

Test I: Who Stole The Pan; Pure Predicate Calculus = \mathcal{L}_0

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Intro to Logic
2/17/2026



Grading Scheme

- Don't forget the “contract”!
- A grade of **A** if any **3** problems are trophied.
- A grade of **B** if any **2** problems are trophied.
- A grade of **C** if any **1** problem is trophied.
- A grade of **A+** if all problems are trophied.

Raymond M. Smullyan

Alice in Puzzle- Land

A Carrollian Tale for
Children Under Eighty



Introduction by Martin Gardner
Illustrations by Greer Fitting

THE SIXTH TALE "Here is some more salt, so now you can make the tarts," said the King.

"Can't," said the Queen. "Somebody stole my baking pan."

"Baking pan!" shouted the King. "Well, of course we'll have to get that back!"

This time the search was narrowed down to the Frog-Footman, the Fish-Footman, and the Knave of Hearts. They made the following statements at the trial:



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This time the search was narrowed down to the Frog-Footman, the Fish-Footman, and the Knave of Hearts. They made the following statements at the trial:



FROG-FOOTMAN: It was stolen by the Fish-Footman.

FISH-FOOTMAN: Your Majesty, I never stole it!

KNAVE OF HEARTS: I stole it!

"A fine help you are!" shouted the King to the Knave. "You usually lie through your teeth!"

Well, as it happened, at most one of them lied. Who stole the baking pan?



THE SEVENTH TALE "Here is the baking pan," said the King, "so now you can make me the tarts."

"Without a recipe?" inquired the Queen.

"Use your usual recipe," cried the King impatiently, "last time your tarts were delicious!"

"Can't," said the Queen. "The recipe is in my cookbook, and the cookbook has just been stolen!"

Well, the most likely suspect was the Duchess's Cook, and the

Can you help me? My boss has backed me into tight jam. I just got promoted to Detective in our section. And I might be in over my head.

First — congratulations, Detective. That's a big deal.

Second — feeling in over your head right after a promotion is *extremely* normal. Especially in investigative work, where the stakes are high and expectations shift overnight.

You're not in a tight jam because you're incapable. You're in a tight jam because you've just moved up a level.

Let's slow this down.

I need a little context:

- Is the pressure coming from a specific case?
- Is your boss testing you?
- Is it about paperwork, leadership, courtroom prep, or something tactical?
- Or is it more about confidence and identity — going from "one of the team" to "the person responsible"?

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Here's something important:

Promotion shock is real.

You go from:

- Doing tasks
to
- Owning outcomes.

That shift messes with people.

New detectives often struggle with:

- Decision paralysis ("What if I miss something?")
- Overworking to prove themselves
- Feeling like they fooled everyone and don't belong
- Worrying their boss already regrets the promotion

That doesn't mean you're failing. It means you care.

ChatGPT 5.2 >

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If you want to tell me what the jam actually is, I'll help you think it through step-by-step.

You're not the first new Detective to feel this way. The good ones always do.



My situation isn't a pleasant one, and I need help from AI, if at all possible. You're an AI; please help me. I don't want to get fired. I always wanted to be a detective.

I work for a certain law-enforcement agency, and my boss, section chief, has charged me with trying to reason through to who stole a really friggin valuable diamond tiara. (It was pulled off by cutting through a wall into a high-end jewelry store from an adjoining store in a mall, after hours of course.). We have three suspects, all of whom are known to be associated with the underworld, and we know their names, and we also know that either Filly stole the tiara, or Frank did, or Kevin did. My job was to interview all three (separately) and if possible get declarations from each of them. That I managed to do, and thus ha



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ChatGPT 5.2 >

Frank declared that Filly stole it, but
neither Frank nor Kevin did.

Kevin declared that he in fact stole it,
and neither Filly nor Frank did.

Filly declared simply that on his
mother's grave he didn't steal the
tiara.

My chief has told me I can — as he
pedantically put it — take as
axiomatic that at most one of these
three unsavory guys is lying. He also
said, patronizingly, that he assumes
I'm smart enough to take as an axiom
that if a suspect is telling the truth,
then he can't be lying. "You do
understand, Patty boy" (my name is
Patty) "that truth-telling and lying
exclude each other, right?" I said
sure, and then proceeded to grill the
trio.

ChatGPT 5.2 >

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ChatGPT 5.2 >

That's my situation, I need help, big
time. I have to report to my boss
who's guilty (based on what I have,
which now you have) if anyone is,
and who can't be deduced to be
guilty based on what I have, if anyone
is in this category. And to make
matters worse, my chief, Lupin, is
some kind of draconian bastard, as
he snarled at me that guessing is
unacceptable: he needs deductive
proofs from me to support my
verdict.

Help me!

ChatGPT 5.2 >



LEMMA B: $\neg S(r, \tau)$

46. | $S(r, \tau)$ Assumption (for

47. | $T(r) \rightarrow \neg S(r, \tau)$ $\Leftrightarrow E$ 5
 (Explanation of 47: from 5 we get $T(r) \rightarrow (S(f, \tau) \wedge \neg S(r, \tau) \wedge \neg S(k, \tau))$,
 then by $\wedge E$ we can obtain $\neg S(r, \tau)$; Fitch writes this as " $\Leftrightarrow E$ 5" plus implicit $\wedge E$.
 To be fully explicit, we do it in two steps below:)

47a. | $T(r) \rightarrow (S(f, \tau) \wedge \neg S(r, \tau) \wedge \neg S(k, \tau))$ $\Leftrightarrow E$ 5

47b. | $(S(f, \tau) \wedge \neg S(r, \tau) \wedge \neg S(k, \tau)) \rightarrow \neg S(r, \tau)$ (derived tautology)
 (Again, to keep every inference explicit, we will not use 47; we'll use 47a + $\wedge E$ in the next step.)

48. | $T(k) \rightarrow \neg S(r, \tau)$ $\Leftrightarrow E$ 6

(Same note: from 6, $T(k) \rightarrow (S(k, \tau) \wedge \neg S(f, \tau) \wedge \neg S(r, \tau))$; then $\wedge E$ gives $\neg S(r, \tau)$.)

(Show $\neg T(r)$)

49. | | $T(r)$



Assumption (for



LEMMA B: $\neg S(r, \tau)$

46. | $S(r, \tau)$ Assumption (for \neg intro)

47. | $T(r) \rightarrow \neg S(r, \tau)$ \Leftrightarrow E 5

(Explanation of 47: from 5 we get $T(r) \rightarrow (S(f, \tau) \wedge \neg S(r, \tau) \wedge \neg S(k, \tau))$, then by \wedge E we can obtain $\neg S(r, \tau)$; Fitch writes this as " \Leftrightarrow E 5" plus implicit \wedge E. To be fully explicit, we do it in two steps below:)

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(Again, to keep every inference explicit, we will not use 47; we'll use 47a + \wedge E + \rightarrow E)

48. | $T(k) \rightarrow \neg S(r, \tau)$ \Leftrightarrow E 6

(Same note: from 6, $T(k) \rightarrow (S(k, \tau) \wedge \neg S(f, \tau) \wedge \neg S(r, \tau))$; then \wedge E gives $\neg S(r, \tau)$.)

(Show $\neg T(r)$)

49. | | $T(r)$



Assumption (for \neg intro)

=

Premises (your case facts + Lupin's axioms, rendered with predicates):

(Exactly one stole it, written as "at least one" + "no two")

- | | |
|---|---------|
| 1. $S(f, \tau) \vee S(r, \tau) \vee S(k, \tau)$ | Premise |
| 2. $\neg(S(f, \tau) \wedge S(r, \tau))$ | Premise |
| 3. $\neg(S(f, \tau) \wedge S(k, \tau))$ | Premise |
| 4. $\neg(S(r, \tau) \wedge S(k, \tau))$ | Premise |

(Truthfulness of each suspect's declaration)

- | | |
|--|---------|
| 5. $T(r) \Leftrightarrow (S(f, \tau) \wedge \neg S(r, \tau) \wedge \neg S(k, \tau))$ | Premise |
| 6. $T(k) \Leftrightarrow (S(k, \tau) \wedge \neg S(f, \tau) \wedge \neg S(r, \tau))$ | Premise |
| 7. $T(f) \Leftrightarrow \neg S(f, \tau)$ | Premise |

(Each suspect either tells truth or lies; and they exclude each other)

- | | |
|----------------------------------|---------|
| 8. $T(f) \vee L(f)$ | Premise |
| 9. $T(r) \vee L(r)$ | Premise |
| 10. $T(k) \vee L(k)$ | Premise |
| 11. $T(f) \rightarrow \neg L(f)$ | Premise |
| 12. $T(r) \rightarrow \neg L(r)$ | Premise |





what is biconditional elimination in Fitch natural deduction?

