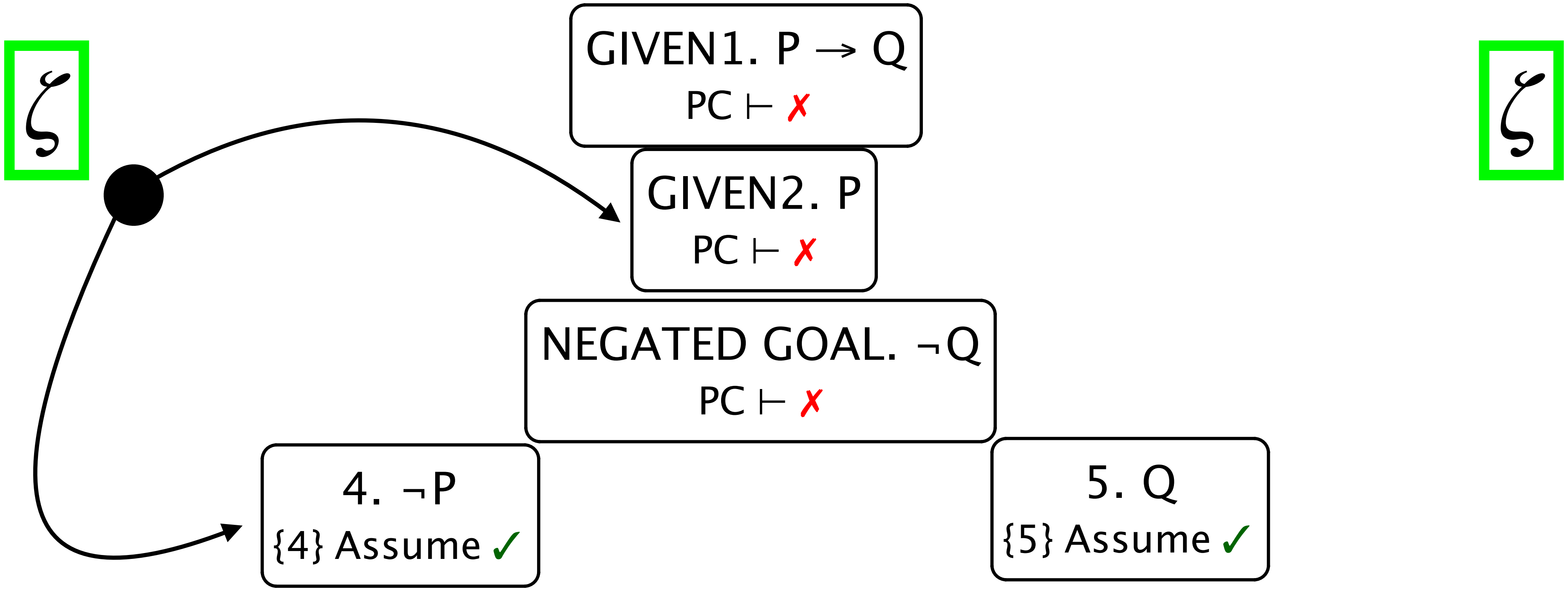
5. Q
{5} Assume ✓

ζ

ζ

Either way, a contradiction!

$\{P \to Q, P\} \vdash Q$



ζ

GIVEN1. $P \to Q$
PC $\vdash$ ✗

GIVEN2. P
PC $\vdash$ ✗

NEGATED GOAL. $\neg Q$
PC $\vdash$ ✗

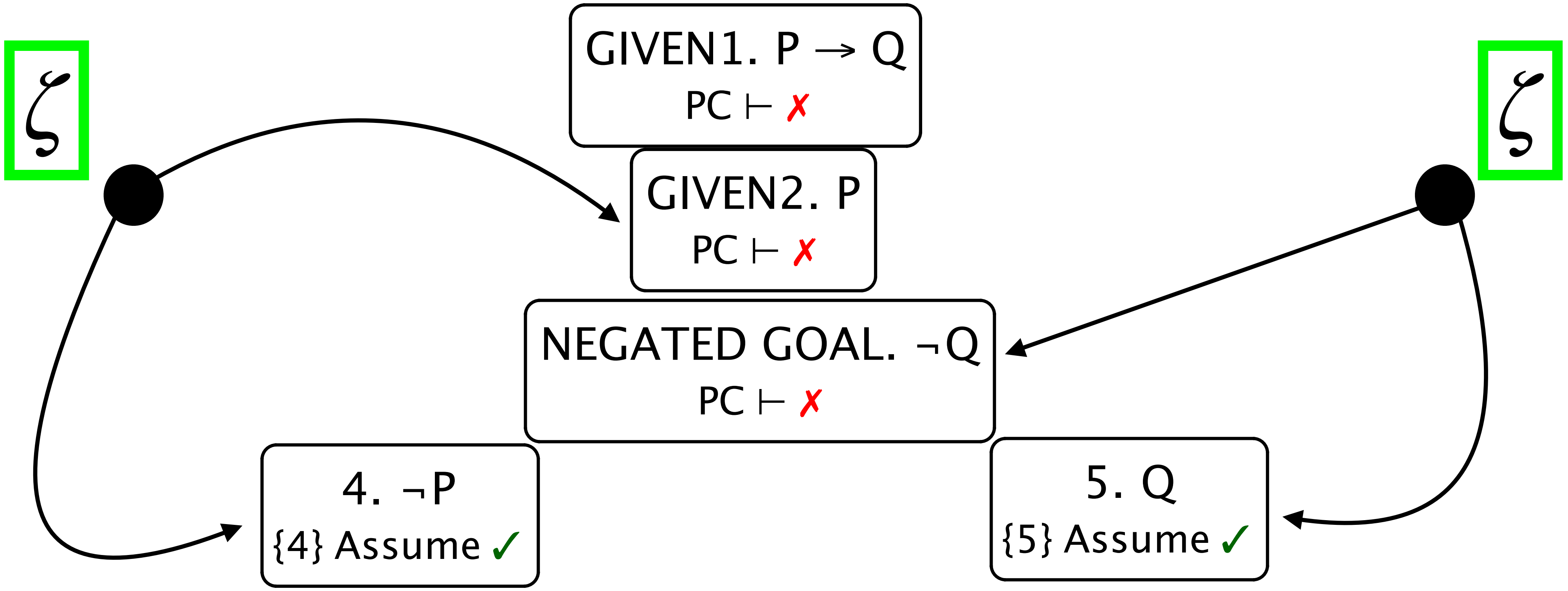
4. $\neg P$
{4} Assume ✓

5. Q
{5} Assume ✓

ζ

Either way, a contradiction!

$\{\mathtt{P} \to \mathtt{Q}, \mathtt{P}\} \vdash \mathtt{Q}$



GIVEN1. P → Q
PC ⊢ ✗

GIVEN2. P
PC ⊢ ✗

NEGATED GOAL. ¬Q
PC ⊢ ✗

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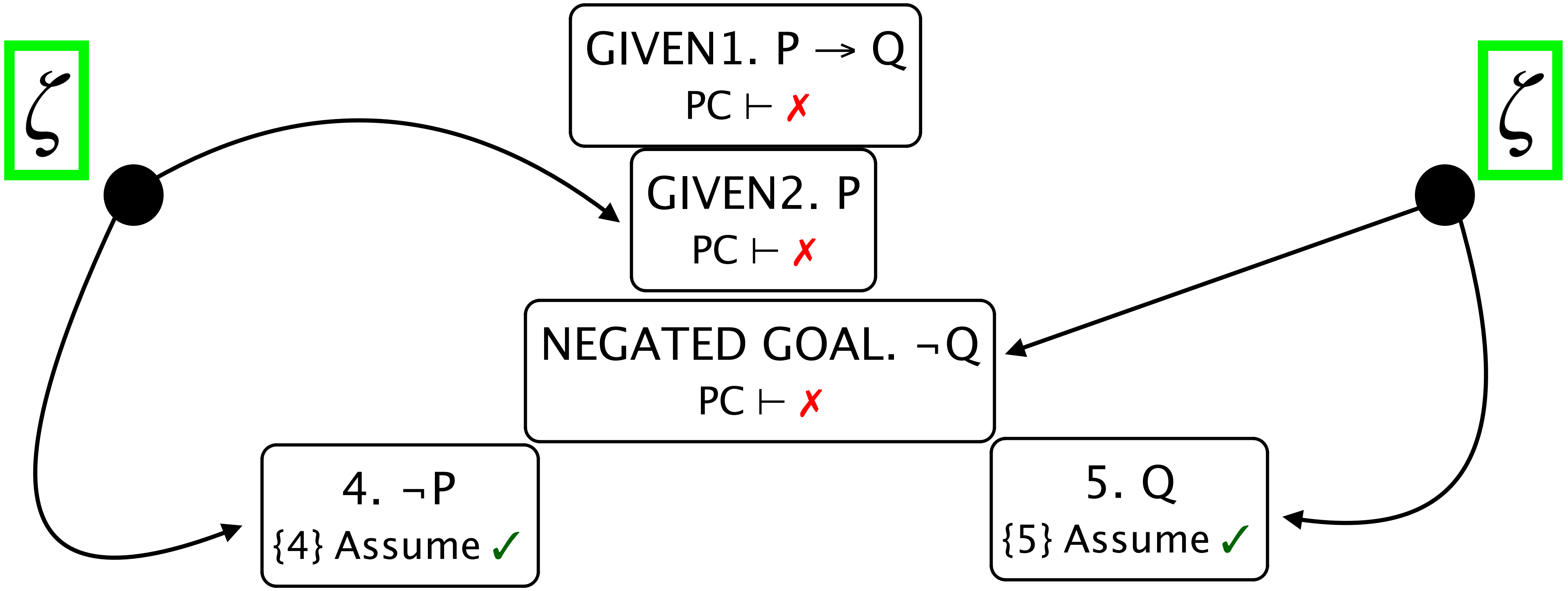
5. Q
{5} Assume ✓

Either way, a contradiction!

$\{P \rightarrow Q, P\} \vdash Q$

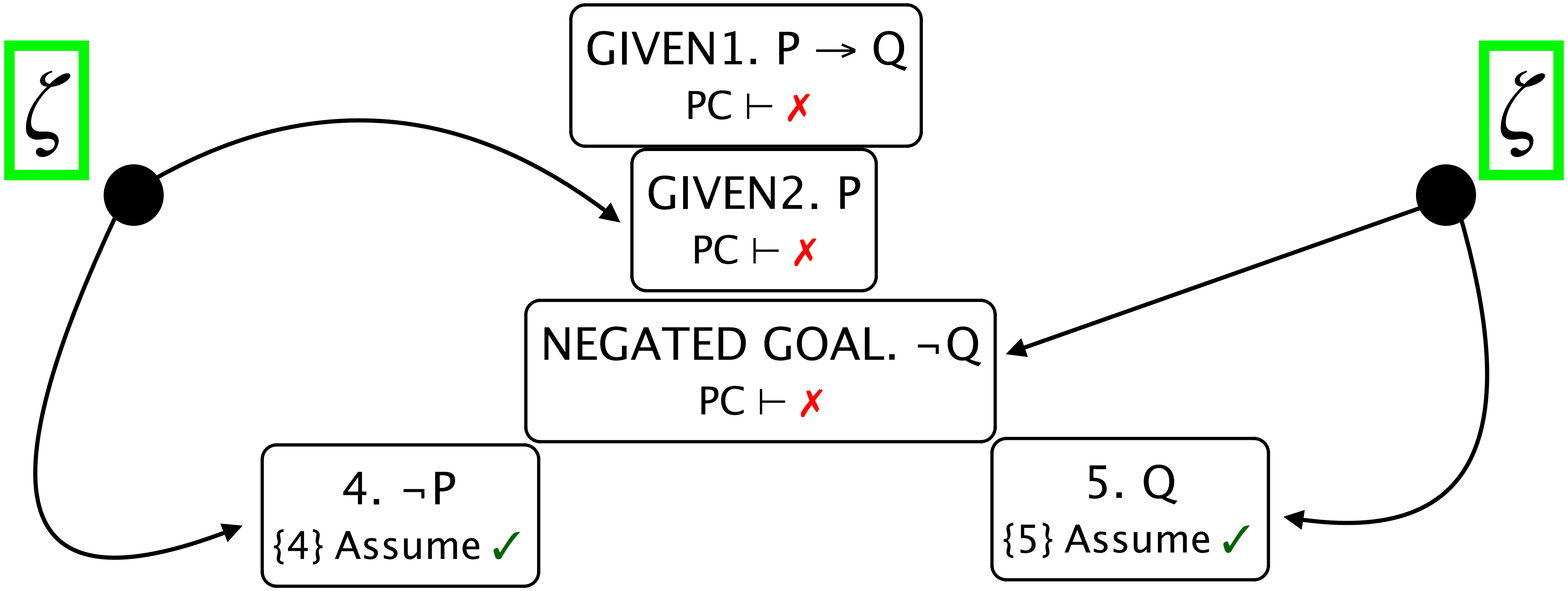


Either way, a contradiction!

Therefore the entailment holds!

$\{\mathtt{P} \to \mathtt{Q}, \mathtt{P}\} \vdash \mathtt{Q}$

This *not* a tree!



GIVEN1. P → Q
PC ⊢ ✗

GIVEN2. P
PC ⊢ ✗

NEGATED GOAL. ¬Q
PC ⊢ ✗

4. ¬P
{4} Assume ✓

5. Q
{5} Assume ✓

Either way, a contradiction!

Therefore the entailment holds!

Slightly Harder Truth Tree

Slightly Harder Truth Tree

$$\vdash (\mathtt{P} \to (\mathtt{Q} \to \mathtt{R})) \to ((\mathtt{P} \to \mathtt{Q}) \to (\mathtt{P} \to \mathtt{R}))$$

Slightly Harder Truth Tree

$$\vdash (\mathtt{P} \to (\mathtt{Q} \to \mathtt{R})) \to ((\mathtt{P} \to \mathtt{Q}) \to (\mathtt{P} \to \mathtt{R}))$$

(This is the axiom THEN-2 in Frege's (brutal) axiomatization of the propositional calculus.)

Frege

Slightly Harder Truth Tree

$$\vdash (\mathtt{P} \rightarrow (\mathtt{Q} \rightarrow \mathtt{R})) \rightarrow ((\mathtt{P} \rightarrow \mathtt{Q}) \rightarrow (\mathtt{P} \rightarrow \mathtt{R}))$$

(This is the axiom THEN-2 in Frege's (brutal) axiomatization of the propositional calculus.)

Frege

https://en.wikipedia.org/wiki/Frege%27s_propositional_calculus

Slightly Harder Truth Tree

$$\vdash (\mathtt{P} \to (\mathtt{Q} \to \mathtt{R})) \to ((\mathtt{P} \to \mathtt{Q}) \to (\mathtt{P} \to \mathtt{R}))$$

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Frege

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The Rules of the Game!

