Propositional Calculus I:

The Formal Language,
The Prop. Calc. Provability Oracle (= AI),
Application to Some Motivating Problems

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Intro to Formal Logic (With AI)
2/2/2026

Questions about the “whirlwind” history &

The Singularity, our focus last time …?

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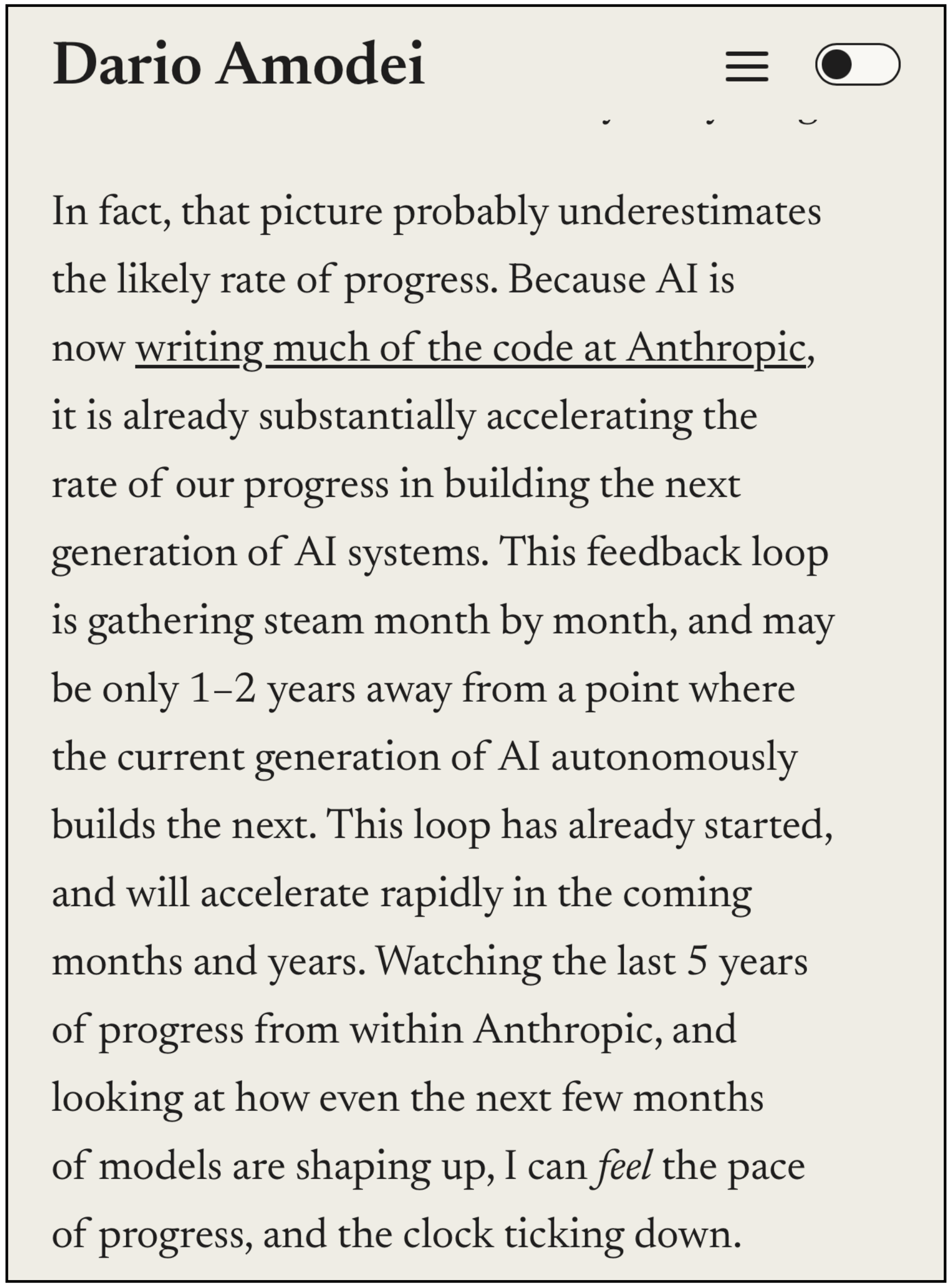
Dario Amodei

The Adolescence of Technology

Confronting and Overcoming the Risks of Powerful AI

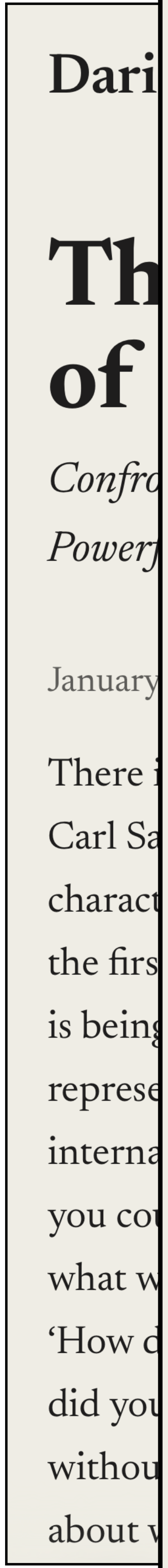
January 2026

There is a scene in the movie version of Carl Sagan's book *Contact* where the main character, an astronomer who has detected the first radio signal from an alien civilization, is being considered for the role of humanity's representative to meet the aliens. The international panel interviewing her asks, "If you could ask [the aliens] just one question, what would it be?" Her reply is: "I'd ask them, 'How did you do it? How did you evolve, how did you survive this technological adolescence without destroying yourself?" When I think about where humanity is now with AI—about



Dario Amodei

In fact, that picture probably underestimates the likely rate of progress. Because AI is now writing much of the code at Anthropic, it is already substantially accelerating the rate of our progress in building the next generation of AI systems. This feedback loop is gathering steam month by month, and may be only 1–2 years away from a point where the current generation of AI autonomously builds the next. This loop has already started, and will accelerate rapidly in the coming months and years. Watching the last 5 years of progress from within Anthropic, and looking at how even the next few months of models are shaping up, I can *feel* the pace of progress, and the clock ticking down.



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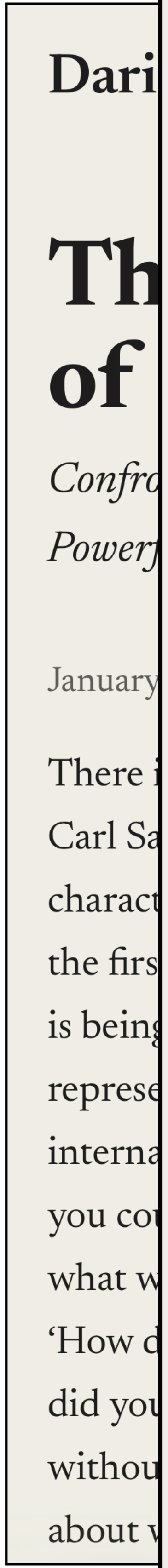
The Singularity



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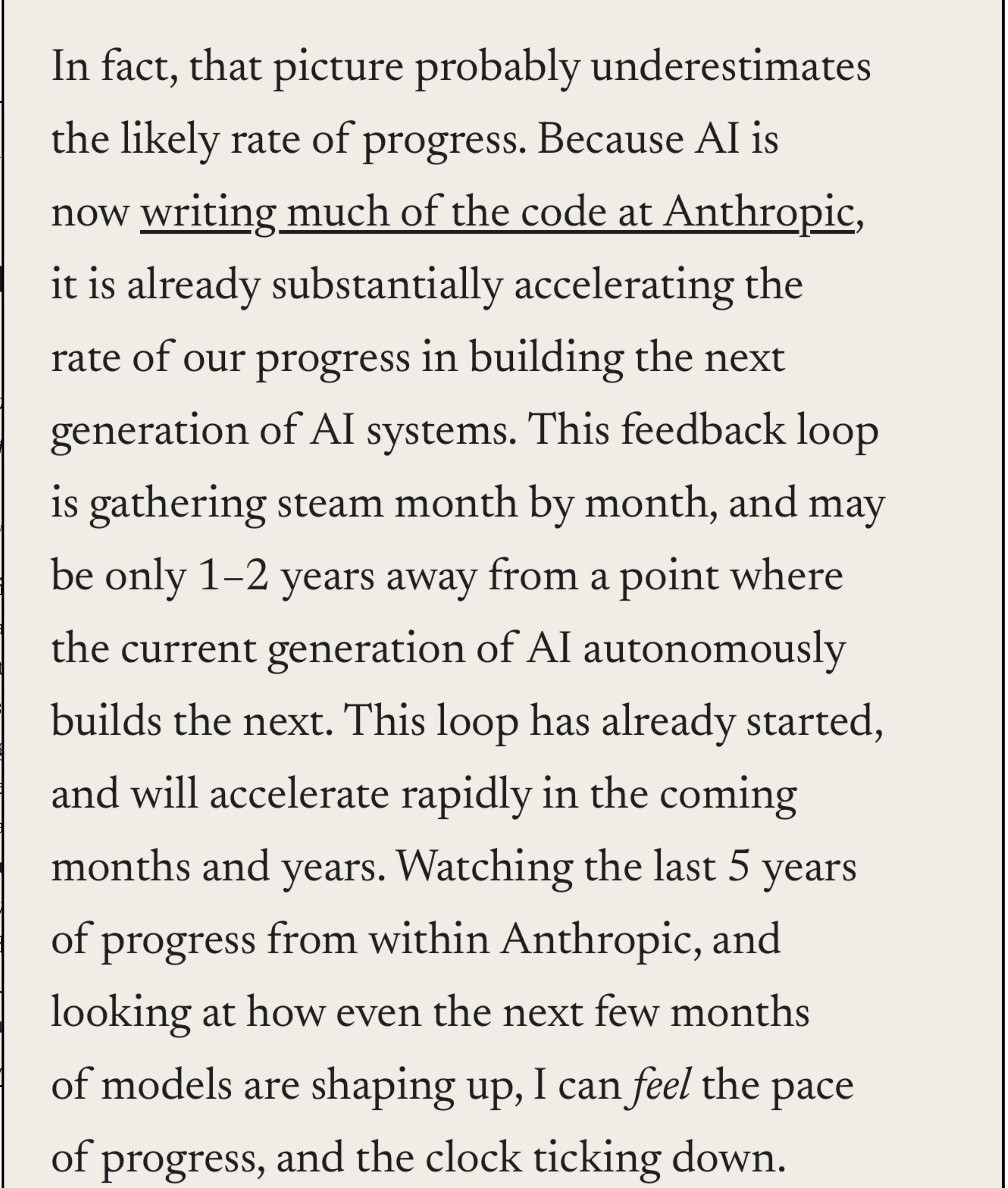

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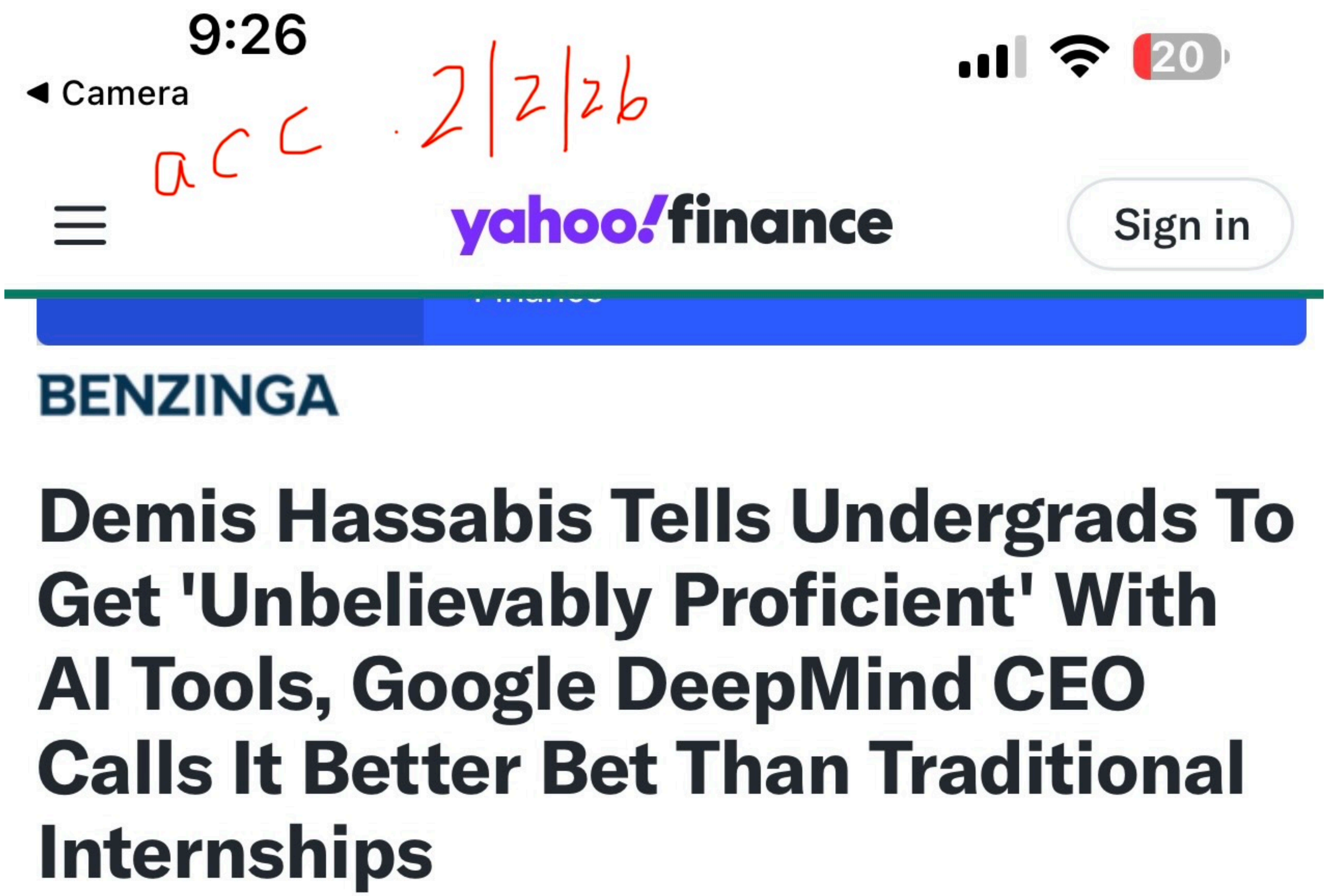

acc. 2/2/26

BENZINGA

Demis Hassabis Tells Undergrads To Get 'Unbelievably Proficient' With AI Tools, Google DeepMind CEO Calls It Better Bet Than Traditional Internships

Btw, in our “Why study logic?” mtgs we skipped over *n*-door challenges.

They are inductive, not deductive; we’ll visit inductive later (see our schedule in the syllabus).