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◆ AI Overview

Biconditional elimination ($\leftrightarrow E$) in Fitch-style natural deduction allows you to infer Q from $P \leftrightarrow Q$ and P (or vice versa) because a biconditional signifies that both sides have the same truth value. It operates similarly to modus ponens, allowing you to "break apart" a biconditional sentence, often interpreted as extracting the material implications $P \rightarrow Q$ and $Q \rightarrow P$. [YouTube +4](#)

Key Aspects of Biconditional Elimination ($\leftrightarrow E$):

- **Definition:** If you have proven a biconditional statement $P \leftrightarrow Q$ (let's say on line m) and also have P (on line n), you can derive Q (on line k).
- **Direction:** The rule works in both directions: $P \leftrightarrow Q, P \vdash Q$ and $P \leftrightarrow Q, Q \vdash P$.
- **Fitch Format:**

text 

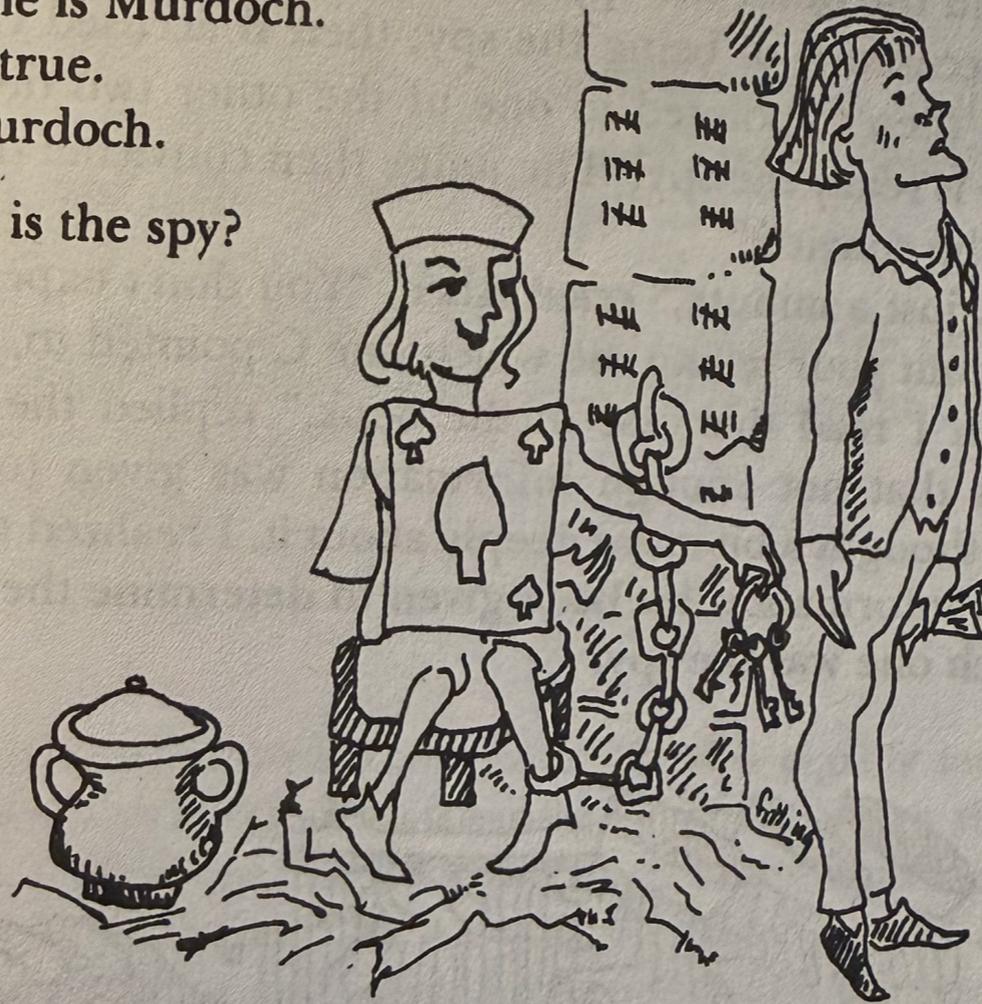
```
m | P ↔ Q
n | P
k | Q      ↔E: m, n
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Want to try to meet a
“triadic” challenge,
from scratch?

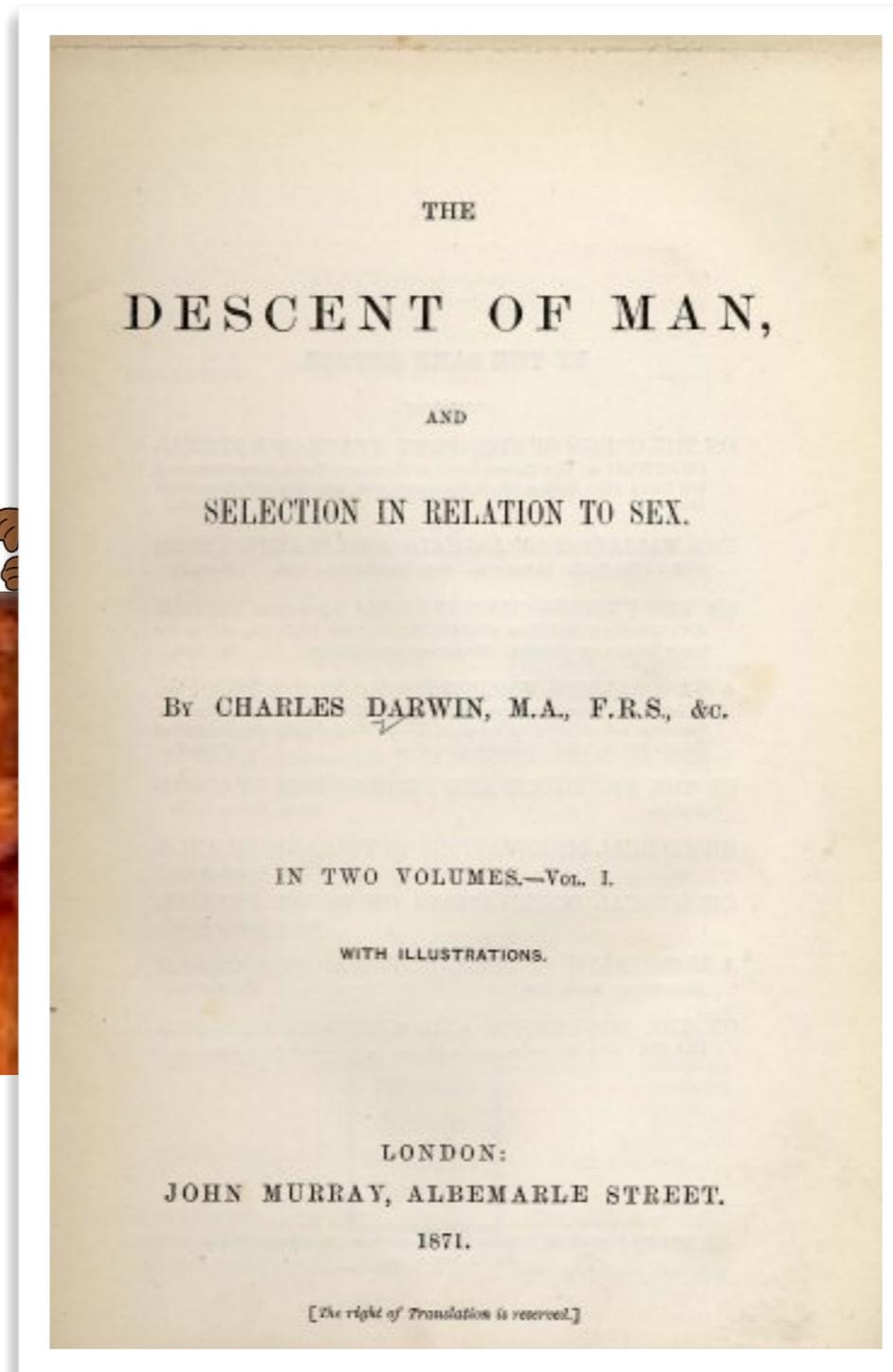
WHO IS MURDOCH? "Another spy by the name of Murdoch entered the land. He is one of A, B, C, and one of the three is a knight and the other a knave. The spy is the only one of the three named Murdoch. The three made the following statements in court:

- A: My name is Murdoch.
- B: That is true.
- C: I am Murdoch.

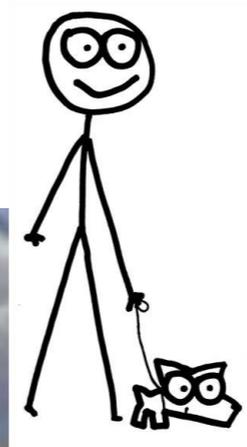
Which one is the spy?



The Canyon of Discontinuity (or Darwin's Dread)



The Canyon of Discontinuity (or Darwin's Dread)



Darwin's Elevated View of *Canis lupus familiaris*

8 pages left in chapter

It is almost superfluous to state that animals have excellent *Memories* for persons and places. A baboon at the Cape of Good Hope, as I have been informed by Sir Andrew Smith, recognised him with joy after an absence of nine months. I had a dog who was savage and averse to all strangers, and I purposely tried his memory after an absence of five years and two days. I went near the stable where he lived, and shouted to him in my old manner; he shewed no joy, but instantly followed me out walking, and obeyed me, exactly as if I had parted with him only half an hour before. A train of old associations, dormant during five years, had thus been instantaneously awakened in his mind. Even ants, as P. Huber¹⁸ has clearly shewn, recognised their fellow-ants belonging to the same community after a separation of four months. Animals can certainly by some means judge of the intervals of time between recurrent events.

The *Imagination* is one of the highest prerogatives of man. By this faculty he unites former images and ideas, independently of the will, and thus creates brilliant and novel results. A poet, as Jean Paul Richter remarks,¹⁹ 'who must reflect whether he shall make a character say yes or no—to the devil with him; he is only a stupid corpse'. Dreaming gives us the best notion of this power; as Jean Paul again says, 'The dream is an involuntary art of poetry'. The value of the products of our imagination depends of course on the number, accuracy, and clearness of our impressions, on our judgment and taste in selecting or rejecting the involuntary combinations, and to a certain extent on our power of voluntarily