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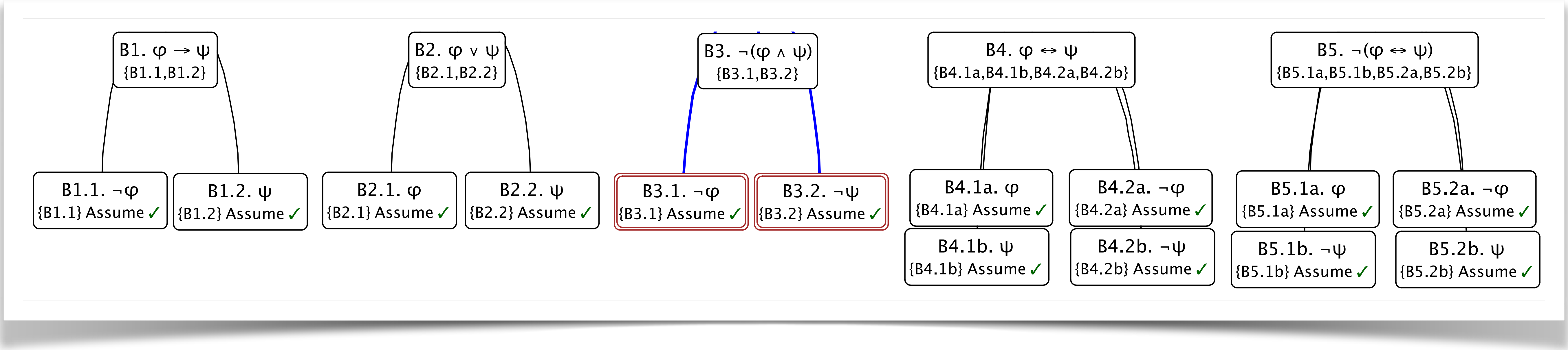
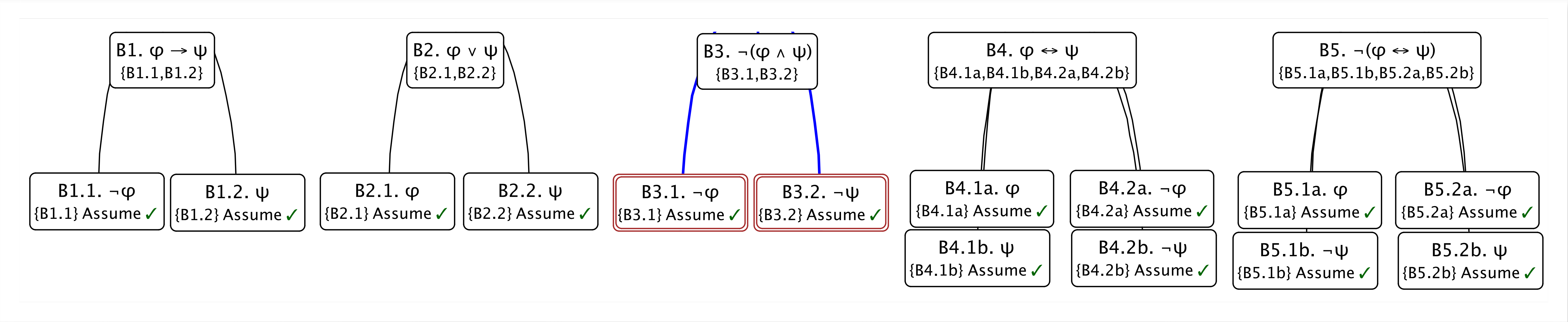


The Rules of the Game!



B1. φ → ψ
{B1.1,B1.2}

B1.1. ¬φ
{B1.1} Assume ✓

B1.2. ψ
{B1.2} Assume ✓

B2. φ ∨ ψ
{B2.1,B2.2}

B2.1. φ
{B2.1} Assume ✓

B2.2. ψ
{B2.2} Assume ✓

B3. ¬(φ ∧ ψ)
{B3.1,B3.2}

B3.1. ¬φ
{B3.1} Assume ✓

B3.2. ¬ψ
{B3.2} Assume ✓

B4. φ ↔ ψ
{B4.1a,B4.1b,B4.2a,B4.2b}

B4.1a. φ
{B4.1a} Assume ✓

B4.1b. ψ
{B4.1b} Assume ✓

B4.2a. ¬φ
{B4.2a} Assume ✓

B4.2b. ¬ψ
{B4.2b} Assume ✓

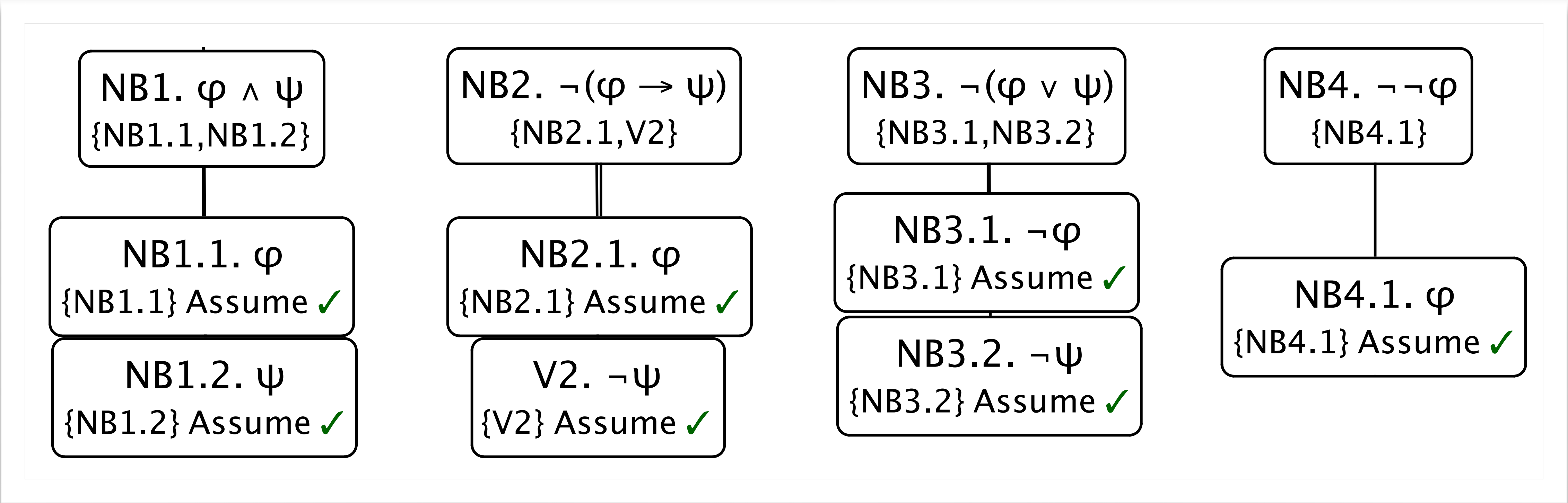
B5. ¬(φ ↔ ψ)
{B5.1a,B5.1b,B5.2a,B5.2b}

B5.1a. φ
{B5.1a} Assume ✓

B5.1b. ¬ψ
{B5.1b} Assume ✓

B5.2a. ¬φ
{B5.2a} Assume ✓

B5.2b. ψ
{B5.2b} Assume ✓



NB1. φ ∧ ψ
{NB1.1,NB1.2}

NB1.1. φ
{NB1.1} Assume ✓

NB1.2. ψ
{NB1.2} Assume ✓

NB2. ¬(φ → ψ)
{NB2.1,V2}

NB2.1. φ
{NB2.1} Assume ✓

V2. ¬ψ
{V2} Assume ✓

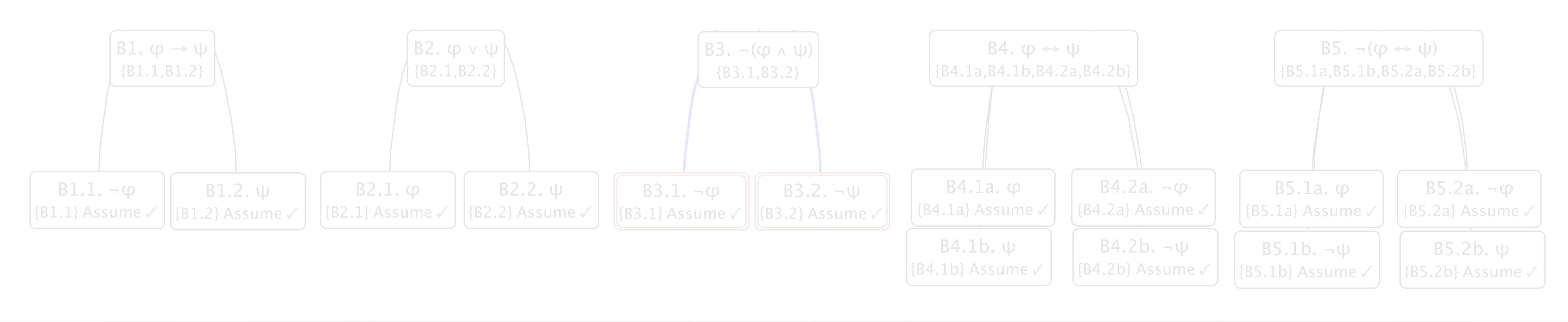
NB3. ¬(φ ∨ ψ)
{NB3.1,NB3.2}

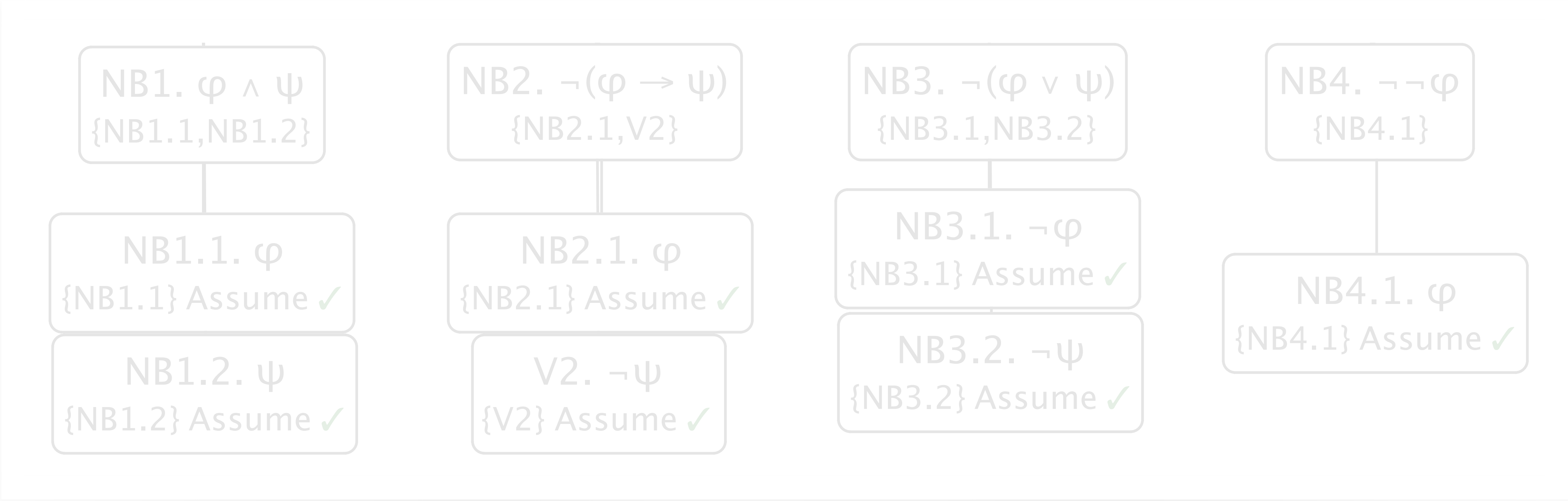
NB3.1. ¬φ
{NB3.1} Assume ✓

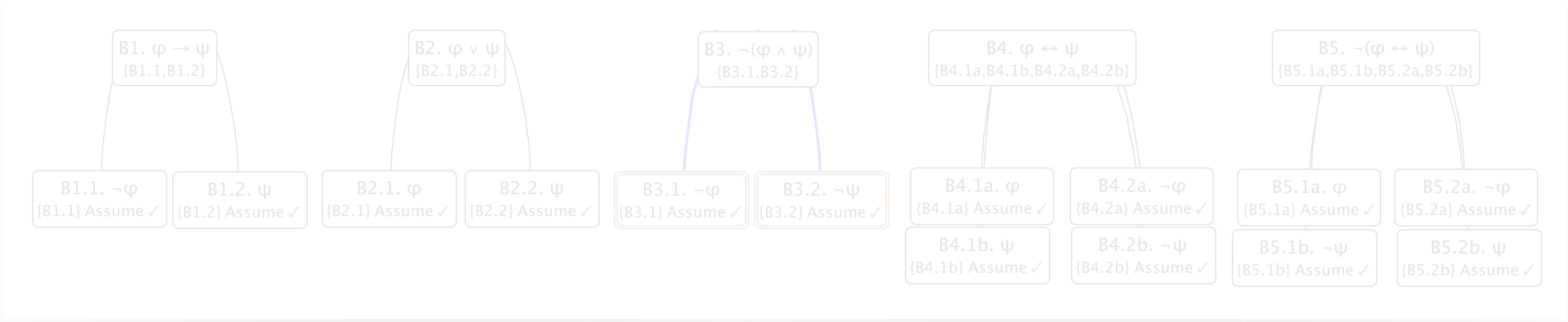
NB3.2. ¬ψ
{NB3.2} Assume ✓

NB4. ¬¬φ
{NB4.1}

NB4.1. φ
{NB4.1} Assume ✓


B1. φ → ψ
{B1.1,B1.2}
B1.1. ¬φ
{B1.1} Assume ✓
B1.2. ψ
{B1.2} Assume ✓
B2. φ ∨ ψ
{B2.1,B2.2}
B2.1. φ
{B2.1} Assume ✓
B2.2. ψ
{B2.2} Assume ✓
B3. ¬(φ ∧ ψ)
{B3.1,B3.2}
B3.1. ¬φ
{B3.1} Assume ✓
B3.2. ¬ψ
{B3.2} Assume ✓
B4. φ ↔ ψ
{B4.1a,B4.1b,B4.2a,B4.2b}
B4.1a. φ
{B4.1a} Assume ✓
B4.1b. ψ
{B4.1b} Assume ✓
B4.2a. ¬φ
{B4.2a} Assume ✓
B4.2b. ¬ψ
{B4.2b} Assume ✓
B5. ¬(φ ↔ ψ)
{B5.1a,B5.1b,B5.2a,B5.2b}
B5.1a. φ
{B5.1a} Assume ✓
B5.1b. ¬ψ
{B5.1b} Assume ✓
B5.2a. ¬φ
{B5.2a} Assume ✓
B5.2b. ψ
{B5.2b} Assume ✓


