Second-Order Logic and the *k*-order Ladder; Second-Order Axiomatized Arithmetic; Astrologic

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Formal Logic (With AI)
3/22/2026

RAIR
Rensselaer AI and Reasoning Lab

Promised Orientation to Our First Finding-Axioms Problem

Promised Orientation to Our First Finding-Axioms Problem

FindingAxiomsEx1p

Do *not* attempt this problem in earnest unless you're completely on top of your other courses, etc.

After opening HyperSlate® to begin tackling the challenge, you only need to shift to use of the PC provability AI/oracle to be granted a trophy. So get that out of the way first, by all means.

The challenge is to engineer an axiom system of your own from the raw material given to you here, and to then show it in flawless operation. More specifically, you must select a proper subset of the 14 givens as your axioms, designate them as such by carefully annotating the relevant labels (as demonstrated in class), and then prove all the other formulae from your selected axioms. You must also introduce a new node at the very top of your workspace, with it's content as

Deadline March 26, 2026 at 11:59 PM EDT

FOL

√ FOL

√ FOL

Epistemic + FOL

$\mathbf{B}_d \mathbf{B}_v \mathbf{B}_d V v$

√ FOL

Epistemic + FOL (for coverage of “killer” robots, later)

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TOL

$\exists X [X(j) \land \neg X(m) \land S(X)]$

√ FOL

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$$\mathbf{B}_d \mathbf{B}_v \mathbf{B}_d V v$$

TOL

$$\exists X [X(j) \wedge \neg X(m) \wedge S(X)]$$

√ FOL

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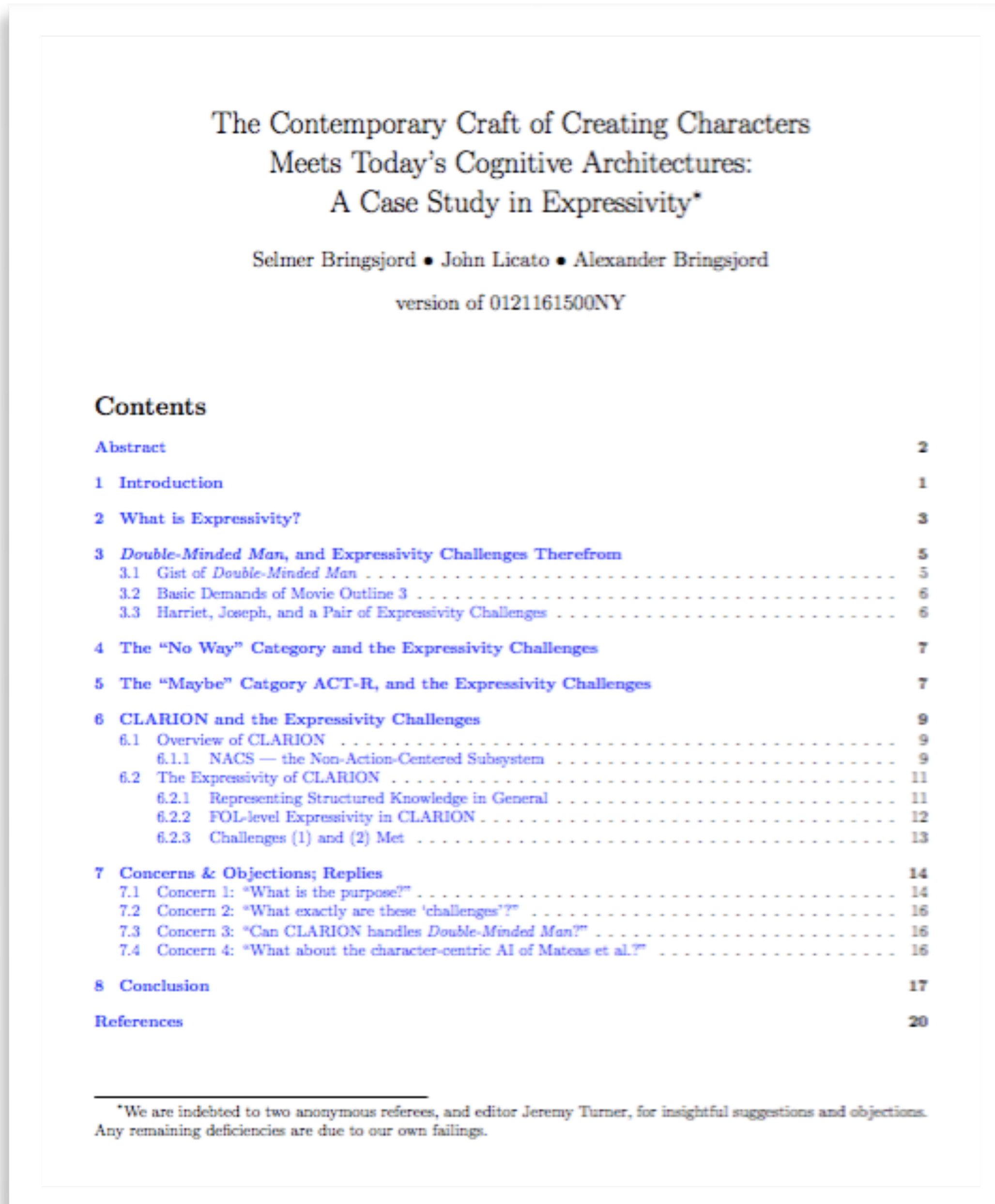
$\mathbf{B}_d \mathbf{B}_v \mathbf{B}_d V v$

TOL

❓ $\exists X[X(j) \land \neg X(m) \land S(X)]$

Double-Minded Man



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Meets Today's Cognitive Architectures:
A Case Study in Expressivity*

Selmer Bringsjord • John Licato • Alexander Bringsjord

version of 0121161500NY

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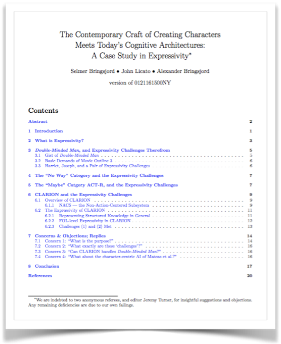
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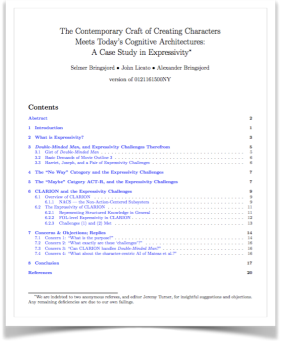

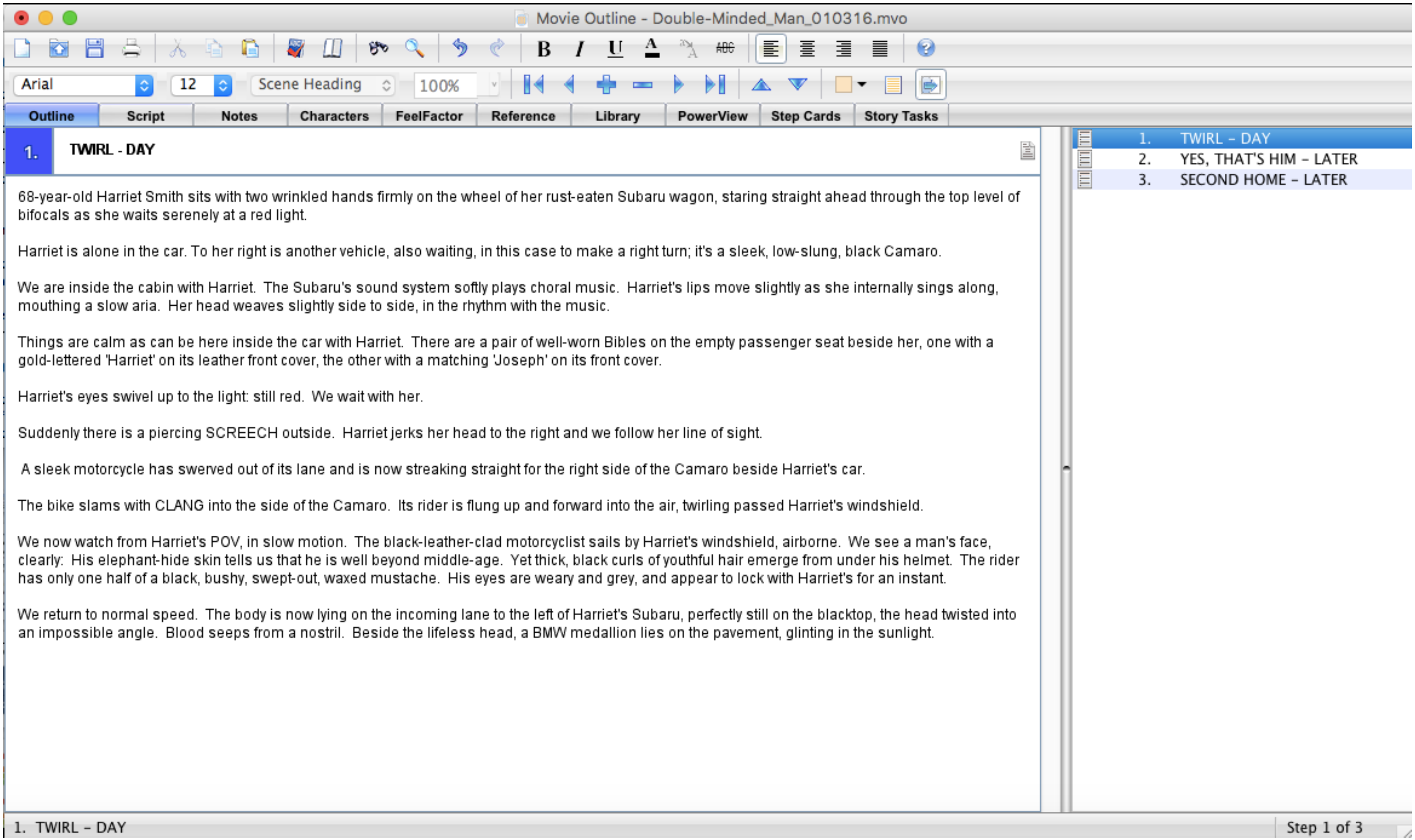

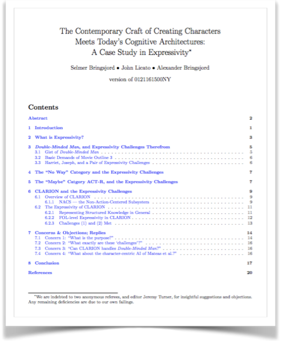

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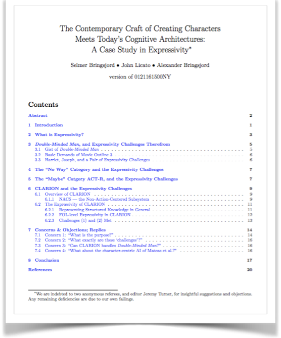