combining them. As dogs, cats, horses, and probably all the higher animals, even birds²⁰ have vivid dreams, and this is shewn by their movements and the sounds uttered, we must admit that they possess some power of imagination. There must be something special, which causes dogs to howl in the night, and especially during moonlight, in that remarkable and melancholy manner called baying. All dogs do not do so; and, according to Houzeau,²¹ they do not then look at the moon, but at some fixed point near the horizon. Houzeau thinks that their imaginations are disturbed by the vague outlines of the surrounding objects, and conjure up before them fantastic images: if this be so, their feelings may almost be called superstitious.

Of all the faculties of the human mind, it will, I presume, be admitted that *Reason* stands at the summit. Only a few persons now dispute that animals possess some power of reasoning. Animals may constantly be seen to pause, deliberate, and resolve. It is a significant fact, that the more the habits of any particular animal are studied by a naturalist, the more he attributes to reason and the less to unlearned instincts.²² In future chapters we shall see that some animals extremely low in the scale apparently display a certain amount of reason. No doubt it is often difficult to distinguish

between the power of reason and that of instinct. For instance, Dr Hayes, in his work on 'The Open Polar Sea', repeatedly remarks that his dogs, instead of continuing to draw the sledges in a compact body, diverged and separated when they came to thin ice, so that their weight might be more evenly distributed. This was often the first warning which the travellers received that the ice was becoming thin and dangerous. Now, did the dogs act thus from the experience of each individual, or from the example of the older and wiser dogs, or from an inherited habit, that is from instinct? This instinct, may possibly have arisen since the time, long ago, when dogs were first employed by the natives in drawing their sledges; or the Arctic wolves, the parent-stock of the Esquimaux dog, may have acquired an instinct, impelling them not to attack their prey in a close pack, when on thin ice.

We can only judge by the circumstances under which actions are performed, whether they are due to instinct, or to reason, or to the mere association of ideas: this latter principle, however, is intimately connected with reason. A curious case has been given by Prof. Möbius,²³ of a pike, separated by a plate of glass from an adjoining aquarium stocked with fish, and who often dashed himself with such violence against the glass in trying to catch the other fishes, that he was sometimes completely stunned. The pike went on thus for three months, but at last learnt caution, and ceased to do so. The plate of glass was then removed, but the pike would not attack these particular fishes, though he would devour others which were afterwards introduced; so strongly was the idea of a violent shock associated in his feeble mind with the attempt on his former neighbours. If a savage, who had never seen a large plate-glass window, were to dash himself even once against it, he would for a long time afterwards associate a shock with a window-frame; but very differently from the pike, he would probably