NB1. φ ∧ ψ
{NB1.1,NB1.2}
NB1.1. φ
{NB1.1} Assume ✓
NB1.2. ψ
{NB1.2} Assume ✓
NB2. ¬(φ → ψ)
{NB2.1,V2}
NB2.1. φ
{NB2.1} Assume ✓
V2. ¬ψ
{V2} Assume ✓
NB3. ¬(φ ∨ ψ)
{NB3.1,NB3.2}
NB3.1. ¬φ
{NB3.1} Assume ✓
NB3.2. ¬ψ
{NB3.2} Assume ✓
NB4. ¬¬φ
{NB4.1}
NB4.1. φ
{NB4.1} Assume ✓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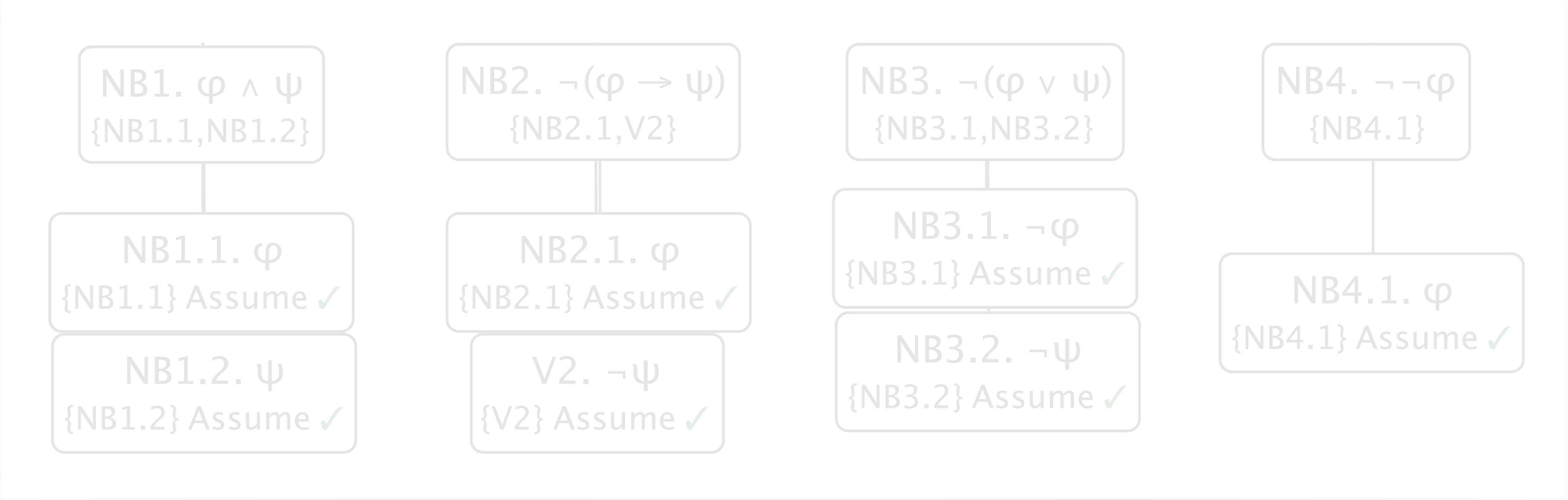
{B1.2} Assume ✓
B2. φ ∨ ψ
{B2.1,B2.2}
B2.1. φ
{B2.1} Assume ✓
B2.2. ψ
{B2.2} Assume ✓
B3. ¬(φ ∧ ψ)
{B3.1,B3.2}
B3.1. ¬φ
{B3.1} Assume ✓
B3.2. ¬ψ
{B3.2} Assume ✓
B4. φ ↔ ψ
{B4.1a,B4.1b,B4.2a,B4.2b}
B4.1a. φ
{B4.1a} Assume ✓
B4.1b. ψ
{B4.1b} Assume ✓
B4.2a. ¬φ
{B4.2a} Assume ✓
B4.2b. ¬ψ
{B4.2b} Assume ✓
B5. ¬(φ ↔ ψ)
{B5.1a,B5.1b,B5.2a,B5.2b}
B5.1a. φ
{B5.1a} Assume ✓
B5.1b. ¬ψ
{B5.1b} Assume ✓
B5.2a. ¬φ
{B5.2a} Assume ✓
B5.2b. ψ
{B5.2b} Assume ✓

Questions?


NB1. φ ∧ ψ
{NB1.1,NB1.2}
NB1.1. φ
{NB1.1} Assume ✓
NB1.2. ψ
{NB1.2} Assume ✓
NB2. ¬(φ → ψ)
{NB2.1,V2}
NB2.1. φ
{NB2.1} Assume ✓
V2. ¬ψ
{V2} Assume ✓
NB3. ¬(φ ∨ ψ)
{NB3.1,NB3.2}
NB3.1. ¬φ
{NB3.1} Assume ✓
NB3.2. ¬ψ
{NB3.2} Assume ✓
NB4. ¬¬φ
{NB4.1}
NB4.1. φ
{NB4.1} Assume ✓

Theorem:

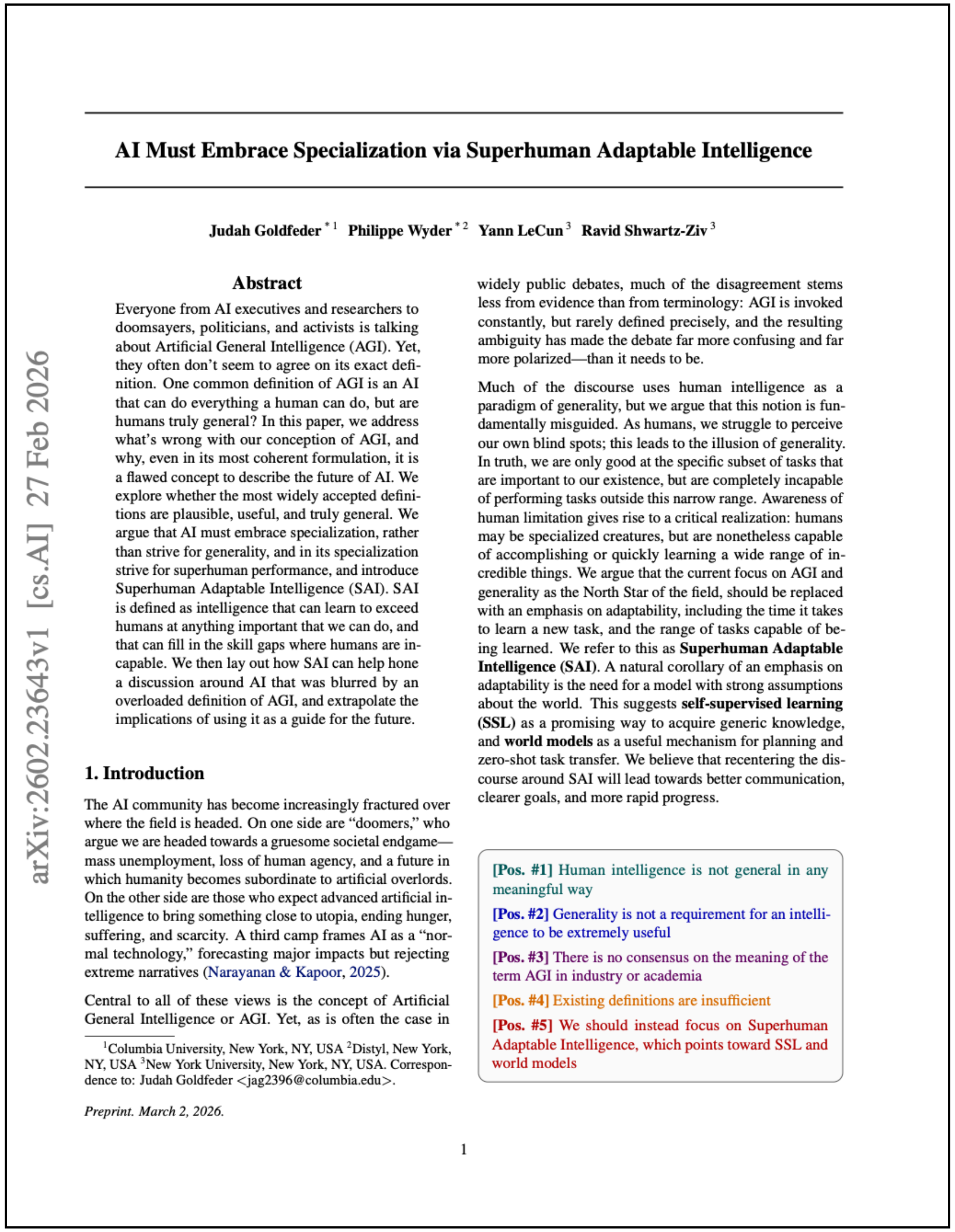
Let ϕ be a theorem in the propositional calculus $= \mathcal{L}_{\text{PC}}$.
Then the truth-tree algorithm will lead to no open branches.

On Measuring The Intelligence of Agents

…

using quantification …

Yet *Another* Concept of Machine Intelligence!



AI Must Embrace Specialization via Superhuman Adaptable Intelligence

Judah Goldfeder * [1] **Philippe Wyder** * [2] **Yann LeCun** [3] **Ravid Shwartz-Ziv** [3]

Abstract

Everyone from AI executives and researchers to doomsayers, politicians, and activists is talking about Artificial General Intelligence (AGI). Yet, they often don't seem to agree on its exact definition. One common definition of AGI is an AI that can do everything a human can do, but are humans truly general? In this paper, we address what's wrong with our conception of AGI, and why, even in its most coherent formulation, it is a flawed concept to describe the future of AI. We explore whether the most widely accepted definitions are plausible, useful, and truly general. We argue that AI must embrace specialization, rather than strive for generality, and in its specialization strive for superhuman performance, and introduce Superhuman Adaptable Intelligence (SAI). SAI is defined as intelligence that can learn to exceed humans at anything important that we can do, and that can fill in the skill gaps where humans are incapable. We then lay out how SAI can help hone a discussion around AI that was blurred by an overloaded definition of AGI, and extrapolate the implications of using it as a guide for the future.

1. Introduction

The AI community has become increasingly fractured over where the field is headed. On one side are "doomers," who argue we are headed towards a gruesome societal endgame—mass unemployment, loss of human agency, and a future in which humanity becomes subordinate to artificial overlords. On the other side are those who expect advanced artificial intelligence to bring something close to utopia, ending hunger, suffering, and scarcity. A third camp frames AI as a "normal technology," forecasting major impacts but rejecting extreme narratives (Narayanan & Kapoor, 2025).

Central to all of these views is the concept of Artificial General Intelligence or AGI. Yet, as is often the case in widely public debates, much of the disagreement stems less from evidence than from terminology: AGI is invoked constantly, but rarely defined precisely, and the resulting ambiguity has made the debate far more confusing and far more polarized—than it needs to be.

Much of the discourse uses human intelligence as a paradigm of generality, but we argue that this notion is fundamentally misguided. As humans, we struggle to perceive our own blind spots; this leads to the illusion of generality. In truth, we are only good at the specific subset of tasks that are important to our existence, but are completely incapable of performing tasks outside this narrow range. Awareness of human limitation gives rise to a critical realization: humans may be specialized creatures, but are nonetheless capable of accomplishing or quickly learning a wide range of incredible things. We argue that the current focus on AGI and generality as the North Star of the field, should be replaced with an emphasis on adaptability, including the time it takes to learn a new task, and the range of tasks capable of being learned. We refer to this as **Superhuman Adaptable Intelligence (SAI)**. A natural corollary of an emphasis on adaptability is the need for a model with strong assumptions about the world. This suggests **self-supervised learning (SSL)** as a promising way to acquire generic knowledge, and **world models** as a useful mechanism for planning and zero-shot task transfer. We believe that recentering the discourse around SAI will lead towards better communication, clearer goals, and more rapid progress.

[Pos. #1] Human intelligence is not general in any meaningful way

[Pos. #2] Generality is not a requirement for an intelligence to be extremely useful

[Pos. #3] There is no consensus on the meaning of the term AGI in industry or academia

[Pos. #4] Existing definitions are insufficient

[Pos. #5] We should instead focus on Superhuman Adaptable Intelligence, which points toward SSL and world models

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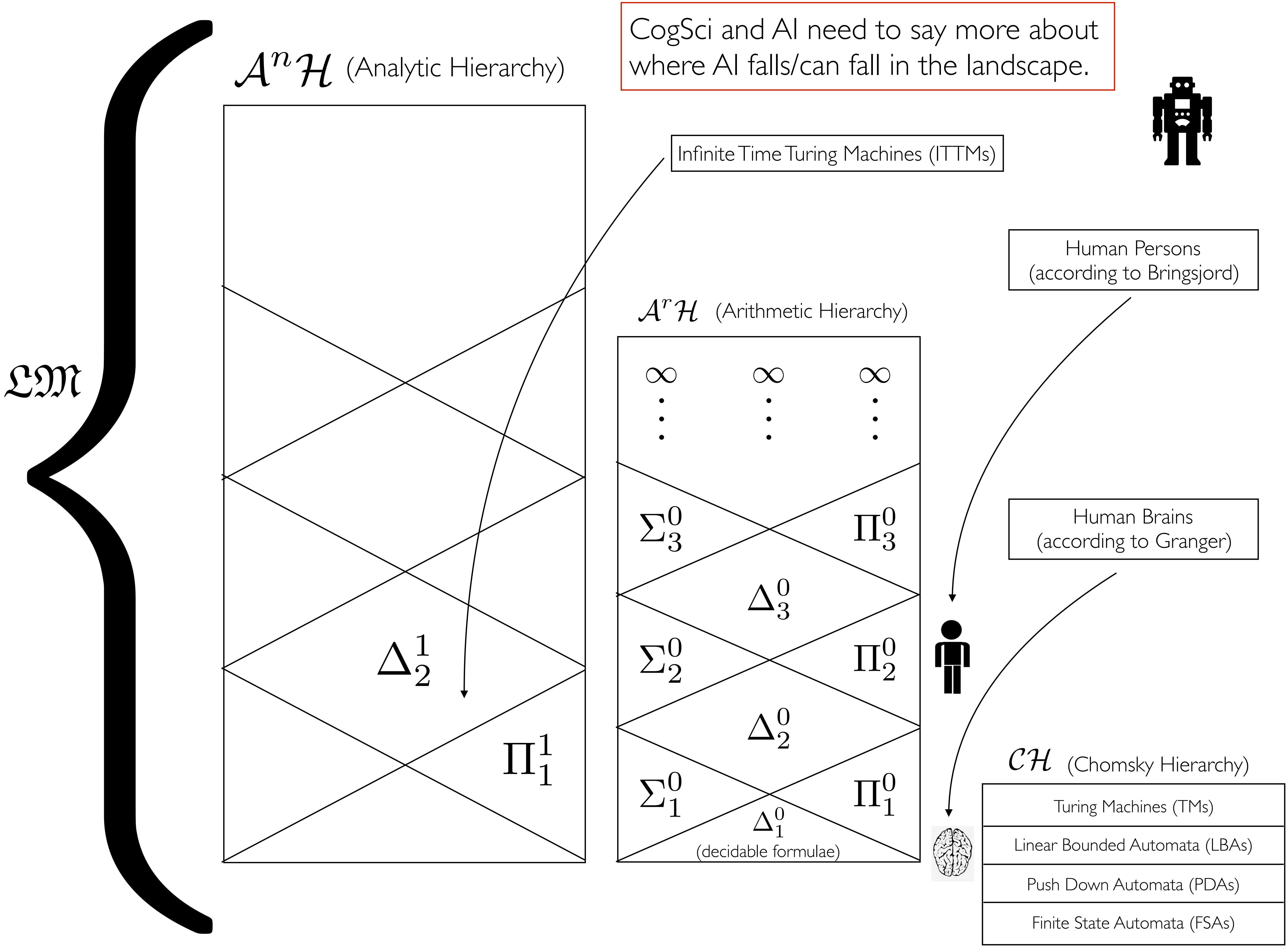
Preprint. March 2, 2026.

https://arxiv.org/pdf/2602.23643

CogSci and AI need to say more about where AI falls/can fall in the landscape.