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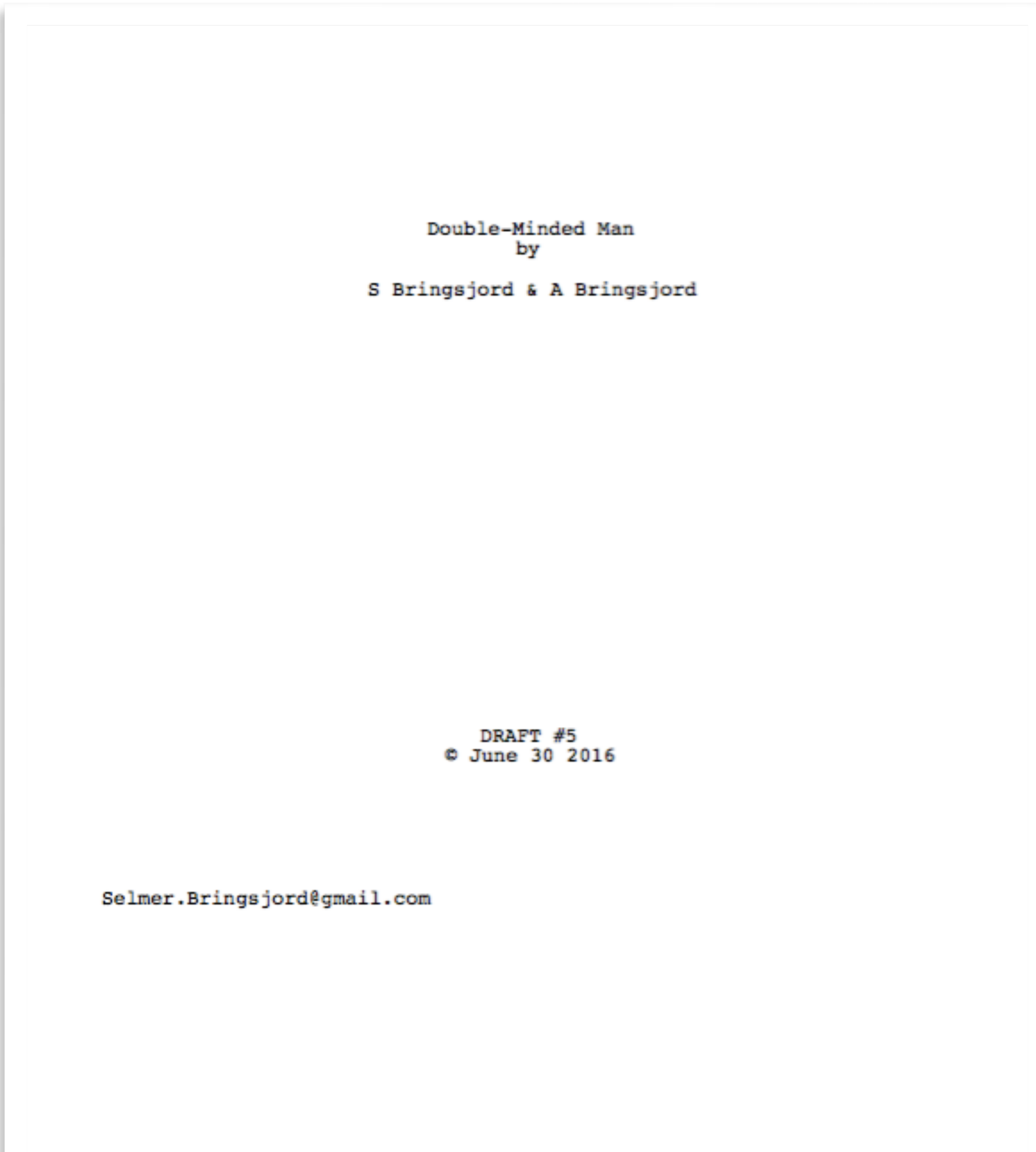

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by

S Bringsjord & A Bringsjord

DRAFT #5

Selmer.Bringsjord@gmail.com

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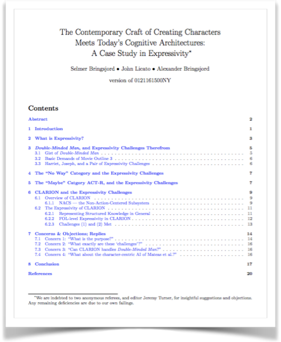

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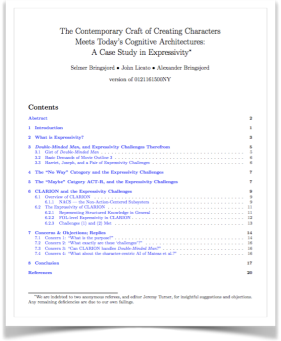

Double-Minded Man

1. TWIRL - DAY

68-year-old Harriet Smith sits with two wrinkled hands firmly on the wheel of her rust-eaten Subaru wagon, staring straight ahead through the top level of bifocals as she waits serenely at a red light.

Harriet is alone in the car. To her right is another vehicle, also waiting, in this case to make a right turn; it's a sleek, low-slung, black Camaro.

We are inside the cabin with Harriet. The Subaru's sound system softly plays choral music. Harriet's lips move slightly as she internally sings along, mouthing a slow aria. Her head weaves slightly side to side, in the rhythm with the music.

Things are calm as can be here inside the car with Harriet. There are a pair of well-worn Bibles on the empty passenger seat beside her, one with a gold-lettered 'Harriet' on its leather front cover, the other with a matching 'Joseph' on its front cover.

Harriet's eyes swivel up to the light: still red. We wait with her.

Suddenly there is a piercing SCREECH outside. Harriet jerks her head to the right and we follow her line of sight.

A sleek motorcycle has swerved out of its lane and is now streaking straight for the right side of the Camaro beside Harriet's car.

The bike slams with CLANG into the side of the Camaro. Its rider is flung up and forward into the air, twirling passed Harriet's windshield.

We now watch from Harriet's POV, in slow motion. The black-leather-clad motorcyclist sails by Harriet's windshield, airborne. We see a man's face, clearly: His elephant-hide skin tells us that he is well beyond middle-age. Yet thick, black curls of youthful hair emerge from under his helmet. The rider has only one half of a black, bushy, swept-out, waxed mustache. His eyes are weary and grey, and appear to lock with Harriet's for an instant.

We return to normal speed. The body is now lying on the incoming lane to the left of Harriet's Subaru, perfectly still on the blacktop, the head twisted into an impossible angle. Blood seeps from a nostril. Beside the lifeless head, a BMW medallion lies on the pavement, glinting in the sunlight.

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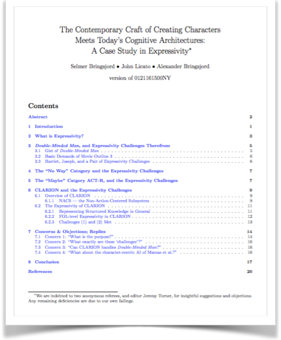

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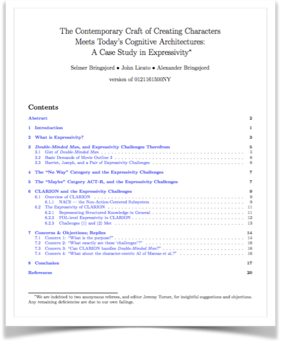