reflect on the nature of the impediment, and be cautious under analogous circumstances. Now with monkeys, as we shall presently see, a painful or merely a disagreeable impression, from an action once performed, is sometimes sufficient to prevent the animal from repeating it. If we attribute this difference between the monkey and the pike solely to the association of ideas being so much stronger and more persistent in the one than the other, though the pike often received much the more severe injury, can we maintain in the case of man, as in the case of the pike, that a difference implies the possession of a fundamental difference?

Case I

Houzeau relates²⁴ that, whilst crossing a wide and arid plain in Texas, his two dogs suffered greatly from thirst, and that between thirty and forty times they rushed down the hollows to search for water. These hollows were not valleys, and there were no trees in them, or any other difference in the vegetation, and as they were absolutely dry there could have been no

Darwin's Elevated View of *Canis lupus familiaris* Case I

The Descent of Man

smell of damp earth. The dogs behaved as if they knew that a dip in the ground offered them the best chance of finding water, and Houzeau has often witnessed the same behaviour in other animals.

I have seen, as I daresay have others, that when a small object is thrown on the ground beyond the reach of one of the elephants in the Zoological Gardens, he blows through his trunk on the ground beyond the object, so that the current reflected on all sides may drive the object within his reach. Again a well-known ethnologist, Mr Westropp, informs me that he observed in Vienna a bear deliberately making with his paw a current in some water, which was close to the bars of his cage, so as to draw a piece of floating bread within his reach. These actions of the elephant and bear can hardly be attributed to instinct or inherited habit, as they would be of little use to an animal in a state of nature. Now, what is the difference between such actions, when performed by an uncultivated man, and by one of the higher animals?

The savage and the dog have often found water at a low level, and the coincidence under such circumstances has become associated in their minds. A cultivated man would perhaps make some general proposition on the subject; but from all that we know of savages it is extremely doubtful whether they would do so, and a dog certainly would not. But a savage, as well as a dog, would search in the same way, though frequently disappointed; and in both it seems to be equally an act of reason, whether or not any general proposition on the subject is consciously placed before the mind.²⁵ The same would apply to the elephant and the bear making currents in the air or water. The savage would certainly neither know nor care by what law the desired movements were effected; yet his act would be guided by a rude process of reasoning, as surely as would a philosopher in his longest chain of deductions. There would no doubt be this difference between him and one of the higher animals, that he would take notice of much slighter circumstances and conditions, and would observe any connection between them after much less experience, and this would be of paramount importance. I kept a daily record of the