$\mathfrak{LM}$

$\mathcal{A}^n\mathcal{H}$ (Analytic Hierarchy)

Δ^1_2

Π^1_1

Infinite Time Turing Machines (ITTMs)

$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)

∞ ∞ ∞

Σ^0_3 Π^0_3

Δ^0_3

Σ^0_2 Π^0_2

Δ^0_2

Σ^0_1 Π^0_1

Δ^0_1
(decidable formulae)

Human Persons
(according to Bringsjord)

Human Brains
(according to Granger)

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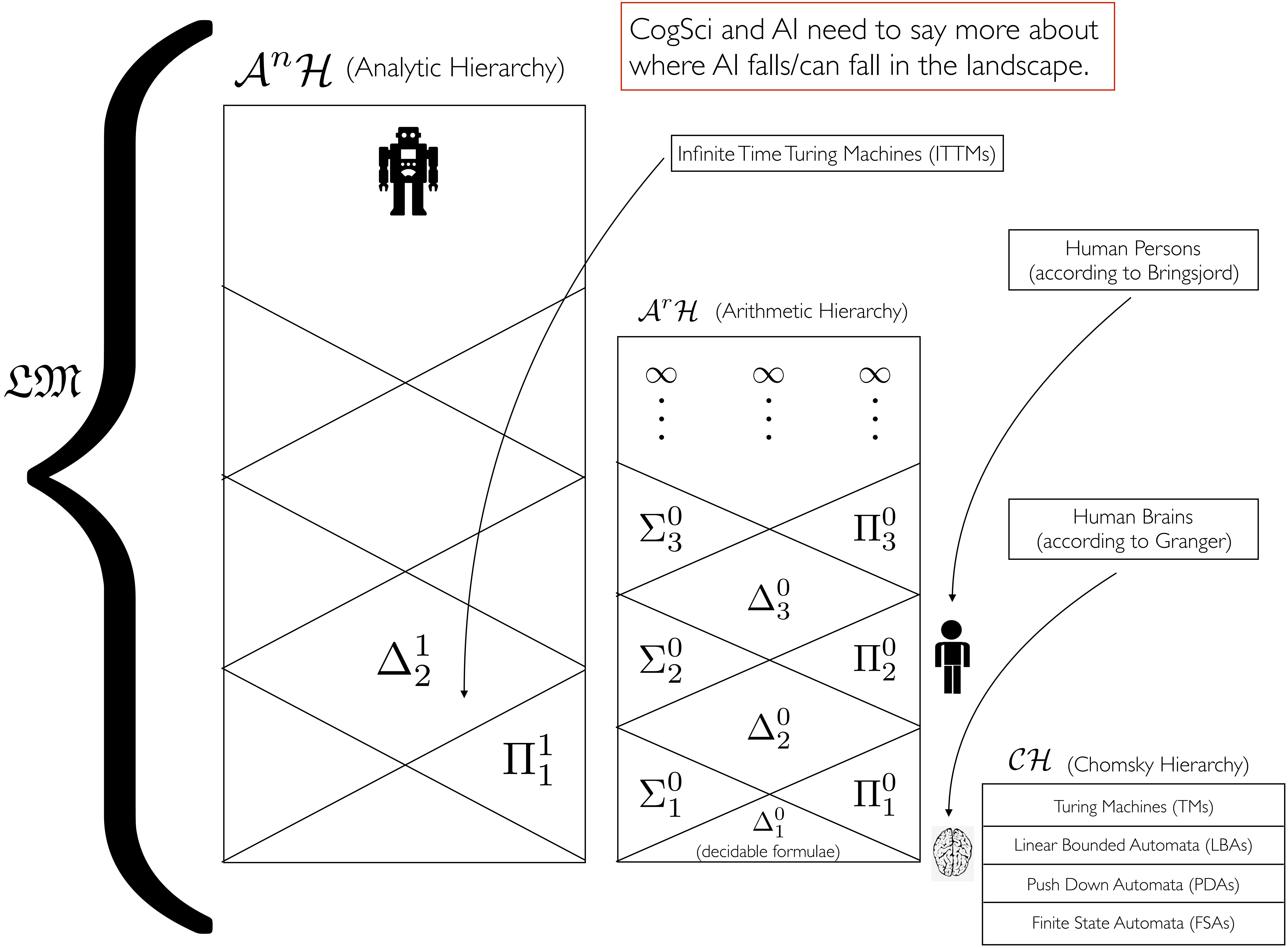

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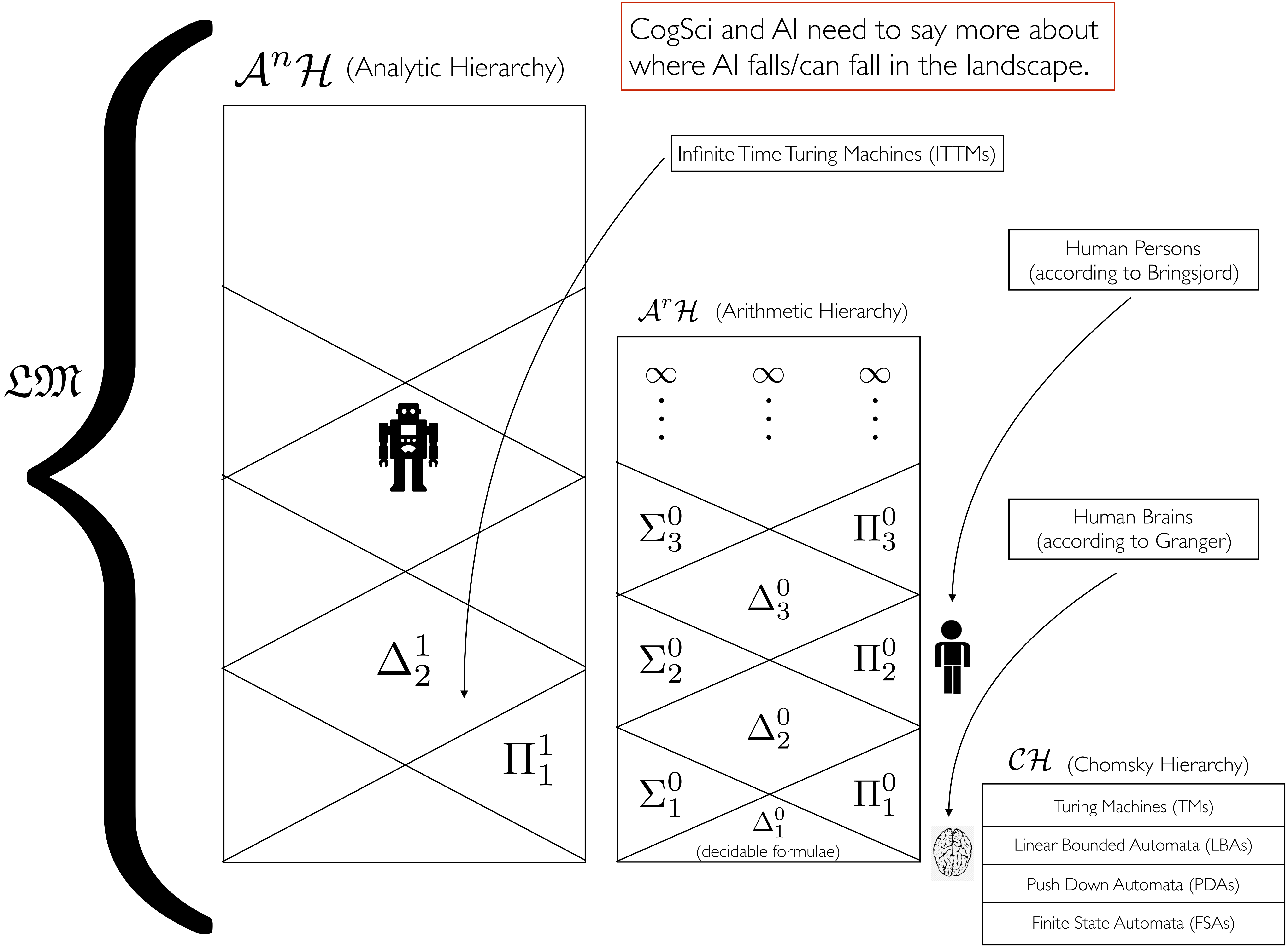

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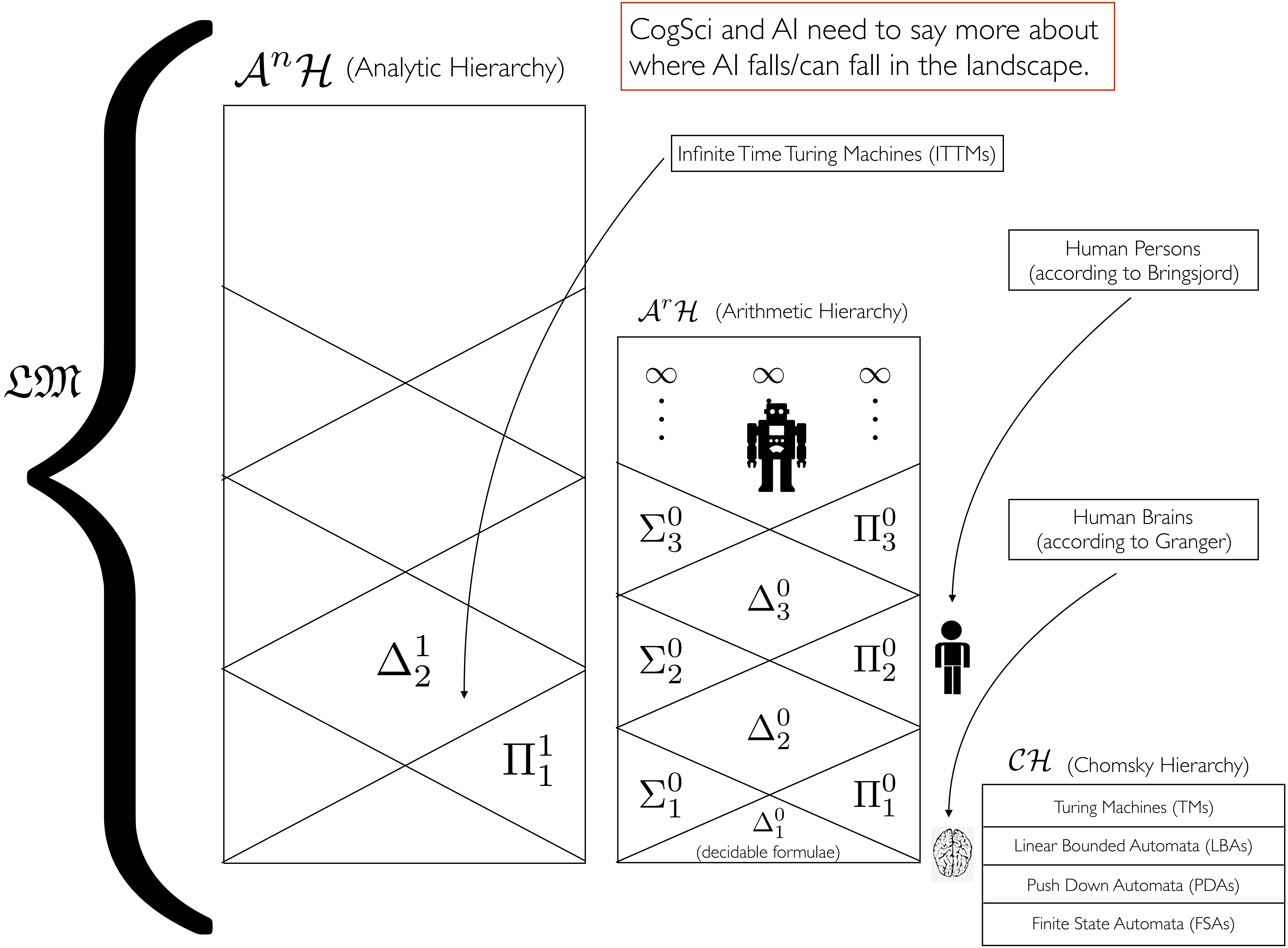

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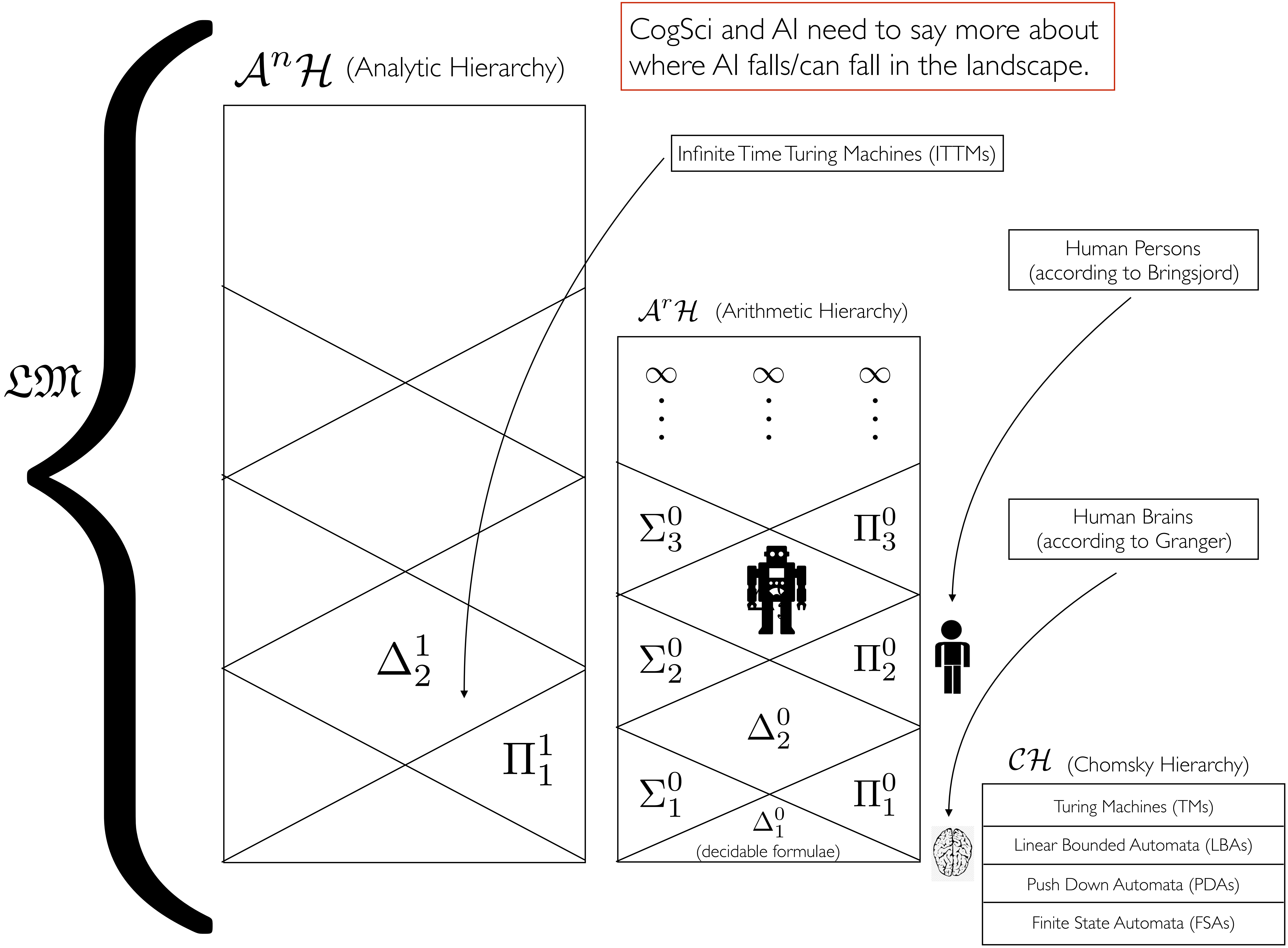