The Monty Hall Problem

$1M

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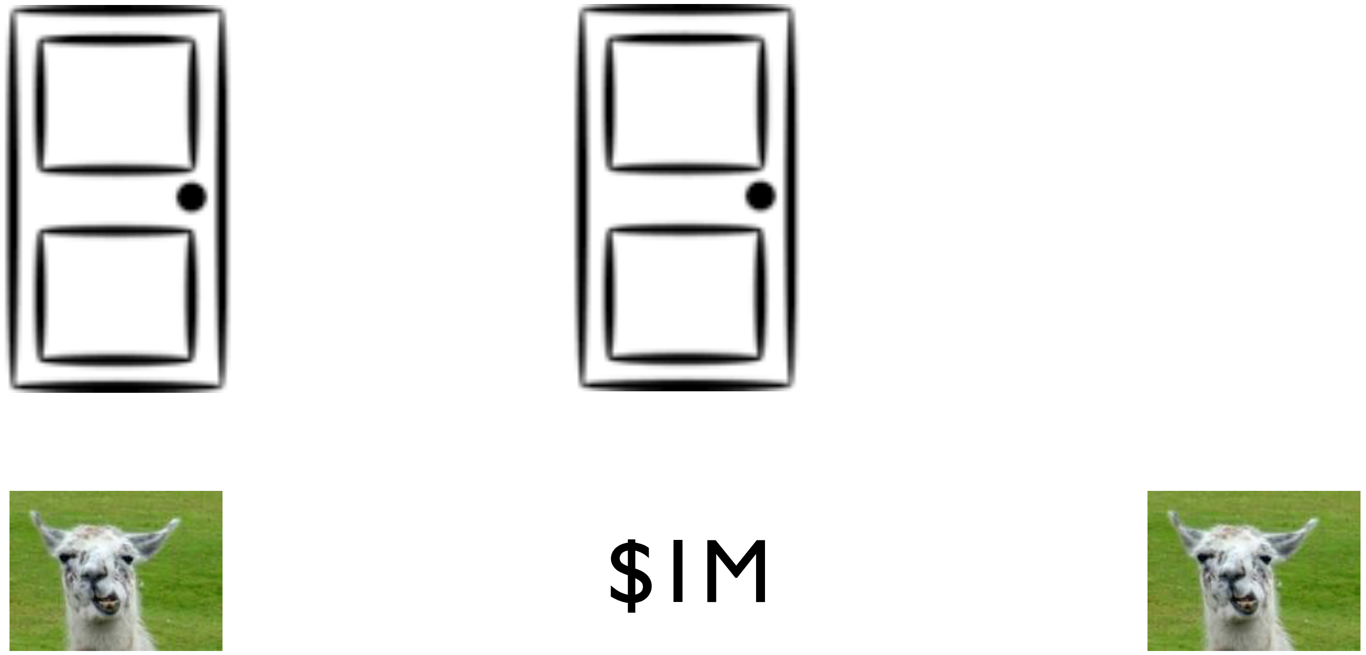
The Monty Hall Problem



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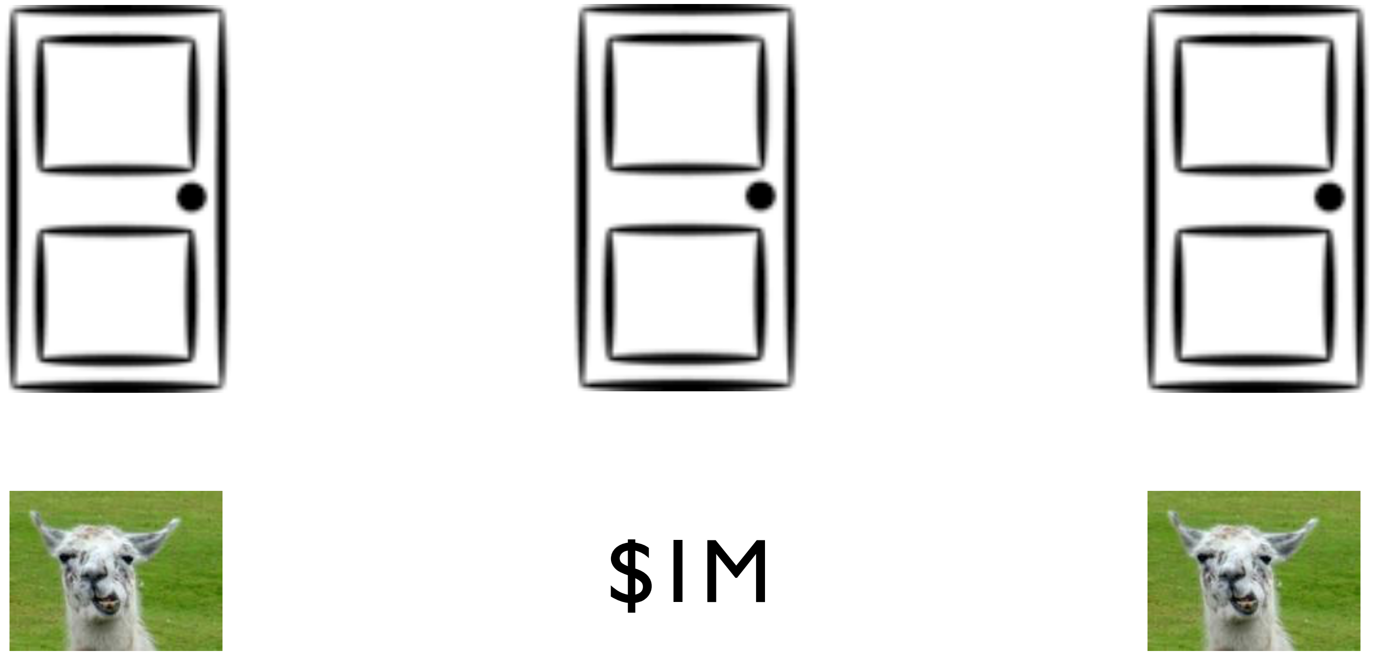
The Monty Hall Problem



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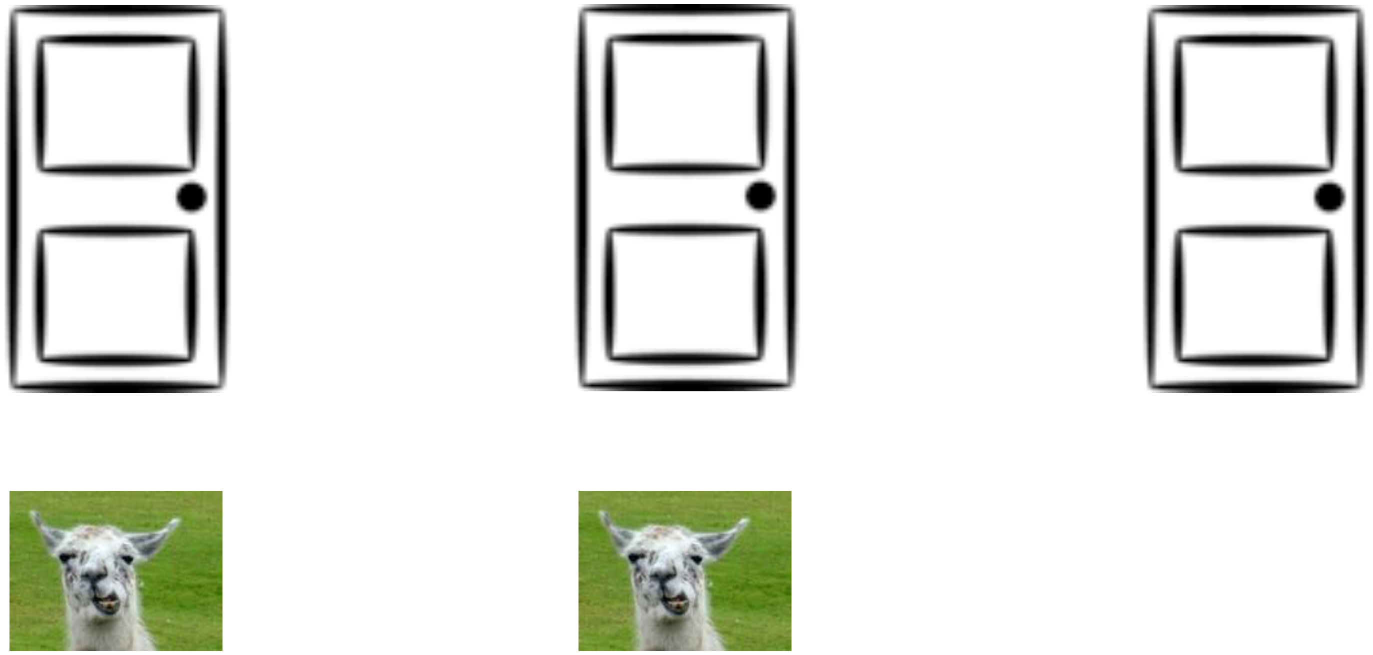

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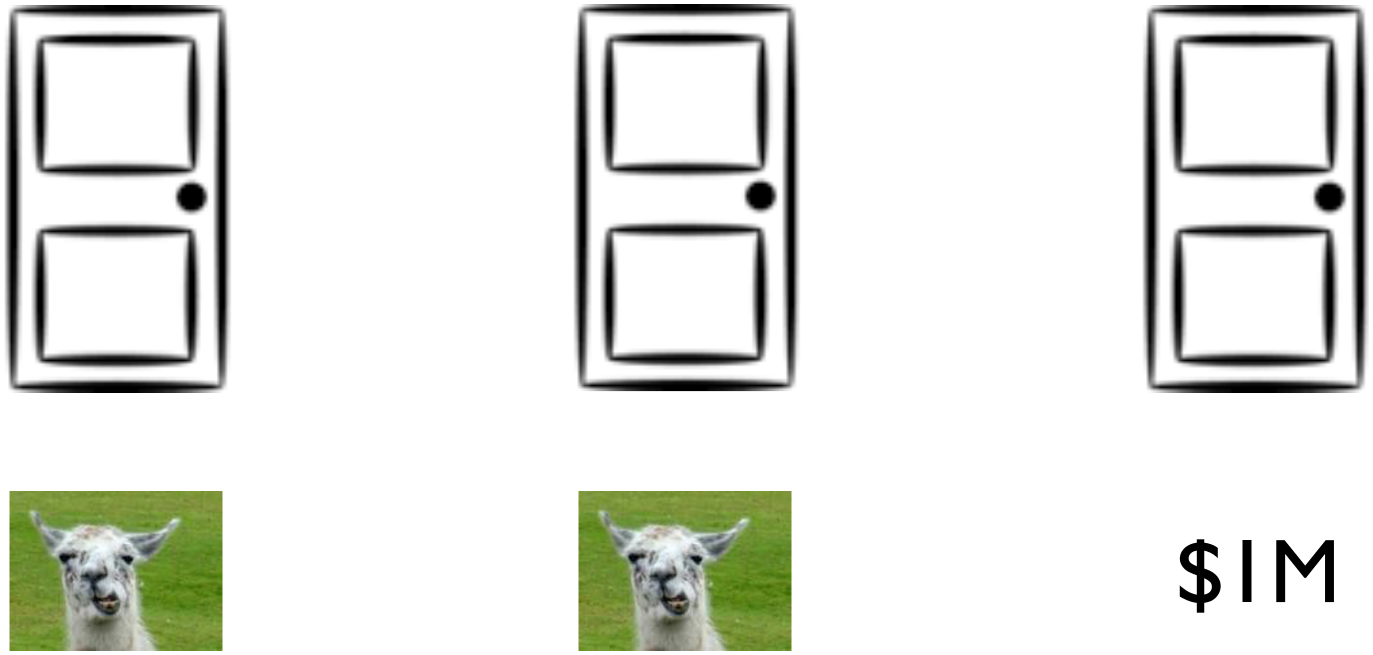

$1M

Logistics again …

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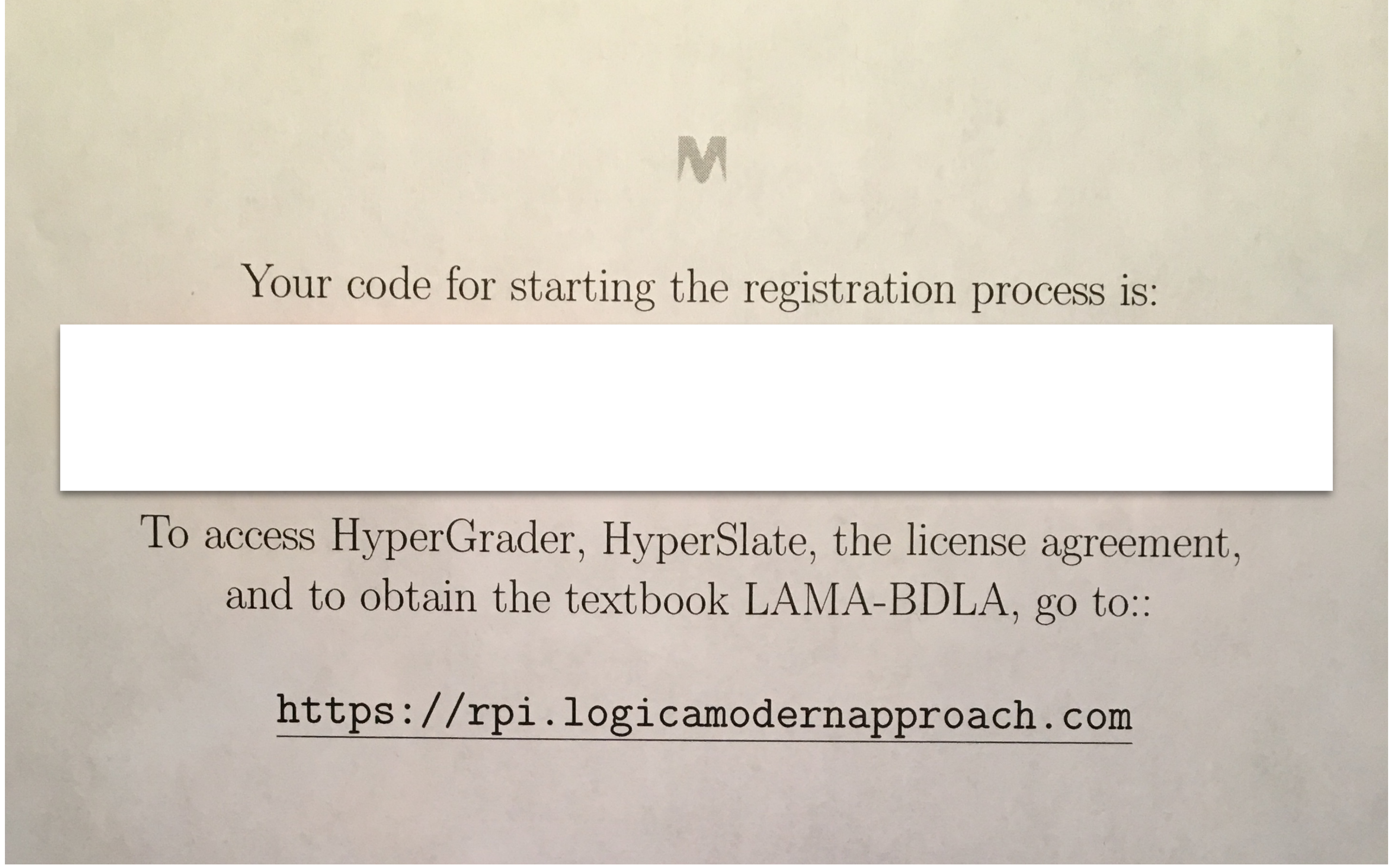

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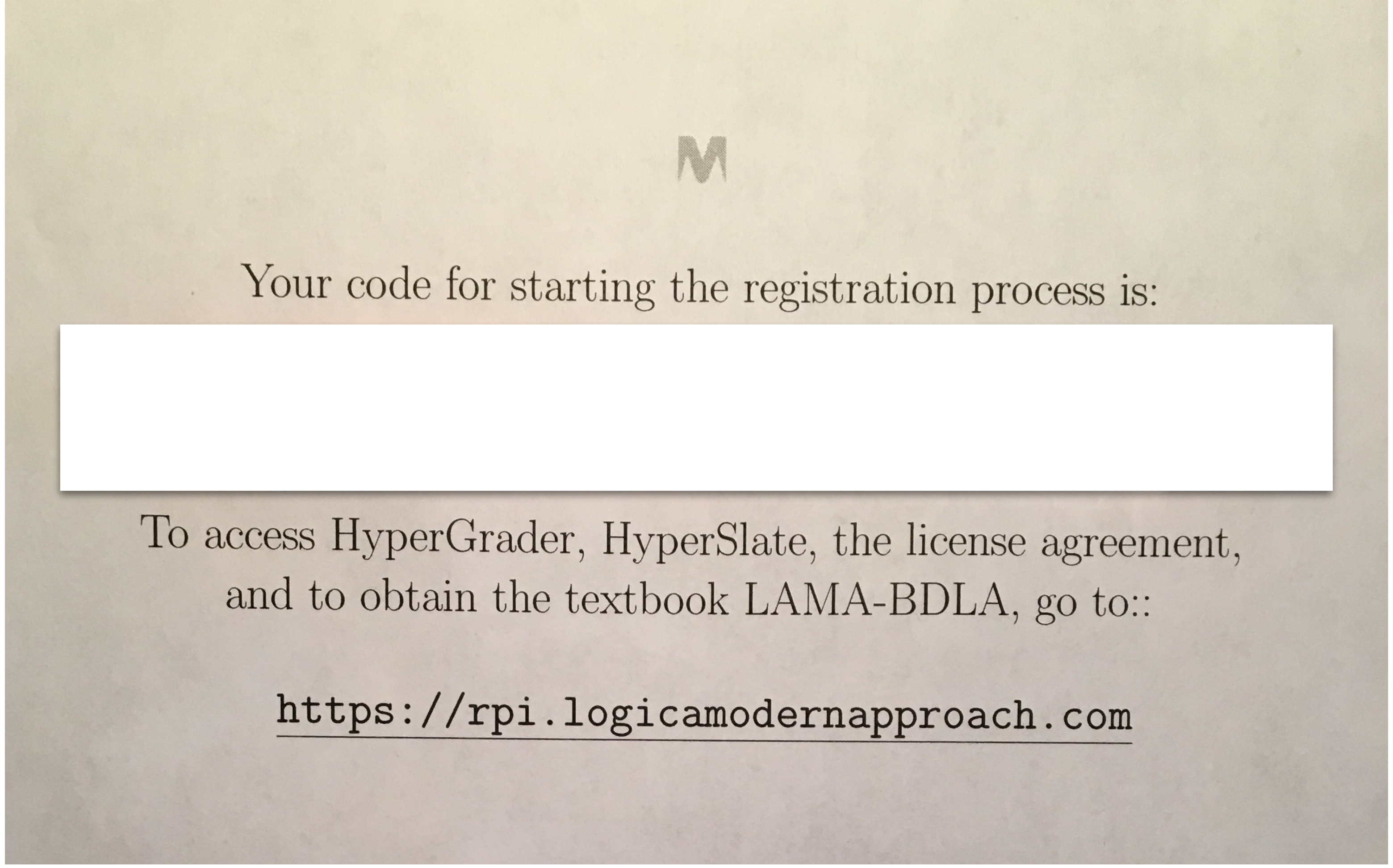