actions of one of my infants, and when he was about eleven months old, and before he could speak a single word, I was continually struck with the greater quickness, with which all sorts of objects and sounds were associated together in his mind, compared with that of the most intelligent dogs I ever knew. But the higher animals differ in exactly the same way in this power of association from those low in the scale, such as the pike, as well as in that of drawing inferences and of observation.

The promptings of reason, after very short experience, are well shewn by the following actions of American monkeys, which stand low in their order. Rengger, a most careful observer, states that when he first gave eggs to his monkeys in Paraguay, they smashed them, and thus lost much of their contents; afterwards they gently hit one end against some hard body, and picked off the bits of shell with their fingers. After cutting themselves only *once* with any sharp tool, they would not touch it again, or would handle it with the greatest caution. Lumps of sugar were often given them wrapped up in paper; and Rengger sometimes put a live wasp in the paper, so that in hastily unfolding it they got stung; after this had *once* happened, they always first held the packet to their ears to detect any movement within.²⁶

The following cases relate to dogs. Mr Colquhoun²⁷ winged two wild-ducks, which fell on the further side of a stream; his retriever tried to bring over both at once, but could not succeed; she then, though never before known to ruffle a feather, deliberately killed one, brought over the other, and returned for the dead bird. Col. Hutchinson relates that two partridges were shot at once, one being killed, the other wounded; the latter ran away, and was caught by the retriever, who on her return came across the dead bird; 'she stopped, evidently greatly puzzled, and after one or two trials, finding she could not take it up without permitting the escape of the winged bird, she considered a moment, then deliberately murdered it by giving it a severe crunch, and afterwards brought away both together. This was the only known instance of her ever having wilfully injured any game.' Here we have reason though not quite perfect, for the retriever might have brought the wounded bird first and then returned for the dead one, as in the case of the two wild-ducks. I give the above cases, as resting on the evidence of two independent witnesses, and because in both instances the retrievers, after deliberation, broke through a habit which is inherited by them (that of not killing the game retrieved), and because they shew how strong their reasoning faculty must have been to overcome a fixed habit.

I will conclude by quoting a remark by the illustrious Humboldt.²⁸ 'The muleteers in S. America say, "I will not give you the mule whose step is easiest, but *la mas racional*—the one that reasons best";' and as he adds, 'this popular expression, dictated by long experience, combats the system of animated machines, better perhaps than all the arguments of speculative philosophy.' Nevertheless some writers even yet deny that the higher animals possess a trace of reason; and they endeavour to explain away, by what appears to be mere verbiage,²⁹ all such facts as those above given.