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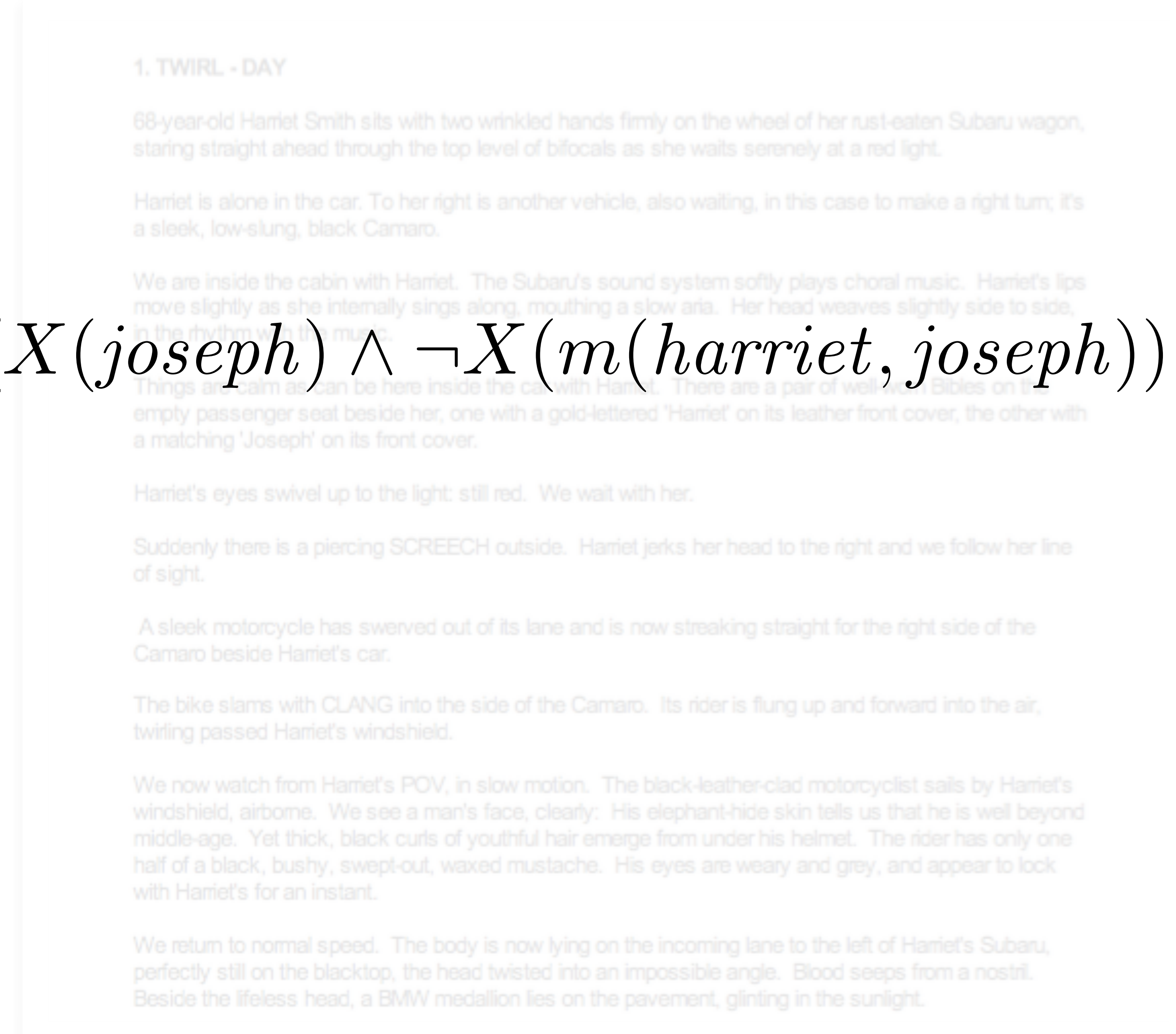

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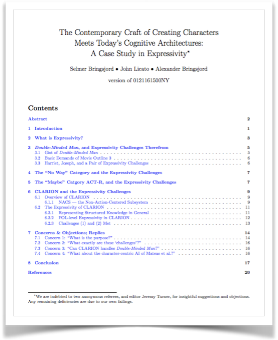
$$\exists X[X(joseph) \wedge \neg X(m(harriet, joseph)) \wedge Sleazy(X)]$$



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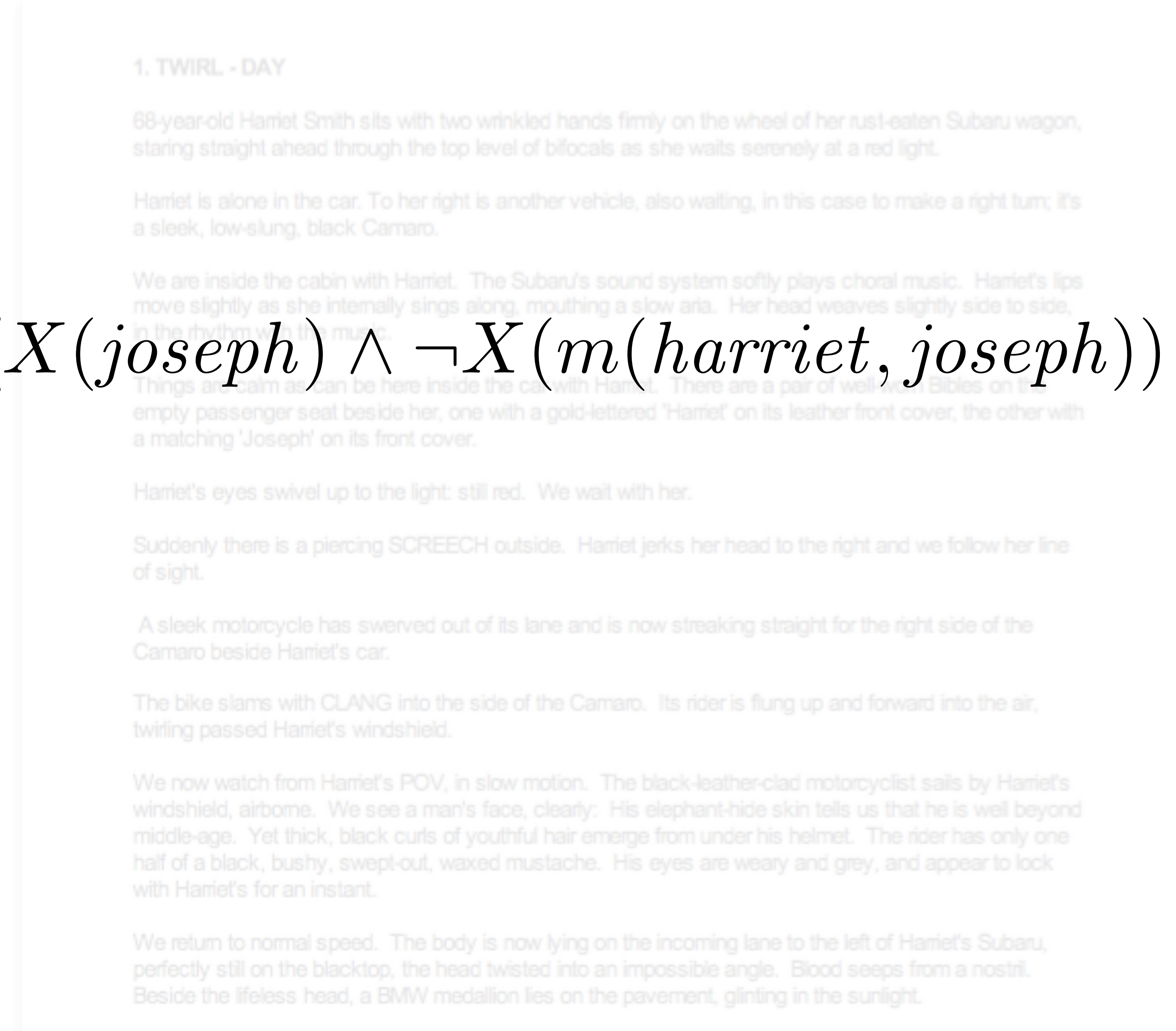

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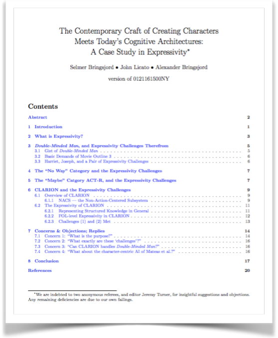
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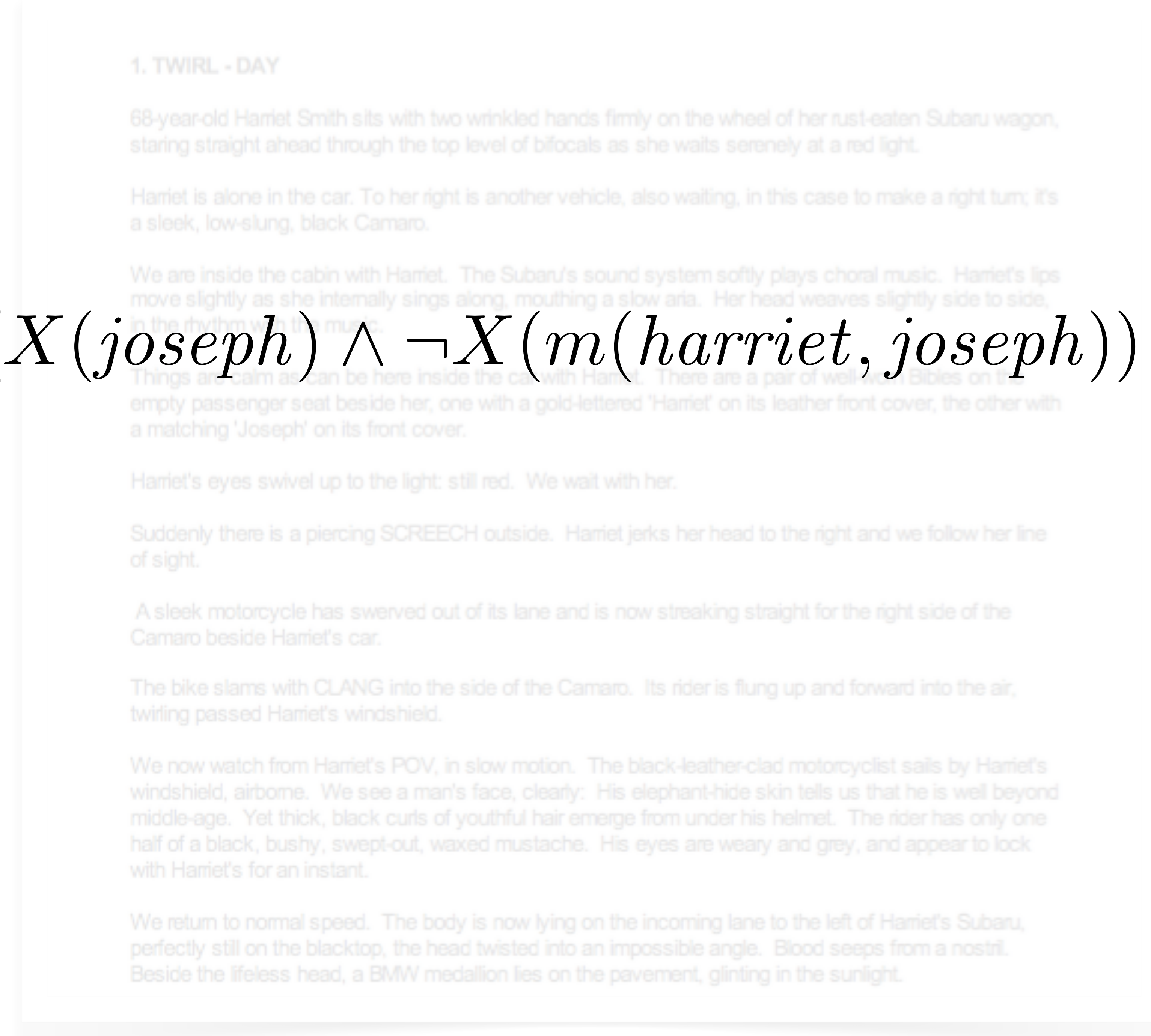
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Climbing the *k*-order Ladder