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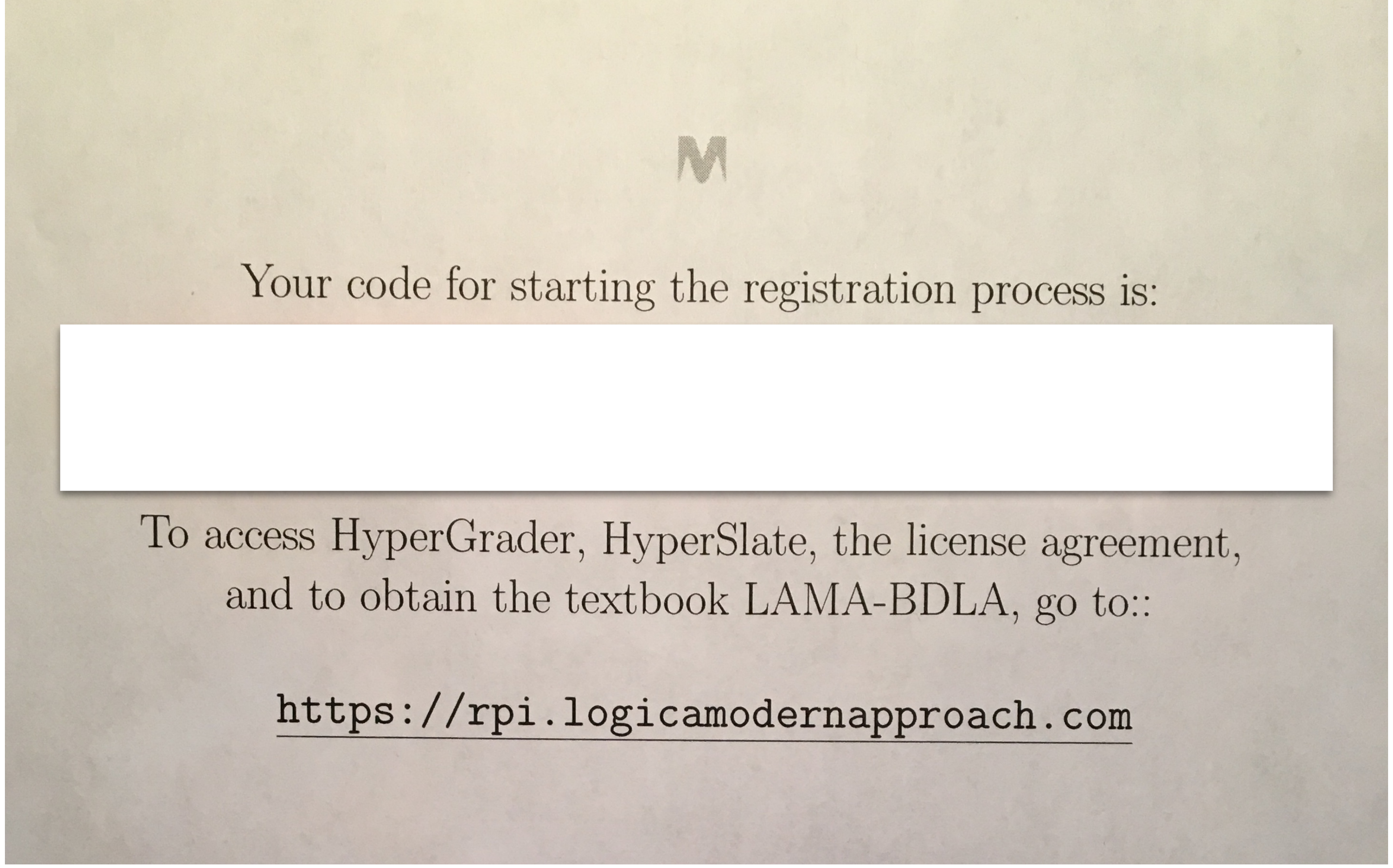

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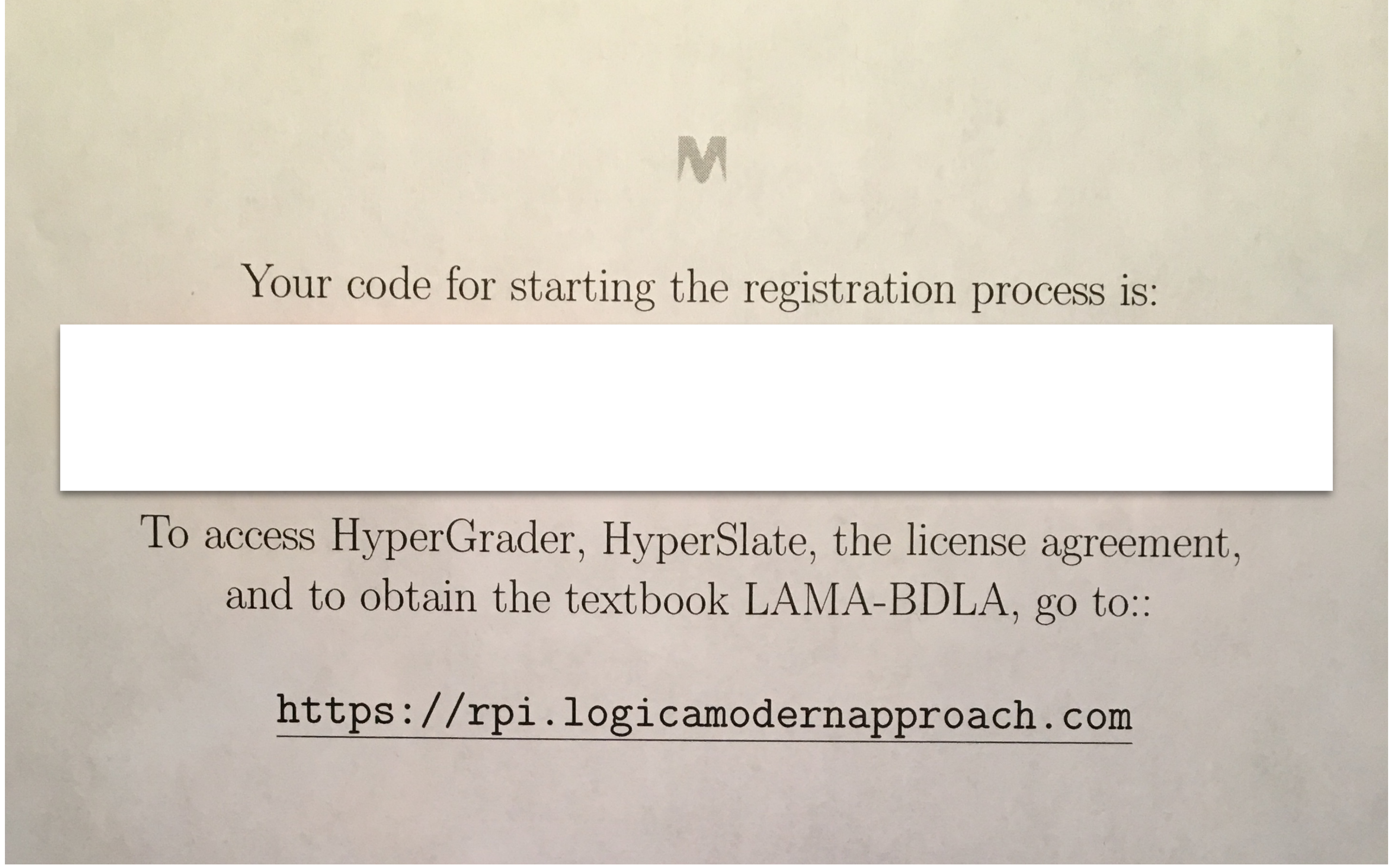

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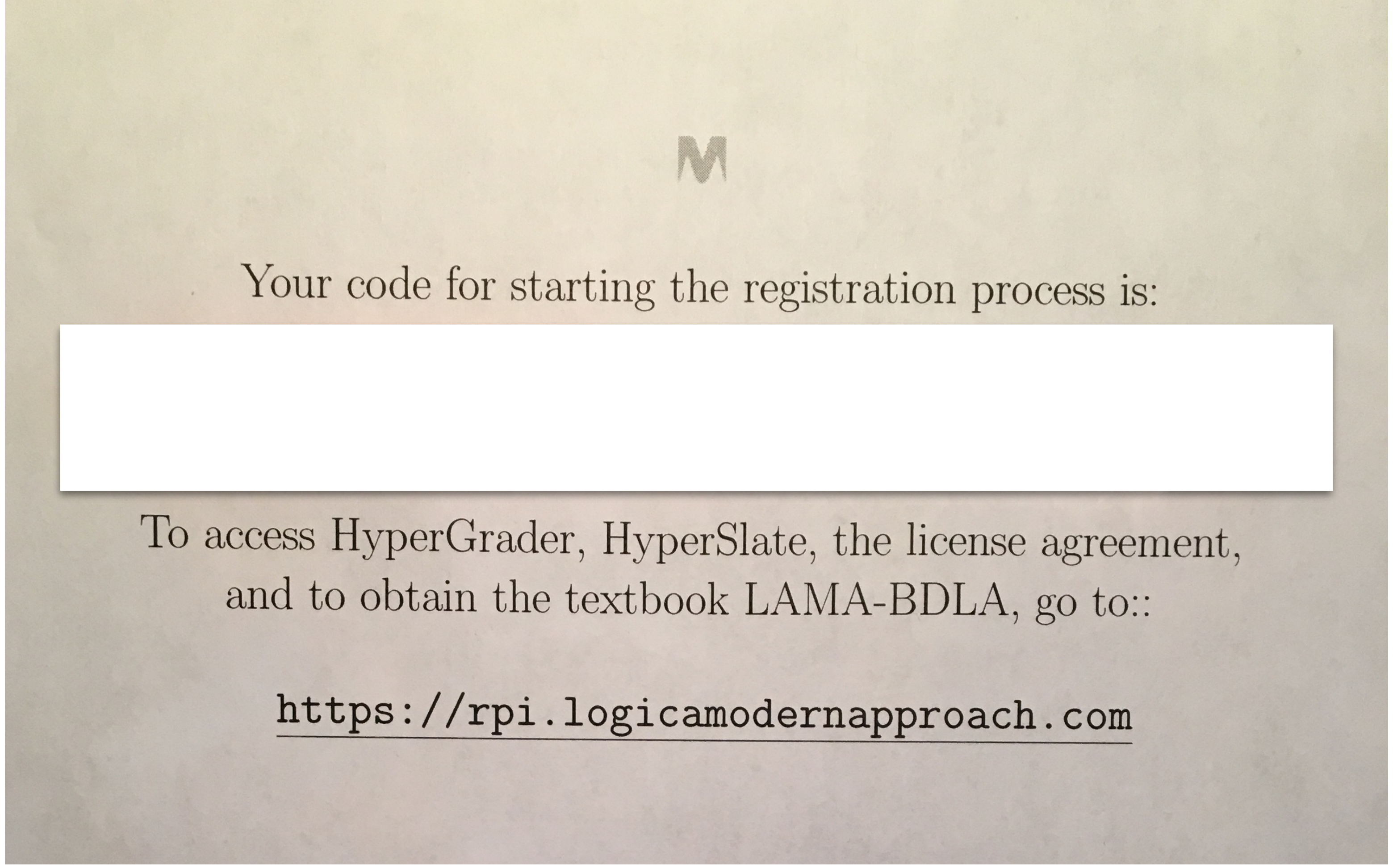

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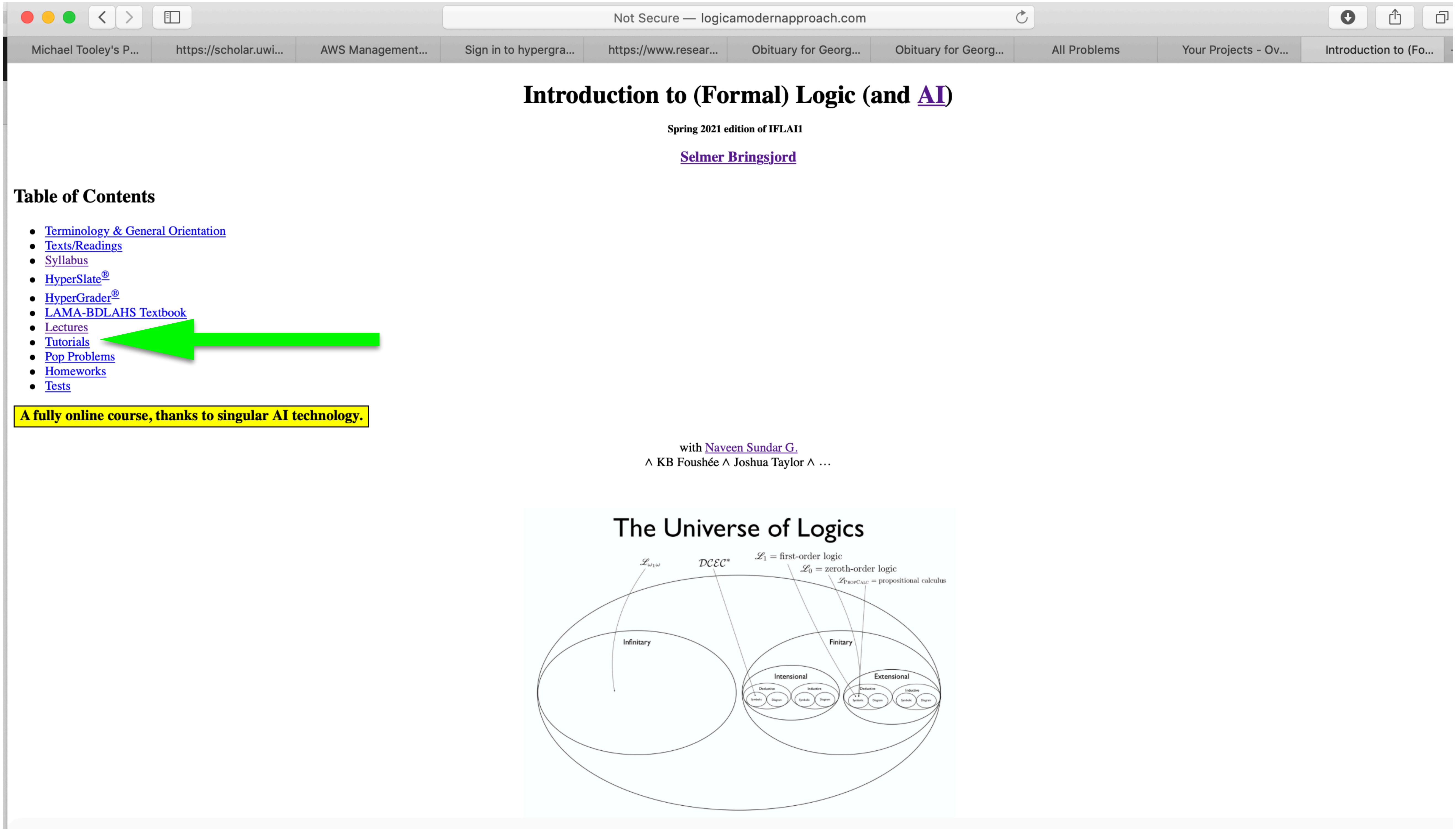

Introduction to (Formal) Logic (and AI)

Spring 2021 edition of IFLAI1

Selmer Bringsjord

Table of Contents

A fully online course, thanks to singular AI technology.

with Naveen Sundar G.
∧ KB Foushée ∧ Joshua Taylor ∧ …

The Universe of Logics
$\mathscr{L}_{\omega_1\omega}$
$\mathcal{DCEC}^*$
$\mathscr{L}_1$ = first-order logic
$\mathscr{L}_0$ = zeroth-order logic
$\mathscr{L}_{\text{PropCalc}}$ = propositional calculus
Infinitary
Finitary
Intensional
Extensional
Deductive
Inductive
Symbolic
Diagram

Micro-homily:

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Having read Chapter 1, now to Chap 2, skipping to ~ p. 34!