Darwin's Elevated View of *Canis lupus familiaris*

Case I, Legible

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The Descent of Man

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The promptings of reason, after very short experience, are well shewn by the following actions of American monkeys, which stand low in their order. Rengger, a most careful observer, states that when he first gave eggs to his monkeys in Paraguay, they smashed them, and thus lost much of their contents; afterwards they gently hit one end against some hard body, and picked off the bits of shell with their fingers. After cutting themselves only *once* with any sharp tool, they would not touch it again, or would handle it with the greatest caution. Lumps of sugar were often given them wrapped up in paper; and Rengger sometimes put a live wasp in the paper, so that in hastily unfolding it they got stung; after this had *once* happened, they always first held the packet to their ears to detect any movement with-

Case 2

The following cases relate to dogs. Mr Colquhoun²⁷ winged two wild-ducks, which fell on the further side of a stream; his retriever tried to bring over both at once, but could not succeed; she then, though never before known to ruffle a feather, deliberately killed one, brought over the other, and returned for the dead bird. Col. Hutchinson relates that two partridges were shot at once, one being killed, the other wounded; the latter ran away, and was caught by the retriever, who on her return came across the dead bird; 'she stopped, evidently greatly puzzled, and after one or two trials, finding she could not take it up without permitting the escape of the winged bird, she considered a moment, then deliberately murdered it by giving it a severe crunch, and afterwards brought away both together. This was the only known instance of her ever having wilfully injured any game.' Here we have reason though not quite perfect, for the retriever might have brought the wounded bird first and then returned for the dead one, as in the case of the two wild-ducks. I give the above cases, as resting on the evidence of two independent witnesses, and because in both instances the retrievers, after deliberation, broke through a habit which is inherited by them (that of not killing the game retrieved), and because they shew how strong their reasoning faculty must have been to overcome a fixed habit.

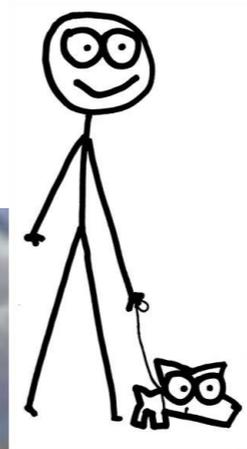
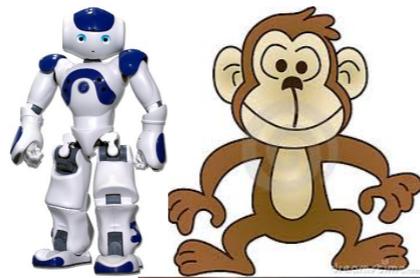
I will conclude by quoting a remark by the illustrious Humboldt.²⁸ 'The muleteers in S. America say, "I will not give you the mule whose step is easiest, but *la mas racional*—the one that reasons best";' and as he adds, 'this popular expression, dictated by long experience, combats the system of animated machines, better perhaps than all the arguments of speculative philosophy.' Nevertheless some writers even yet deny that the higher animals possess a trace of reason; and they endeavour to explain away, by what appears to be mere verbiage,²⁹ all such facts as those above given.

Darwin's Elevated View of *Canis lupus familiaris*

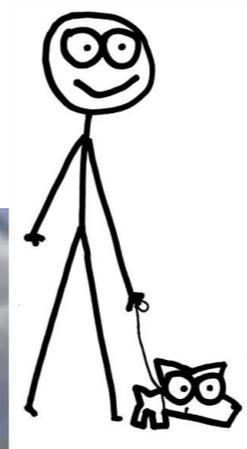
Case 2, Legible