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a is a llama, as is b, a likes b, and the father of a is a llama as well.

Climbing the *k*-order Ladder

$$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

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Climbing the *k*-order Ladder

Things *x* and *y*, along with the father of *x*,
share a certain property (and *x* likes *y*).

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Climbing the *k*-order Ladder

$$\exists x \exists y \exists R[R(x) \wedge R(y) \wedge Likes(x,y) \wedge R(fatherOf(x))]$$

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Climbing the *k*-order Ladder

Things x and y, along with the father of x, share a certain property; and, x R^2s y, where R^2 is a positive property.

SOL $\exists x \exists y \exists R[R(x) \land R(y) \land Likes(x, y) \land R(fatherOf(x))]$

Things x and y, along with the father of x, share a certain property (and x likes y).

FOL $\exists x[Llama(x) \land Llama(b) \land Likes(x, b) \land Llama(fatherOf(x))]$

There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

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Climbing the *k*-order Ladder

$$\exists x, y\ \exists R, R^2[R(x) \land R(y) \land R^2(x,y) \land Positive(R^2) \land R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property; and, *x* R^2s *y*, where R^2 is a positive property.

SOL $$\exists x \exists y \exists R[R(x) \land R(y) \land Likes(x,y) \land R(fatherOf(x))]$$

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a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

TOL $\mathscr{L}_3$

$$\exists x, y\ \exists R, R^2[R(x) \land R(y) \land R^2(x,y) \land Positive(R^2) \land R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property; and, *x* R^2s *y*, where R^2 is a positive property.

SOL $\mathscr{L}_2$

$$\exists x \exists y \exists R[R(x) \land R(y) \land Likes(x,y) \land R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property (and *x* likes *y*).

FOL $\mathscr{L}_1$

$$\exists x[Llama(x) \land Llama(b) \land Likes(x,b) \land Llama(fatherOf(x))]$$

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $\mathscr{L}_0$

$$Llama(a) \land Llama(b) \land Likes(a,b) \land Llama(fatherOf(a))$$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

$\vdots$

TOL $\mathscr{L}_3$

$$\exists x, y\ \exists R, R^2[R(x) \land R(y) \land R^2(x,y) \land Positive(R^2) \land R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property; and, *x* R^2s *y*, where R^2 is a positive property.

SOL $\mathscr{L}_2$

$$\exists x \exists y \exists R[R(x) \land R(y) \land Likes(x,y) \land R(fatherOf(x))]$$

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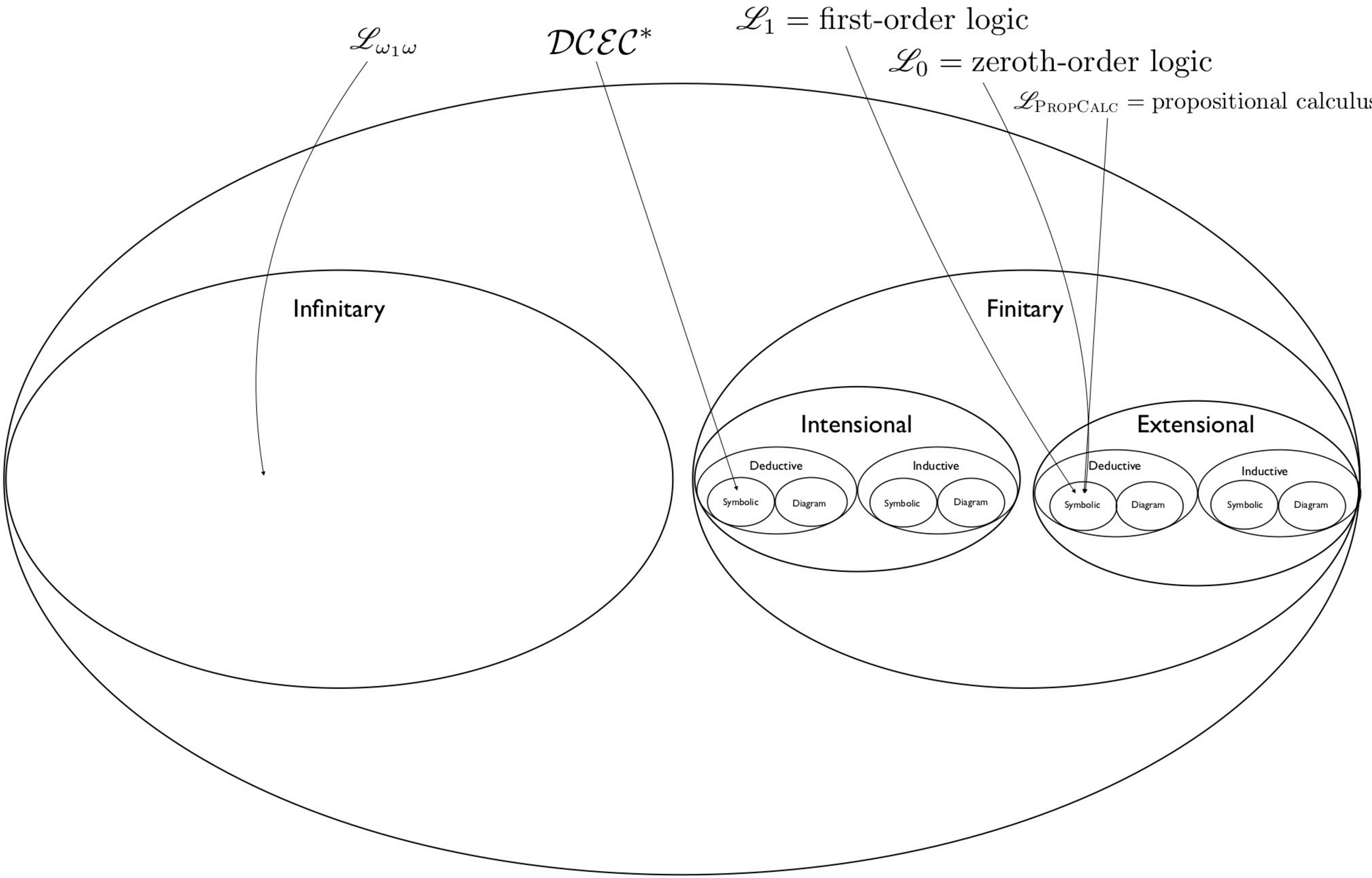
$$Llama(a) \land Llama(b) \land Likes(a,b) \land Llama(fatherOf(a))$$