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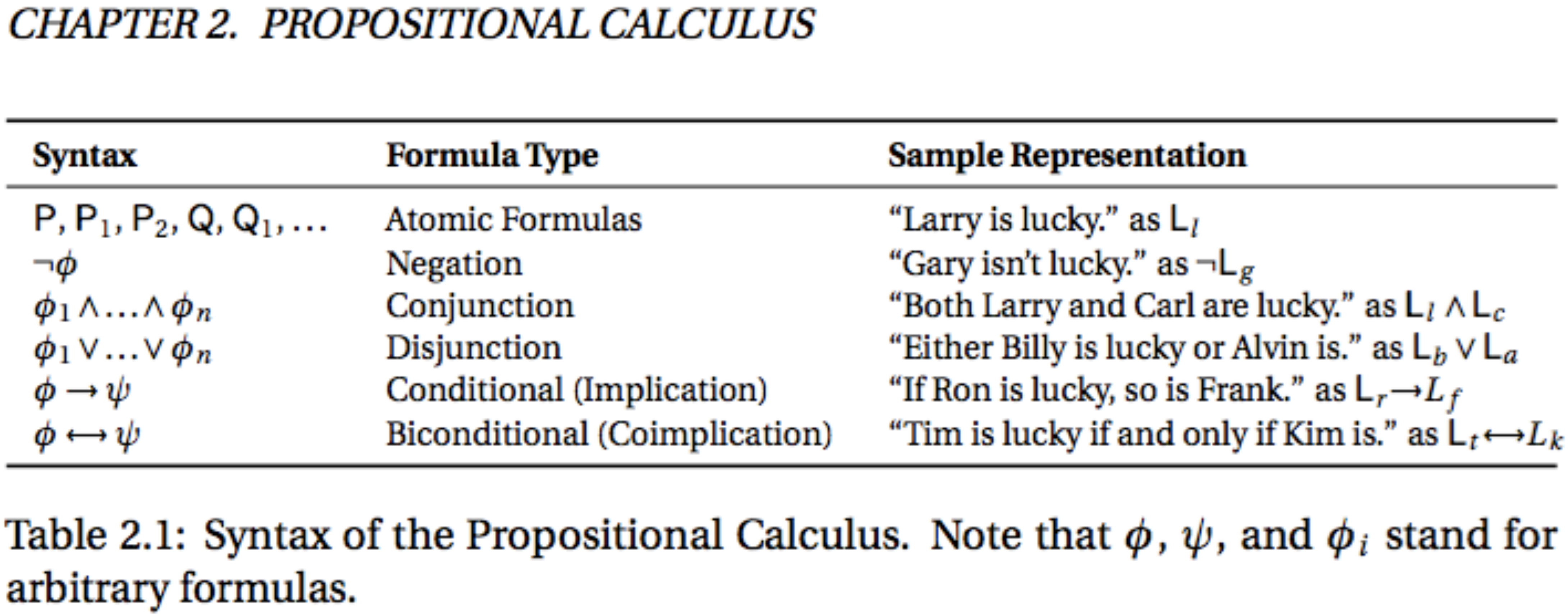
“What category of English sentences does logic focus on?”

The Formal Language

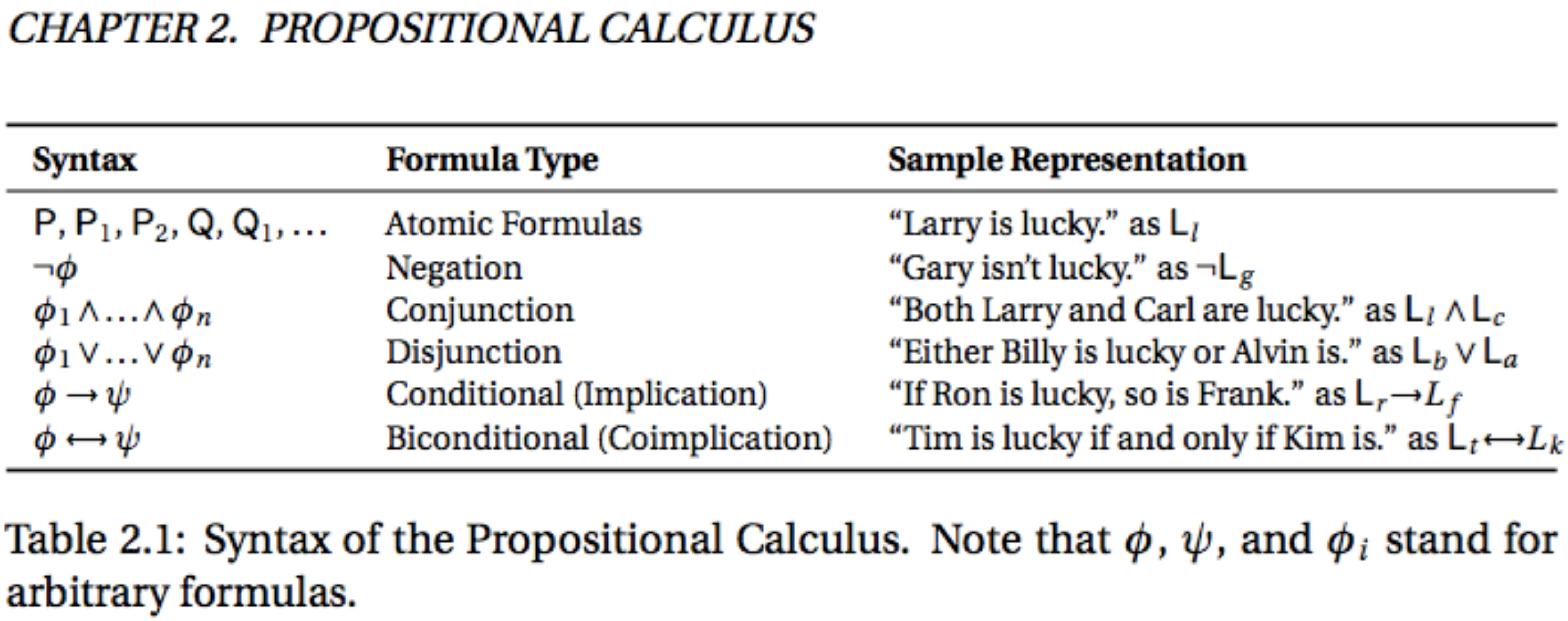


CHAPTER 2. PROPOSITIONAL CALCULUS

Syntax	Formula Type	Sample Representation
$\mathsf{P}, \mathsf{P}_1, \mathsf{P}_2, \mathsf{Q}, \mathsf{Q}_1, \ldots$	Atomic Formulas	"Larry is lucky." as L_l
$\neg\phi$	Negation	"Gary isn't lucky." as $\neg\mathsf{L}_g$
$\phi_1 \wedge \ldots \wedge \phi_n$	Conjunction	"Both Larry and Carl are lucky." as $\mathsf{L}_l \wedge \mathsf{L}_c$
$\phi_1 \vee \ldots \vee \phi_n$	Disjunction	"Either Billy is lucky or Alvin is." as $\mathsf{L}_b \vee \mathsf{L}_a$
$\phi \rightarrow \psi$	Conditional (Implication)	"If Ron is lucky, so is Frank." as $\mathsf{L}_r \rightarrow L_f$
$\phi \longleftrightarrow \psi$	Biconditional (Coimplication)	"Tim is lucky if and only if Kim is." as $\mathsf{L}_t \longleftrightarrow L_k$

Table 2.1: Syntax of the Propositional Calculus. Note that ϕ, ψ, and ϕ_i stand for arbitrary formulas.

The Formal Language



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Exercise: Is this language Roger-decidable? Prove it!

The Formal Language

(presented as formal grammar)

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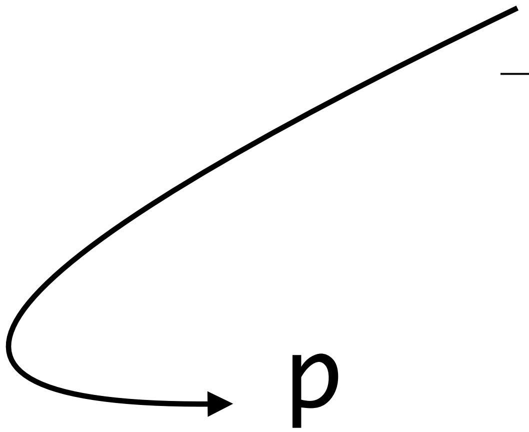

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As S-expressions

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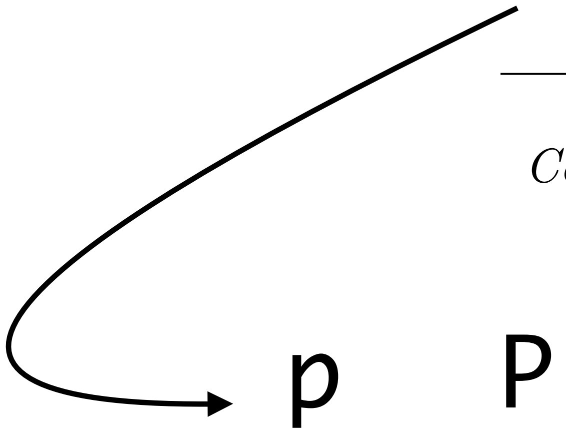

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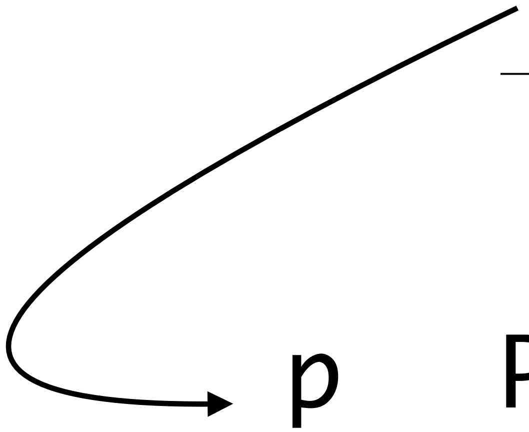