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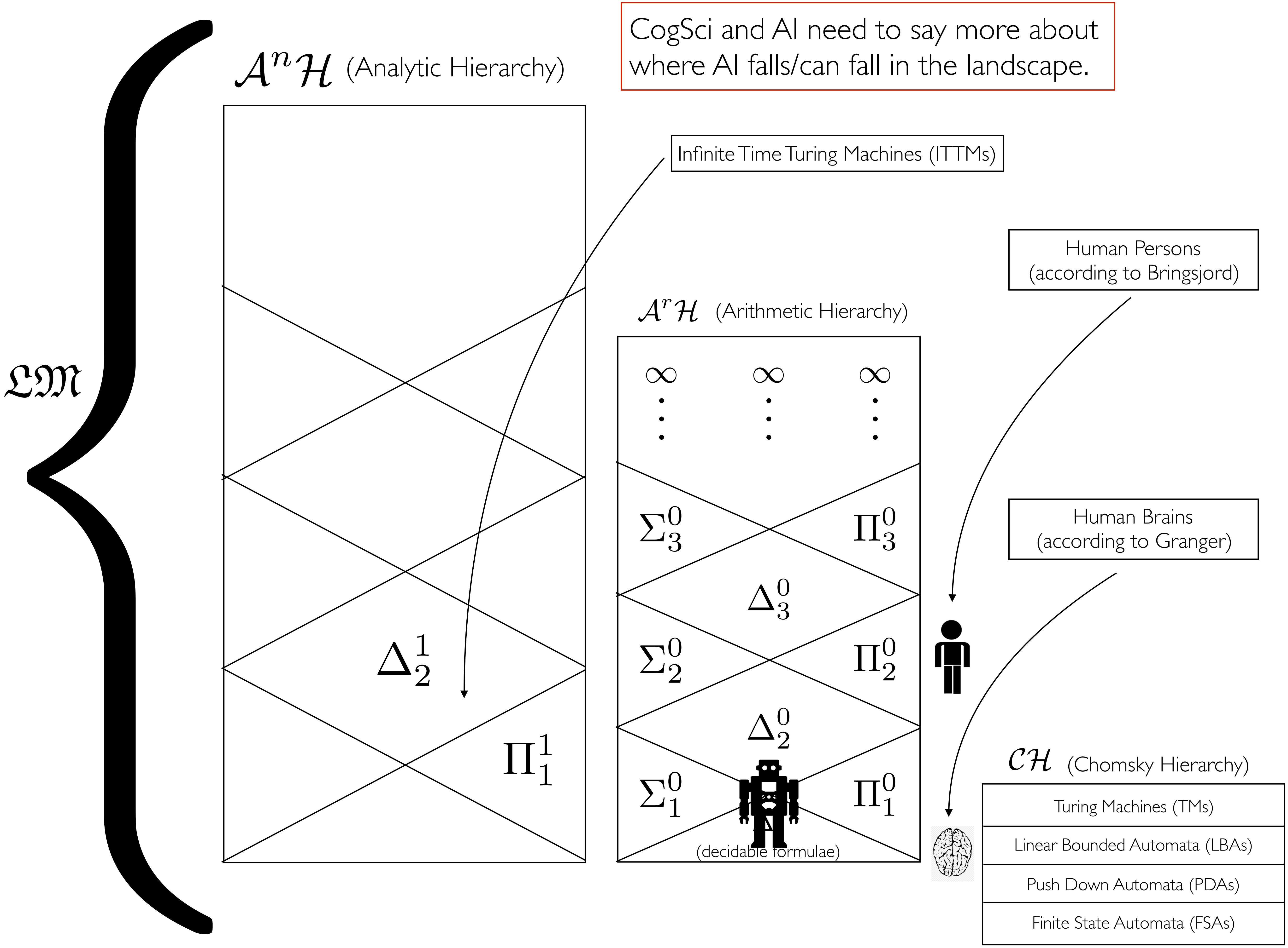

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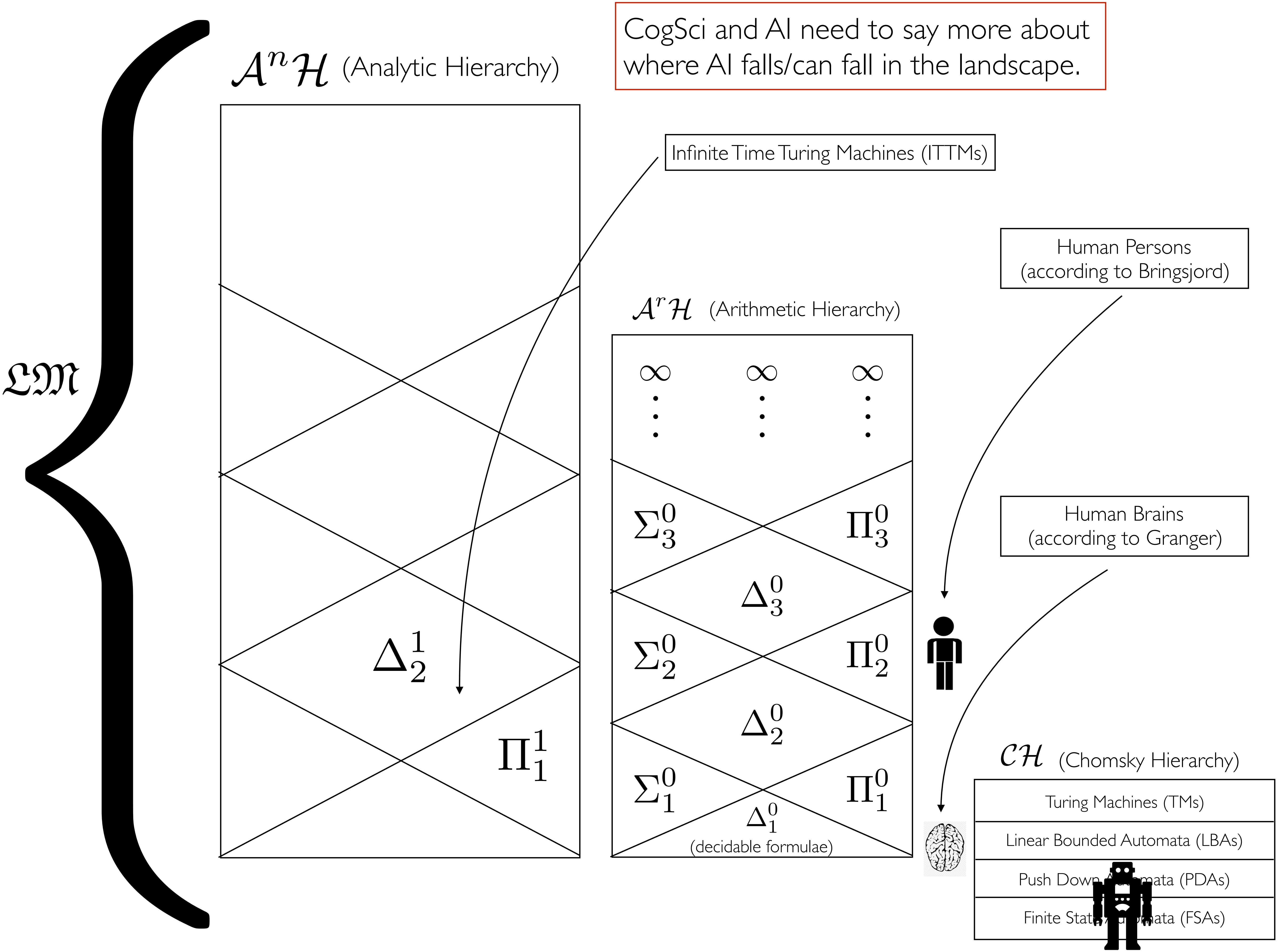

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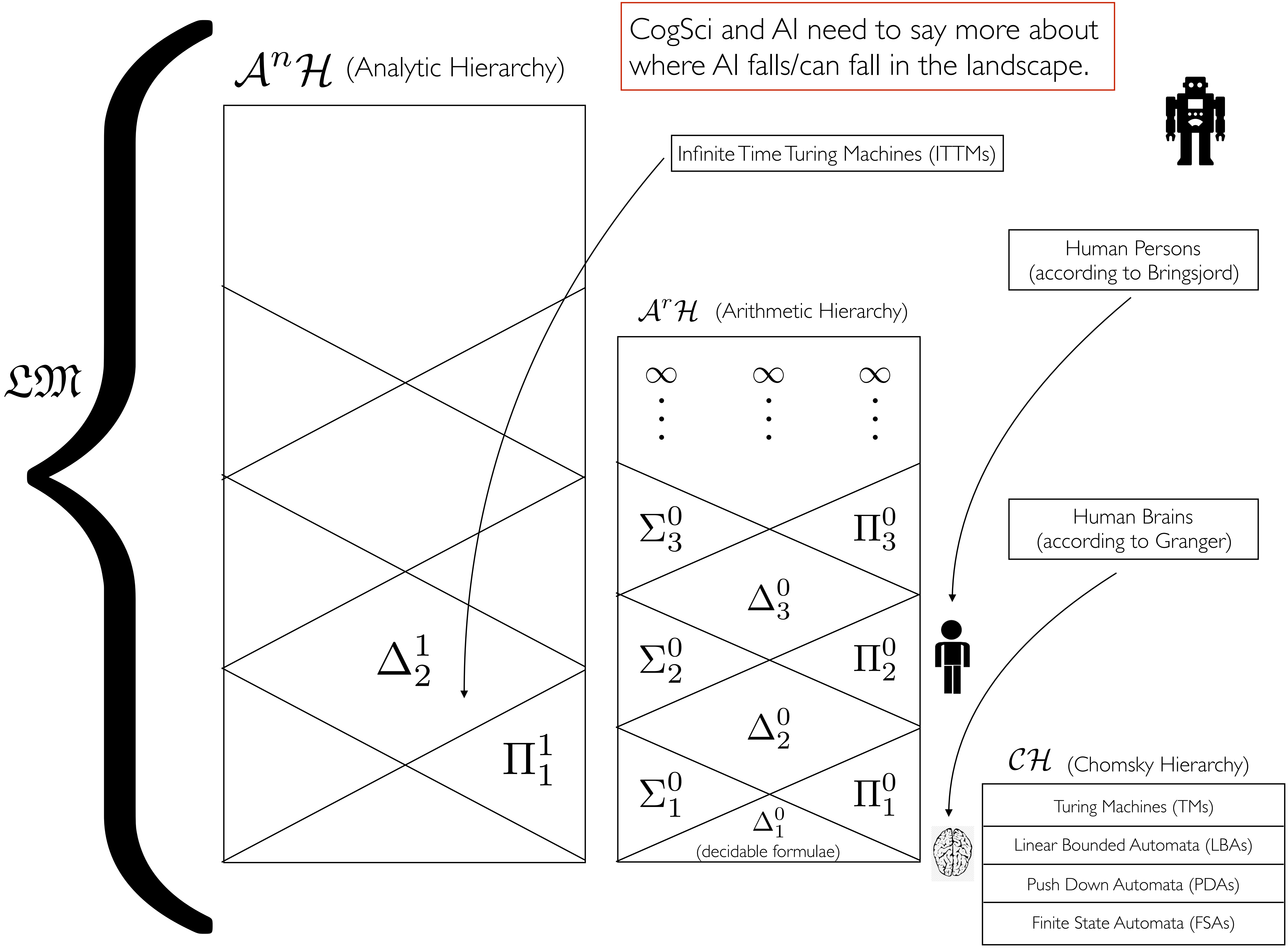