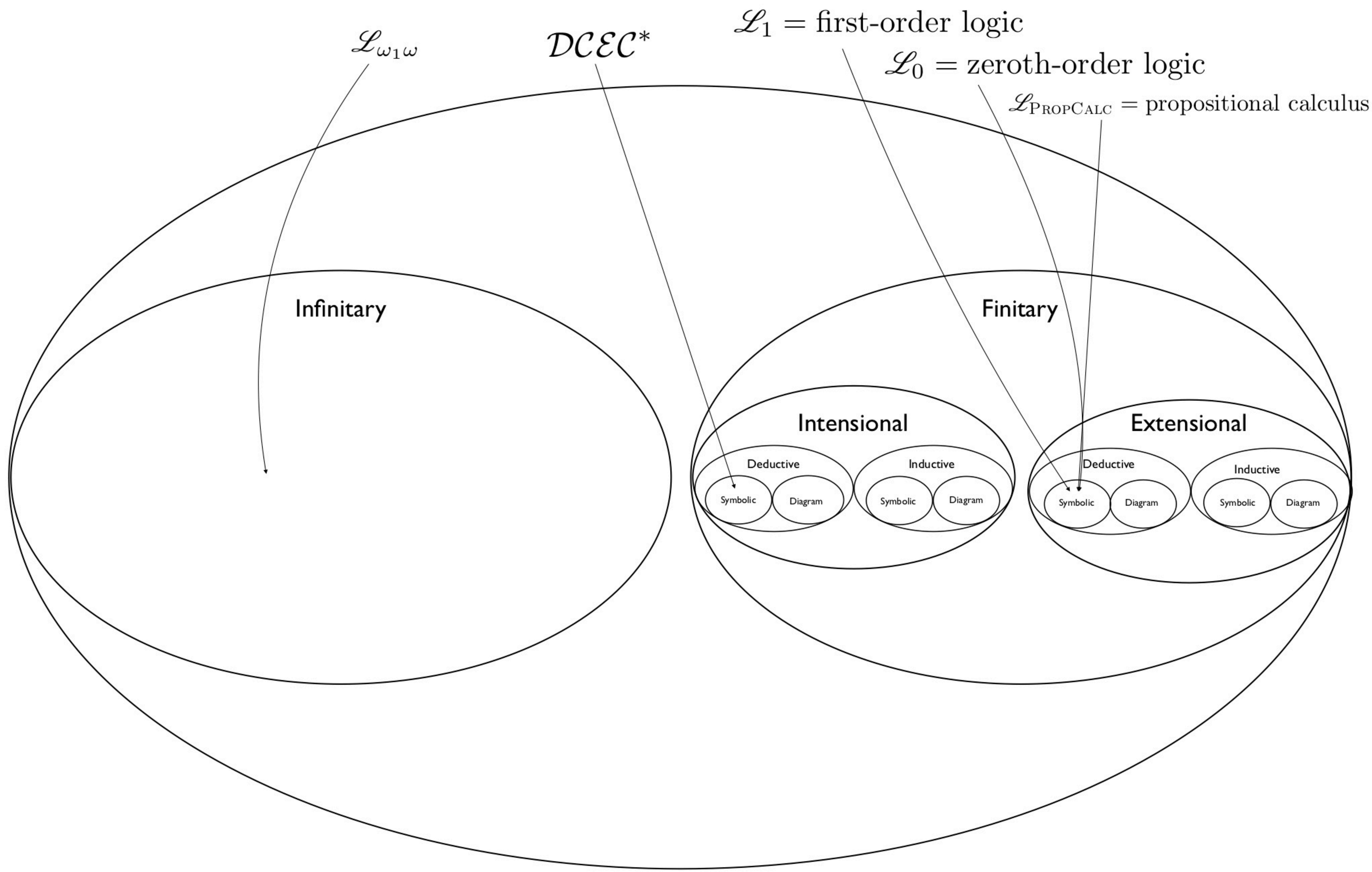
a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

The Universe of Logics

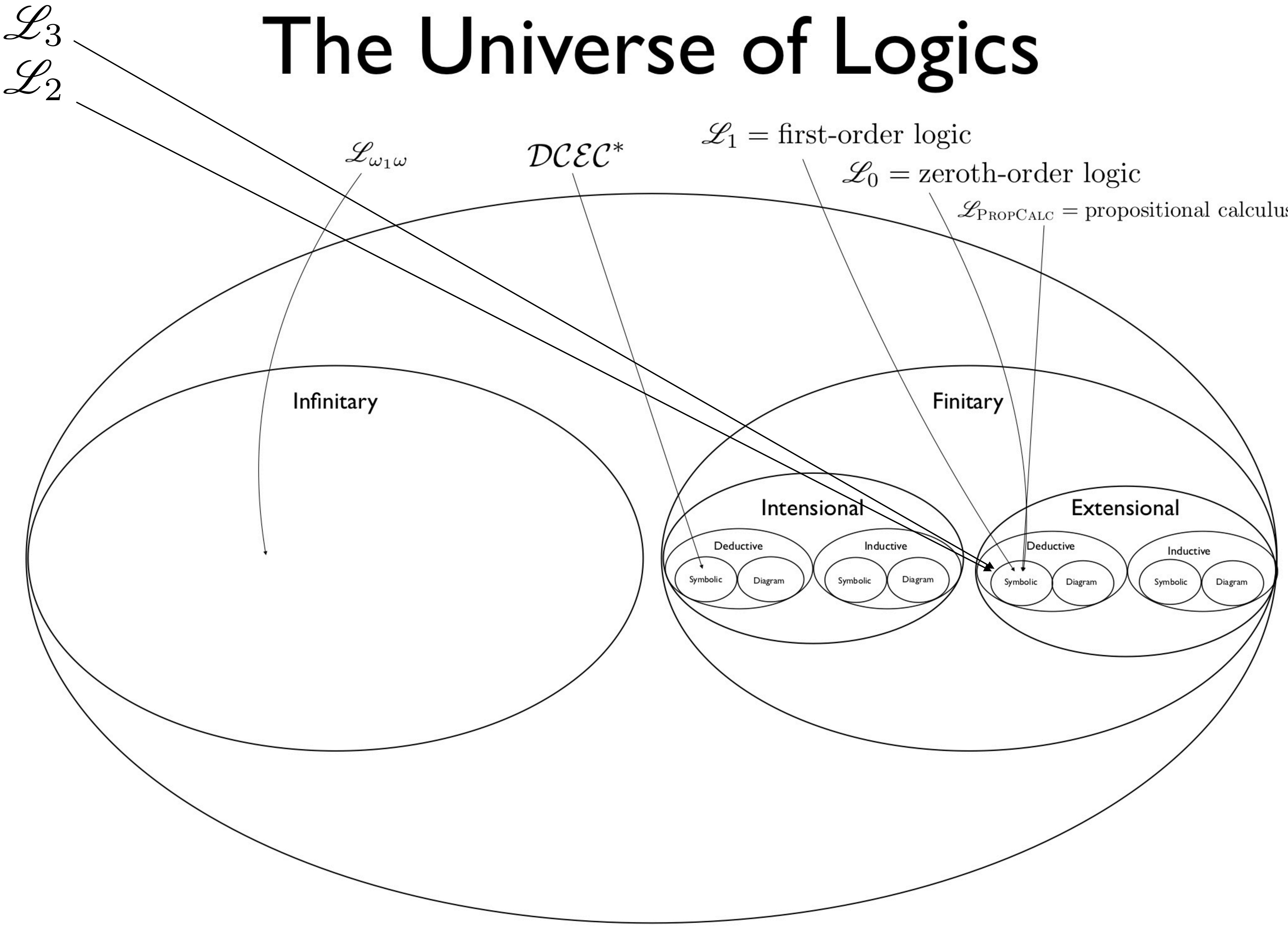


$\mathscr{L}_3$
$\mathscr{L}_2$

The Universe of Logics



The Universe of Logics

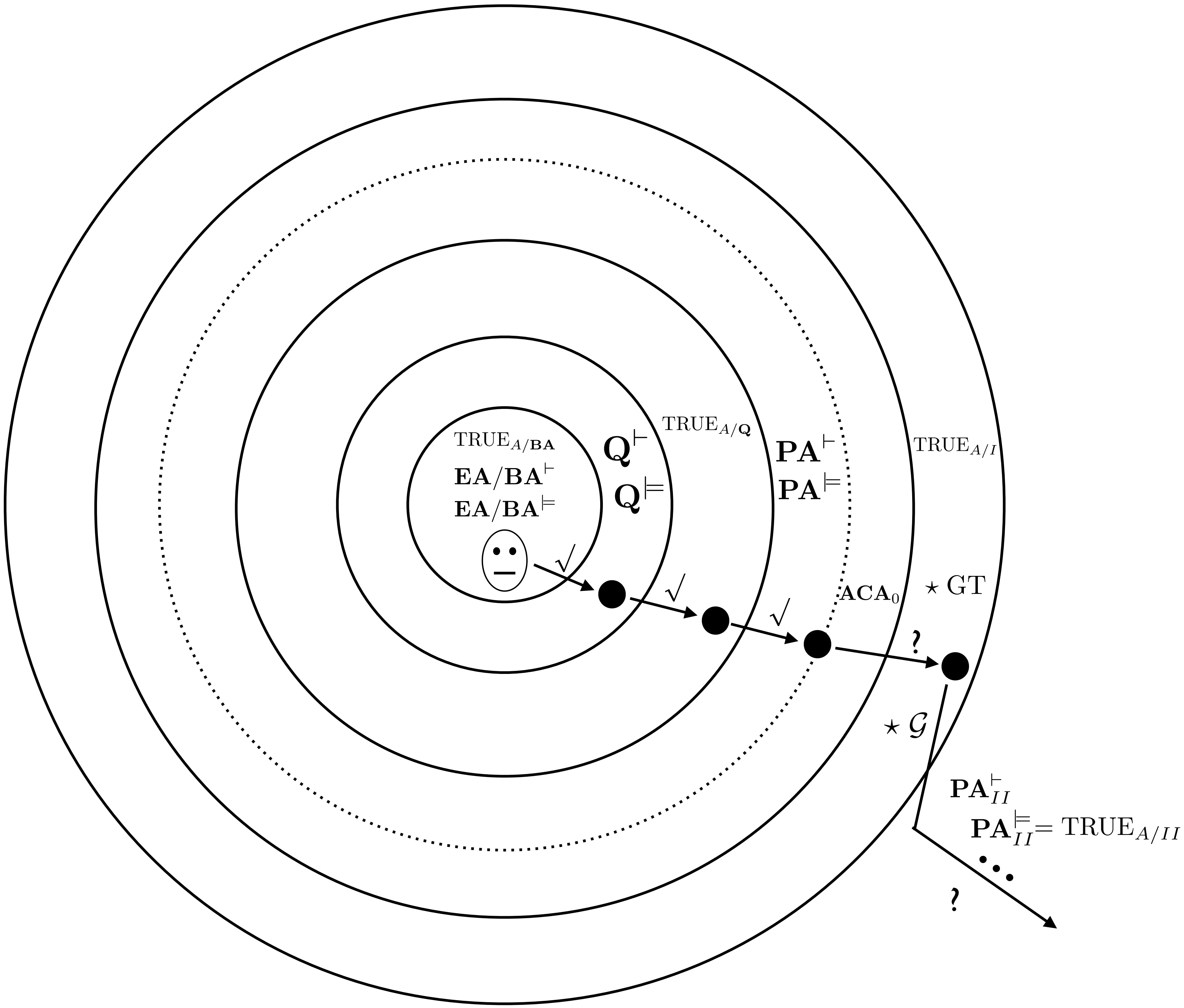


“Higher-Order” Remarks, Glimpse ahead to Gödel

. . .