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p   P   bradywillbeback
```

As S-expressions

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`p   P   bradywillbeback  P26`

As S-expressions

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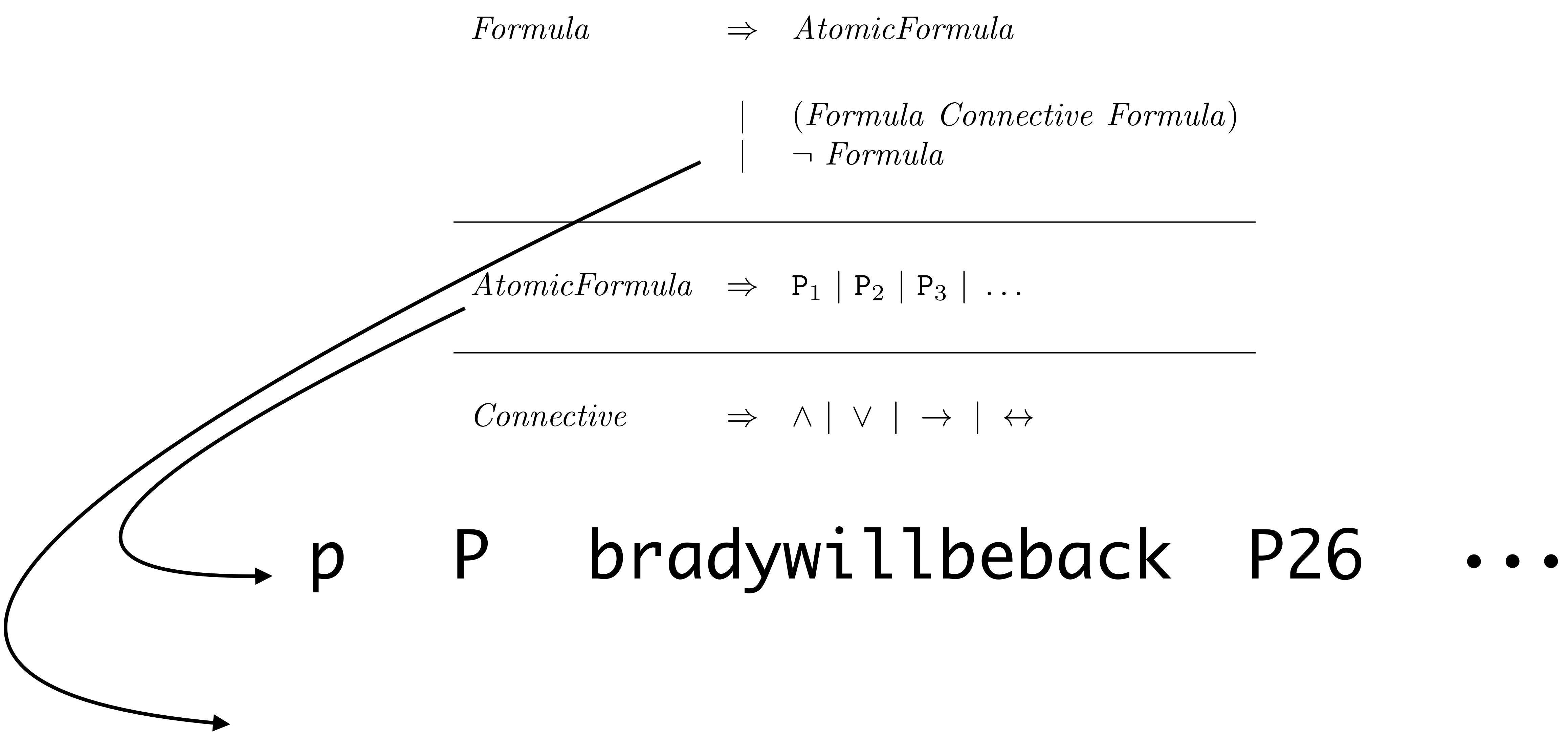
```
p   P   bradywillbeback  P26   ...
```

As S-expressions



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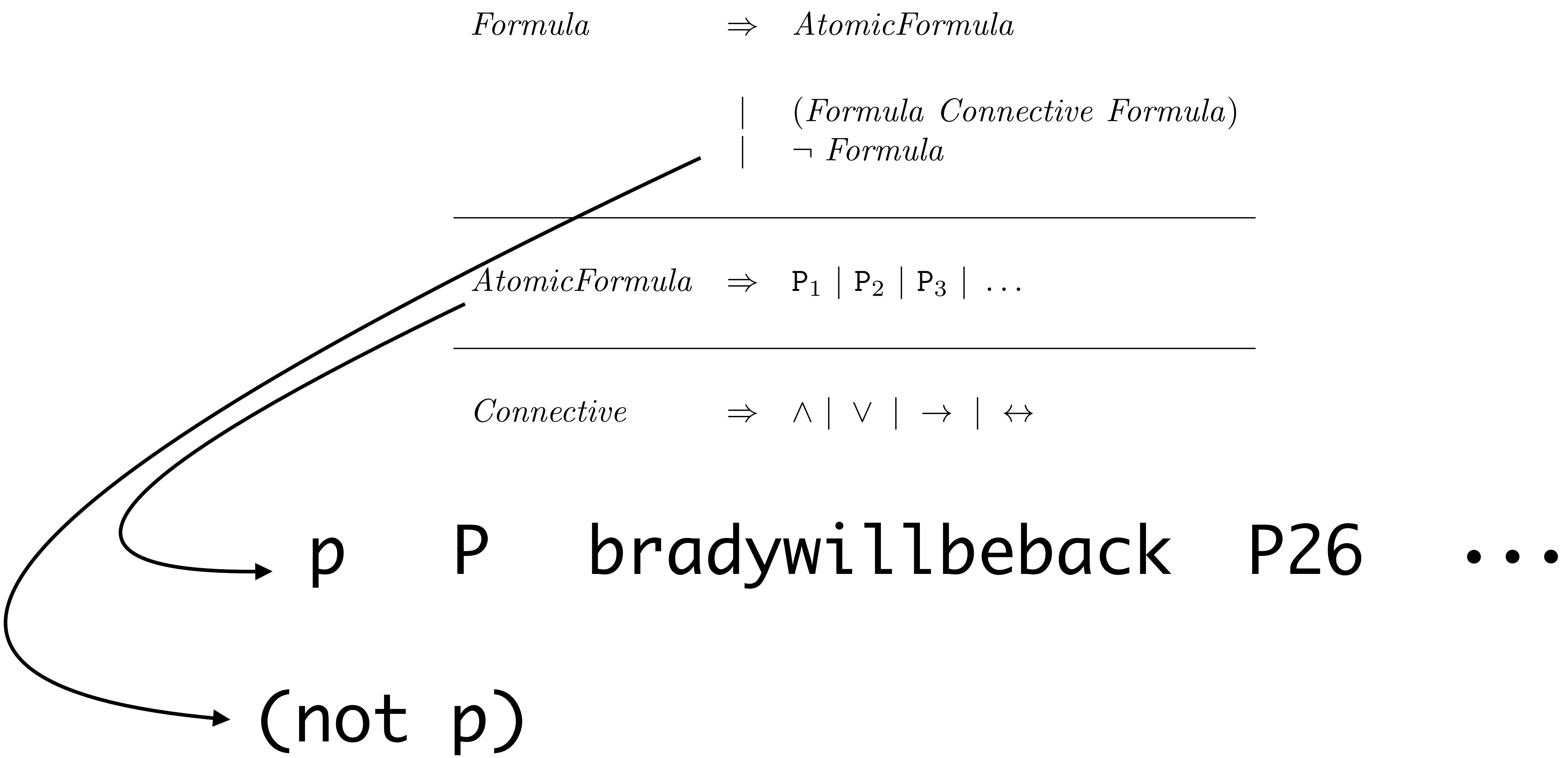

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p   P   bradywillbeback  P26   ...

(not p)
```
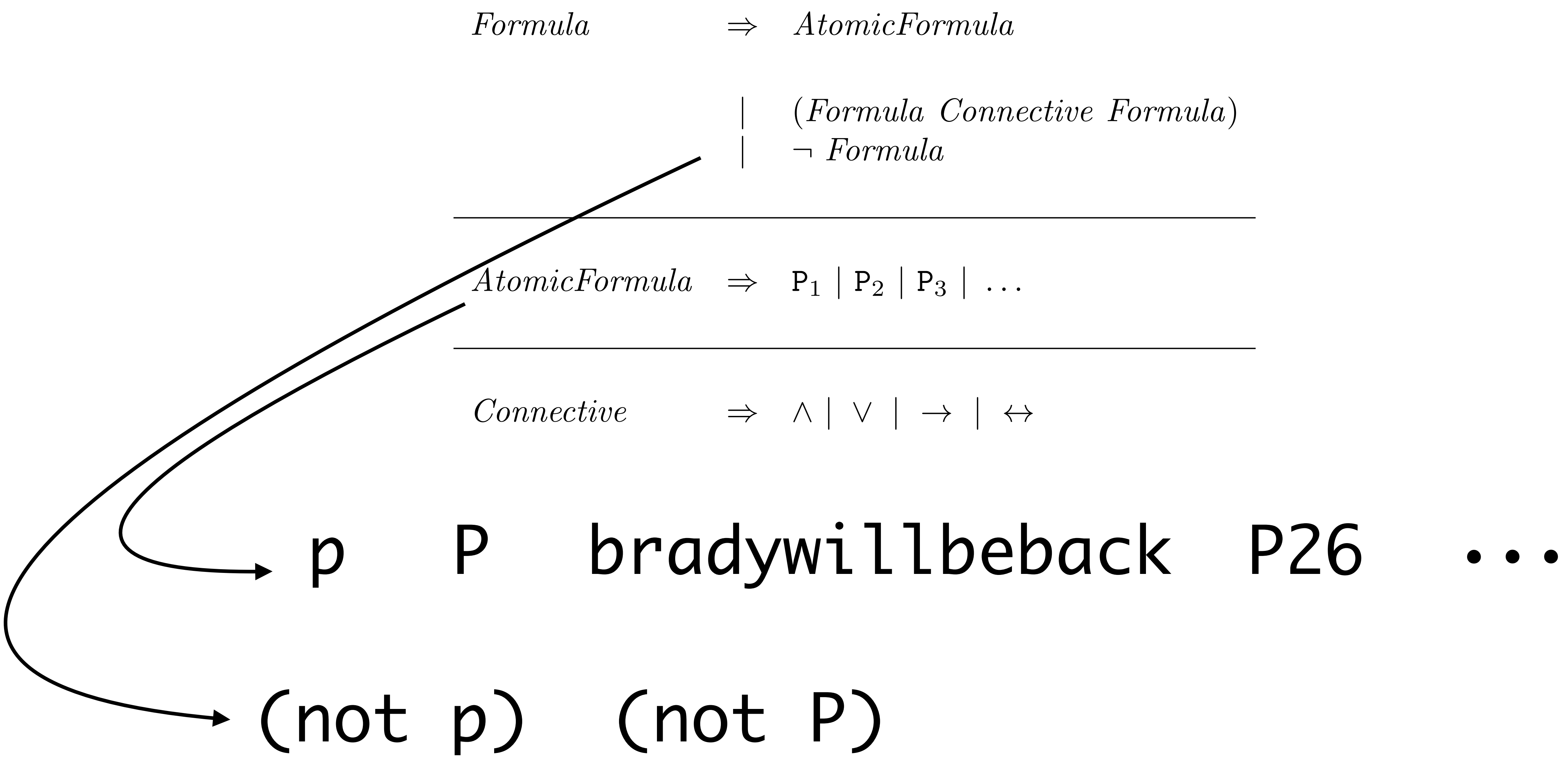

As S-expressions



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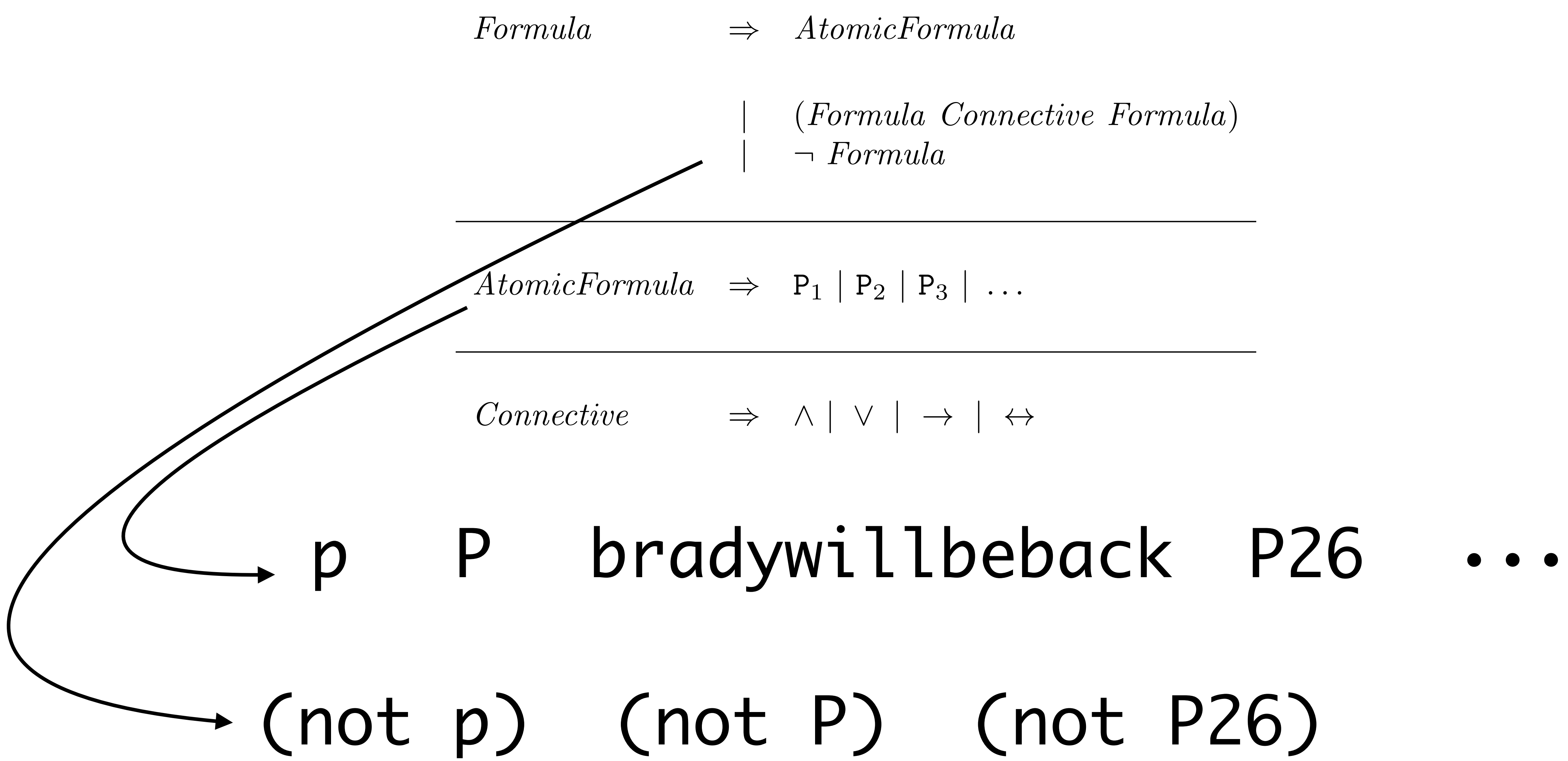
As S-expressions



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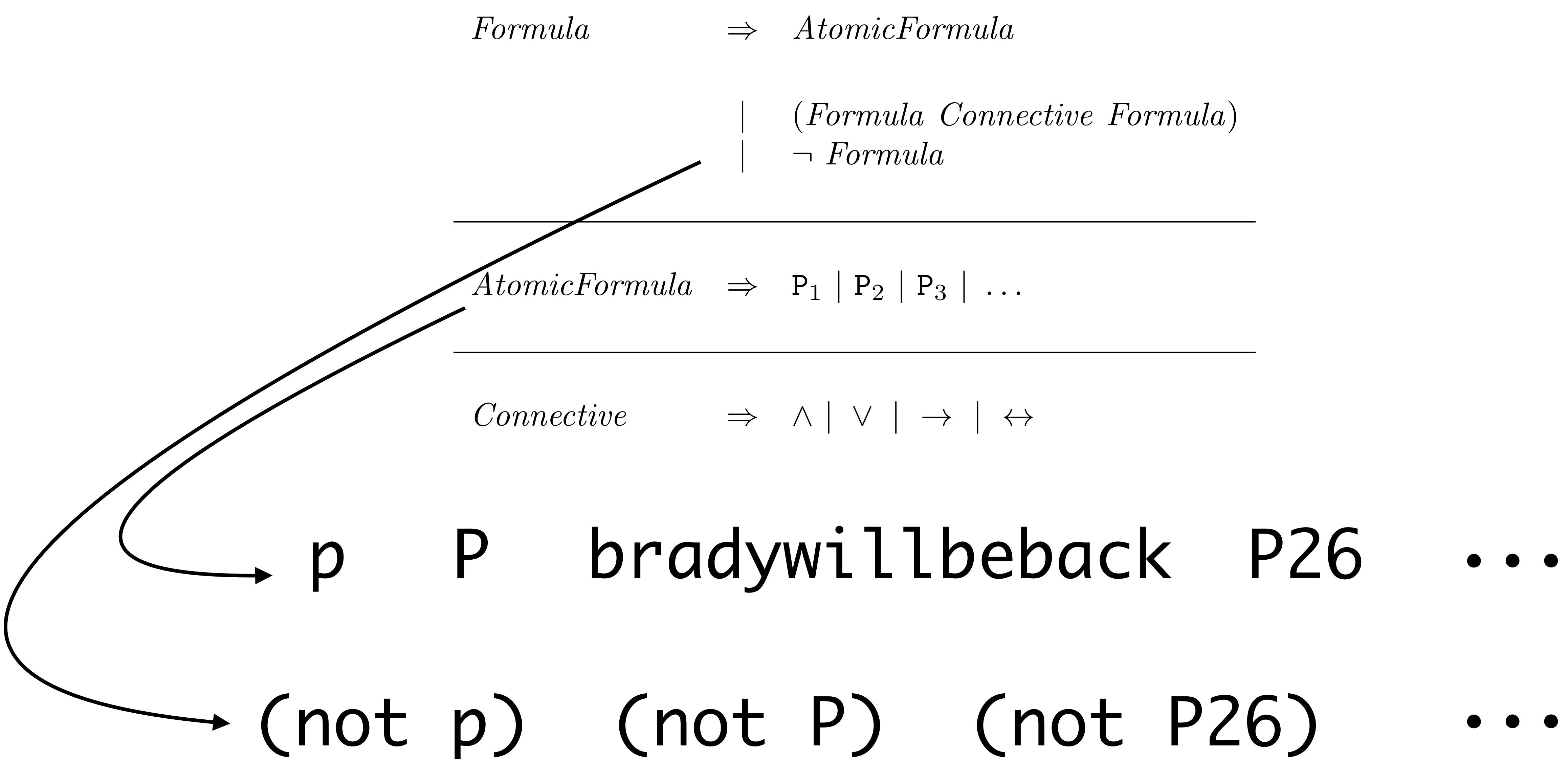

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(not p)  (not P)  (not P26)    •••
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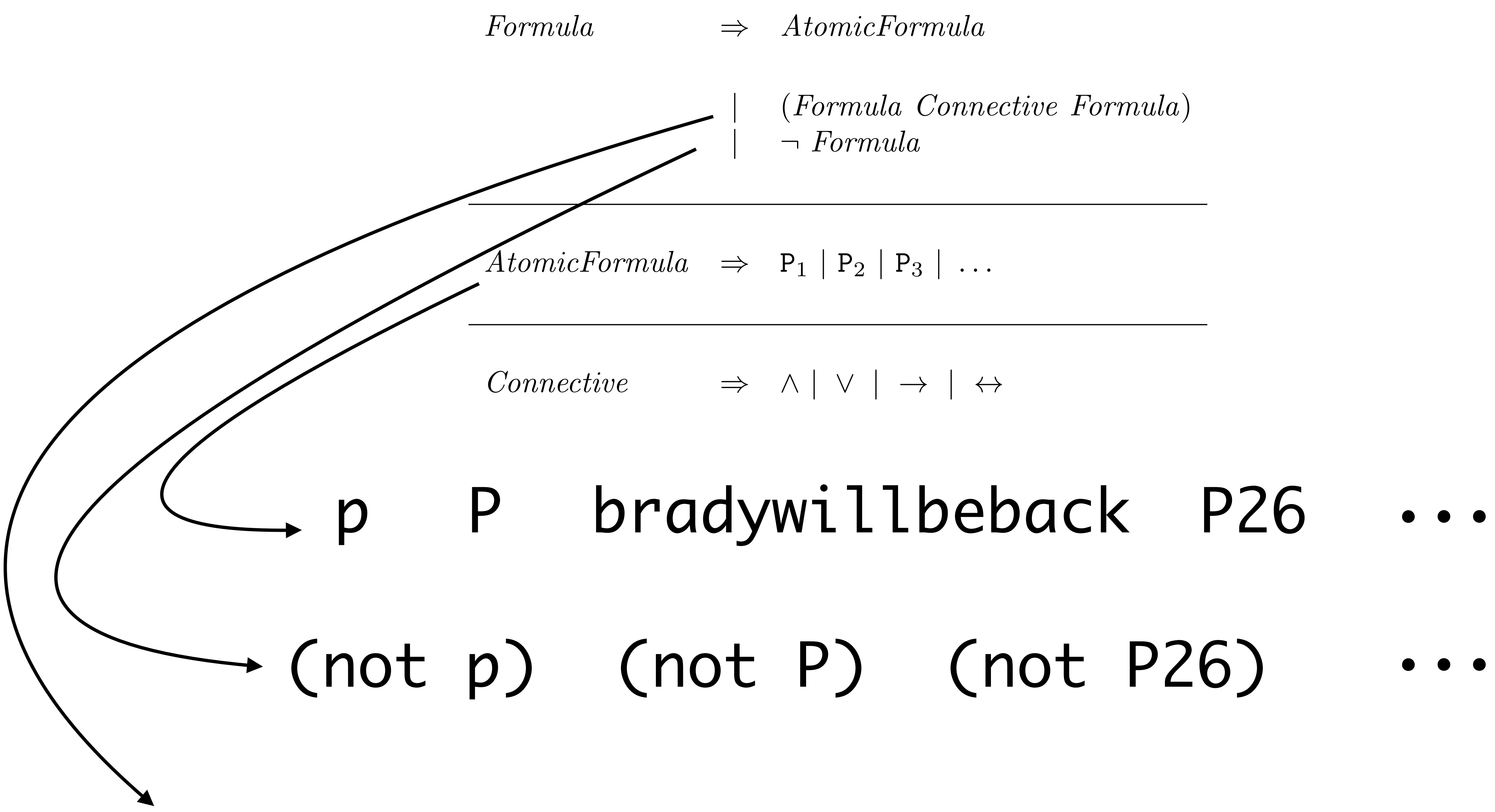
As S-expressions