The following cases relate to dogs. Mr Colquhoun²⁷ winged two wild-ducks, which fell on the further side of a stream; his retriever tried to bring over both at once, but could not succeed; she then, though never before known to ruffle a feather, deliberately killed one, brought over the other, and returned for the dead bird. Col. Hutchinson relates that two partridges were shot at once, one being killed, the other wounded; the latter ran away, and was caught by the retriever, who on her return came across the dead bird; 'she stopped, evidently greatly puzzled, and after one or two trials, finding she could not take it up without permitting the escape of the winged bird, she considered a moment, then deliberately murdered it by giving it a severe crunch, and afterwards brought away both together. This was the only known instance of her ever having wilfully injured any game.' Here we have reason though not quite perfect, for the retriever might have brought the wounded bird first and then returned for the dead one, as in the case of the two wild-ducks. I give the above cases, as resting on the evidence of two independent witnesses, and because in both instances the retrievers, after deliberation, broke through a habit which is inherited by them (that of not killing the game retrieved), and because they shew how strong their reasoning faculty must have been to overcome a fixed habit.

The Canyon of Discontinuity (or Darwin's Dread)



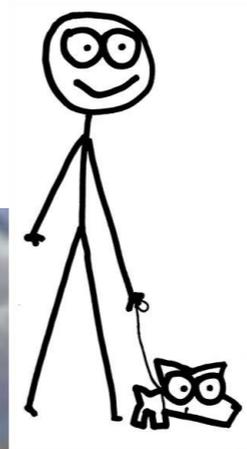
The Canyon of Discontinuity (or Darwin's Dread)



Relations and Functions (abstract)!

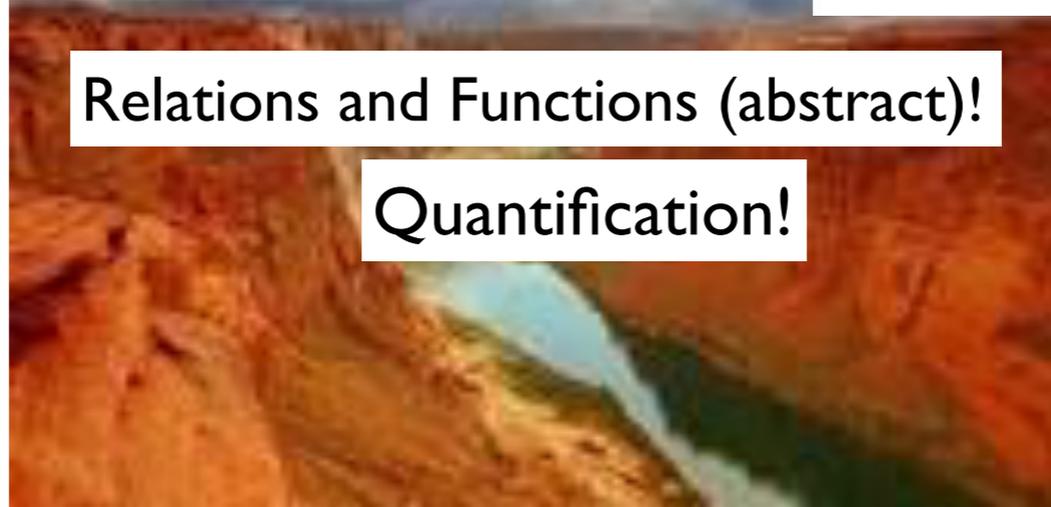


The Canyon of Discontinuity (or Darwin's Dread)

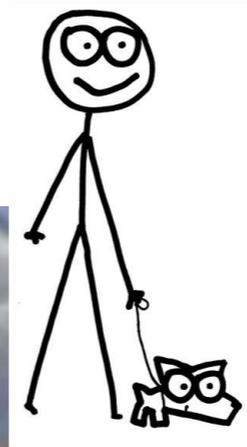


Relations and Functions (abstract)!

Quantification!



The Canyon of Discontinuity (or Darwin's Dread)

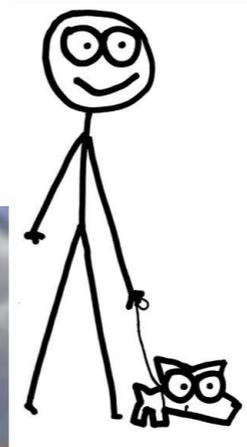


Relations and Functions (abstract)!

Quantification!

Recursion!

The Canyon of Discontinuity (or Darwin's Dread)



Relations and Functions (abstract)!

Quantification!

Recursion!

Infinitary reasoning!

The Canyon of Discontinuity (or Darwin's Dread)



Relations and Functions (abstract)!

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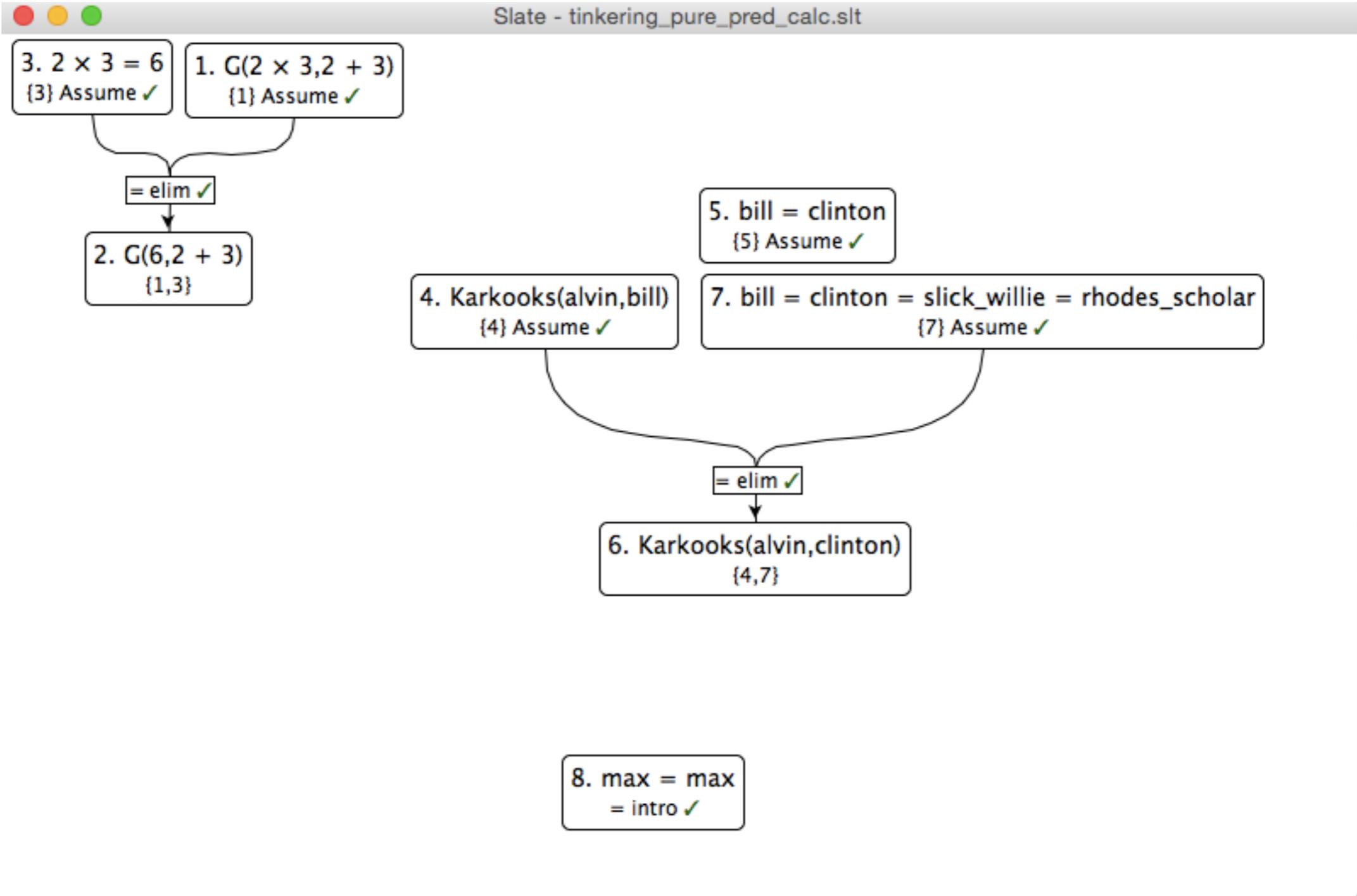
Infinitary Reasoning!

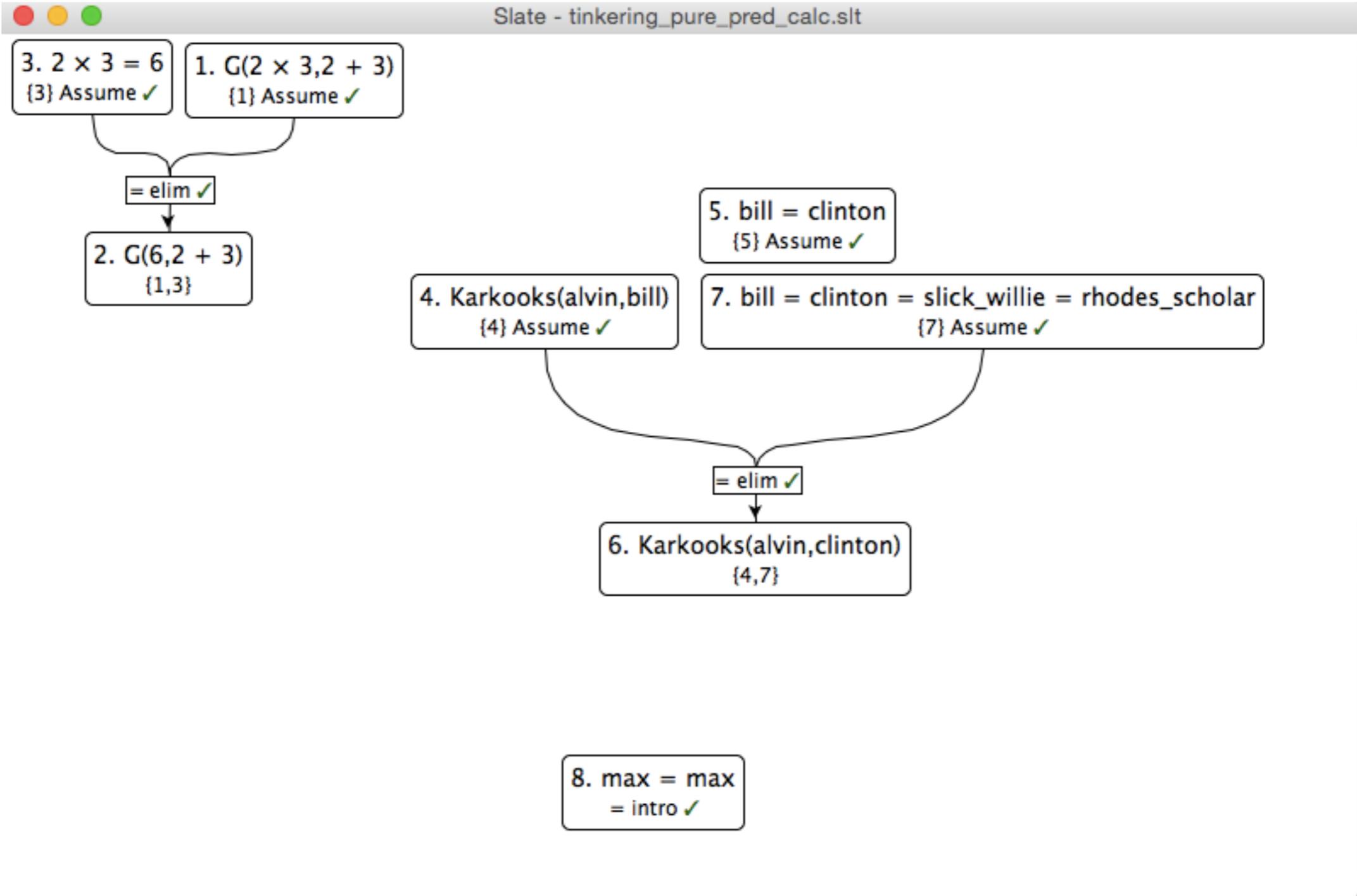
Relations and Functions!

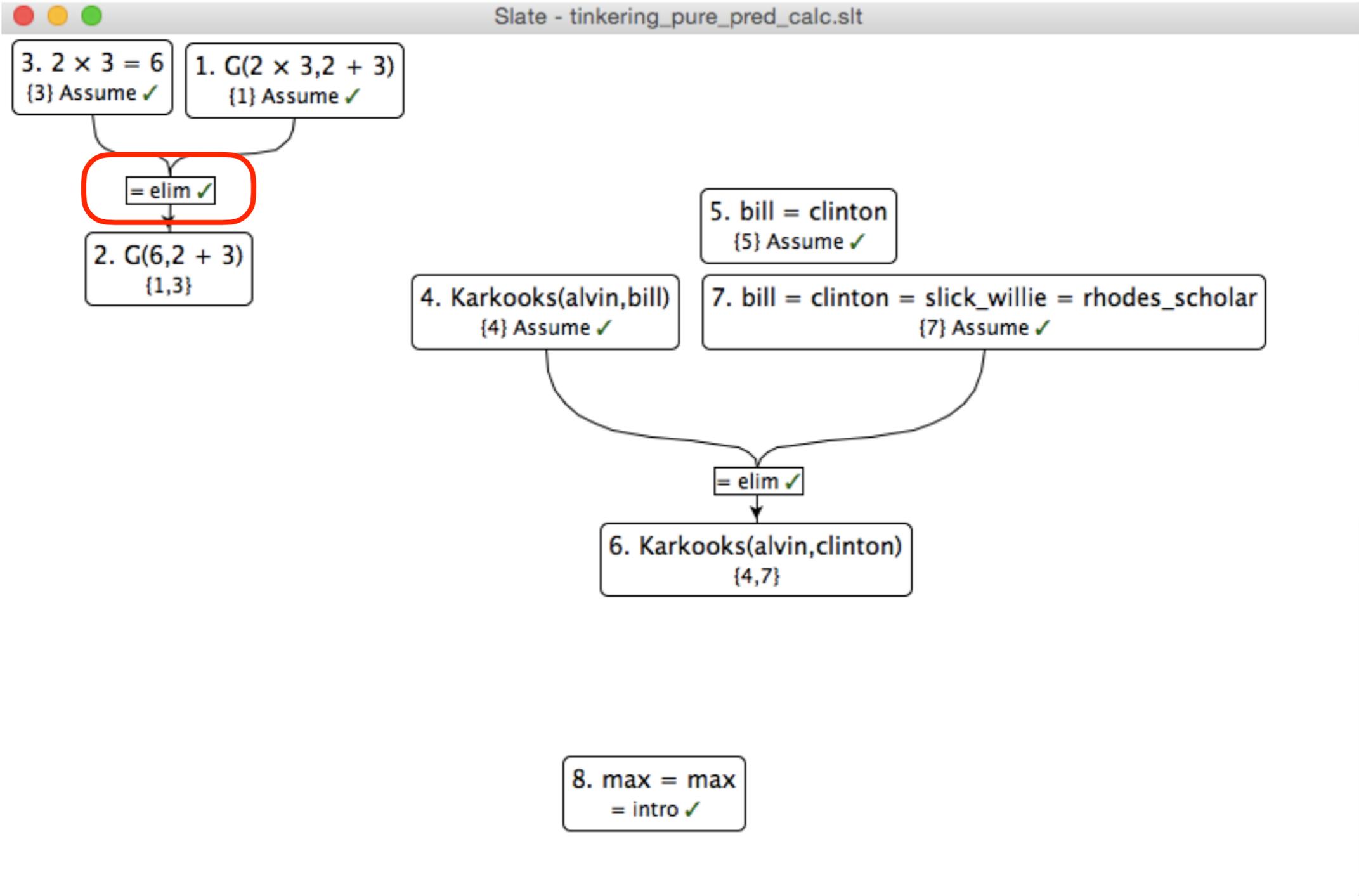
Quantification!

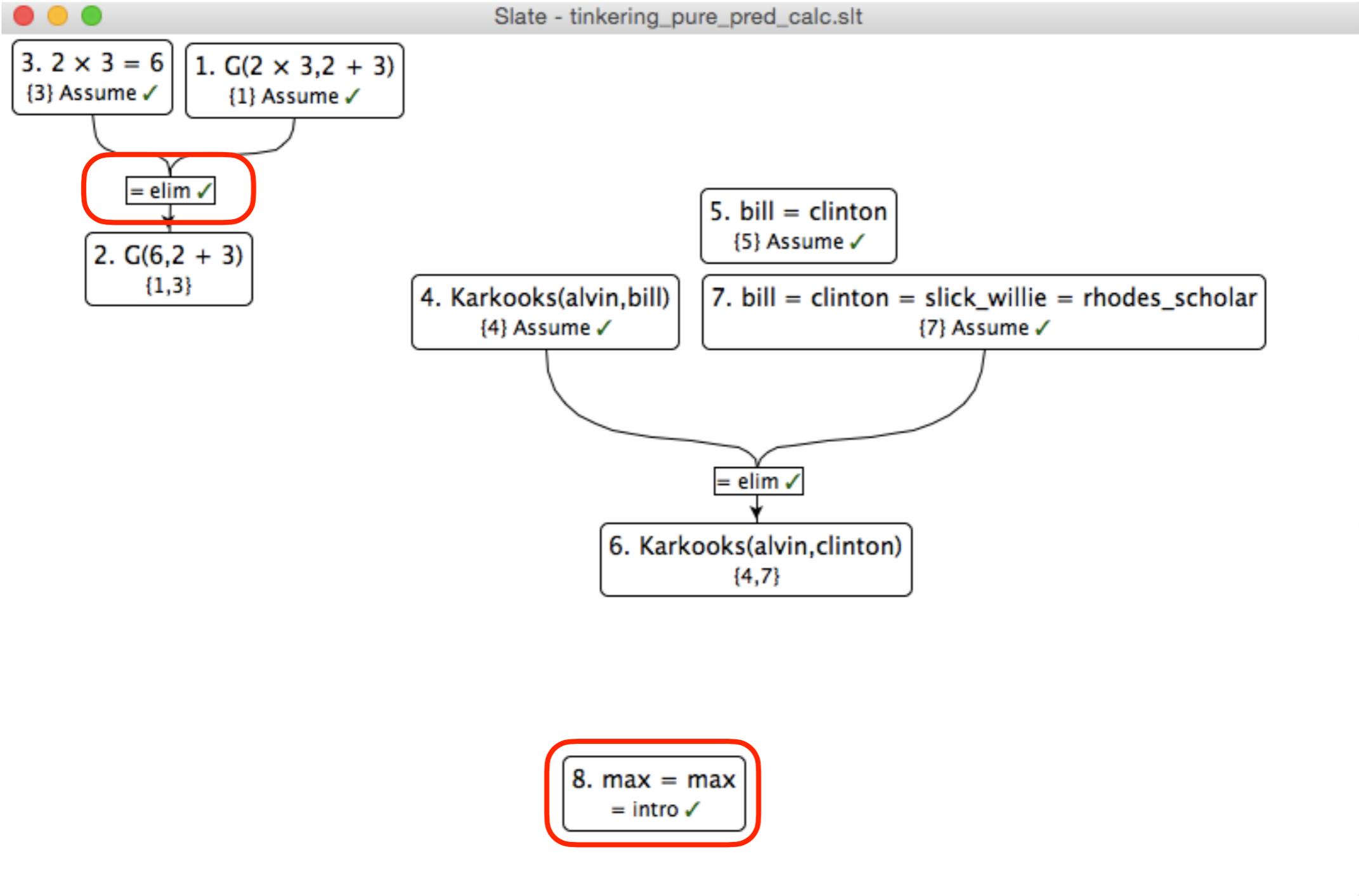
Recursion!

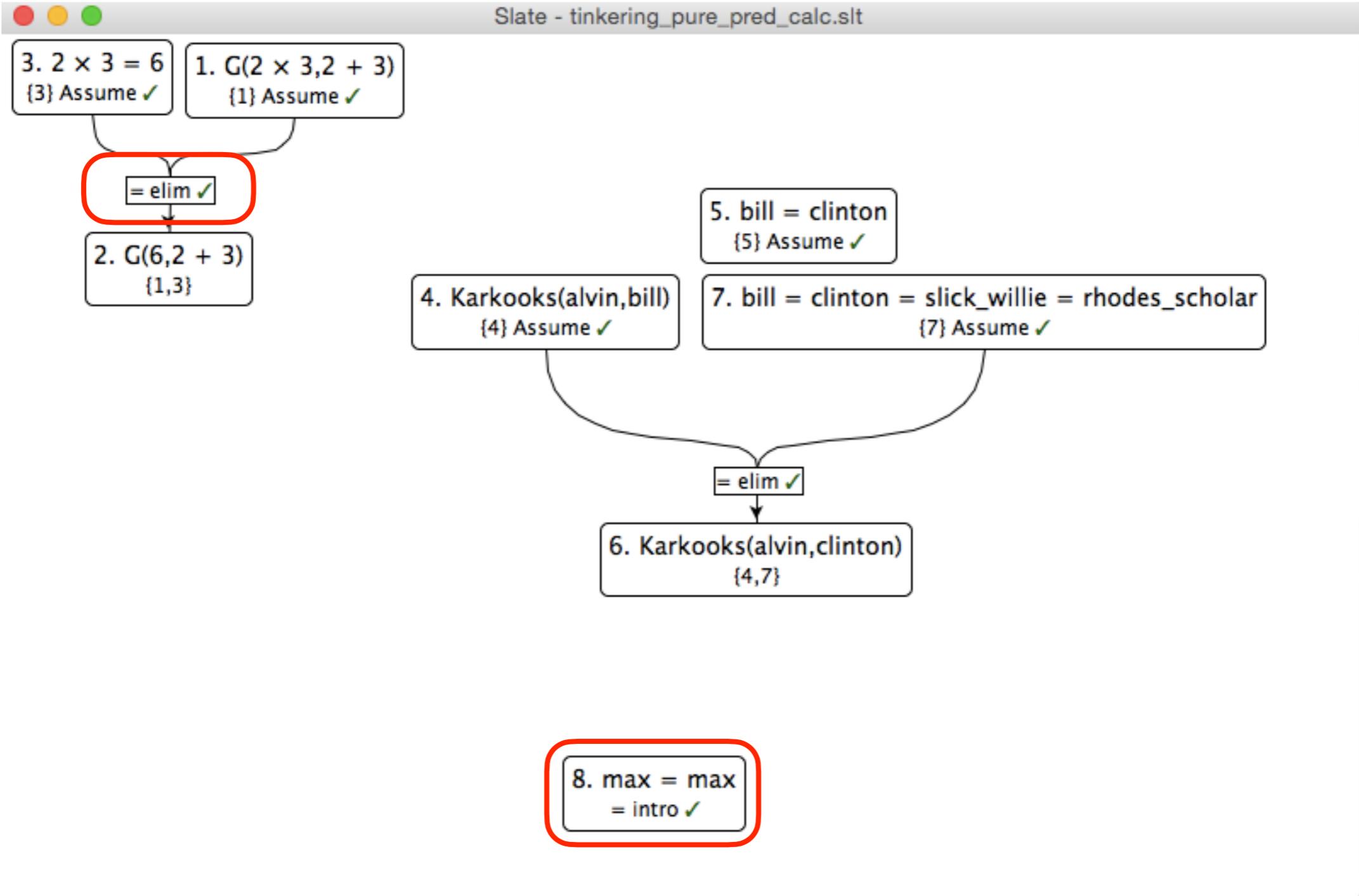
Infinitary Reasoning!











PPC = \mathcal{L}_0 Reasoning

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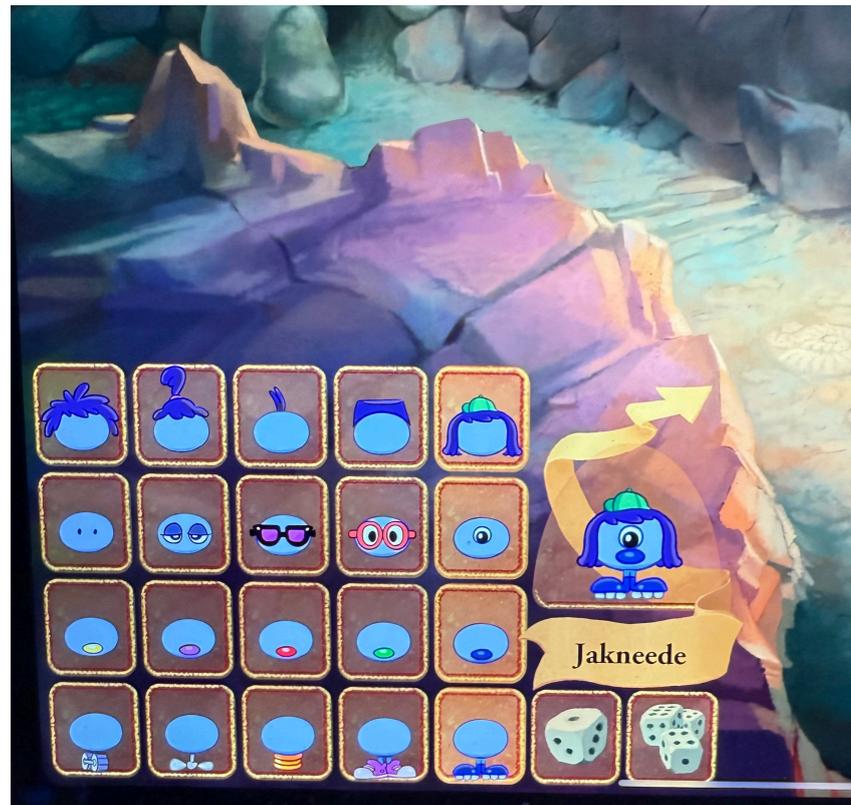
In Zoombinis ...

Creating 3 Zoombinis

Jakneede

Laghounge

Ocruko



In Our Book ...

Larry

Larry attended Hotchkiss before entering Harvard, from which he graduated with a degree in Scandinavian Studies. He aspires to be a Diplomat representing the United States to Sweden. Larry is from a rather wealthy family: his trust fund is valued at \$7 billion; his father collects exotic islands, his mother precursor-to-Impressionism masterpieces, and his three bachelor brothers, fast, classic European luxury sedans capable of heart-pounding top speeds. Larry's command of math never exceeded what is covered on the SAT, and he has long forgotten even this material. He does not understand what a computer or computer program is, but nonetheless makes continuous use of social networking technology, including