Astrologic

(Aliens & Angels on the Same "Race Track")



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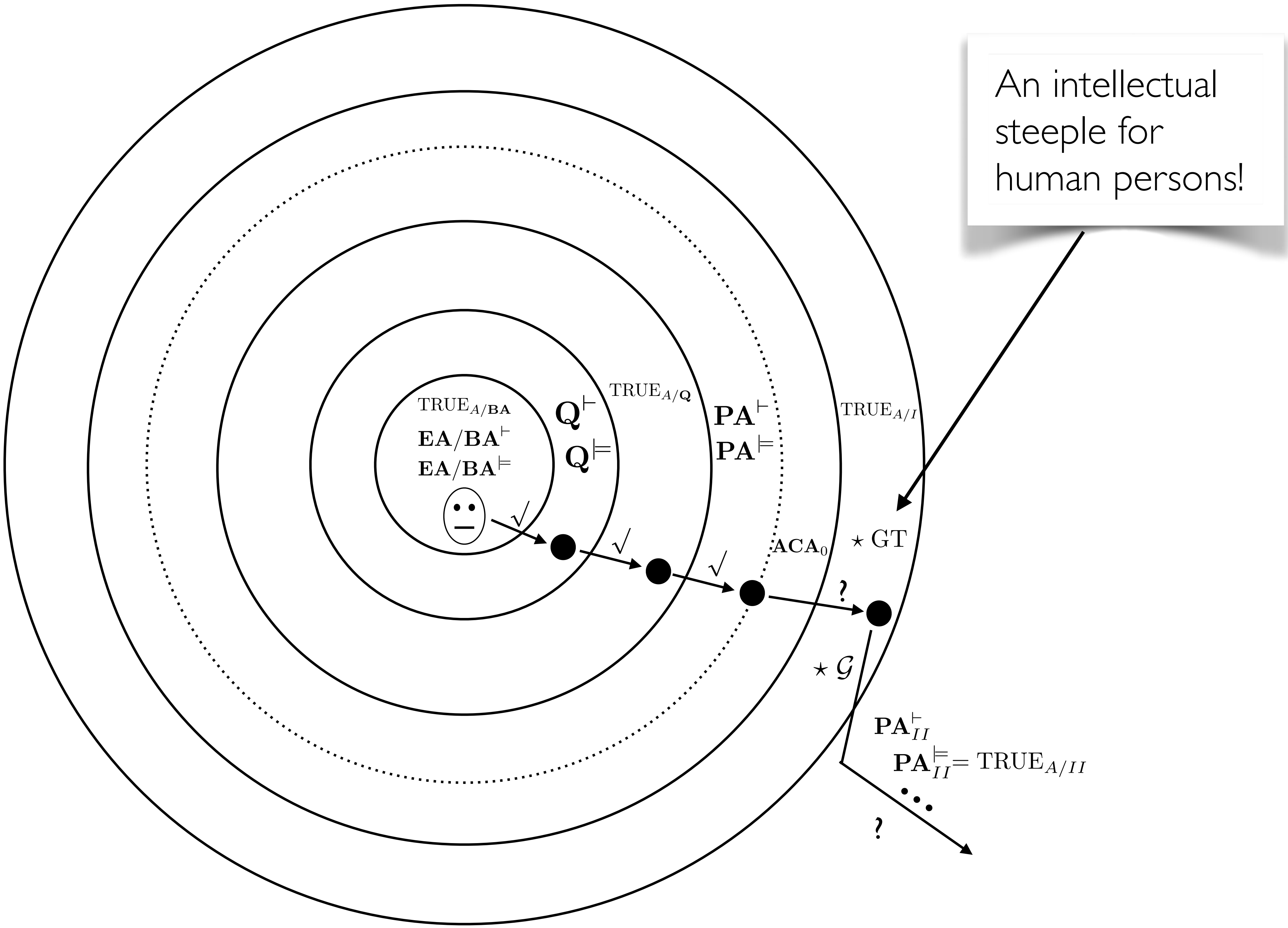

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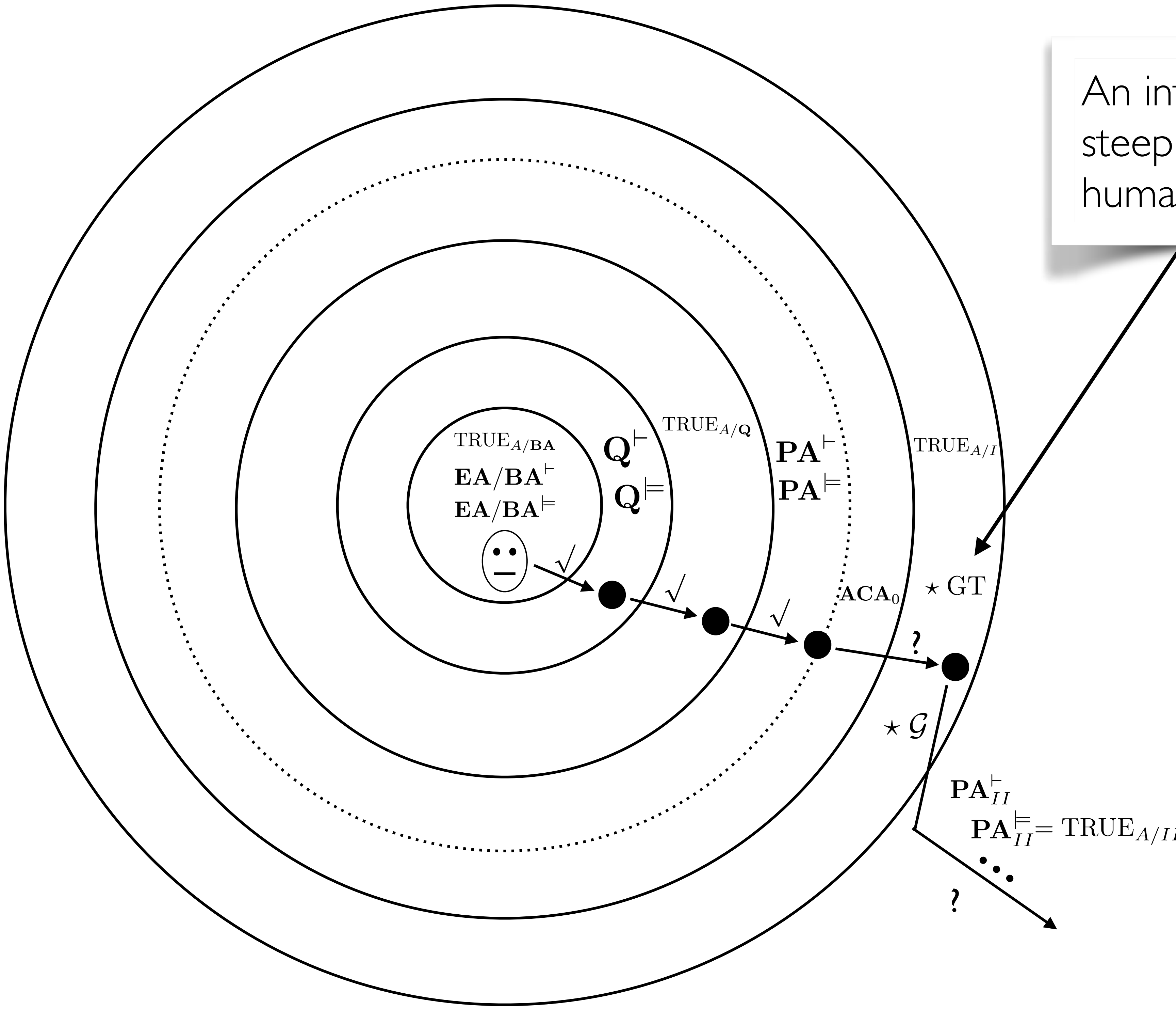

An intellectual steeple for human persons!