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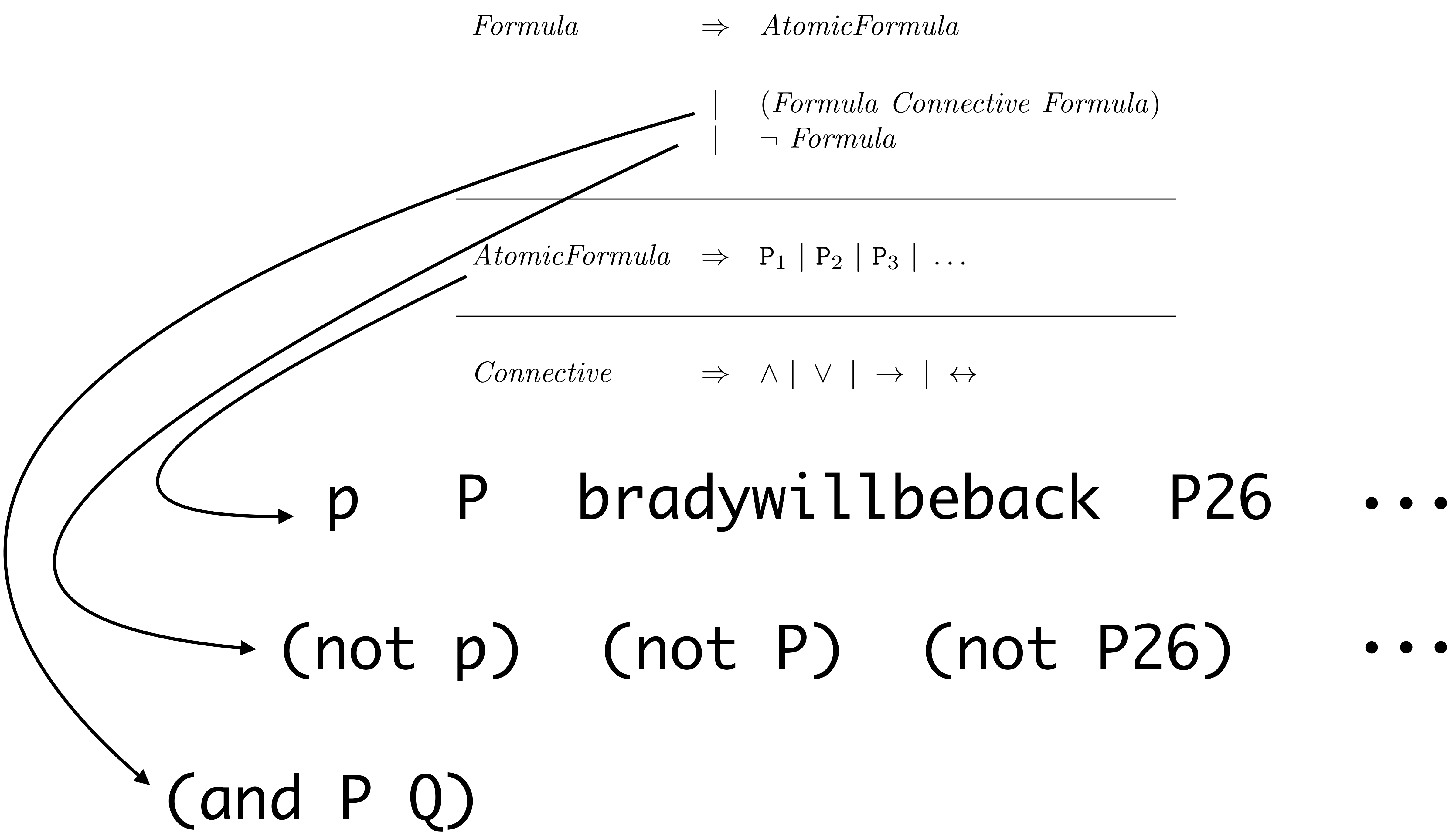
$$

p P bradywillbeback P26 ...

(not p) (not P) (not P26) ...

As S-expressions



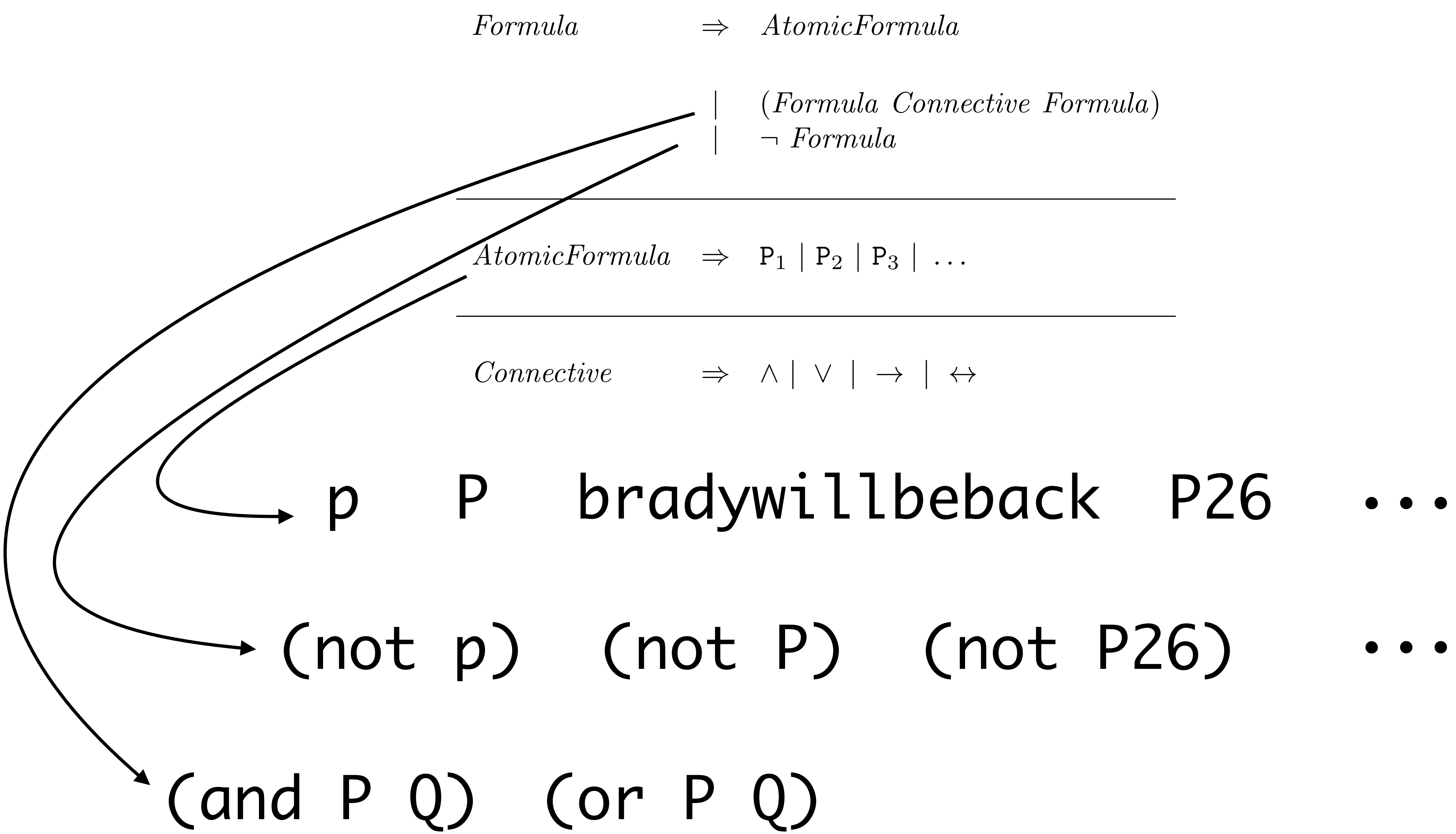
$$\begin{array}{lcl} \mathit{Formula} & \Rightarrow & \mathit{AtomicFormula} \\ & | & (\mathit{Formula}\ \mathit{Connective}\ \mathit{Formula}) \\ & | & \neg\ \mathit{Formula} \\ \hline \mathit{AtomicFormula} & \Rightarrow & \mathtt{P}_1 \mid \mathtt{P}_2 \mid \mathtt{P}_3 \mid \ldots \\ \hline \mathit{Connective} & \Rightarrow & \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow \end{array}$$

```
p  P  bradywillbeback  P26  ...

(not p)  (not P)  (not P26)   ...

(and P Q)
```

As S-expressions



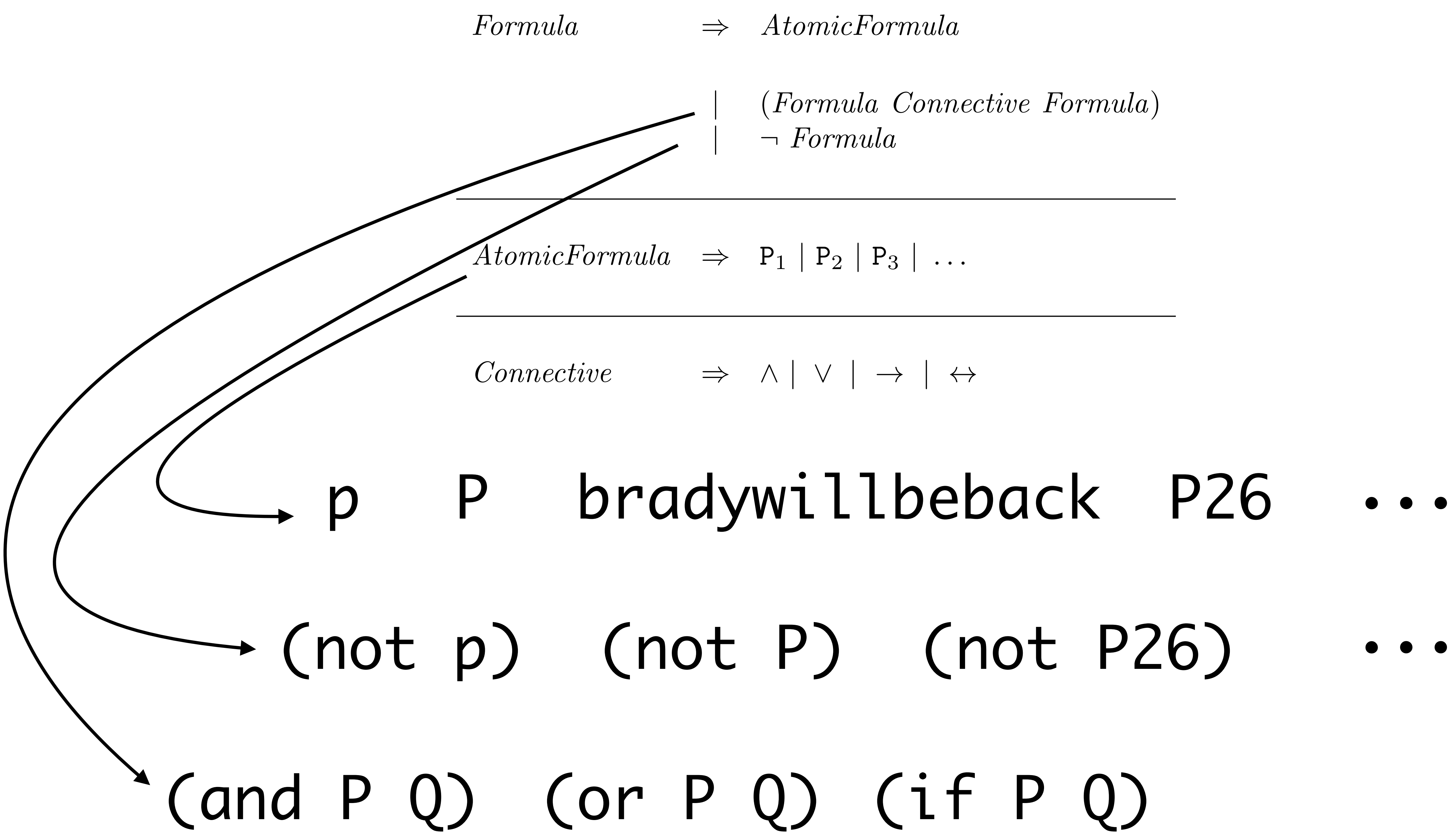
$$\begin{array}{lll} \mathit{Formula} & \Rightarrow & \mathit{AtomicFormula} \\ & \mid & (\mathit{Formula}\ \ \mathit{Connective}\ \ \mathit{Formula}) \\ & \mid & \neg\ \mathit{Formula} \\ \hline \mathit{AtomicFormula} & \Rightarrow & \mathtt{P}_1 \mid \mathtt{P}_2 \mid \mathtt{P}_3 \mid \ldots \\ \hline \mathit{Connective} & \Rightarrow & \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow \end{array}$$

```
p   P   bradywillbeback  P26   ...

(not p)  (not P)  (not P26)    ...

(and P Q)  (or P Q)
```

As S-expressions



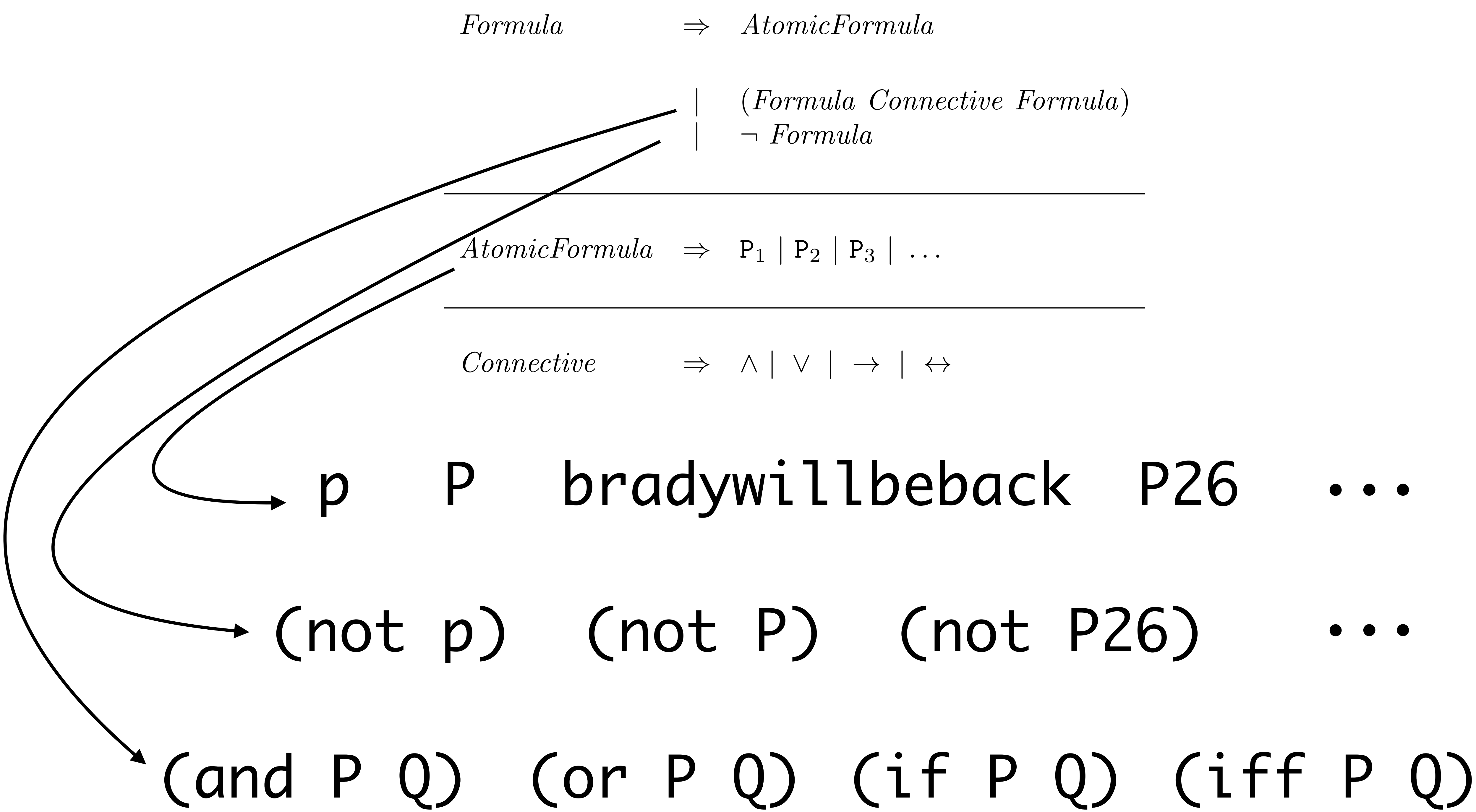
$$\begin{array}{lcl} \textit{Formula} & \Rightarrow & \textit{AtomicFormula} \\ & | & (\textit{Formula}\ \textit{Connective}\ \textit{Formula}) \\ & | & \neg\ \textit{Formula} \\ \hline \textit{AtomicFormula} & \Rightarrow & \texttt{P}_1 \mid \texttt{P}_2 \mid \texttt{P}_3 \mid \ldots \\ \hline \textit{Connective} & \Rightarrow & \land \mid \lor \mid \to \mid \leftrightarrow \end{array}$$

```
p  P  bradywillbeback  P26  ...

(not p)  (not P)  (not P26)   ...

(and P Q)  (or P Q) (if P Q)
```