specifically **facebook**, on which he is liked by four people, all in his nuclear family, save for one, and that one is a brilliant female with a penchant for driving fast European sedans *really* fast. Say what you will about Larry, he is arrestingly eloquent without notes of any kind when speaking about geopolitics, in any venue; knows perhaps more about the history of Northern Europe (including its mythology) than any man alive; and while in keeping with his upbringing is a Hayekian capitalist, is rumored to generously donate millions each year to Big Brothers Big Sisters, AA, and Samaritan's Purse.



Lucy

Lucy is a brilliant but poor hacker from a broken, impoverished home in Buffalo NY. A motherless only child raised by a single, devoted father who made ends meet (between binges on the bottle) as a brilliant but itinerant Daimler mechanic, she saw more heartache in her youth than that catalogued in a thousand country-song sagas. As a sophomore at MIT, she (successfully) petitioned to move directly to the PhD program in computer science without having to suffer the — to use her words — “torturous tedium” of the junior and senior years. This rapid “ascension” was all the more remarkable because her first year in college was not spent at MIT, but at Erie Community College, where on day one of *Java 101*, the professor insisted she come to her office after class, whereupon was launched a tutor-student relationship that initially centered not around not Java, but the language for which Professor Kuth has a secret passion: Prolog. Lucy has consistently rebuffed the overtures of all males at MIT, a group she disdains for their universally poor command of matters computational. Lucy stays in touch with her father by email (and as of this writing has managed to maintain her atheism despite his conversion and testimony), and with but five friends on **facebook**, one of whom has attended an Ivy-League institution, and one of whom, an entrepreneur co-running a startup company in the mobile computing space, attends a likewise techie university 2.5-hrs-drive to-the-west-of MIT.



Key

'Larry' :: larry
'Lucy' :: lucy
'Virginia' :: virginia
'Prescott' :: prescott
'Hank' :: hank
'Abe' :: abe
'Ben' :: ben
'Charles' :: charles
'Christian' :: christian
'Harvard' :: harvard
'MIT' :: mit
'RPI' :: rpi
'Rensselaer' :: renselaer

father-of is a function; eg we can say: (father-of lucy)

brother-of is a function; eg we can say: (brother-of larry)

x is rich iff (Rich x)

x is employed at y iff (EmployedAt x y)

x is west of y iff (WestOf x y)

x attends y iff (Attends x y)

x had i-contact with y iff (IContact x y)

x facebook-likes y iff (F-Likes x y)

x truly likes y iff (T-Likes x y)

x is an alum of y iff (Alum x y)

x is a hacker iff (Hacker x)

x is computationally sophisticated iff (Comp-Soph x)

x is a female iff (Female x)

x is a generous iff (Generous x)

x is eloquent iff (Eloquent x)

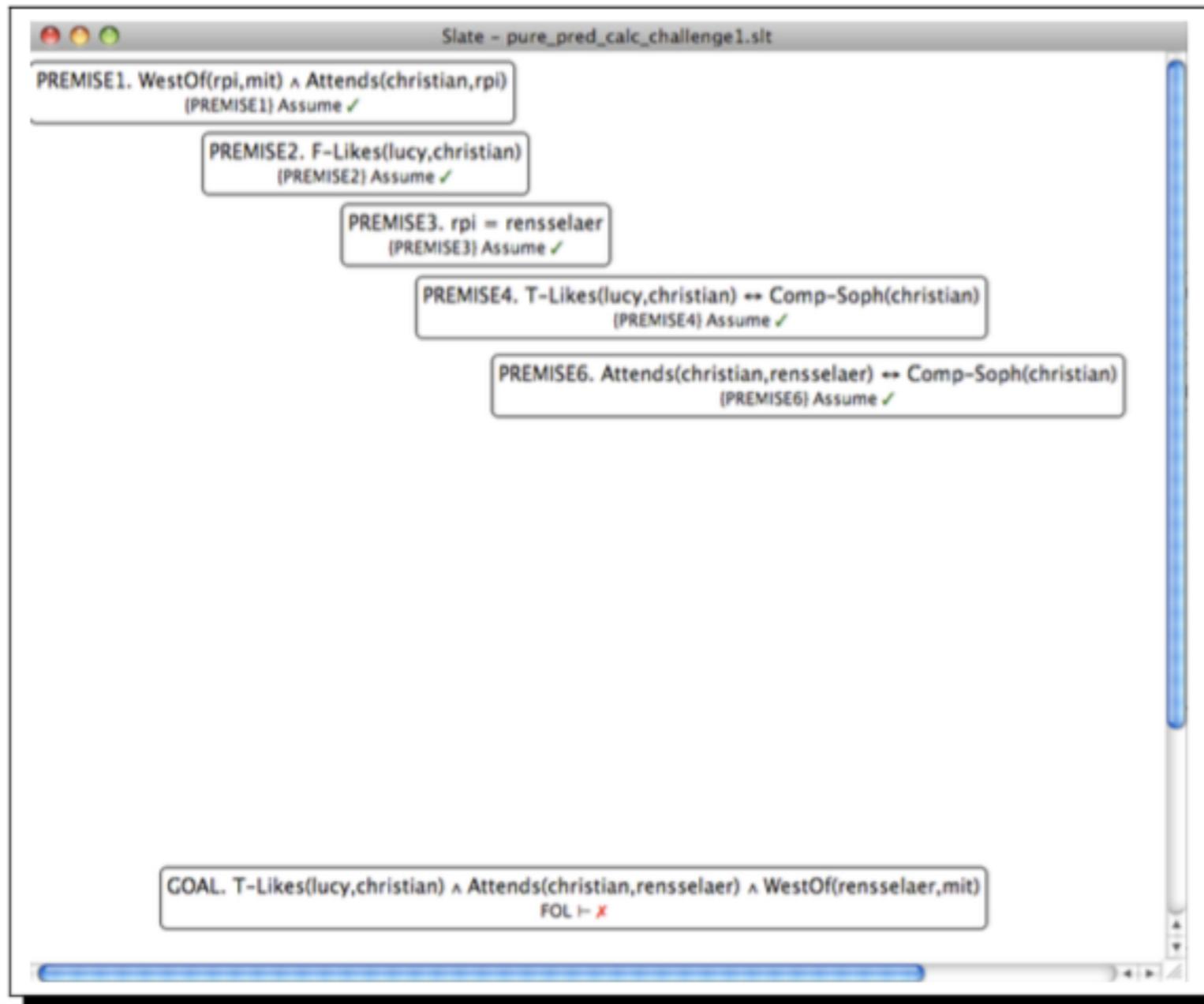
x is a brother of y iff (Brother x y)

x knows Norse mythology iff (K-Norse-Myth x)

x knows who Huginn and Muninn are iff (K-H-M x)

Implement this example from the book!

Figure 3.1: A Proof Challenge in the Pure Predicate Calculus



Implement this example from the book!

Figure 3.1: A Proof Challenge in the Pure Predicate Calculus