Astrologic

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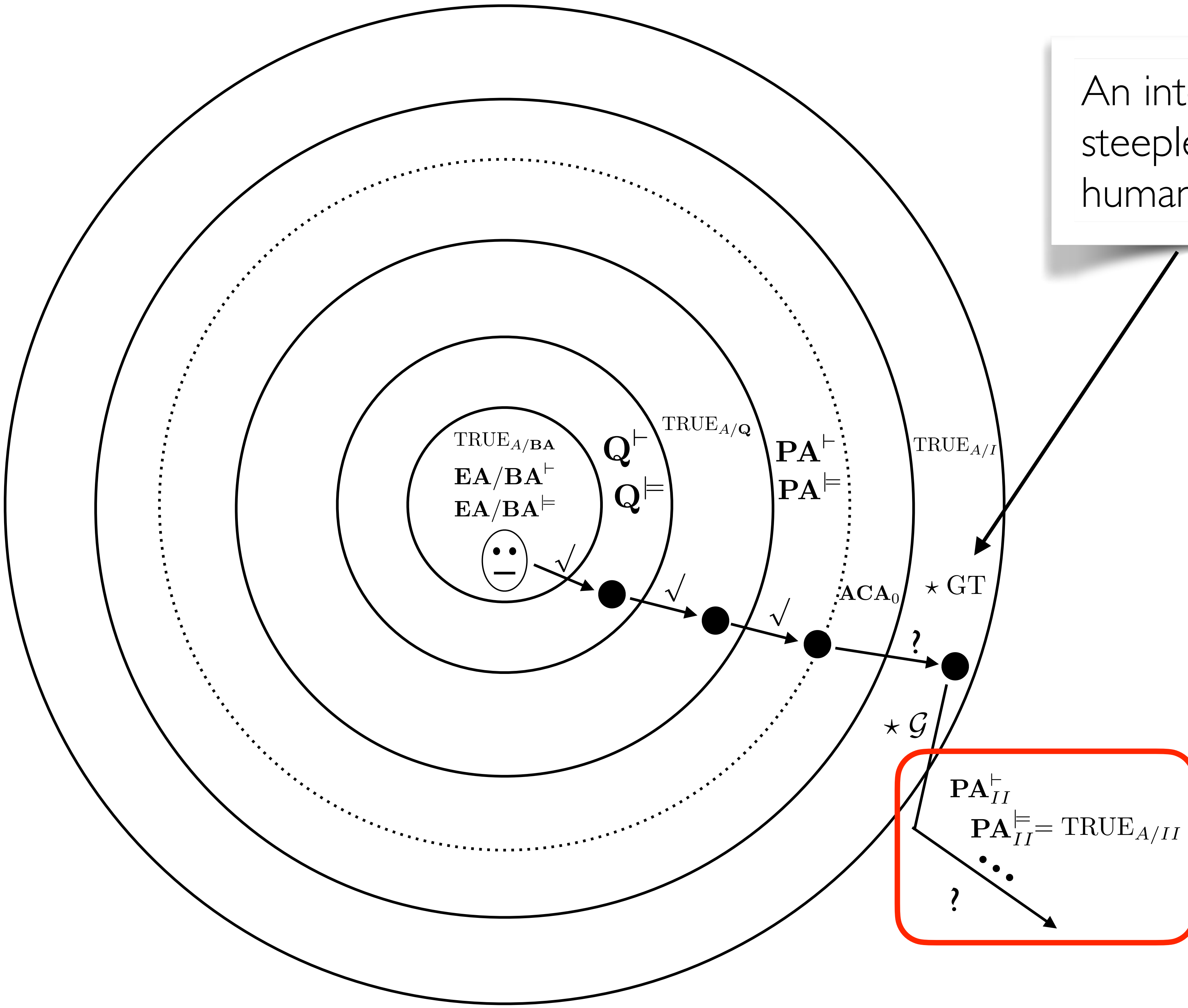

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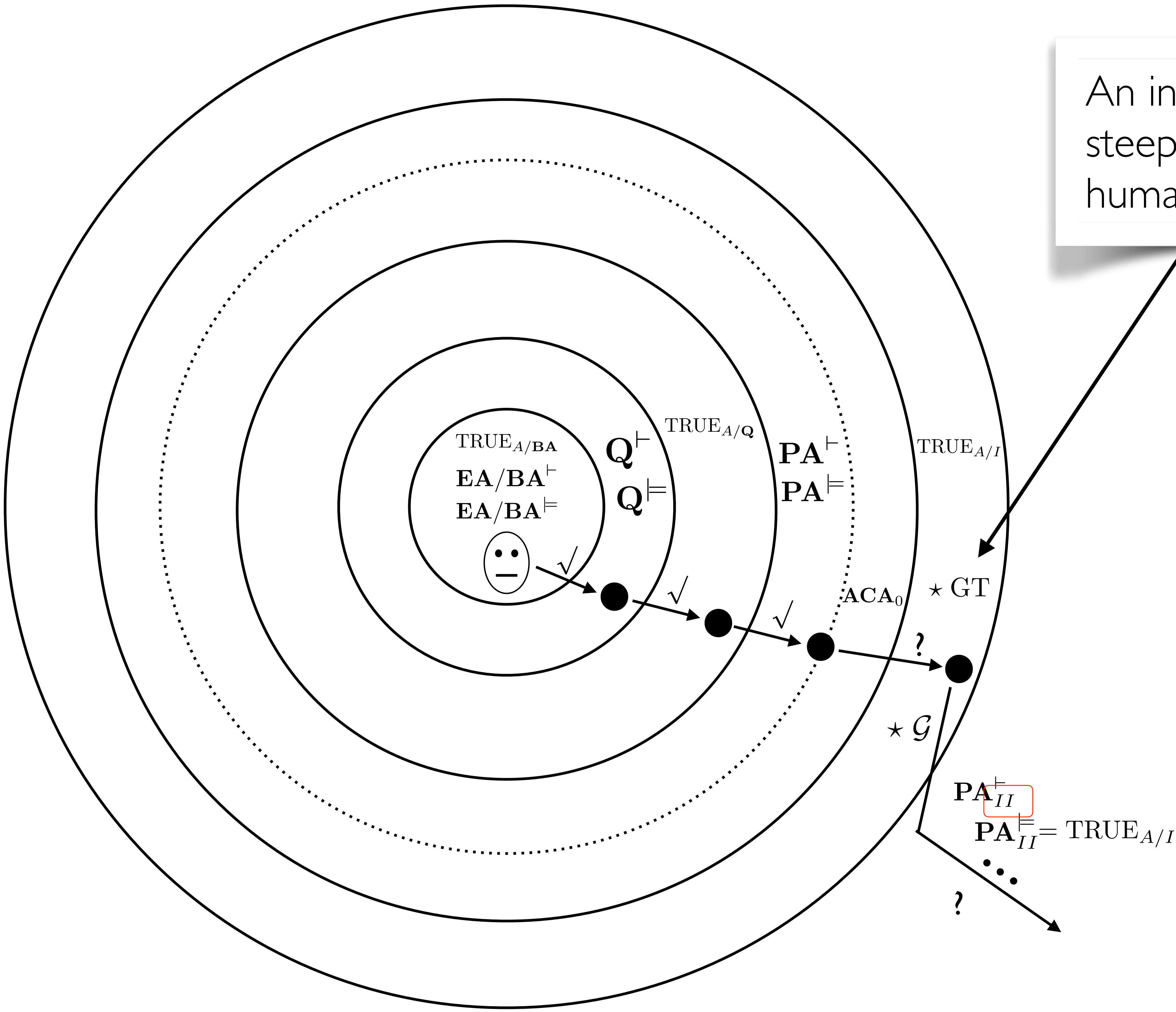

An intellectual steeple for human persons!

Astrologic

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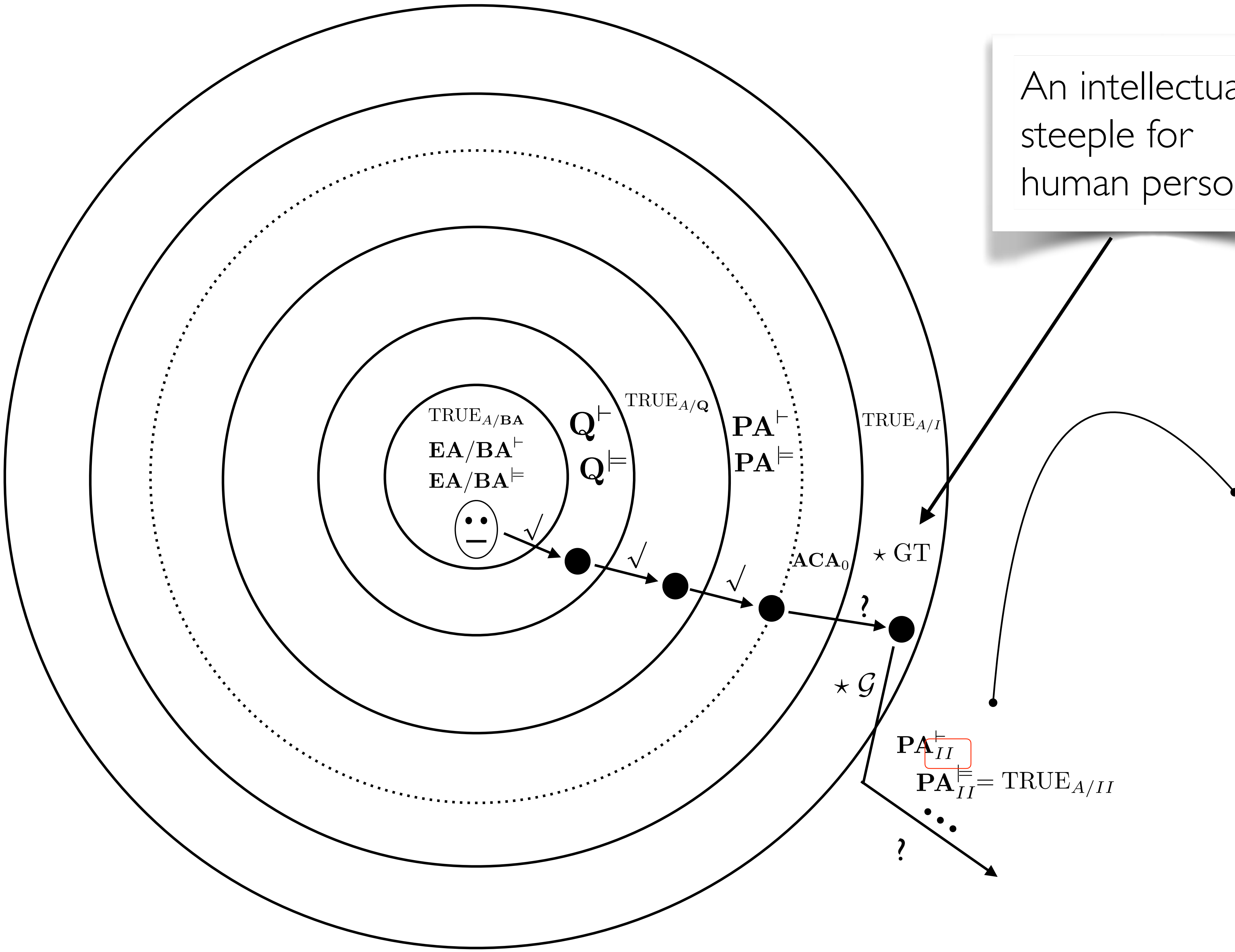