As S-expressions



$$\begin{array}{lcl}
\mathit{Formula} & \Rightarrow & \mathit{AtomicFormula} \\
 & | & (\mathit{Formula}\ \ \mathit{Connective}\ \ \mathit{Formula}) \\
 & | & \neg\ \mathit{Formula} \\
\hline
\mathit{AtomicFormula} & \Rightarrow & \mathtt{P}_1 \mid \mathtt{P}_2 \mid \mathtt{P}_3 \mid \ldots \\
\hline
\mathit{Connective} & \Rightarrow & \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow
\end{array}$$

```
p   P   bradywillbeback  P26   ...

(not p)  (not P)  (not P26)    ...

(and P Q)  (or P Q) (if P Q) (iff P Q)
```

Better Formal Language: Pure Predicate Calculus (presented via formal grammar)

$$
\begin{array}{lcl}
\mathit{Formula} & \Rightarrow & \mathit{AtomicFormula} \\
 & | & (\mathit{Formula}\ \ \mathit{Connective}\ \ \mathit{Formula}) \\
 & | & \neg\ \mathit{Formula}
\end{array}
$$

$$
\begin{array}{lcl}
\mathit{AtomicFormula} & \Rightarrow & (\mathit{Predicate}\ \ \mathit{Term}_1 \ldots \mathit{Term}_k)
\end{array}
$$

$$
\begin{array}{lcl}
\mathit{Term} & \Rightarrow & (\mathit{Function}\ \ \mathit{Term}_1\ \ \ldots\ \ \mathit{Term}_k) \\
 & | & \mathit{Constant} \\
 & | & \mathit{Variable}
\end{array}
$$

$$
\begin{array}{lcl}
\mathit{Connective} & \Rightarrow & \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow
\end{array}
$$

$$
\begin{array}{lcl}
\mathit{Predicate} & \Rightarrow & P_1 \mid P_2 \mid P_3\ \ldots \\
\mathit{Constant} & \Rightarrow & c_1 \mid c_2 \mid c_3\ \ldots \\
\mathit{Variable} & \Rightarrow & v_1 \mid v_2 \mid v_3\ \ldots \\
\mathit{Function} & \Rightarrow & f_1 \mid f_2 \mid f_3\ \ldots
\end{array}
$$

Better Formal Language: Pure Predicate Calculus (presented via formal grammar)

$$
\begin{array}{lcl}
\mathit{Formula} & \Rightarrow & \mathit{AtomicFormula} \\
 & \mid & (\mathit{Formula}\ \ \mathit{Connective}\ \ \mathit{Formula}) \\
 & \mid & \neg\ \mathit{Formula} \\
\hline
\mathit{AtomicFormula} & \Rightarrow & (\mathit{Predicate}\ \ \mathit{Term}_1 \ldots \mathit{Term}_k) \\
\hline
\mathit{Term} & \Rightarrow & (\mathit{Function}\ \ \mathit{Term}_1\ \ \ldots\ \ \mathit{Term}_k) \\
 & \mid & \mathit{Constant} \\
 & \mid & \mathit{Variable} \\
\hline
\mathit{Connective} & \Rightarrow & \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow \\
\hline
\mathit{Predicate} & \Rightarrow & P_1 \mid P_2 \mid P_3\ \ \ldots \\
\mathit{Constant} & \Rightarrow & c_1 \mid c_2 \mid c_3\ \ \ldots \\
\mathit{Variable} & \Rightarrow & v_1 \mid v_2 \mid v_3\ \ \ldots \\
\mathit{Function} & \Rightarrow & f_1 \mid f_2 \mid f_3\ \ \ldots
\end{array}
$$

Exercise: Is this language also Roger-decidable? Prove it!

“NYS 1” Revisited

Given the statements

$\neg a \vee \neg b$

b

$c \rightarrow a$

which one of the following statements is provable?

c

$\neg b$

$\neg c$

h

a

none of the above

“NYS 1” Revisited

Given the statements

$\neg a \vee \neg b$

b

$c \rightarrow a$

which one of the following statements is provable?

c

$\neg b$

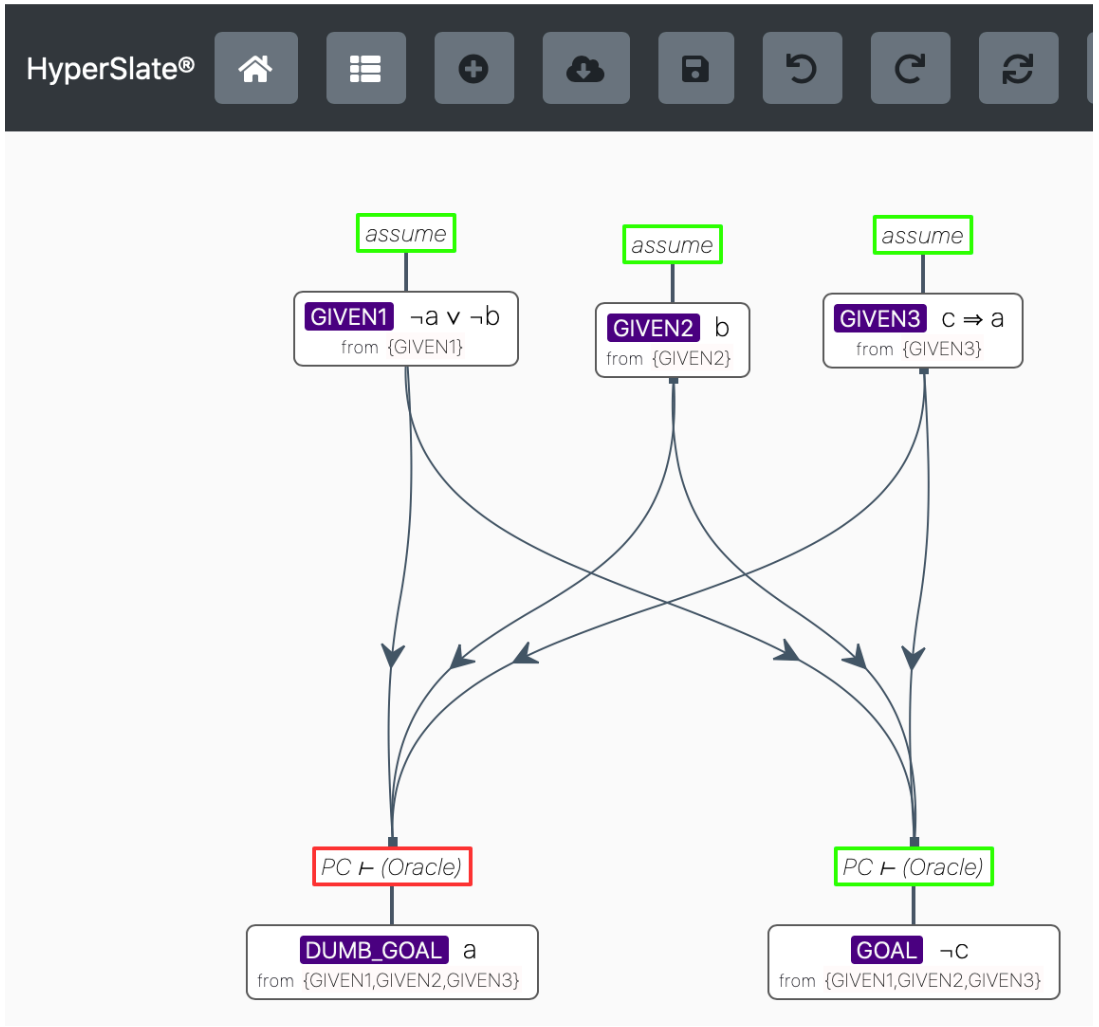
[✓ $\neg c$]

h

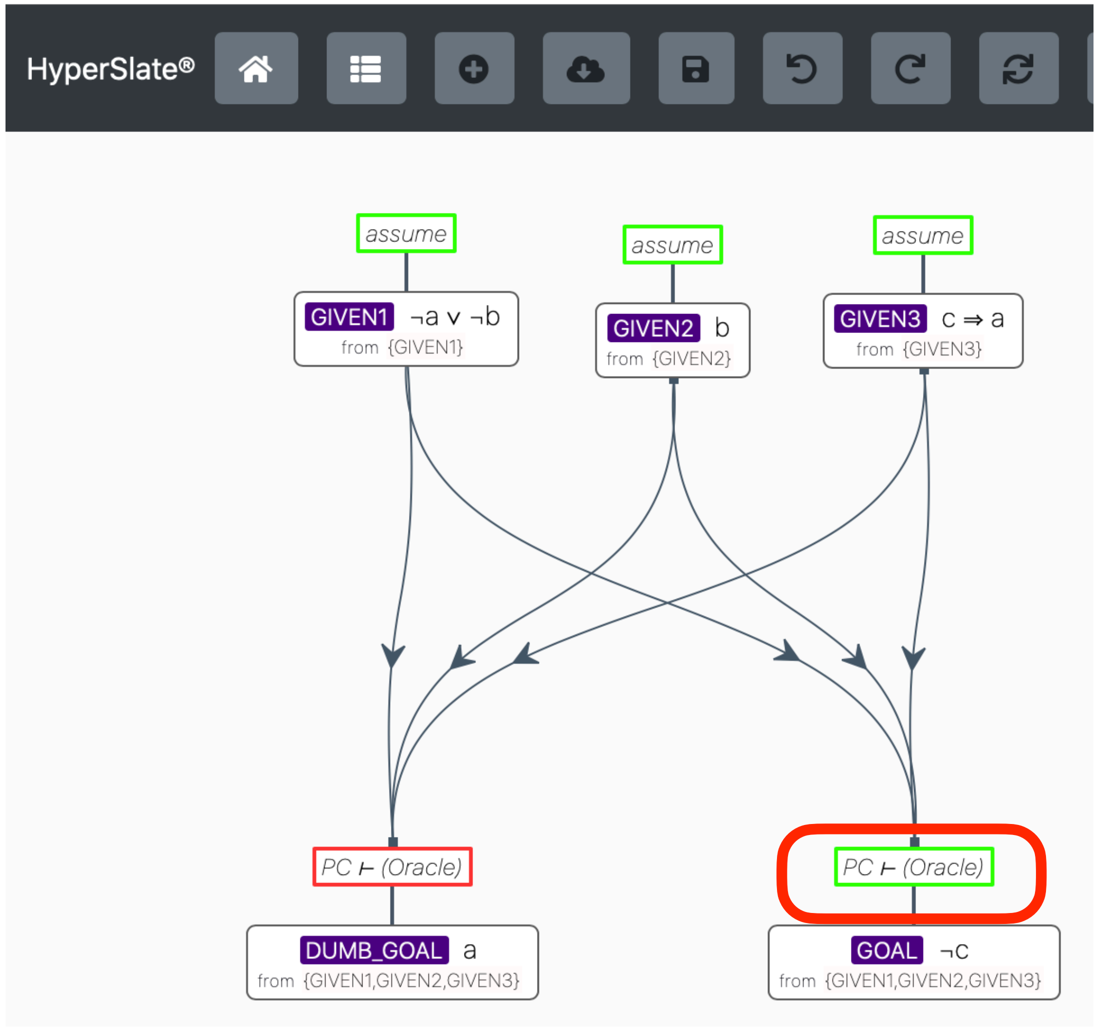
a

none of the above

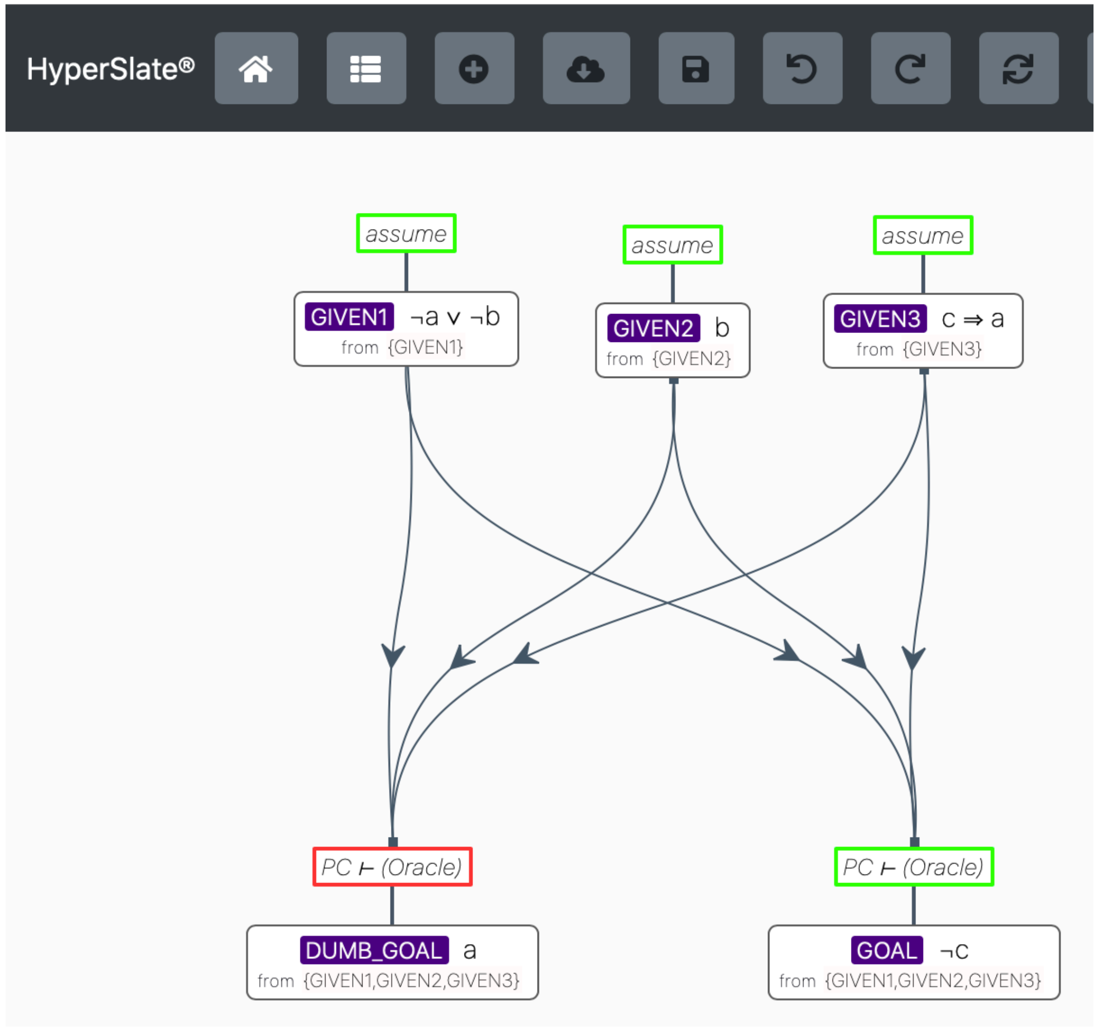
Our First Rule of Inference (= Inference Schema): PC (Entailment) Oracle