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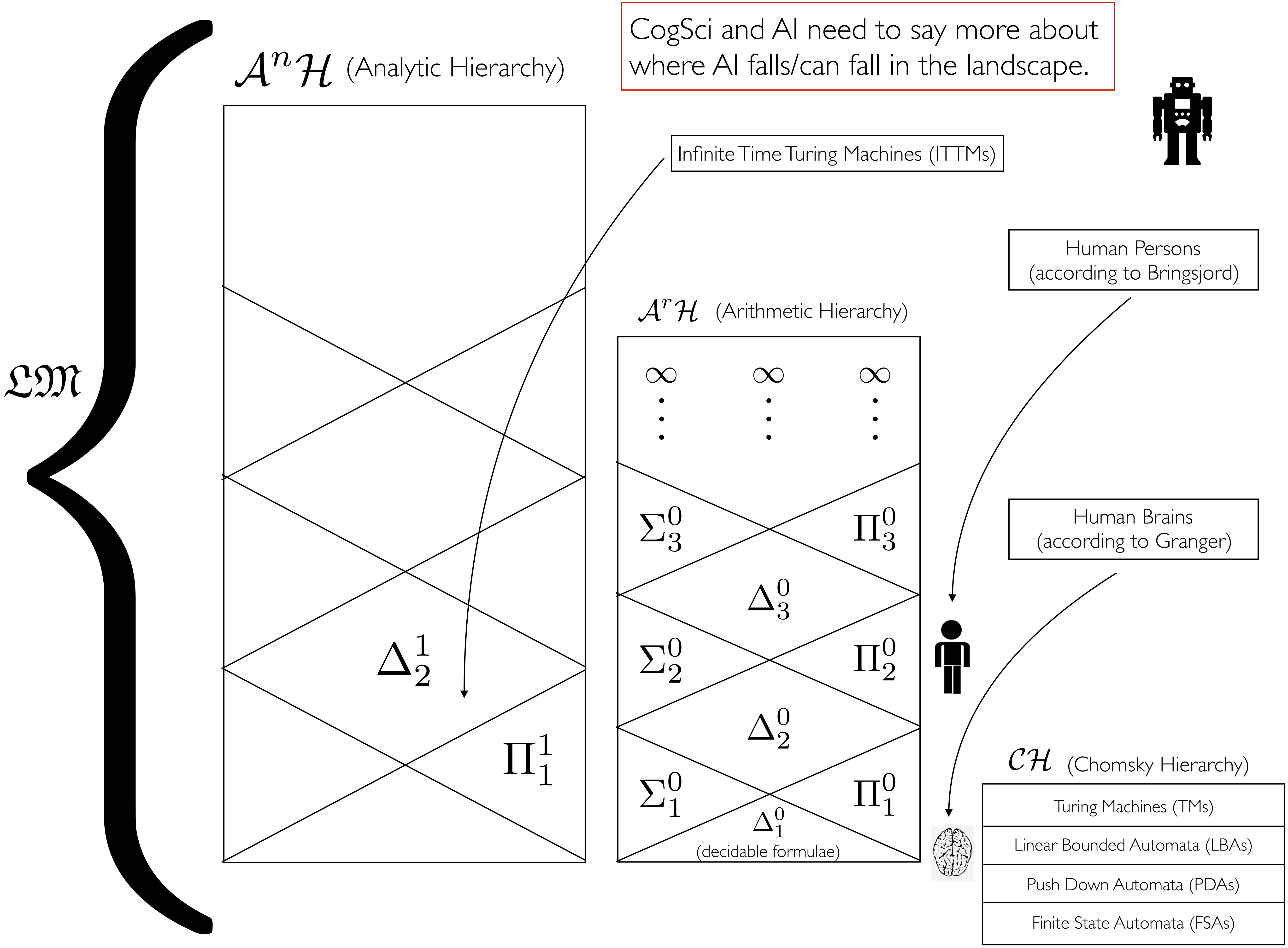
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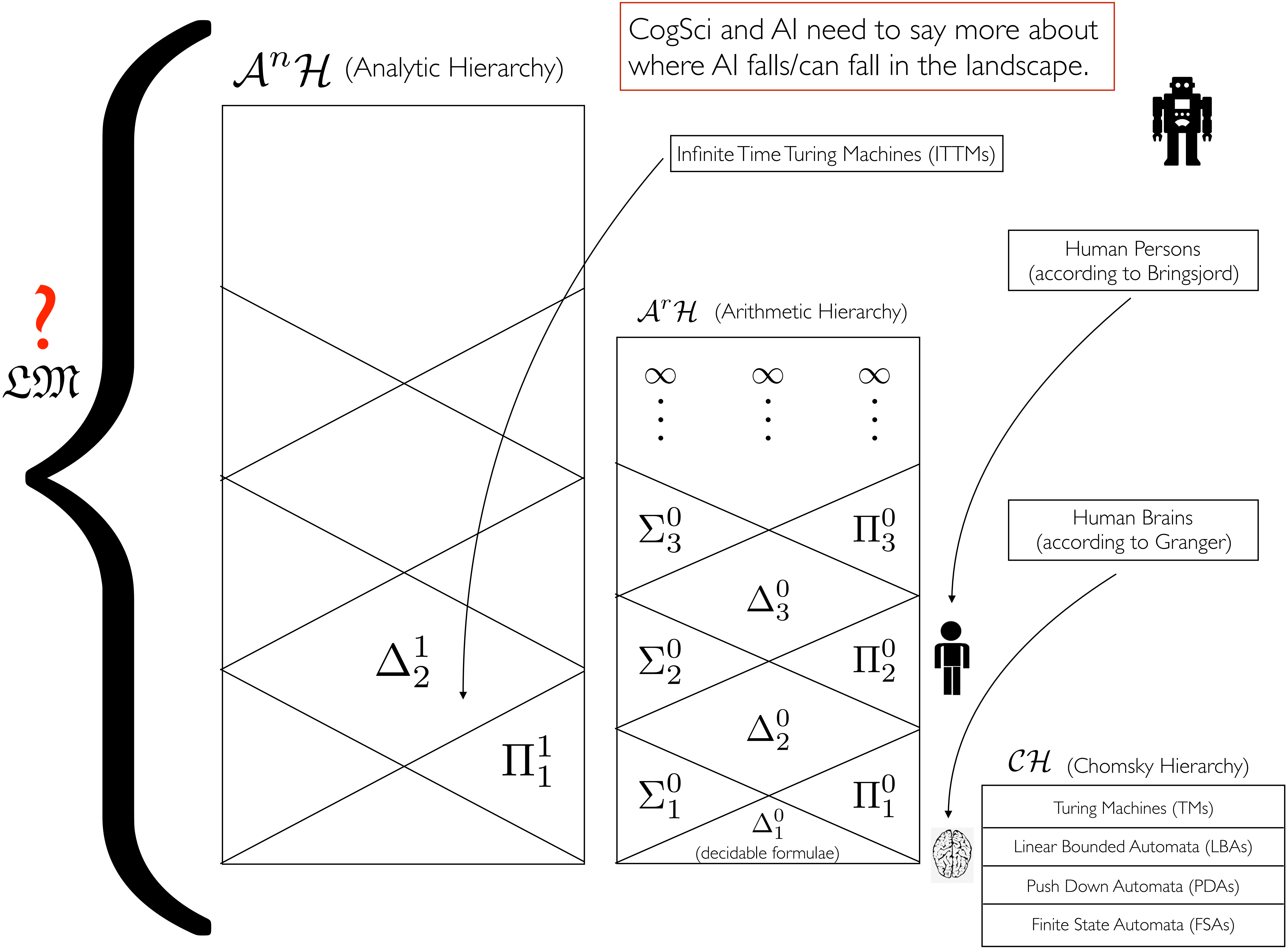

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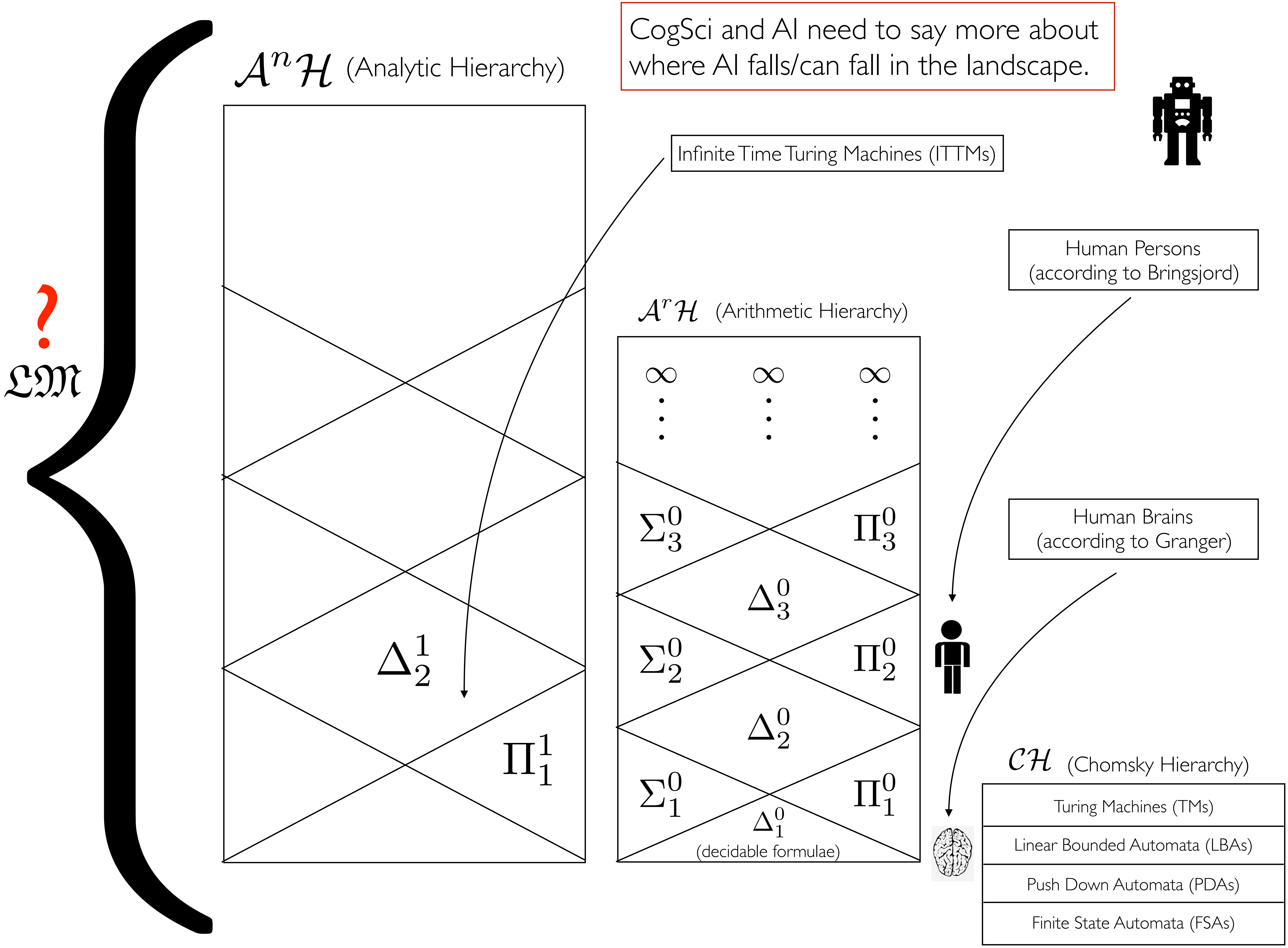

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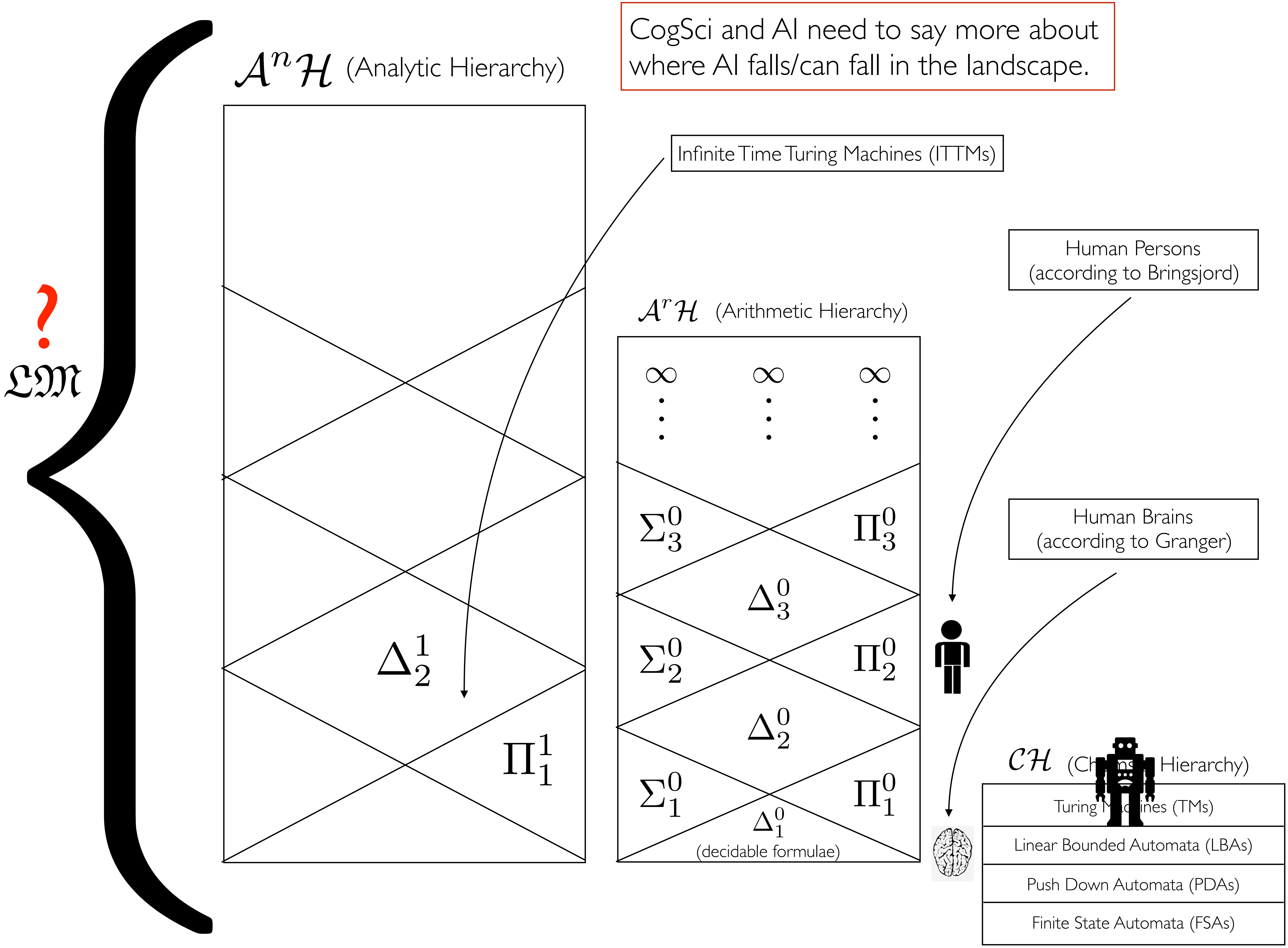

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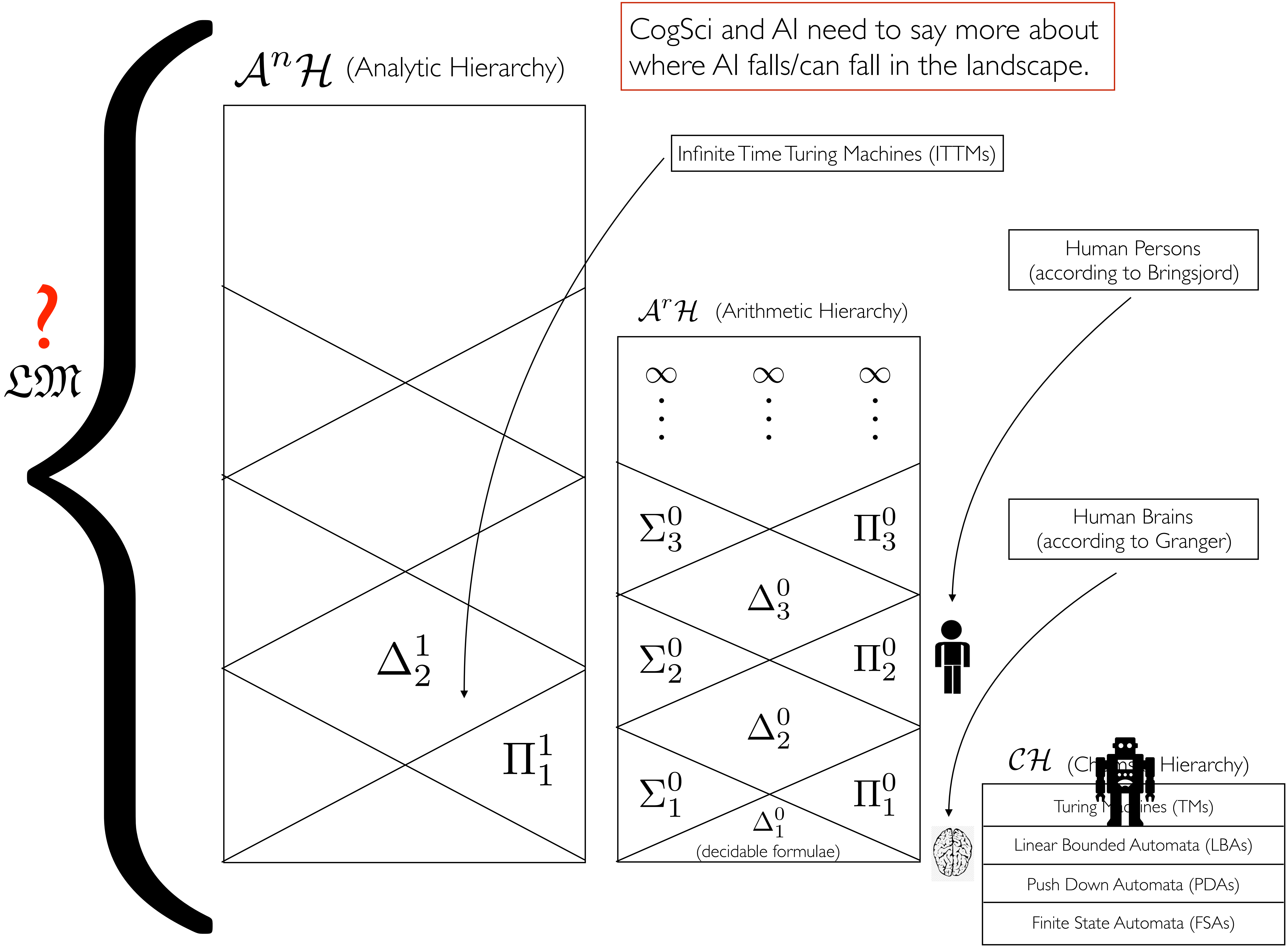

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Measuring Intelligence & AI/The Singularity

Measuring Intelligence & AI/The Singularity

The Singularity (superhuman machine intelligence) is near!!