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Astrologic

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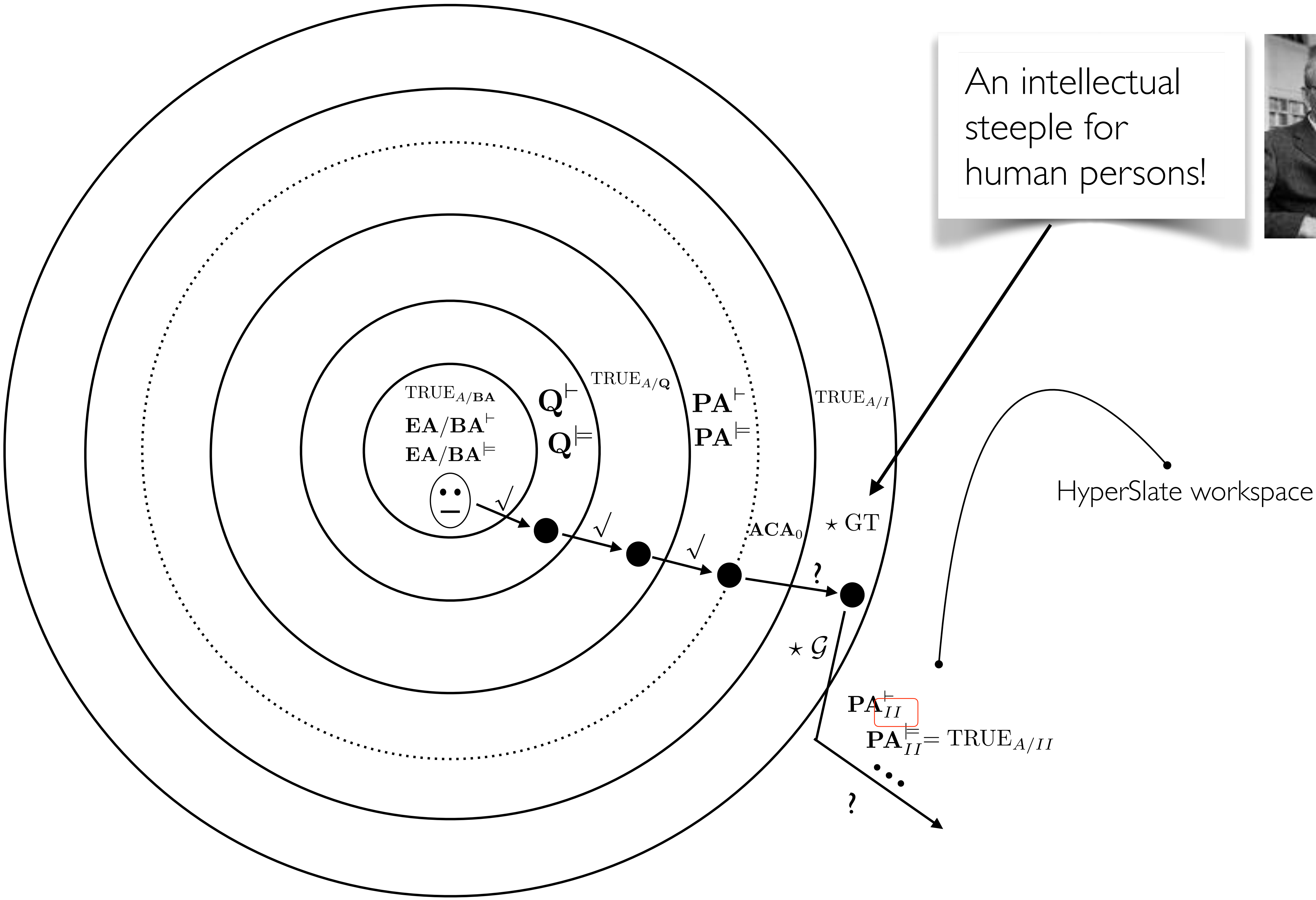

An intellectual steeple for human persons!

(First-Order) Peano Arithmetic = PA_1 = Z_1

$$
\left.\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
\text{A2} & \forall x \forall y(s(x) = s(y) \rightarrow x = y) \\
\text{A3} & \forall x(x \neq 0 \rightarrow \exists y(x = s(y)) \\
\text{A4} & \forall x(x + 0 = x) \\
\text{A5} & \forall x \forall y(x + s(y) = s(x + y)) \\
\text{A6} & \forall x(x \times 0 = 0) \\
\text{A7} & \forall x \forall y(x \times s(y) = (x \times y) + x)
\end{array}\right\} \mathrm{Q}
$$

And, every sentence that is the universal closure of an instance of

$$([\phi(0) \wedge \forall x(\phi(x) \rightarrow \phi(s(x))] \rightarrow \forall x \phi(x))$$

where $\phi(x)$ is open wff with variable x, and perhaps others, free.

(First-Order) Peano Arithmetic = PA_1 = Z_1

$$\left.\begin{array}{ll}
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\end{array}\right\} \text{Q}$$

And, every sentence that is the universal closure of an instance of

$$([\phi(0) \land \forall x(\phi(x) \rightarrow \phi(s(x))] \rightarrow \forall x \phi(x)) \quad \infty$$

where $\phi(x)$ is open wff with variable x, and perhaps others, free.

$PA_2 = Z_2$

$$
\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
\text{A2} & \forall x \forall y(s(x) = s(y) \rightarrow x = y) \\
\text{A3} & \forall x(x \neq 0 \rightarrow \exists y(x = s(y)) \\
\text{A4} & \forall x(x + 0 = x) \\
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\text{A6} & \forall x(x \times 0 = 0) \\
\text{A7} & \forall x \forall y(x \times s(y) = (x \times y) + x)
\end{array}
$$

(if we drop any restriction C, we have full second-order arithmetic.)

$PA_2 = Z_2$

A1	$\forall x(0 \neq s(x))$
A2	$\forall x \forall y(s(x) = s(y) \rightarrow x = y)$
A3	$\forall x(x \neq 0 \rightarrow \exists y(x = s(y)))$
A4	$\forall x(x + 0 = x)$
A5	$\forall x \forall y(x + s(y) = s(x + y))$
A6	$\forall x(x \times 0 = 0)$
A7	$\forall x \forall y(x \times s(y) = (x \times y) + x)$

Induction Axiom $\forall X([X(0) \wedge \forall x(X(x) \rightarrow X(s(x))] \rightarrow \forall x X(x))$

(if we drop any restriction C, we have full second-order arithmetic.)

$PA_2 = Z_2$

$$
\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
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\text{A7} & \forall x \forall y(x \times s(y) = (x \times y) + x)
\end{array}
$$

Induction Axiom $\forall X([X(0) \wedge \forall x(X(x) \rightarrow X(s(x))] \rightarrow \forall x X(x))$

Comprehension Schema $\exists X(\forall x X(x) \leftrightarrow \phi(x))$ where $\phi(x) \in C$

(if we drop any restriction C, we have full second-order arithmetic.)

New!

ACA_0

$$
\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
\text{A2} & \forall x \forall y(s(x) = s(y) \to x = y) \\
\text{A3} & \forall x(x \neq 0 \to \exists y(x = s(y))) \\
\text{A4} & \forall x(x + 0 = x) \\
\text{A5} & \forall x \forall y(x + s(y) = s(x + y)) \\
\text{A6} & \forall x(x \times 0 = 0) \\
\text{A7} & \forall x \forall y(x \times s(y) = (x \times y) + x)
\end{array}
$$

Comprehension Schema

$$\exists X [\forall x \; X(x) \leftrightarrow \phi(x)] \qquad \text{where } \phi(x) \text{ has no second-order quantifiers}$$

New!

ACA_0

$$
\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
\text{A2} & \forall x \forall y(s(x) = s(y) \rightarrow x = y) \\
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\end{array}
$$

Induction Axiom $\forall X([X(0) \wedge \forall x(X(x) \rightarrow X(s(x))] \rightarrow \forall x X(x))$

Comprehension Schema $\exists X[\forall x\ X(x) \leftrightarrow \phi(x)]$ where $\phi(x)$ has no second-order quantifiers

Comprehension inference schema is available in HS®! — But order of biconditional matters!

Later: Bonus "Explosion" Challenge: $\exists Y \, \forall X \, \forall z [Yz \leftrightarrow Xz] \vdash \zeta$

$20 Challenge; Must sign up & get approved to tackle beforehand by Selmer:
Long reasonable proof in **PA**; short counterpart in **PA2**.

Slutten