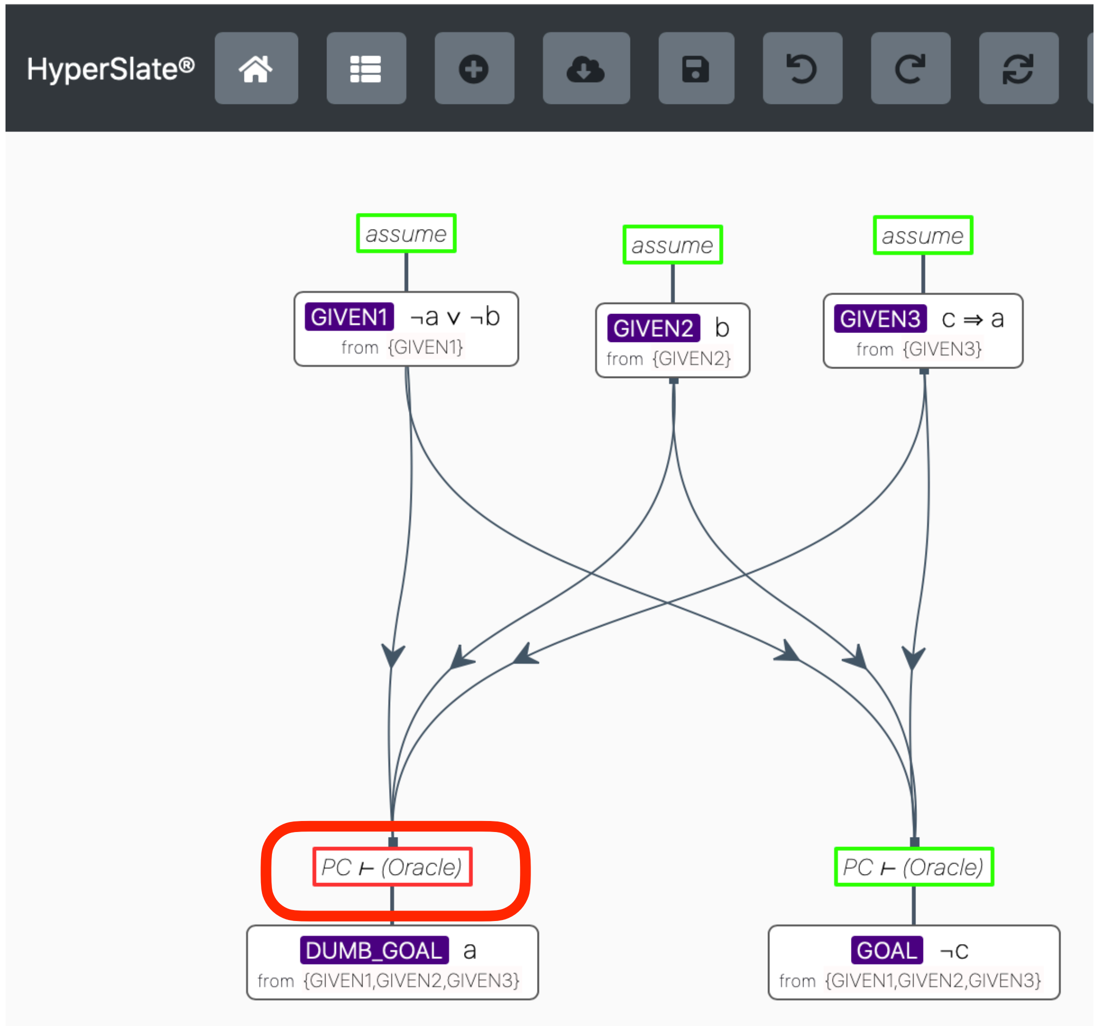
Our First Rule of Inference (= Inference Schema): PC (Entailment) Oracle



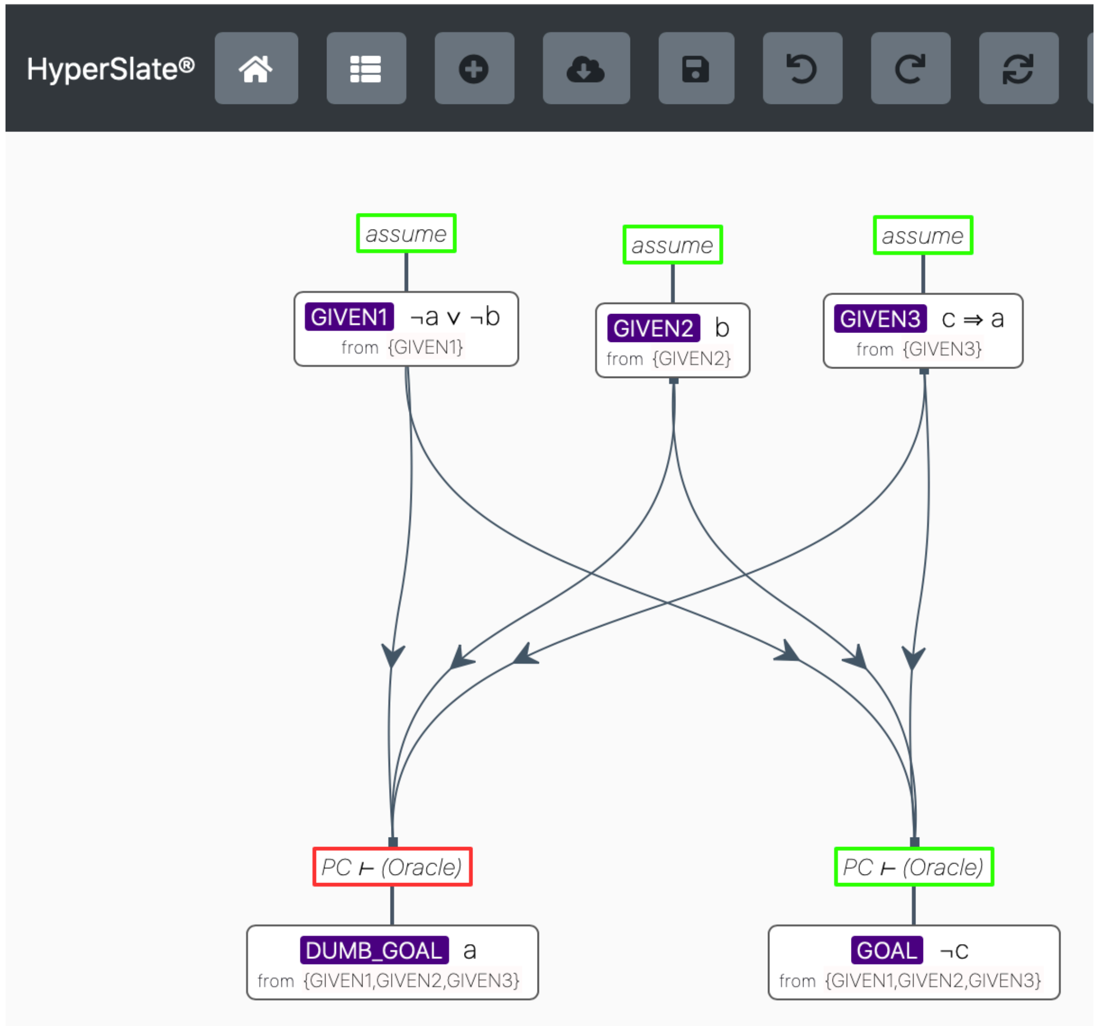
Our First Rule of Inference (= Inference Schema): PC (Entailment) Oracle



Our First Rule of Inference (= Inference Schema): PC (Entailment) Oracle



Our First Rule of Inference (= Inference Schema): PC (Entailment) Oracle



GIVEN1 ¬a ∨ ¬b
from {GIVEN1}

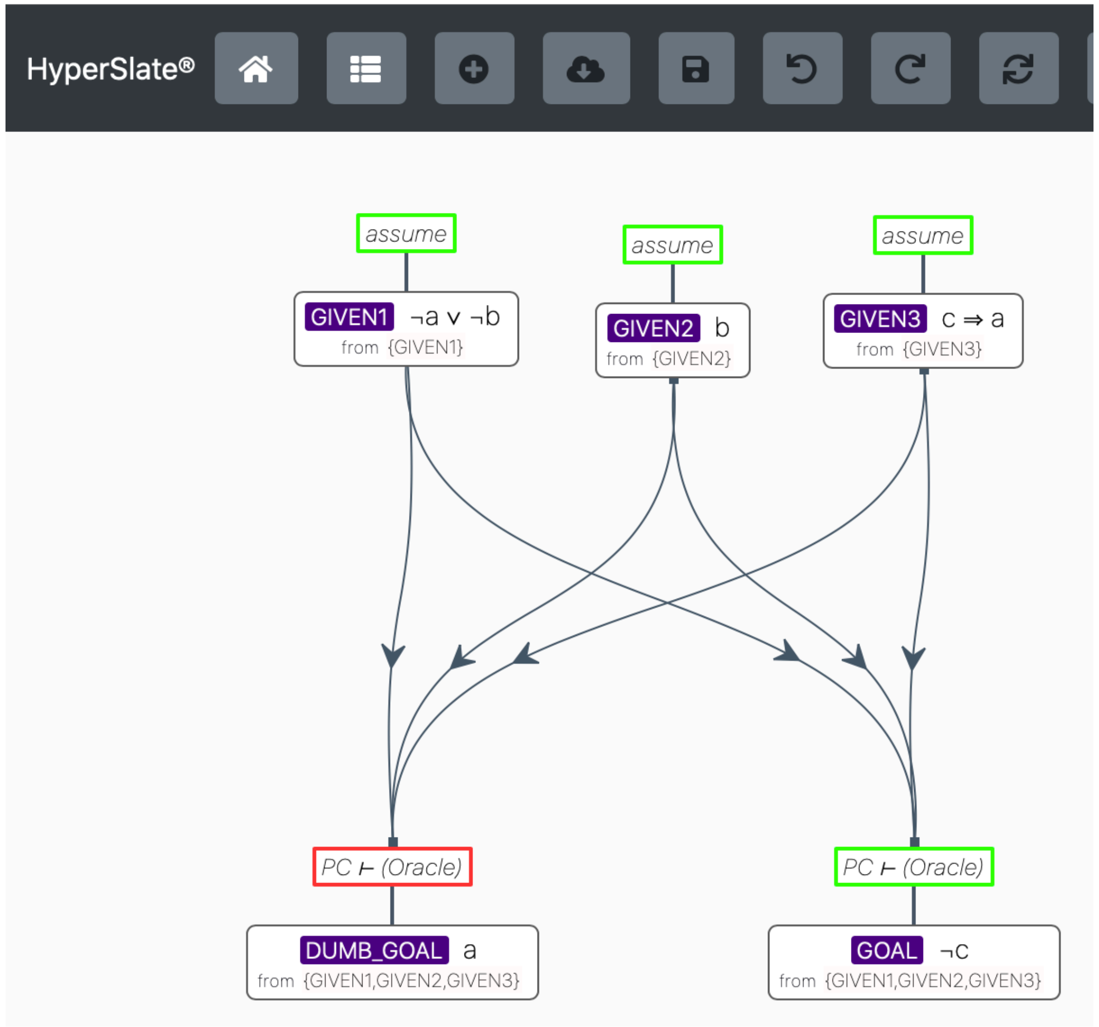
GIVEN2 b
from {GIVEN2}

GIVEN3 c ⇒ a
from {GIVEN3}

DUMB_GOAL a
from {GIVEN1,GIVEN2,GIVEN3}

GOAL ¬c
from {GIVEN1,GIVEN2,GIVEN3}

Our First Rule of Inference (= Inference Schema): PC (Entailment) Oracle



“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements are provable?

$\neg c$

e

h

$\neg a$

all of the above

“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements are provable?

$\neg c$

e

h

$\neg a$

[✓ all of the above]

“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

Show in HyperSlate® that each of the first four options can be proved using the PC entailment oracle.

which one of the following statements are provable?

$\neg c$

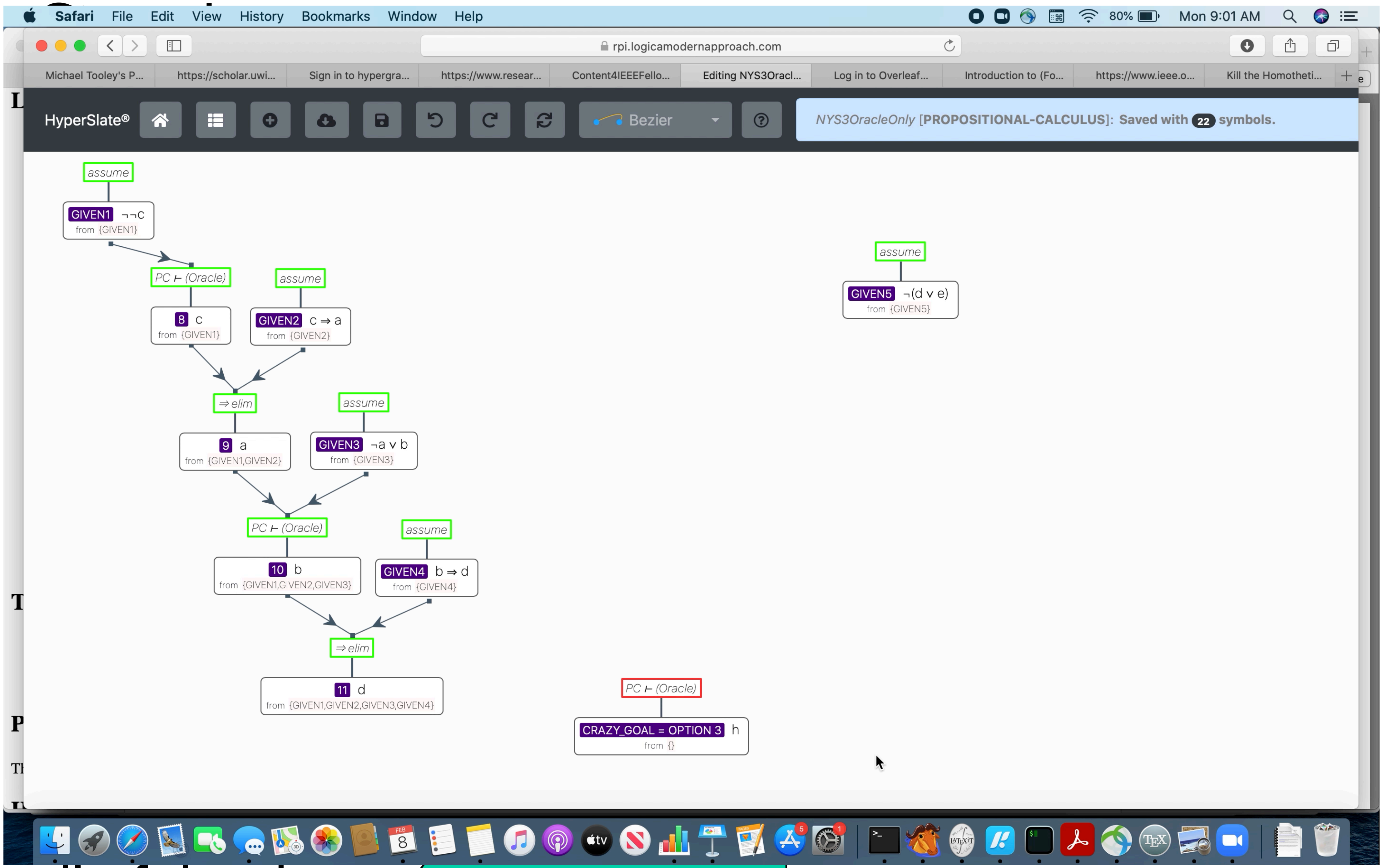
e

h

$\neg a$

all of the above ⬅

"NYS 3" Revisited



NYS3OracleOnly [PROPOSITIONAL-CALCULUS]: Saved with 22 symbols.

all of the above

Det er en ære å lære formell logikk!

Det er en ære å lære formell logikk!

Interactive Part II of Class? …