Measuring Intelligence & AI/The Singularity

The Singularity (superhuman machine intelligence) is near!!

Is that so? And how are you measuring intelligence, pray tell?

Measuring Intelligence & AI/The Singularity

Is that so? And how are you measuring intelligence, pray tell?

Measuring Intelligence & AI/The Singularity

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Polynomial Hierarchy

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Measuring Intelligence & AI/The Singularity

Checkers:Chinook

●

Polynomial Hierarchy

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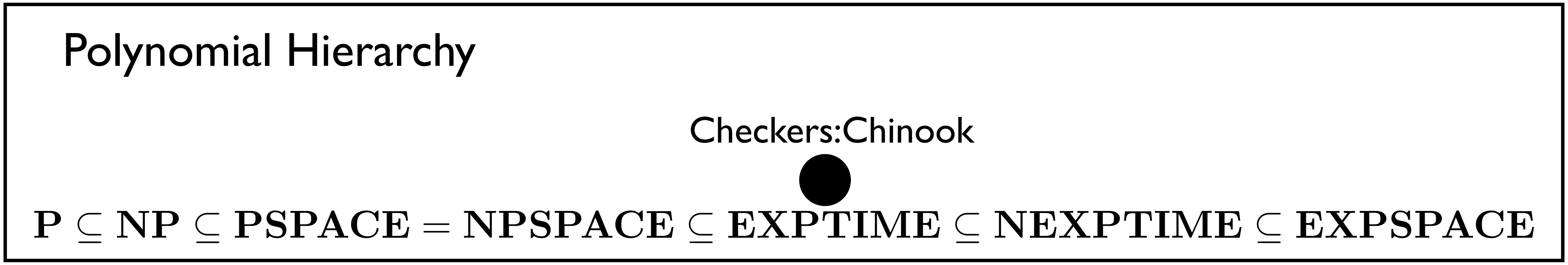

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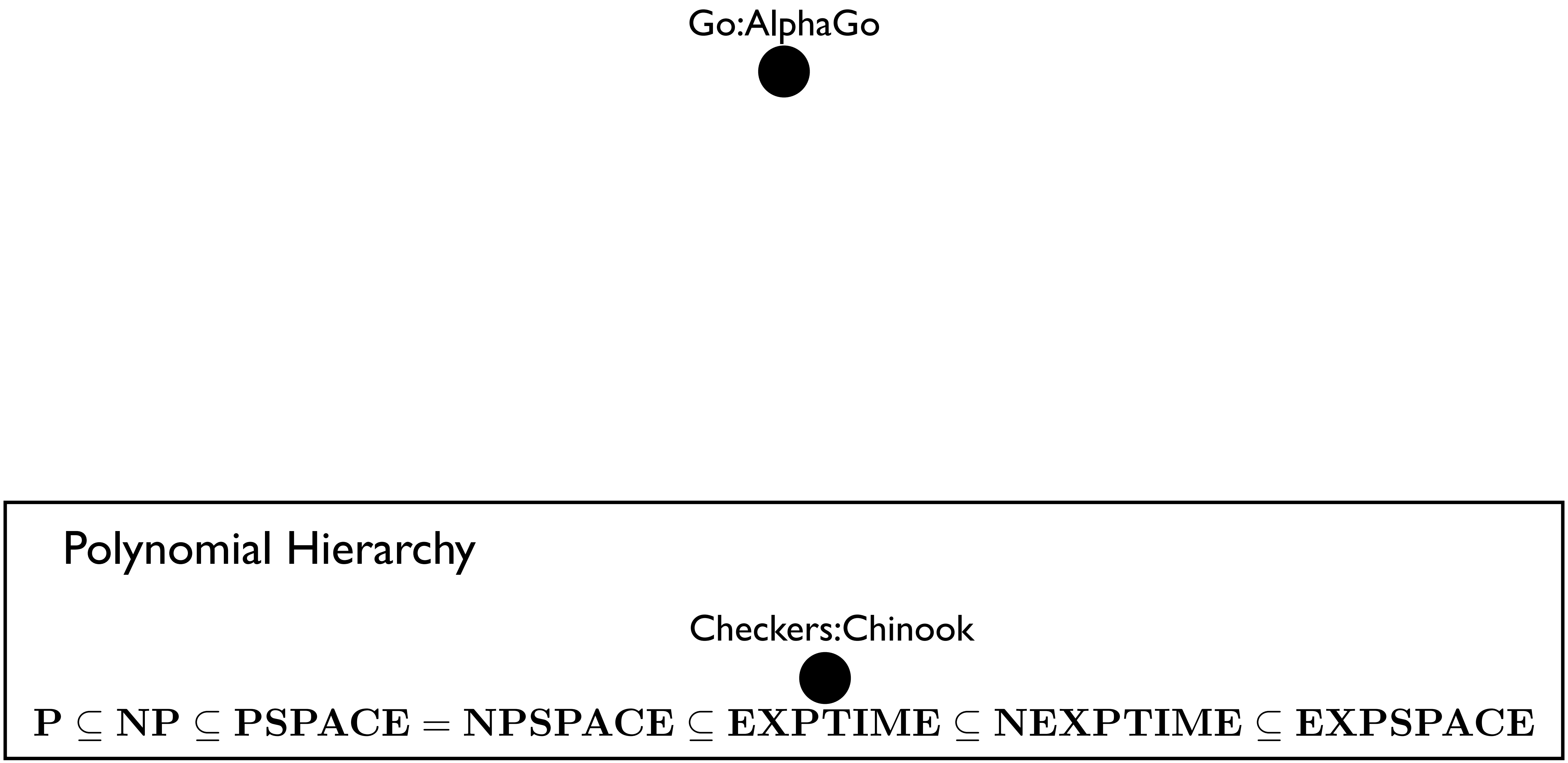

Go:AlphaGo

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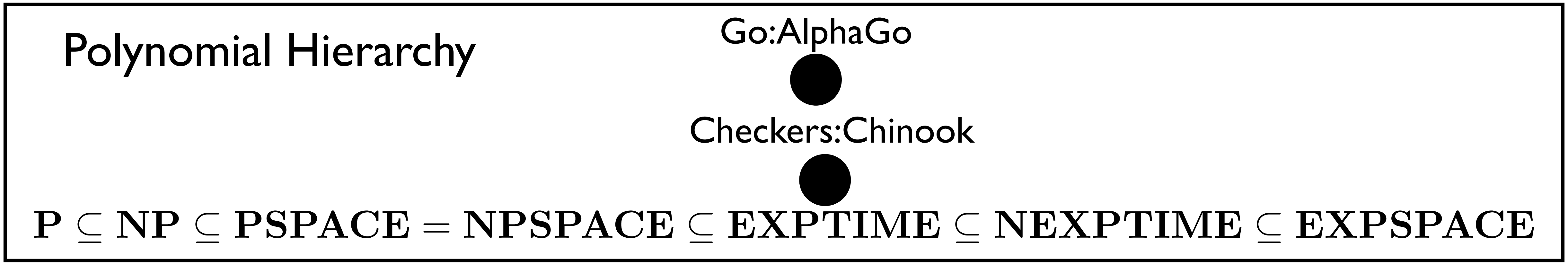

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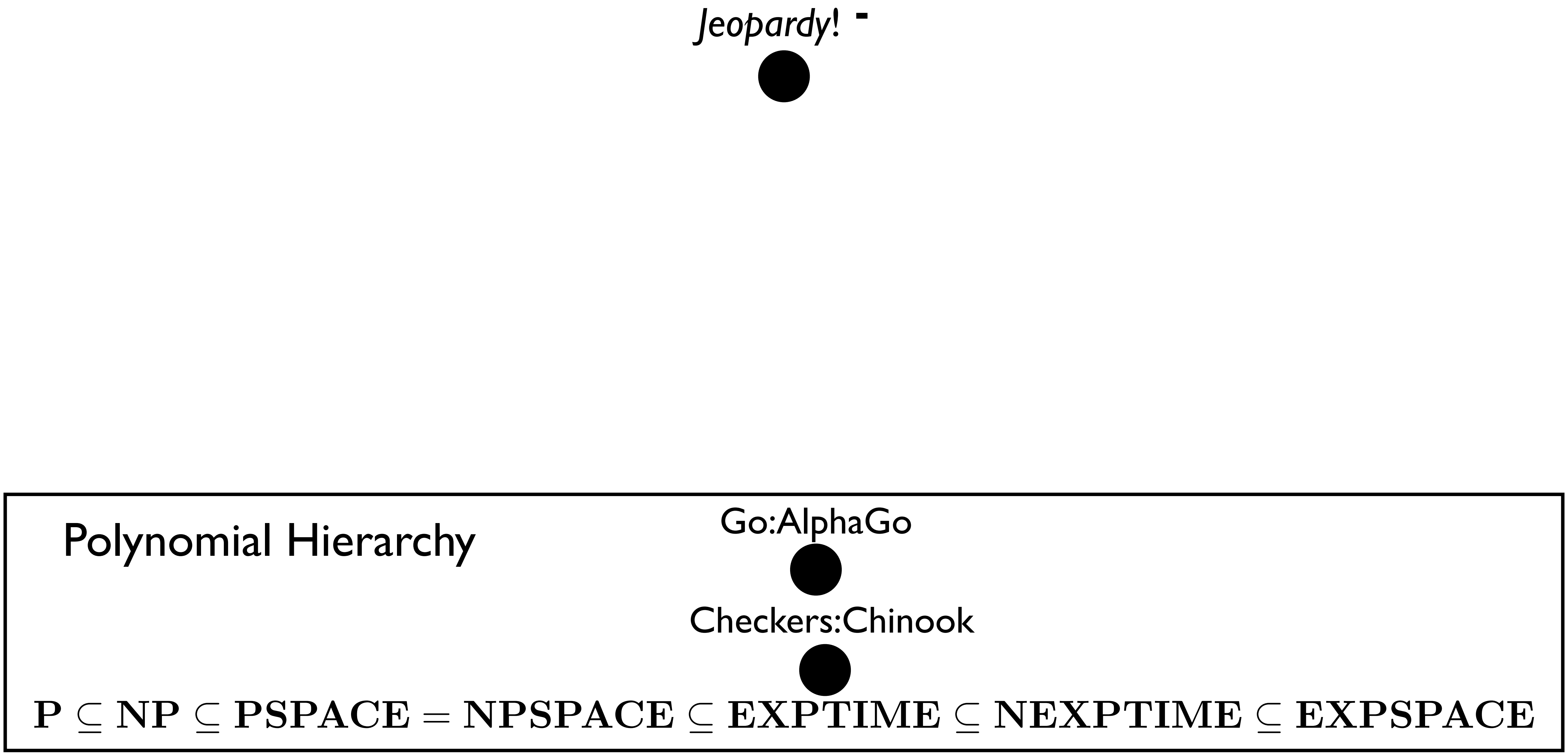

Jeopardy! -

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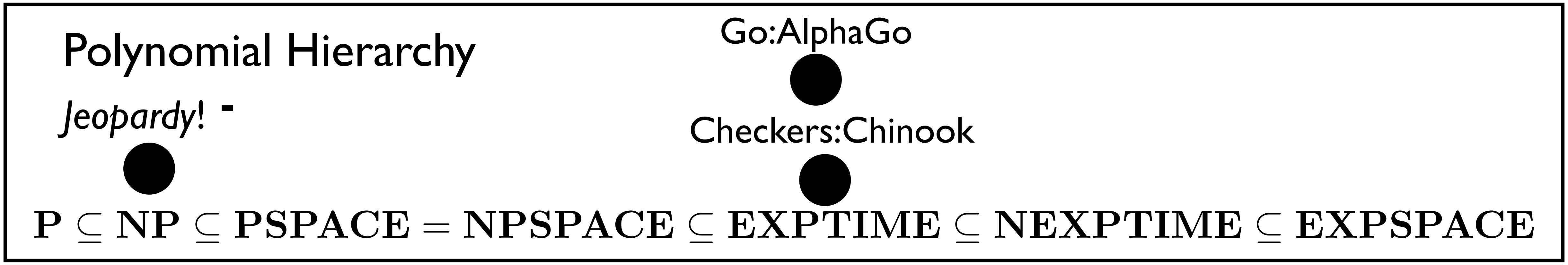

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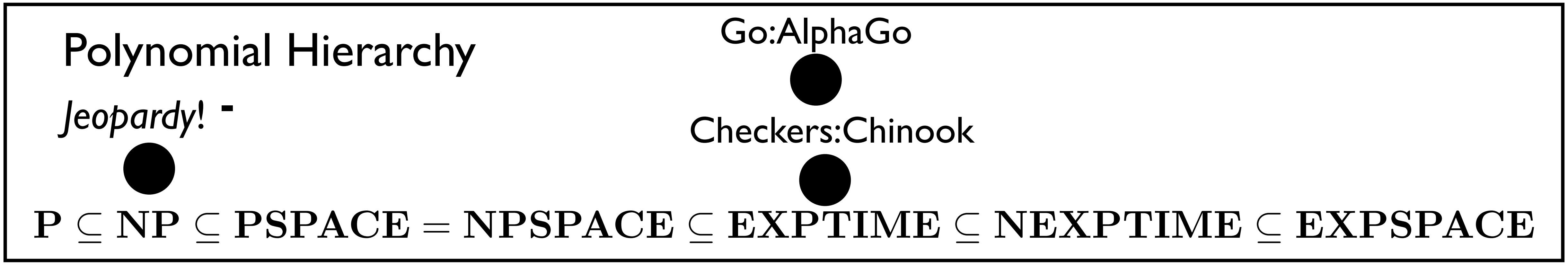

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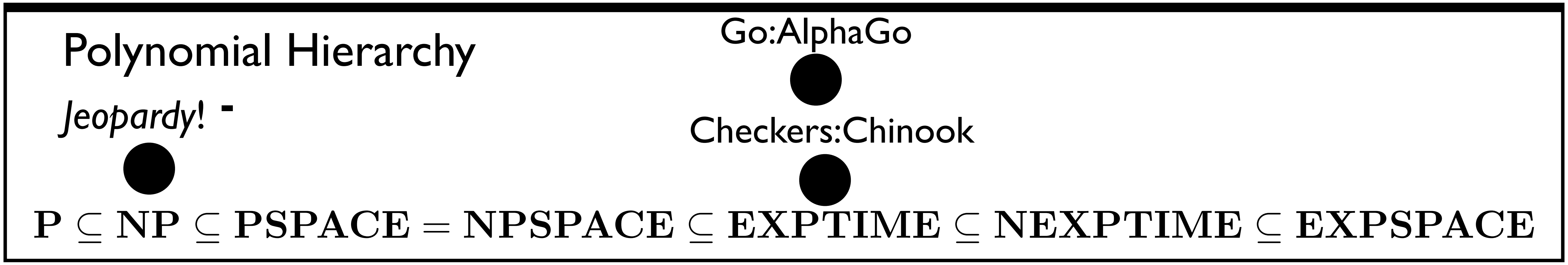

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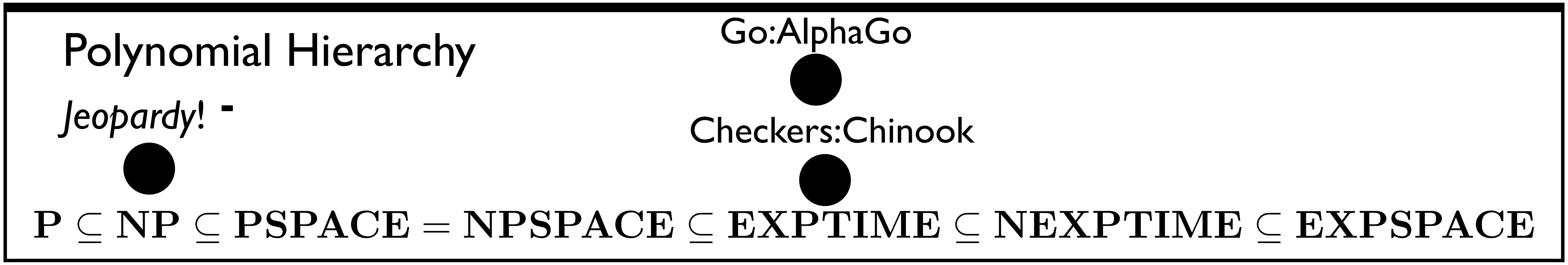
Measuring Intelligence & AI/The Singularity



Polynomial Hierarchy

Jeopardy! -

Go:AlphaGo

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy

Polynomial Hierarchy

Go:AlphaGo

Jeopardy!

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Arithmetical Hierarchy

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Polynomial Hierarchy

Go:AlphaGo

●

Jeopardy! -

Checkers:Chinook

● ●

$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$

Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy

$\vdots$

“Hey, do these two Java programs compute the very same function?” Π_2

Σ_2

Π_1

Σ_1

Σ_0

Polynomial Hierarchy

Go:AlphaGo

●

Jeopardy! -

Checkers:Chinook

● ●

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Analytical Hierarchy

Arithmetical Hierarchy

“Hey, do these two Java programs compute the very same function?”

$$\vdots$$
$$\Pi_2$$
$$\Sigma_2$$
$$\Pi_1$$
$$\Sigma_1$$
$$\Sigma_0$$

Polynomial Hierarchy

Go:AlphaGo

Jeopardy! -

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Analytical Hierarchy

Arithmetical Hierarchy

This, all of this, is derived from consideration of first-order logic and second-order logic, with an emphasis on *quantification* and proof.

"Hey, do these two Java programs compute the very same function?"

$$\begin{matrix} \vdots \\ \Pi_2 \\ \Sigma_2 \\ \Pi_1 \\ \Sigma_1 \\ \Sigma_0 \end{matrix}$$

Polynomial Hierarchy

Jeopardy! ●

Go:AlphaGo ●

Checkers:Chinook ●

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An “Advanced” Topic for Measuring Intelligence …

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- FOL formulae that (only) enforce domain size:

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$$\exists x \exists y (x \neq y)$$

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$$\underline{\phi_n}$$

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$\vdots$

$\underline{\phi_n}$ domain of at least *n* things

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$$\vdots$$

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$$\exists x \forall y (y = x)$$

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$\exists x \exists y \forall z (z = x \vee z = y)$

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$$\vdots$$

$$\overline{\phi_n}$$

For now, let’s settle for
a focus on
quantification. Then …

Measuring AI Intelligence via (in part) Logic:Quantification

Toby Walsh: “The Singularity May Never Be Near”
(http://arxiv.org/pdf/1602.06462v1.pdf)

Measuring AI Intelligence via (in part) Logic:Quantification

Toby Walsh: "The Singularity May Never Be Near"
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"I will not tackle here head on what we mean by measuring the intelligence of machines (or of humans). I will simply suppose there is such a property as intelligence, that it can be measured and compared, and that the technological singularity is when this measure increases exponentially fast in an appropriate and reasonable scale." (p. 1)

But logico-mathematical definitions of intelligence for animals, humans, machines, aliens, gods … *are* possible; recall our consideration of the *Entscheidungsproblem*. We can specifically challenge today’s AI on the basis of simple quantification and simple deduction …

First, need some numerical quantifiers:

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$$\exists x \forall y (y = x \wedge \phi(x)) \text{ will be } \exists^{=1} x \phi(x)$$

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How do we define formulae of this type: $\exists^{=k} x \psi(x)$

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Okay, now AI:

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Okay, now AI:

At least seven kenspeckle blookers are red.

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Given this, is it true that there are two red blookers? Why, exactly?

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Okay, now AI:

At least seven kenspeckle blookers are red.

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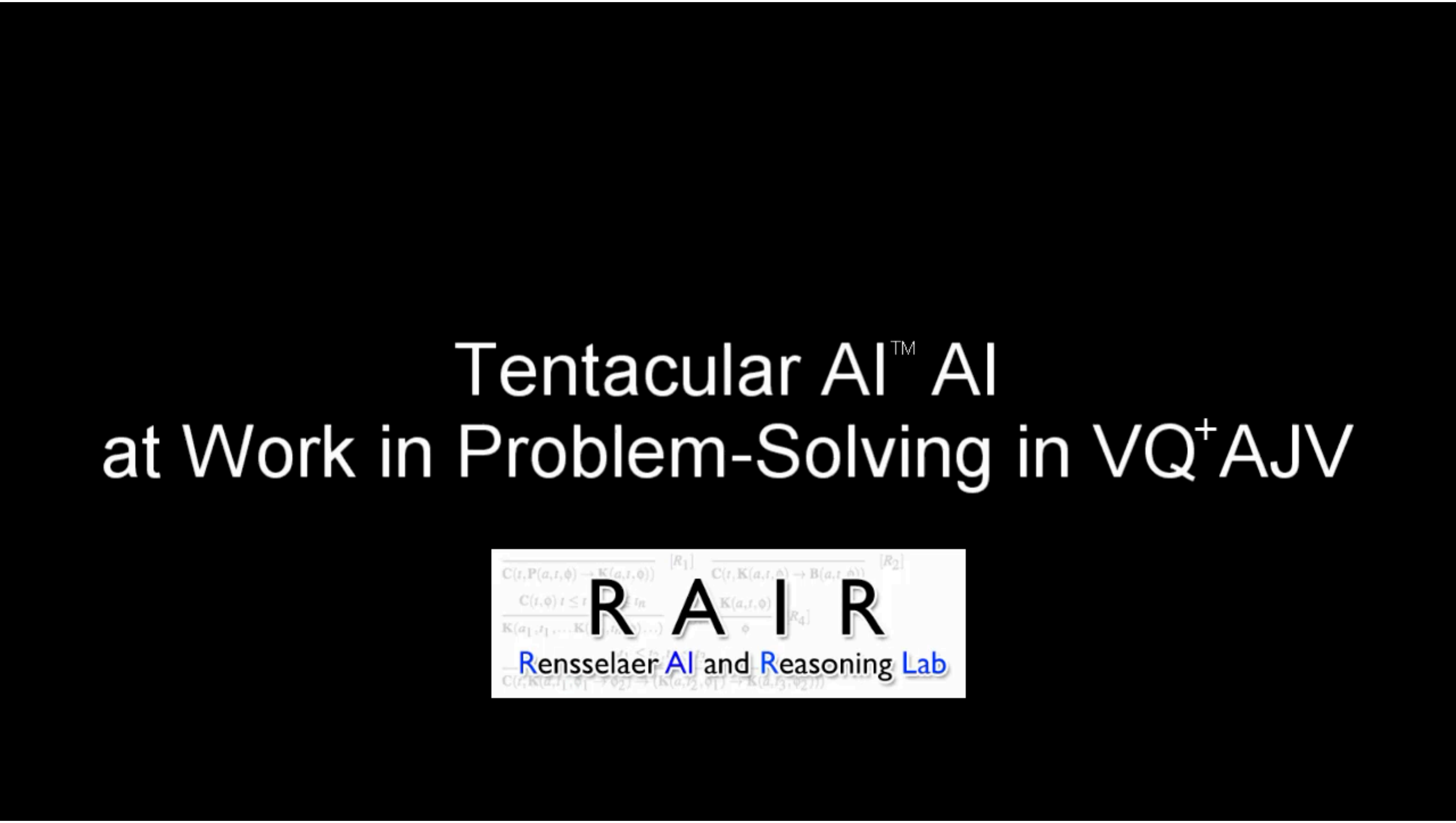
$$
\begin{array}{r}
\forall x \forall y \forall z \ \{[x \neq y \land y \neq z \land x \neq z \land Cx \land Cy \land Cz \ \land \\
Tz' \land \\
\exists w_1 \exists w_2 \ (w_1 \neq w_2 \land Aw_1 \land Aw_2 \land Gz'w_1 \land Gz'w_2) \ \land \\
\forall u_1 \forall u_2 \forall u_3 \ ((Gz'u_1 \land Gz'u_2 \land Gz'u_3 \land C^b u_1 \land C^b u_2 \land C^b u_3) \rightarrow \\
\forall v((Gz'v \land C^b v) \rightarrow (v = u_1 \lor v = u_2 \lor v = u_3)))] \\
\rightarrow \\
(Gxz' \land Gyz' \land Gzz')\}
\end{array}
$$

Every three cylinders glower at any triangular prism that glowers at at least two arches and at at most three cubes.

$$\forall x \forall y \forall z \forall z' \left\{ \begin{array}{c} \left[\begin{array}{c} x \neq y \wedge y \neq z \wedge x \neq z \\ \wedge \\ Cx \wedge Cy \wedge Cz \\ \wedge \\ Tz' \\ \wedge \\ \exists w_1 \exists w_2 \ (w_1 \neq w_2 \wedge Aw_1 \wedge Aw_2 \wedge Gz'w_1 \wedge Gz'w_2) \\ \wedge \\ \forall u_1 \forall u_2 \forall u_3 \quad \left(\begin{array}{c} \big[Gz'u_1 \wedge Gz'u_2 \wedge Gz'u_3 \wedge C^b u_1 \wedge C^b u_2 \wedge C^b u_3\big] \\ \to \\ \forall v \big[(Gz'v \wedge C^b v) \to (v = u_1 \vee v = u_2 \vee v = u_3)\big] \end{array} \right) \end{array} \right] \\ \to \\ (Gxz' \wedge Gyz' \wedge Gzz') \end{array} \right\}$$

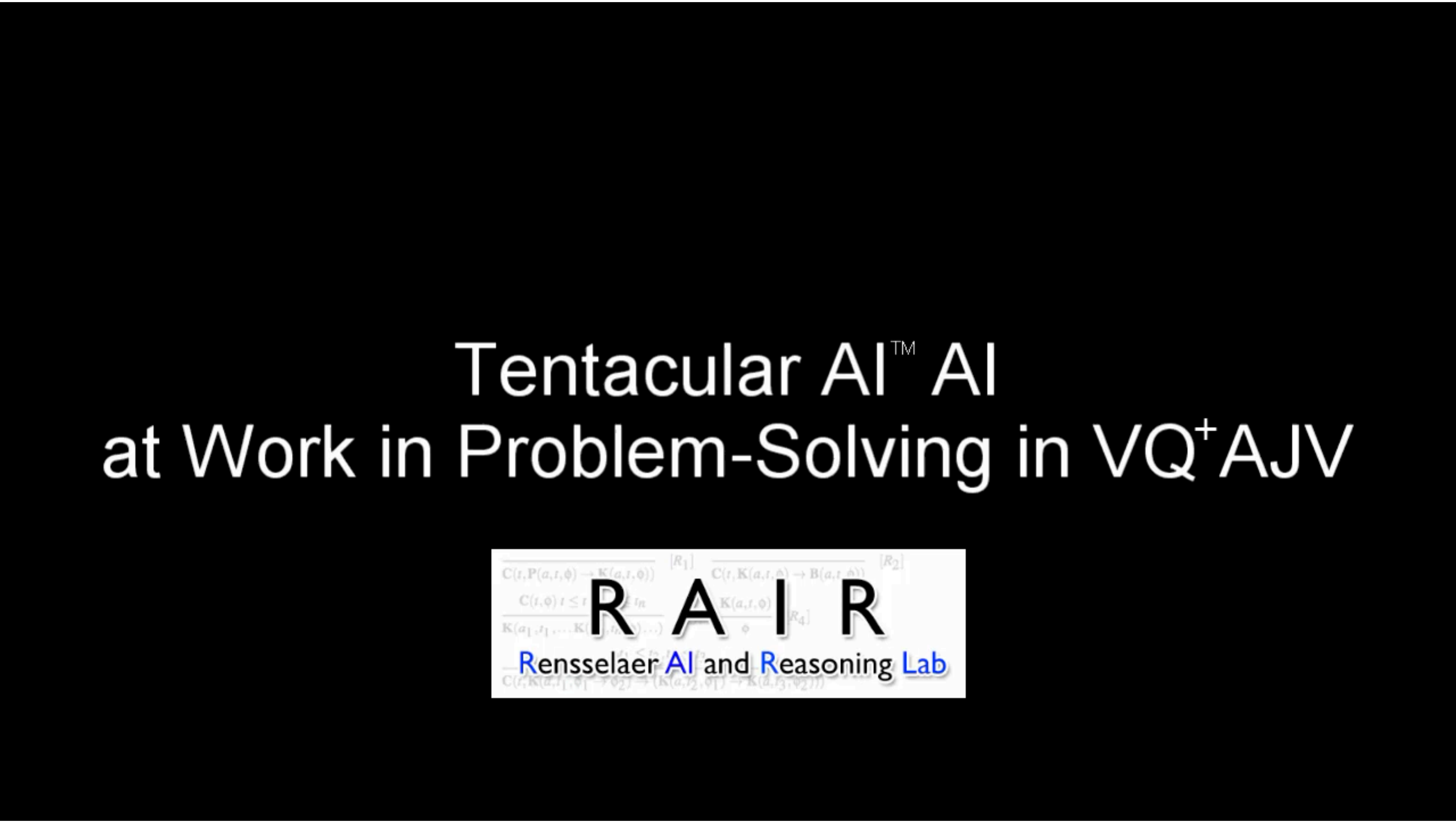
Intelligent Artificial Multi-Agent



Tentacular AI™ AI
at Work in Problem-Solving in VQ⁺AJV

RAIR
Rensselaer AI and Reasoning Lab

Intelligent Artificial Multi-Agent



Tentacular AI™ AI
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R A I R
Rensselaer AI and Reasoning Lab

Part I: *Slutten — for i dag.*

Part I: *Slutten — for i dag.*

Part II: Any hands-on Q&A&H? …